CAREER POINT

JEE Main Online Exam 2019

Question & Solutions

10th January 2019 | Shift - II

PHYSICS

Q.1 Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be -

(1) $\frac{Q^2}{4\pi\varepsilon_0}$ (2) $\frac{Q^2}{2\sqrt{2}\,\pi\varepsilon_0}$ (3) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

Ans. **[4]**

Sol. $W = q\Delta V = q\,[V_0 - V_\infty] = qV_0$

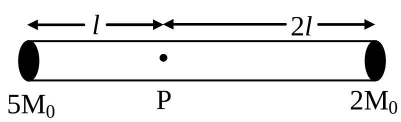
Potential at origin = V_0

$$V_0 = 2.\frac{kQ}{2} + 2.\frac{kQ}{2\sqrt{5}}$$

$$= kQ\left(1+\frac{1}{\sqrt{5}}\right)$$

$$\therefore \text{Word done} = kqQ\left(1+\frac{1}{\sqrt{5}}\right) = \frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$$

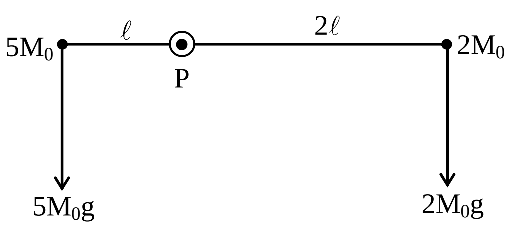
Q.2 A rigid massless rod of length 3*l* has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be -



(1) $\frac{g}{13l}$ (2) $\frac{g}{2l}$ (3) $\frac{g}{3l}$ (4) $\frac{7g}{3l}$

Ans. **[1]**

Sol.



τ about P = $5M_0g\ell - 2M_0g \times 2\ell$

$= M_0g\ell$

τ about P = $I_P\alpha$

$M_0g\ell = [5M_0\ell^2 + 2M_0 \times 4\ell^2]\ \alpha$

$M_0g\ell = 13M_0\ell^2\alpha$

$\alpha = \dfrac{g}{13\ell}$

Q.3 A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

(1) 4 (2) 7 (3) 6 (4) 5

Ans. **[3]**

Sol. $f = (2n-1)\dfrac{v}{4\ell}$

$\dfrac{v}{4\ell} = 1500$

$f = (2n - 1) \times 1500$

f_{max} for audible frequency = 20,000

$(2n - 1) \times 1500 < 20,000$

$2n - 1 < \dfrac{200}{15}$

$2n - 1 < \dfrac{40}{3}$

$2n - 1 < 13.33$

$2n < 14.33$

$n < 7.16$

$n = 7$

Number of overtone = n – 1 = 7 – 1 = 6

Q.4 Consider the nuclear fission

$Ne^{20} \rightarrow 2He^4 + C^{12}$

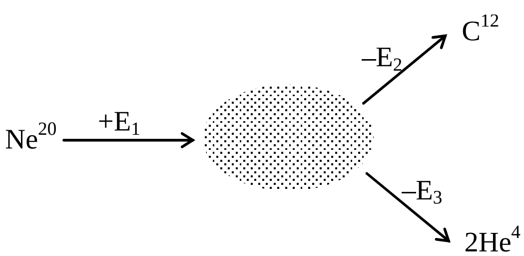
Given that the binding energy/ nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement -

(1) 8.3 MeV energy will be released (2) energy of 11.9 MeV has to be supplied

(3) energy of 12.4 MeV will be supplied (4) energy of 3.6 MeV will be released

Ans. **[2]**

Sol.



$E_{supplied} = E_1 = 20 \times BE_N$ of Ne^{20}

$E_{released} = 12\ BE_N$ of $C^{12} + 2 \times 4 \times BE_N$ of He^4

Supplied energy $= 20\ BE_N$ of $Ne^{20} - 12 \times BE_N$ of $C^{12} - 2 \times 4 \times BE_N$ of He^4

$= 11.9$ MeV

Q.5 A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photoelectrons per second and their maximum energy, respectively, will be -

(1) 10^{14} and 10 eV (2) 10^{12} and 5 eV (3) 10^{11} and 5 eV (4) 10^{10} and 5 eV

Ans. **[4]**

Sol. Energy incident on plate per second = IA

$= 16 \times 10^{-3} \times 1 \times 10^{-4}$

$= 16 \times 10^{-7}$ watt

$KE_{max} = hf - \phi$

$= 10 - 5 = 5$ eV

$$\frac{\dot{N}hc}{\lambda} = 1.6 \times 10^{-7}$$

$$\dot{N} \times 10 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-7}$$

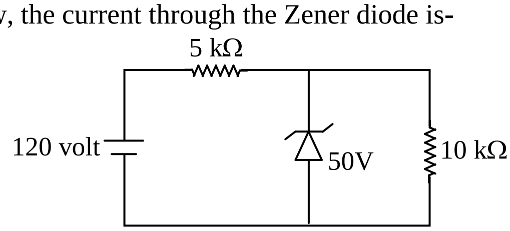
$$\dot{N} \times 10^{-18} = 10^{-7}$$

$$\dot{N} = 10^{11}$$

10% of incident photon emit electron

$\therefore$ No of emitted electrons per second = $\dot{N} \times \frac{10}{100} = 10^{10}$.

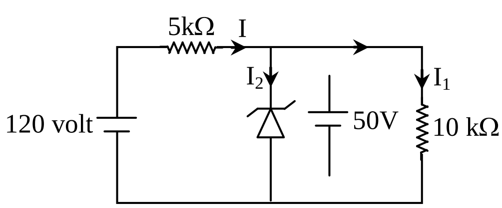
Q.6 For the circuit shown below, the current through the Zener diode is-



(1) 5 mA (2) zero (3) 14 mA (4) 9 mA

Ans. **[3]**

Sol.



$V_1 = 120 - 50 = 70$

$I \times 5 \times 10^3 = 70$

$I = 14$ mA

$V_1 = 50 = I_1 \times 10 \times 10^3$

$I_1 = 5$ mA

$I_2 = I - I_1 = 14 - 5 = 9$ mA

Q.7 Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is -

(1) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (2) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (3) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (4) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

Ans. **[3]**

Sol. $|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$

$A^2 + B^2 + 2AB\cos\theta = n^2 (A^2 + B^2 - 2AB\cos\theta)$

$2AB\cos\theta\,(1 + n^2) = (A^2 + B^2)(n^2 - 1)$

$$\cos\theta = \frac{A^2+B^2}{2AB}\,\frac{(n^2-1)}{(n^2+1)}$$

As A = B

$$\therefore \cos\theta = \frac{B^2+B^2}{2B^2}\left(\frac{n^2-1}{n^2+1}\right)$$

$$\cos\theta = \frac{n^2-1}{n^2+1}$$

$$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

Q.8 The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

(1) $4264.4 \pm 81.0\ cm^3$ (2) $4264 \pm 81\ cm^3$ (3) $4300 \pm 80\ cm^3$ (4) $4260 \pm 80\ cm^3$

Ans. **[4]**

Sol. $$V = \frac{\pi D^2 h}{4}$$

$$V = \frac{\pi}{4} \times (12.6)^2 \times 34.2$$

V = 4262.229

Answer should be in three significant numbers

As D & h both have 3 significant figures

$\therefore$ V = 4260

$$\frac{dV}{V} = 2\frac{dD}{D} + \frac{dh}{h}$$

$$dV = \left[2\times\frac{0.1}{12.6}+\frac{0.1}{34.2}\right] \times 4260$$

dV = 80

$\therefore$ V = $4260 \pm 80\ cm^3$

Q.9 A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is -

(1) 11×10^{-5} W (2) 11×10^{-3} W (3) 11×10^{5} W (4) 11×10^{-4} W

Ans. **[1]**

Sol. $i^2R = P$

$4 \times 10^{-6} \times R = 4.4$

$R = 1.1 \times 10^6\ \Omega$

$$P = \frac{11^2}{R}$$

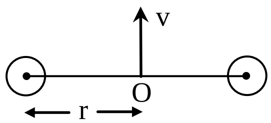
$$= \frac{11\times11\times10}{11\times10^6} = 11 \times 10^{-5}$$

Q.10 Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is - (Take Gravitational constant; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 2.4×10^4 m/s (2) 1.4×10^5 m/s (3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

Ans. **[4]**

Sol.



When total mechanical energy of meteorite become 0

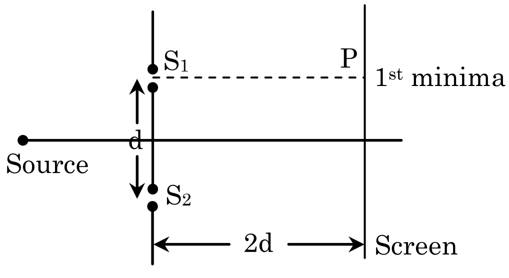
then meteorite will escape out

$$\frac{1}{2}mv^2 + m\left[-\frac{GM}{r} - \frac{GM}{r}\right] = 0$$

$$v^2 = \frac{4GM}{r} = \frac{4\times6.67\times3\times10^{-11}\times10^{31}}{10^{11}}$$

$v = 2.8 \times 10^5$ m/s

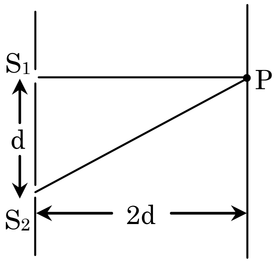
Q.11 Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



(1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{2(\sqrt{5}-2)}$ (3) $\frac{\lambda}{(5-\sqrt{2})}$ (4) $\frac{\lambda}{(\sqrt{5}-2)}$

Ans. **[2]**

Sol.



At point P $\Delta x = \sqrt{5}\,d - 2d$

$\Delta x = d(\sqrt{5} - 2)$

For first minima $\Delta x = \frac{\lambda}{2}$

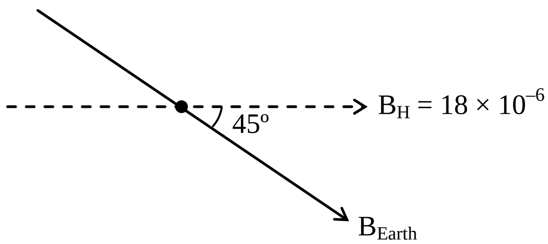
$d = \frac{\lambda}{2(\sqrt{5}-2)}$

Q.12 At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is -

(1) 3.6×10^{-5} N (2) 1.8×10^{-5} N (3) 1.3×10^{-5} N (4) 6.5×10^{-5} N

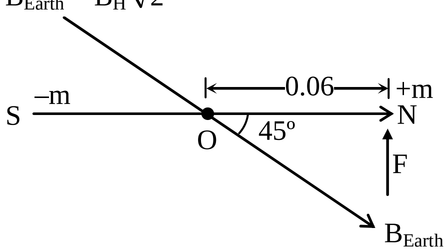
Ans. **[4]**

Sol.



$B_H = B_{Earth} \cos 45^o$

$B_{Earth} = B_H \sqrt{2}$



$Z_{abt_0} = 0$

$MB_{Earth} \sin 45^o - F \times 0.06 = 0$

$1.8 \times 0.12\, B_H \frac{1}{\sqrt{2}} \times \sqrt{2} - F \times 0.06 = 0$

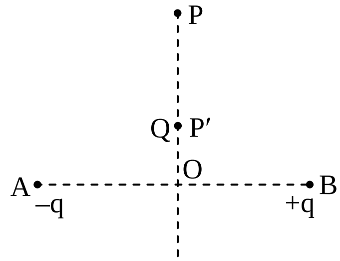
$F = 1.8 \times 2 \times B_H$

$= 3.6 \times 18 \times 10^{-6}$

$= 64.8 \times 10^{-6}$

$= 6.5 \times 10^{-5}$ N

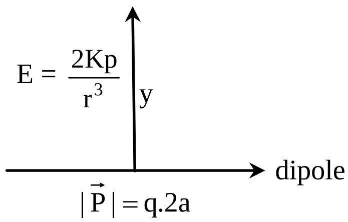
Q.13 Charges –q and +q located at A and B, respectively, constitude an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P′ such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to - $\left(\frac{y}{3} >> 2a\right)$



(1) 9F (2) 3F (3) F/3 (4) 27F

Ans. **[1]**

Sol.



F = QE

$$F = Q.\frac{2Kp}{y^3}$$

$$\frac{F}{F'} = \frac{Q.\frac{2Kp}{y^3}}{Q.\frac{2Kp}{(y/3)^3}}$$

$$\frac{F}{F'} = \frac{1}{27}$$

F′ = 27 F

Q.14 The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is -

(1) 740 J (2) 637.5 J (3) 540 J (4) 437.5 J

Ans. **[4]**

Sol. Induced emf = $L\frac{di}{dt}$

$$= \frac{L \times (25 - 10)}{1}$$

$$= L \times 15$$

L × 15 = 25

$$L = \frac{25}{15}$$

Change in energy of inductor = $\frac{1}{2}L\left[25^2 - 10^2\right]$

$= \frac{1}{2} \times \frac{25}{15} \times 15 \times 35$

$= \frac{25 \times 35}{2}$

= 437.5 Joule

Q.15 Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to – (Gas constant R = 8.31 J/mol.K)

(1) 581 J (2) 73 J (3) 146 J (4) 291 J

Ans. **[4]**

Sol. μ = 0.5 mole; P = 1 atm

T_1 = 20°C ; T_2 = 90°C

$W = P[V_2 - V_1] = \mu R(T_2 - T_1) = \frac{1}{2} \times 8.31 \times 70 = 291$ J

Q.16 Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ?

(1) 10^4 J (2) 10^3 J (3) 10^5 J (4) 10^6 J

Ans. **[1]**

Sol. $P = \frac{\rho RT}{M}$

$4 \times 10^4 = \frac{8}{4 \times 1.6 \times 10^{-27}} \times 8.314T$

$E = \frac{3}{2}PV = \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8.4} = 1.5 \times 10^4$

Q.17 The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(1) 2900 kHz (2) 2750 kHz (3) 2250 kHz (4) 2000 kz

Ans. **[4]**

Sol. Amplitude modulated wave consist of three waves f_c, $f_c - f_m - f_c + f_m$

$f_c - f_m$ and $f_c + f_m$ are side band frequency

$f_c = 250 \times 10$

= 250 kHz

Left Side Band = LSB = $f_c - f_m$ = 2500 – 250

= 2250 kHz

Right Side Band = RSB = $f_c + f_m$ = 2500 + 250

= 2750 kHz

One new signal is of 250 Hz

For this carrier wave should be taken in such a way that.

LSB, RSB, f_c do not overlap with previous frequencies.

$\therefore f_c = 2000$

as LSB = 1750 kHz

RSB = 2250 kHz

NTA has given answer (1) but answer should be (4).

Q.18 The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression $\vec{E}(x,y) = 10\hat{j}\cos[(6x+8z)]$. The magnetic field $\vec{B}(x,z,t)$ is given by – (c is the velocity of light)

(1) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x+8z-10ct)]$

(2) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z-10ct)]$

(3) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x-8z+10ct)]$

(4) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z+10ct)]$

Ans. **[2]**

Sol. $E = 10\hat{j}\cos(6x + 8z)$

Phase angle of E at t = 0 is 6x + 8z.

As E and B oscillate in same phase $\therefore$ at t = 0

Phase angle of B must be 6x + 8z

Direction of wave propagation = $\frac{6\hat{j}+8\hat{k}}{\sqrt{36+64}}$

$= \frac{6\hat{j}+8\hat{k}}{10}$

Let $\vec{B} = a\hat{i}+b\hat{j}+d\hat{k}$ and unit vector in direction of propagation of EM wave is $\frac{\vec{E}\times\vec{B}}{|\vec{E}||\vec{B}|}$

$$\frac{\vec{E}\times\vec{B}}{|\vec{E}||\vec{B}|} = \frac{6\hat{i}+8\hat{k}}{10}$$

$$\frac{10d\hat{i}+\hat{k}(-10a)}{10\times\sqrt{a^2+b^2+a^2}} = \frac{6\hat{i}+8\hat{k}}{10}$$

$|\vec{B}| = \frac{E}{c} = \frac{10}{c}$

$c\frac{[10d\hat{i}-10a\hat{k}]}{10\times10} = \frac{6\hat{i}+8\hat{k}}{10}$

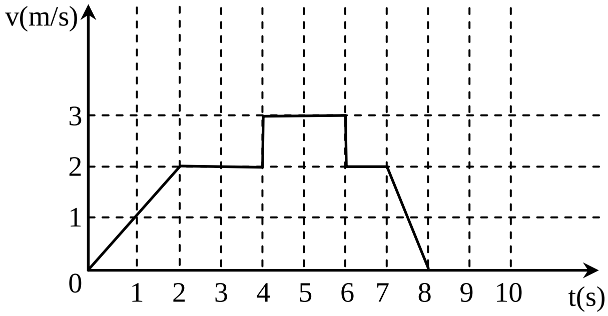
$c|d\hat{i}-a\hat{k}| = 6\hat{i}+8\hat{k}$

$d = \frac{6}{c}$

$a = -\frac{8}{c} \quad \therefore \vec{B} = -\frac{8}{c}\hat{i}+\frac{6}{c}\hat{k}$

$\therefore B = \frac{6\hat{k}-8\hat{i}}{c}\cos(6x + 8z - 10ct)$

Q.19 A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?



(1) 3 m (2) 9 m (3) 10 m (4) 6 m

Ans. **[2]**

Sol. Area under v-t curve gives displacement.

$A = 2 + 4 + 3 = 9 = \Delta S$

$\Delta S = x_f - x_i = 9 - 0 = 9$

So, final position is x = 9 m.

Q.20 Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is -

(1) 90º (2) 60º (3) 30º (4) 120º

Ans. **[2]**

Sol. $R_1 = \sqrt{(2F)^2 + (3F)^2 + 2.2F.3F\cos\theta}$

$R_2 = \sqrt{(2F)^2 + (6F)^2 + 2.2F.6F\cos\theta}$

If $R_2 = 2R_1$

$\sqrt{(2F)^2 + (3F)^2 + 2.2F.3F\cos\theta} = 2\sqrt{(2F)^2 + (6F)^2 + 2.2F.6F\cos\theta}$

$\cos\theta = -\frac{1}{2} = \cos 120°$

$\theta = 120°$

Q.21 The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separateds two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus -

(1) 2 cm (2) 3.1 cm (3) 4.0 cm (4) 1 cm

Ans. **[2]**

Sol. $\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R}$

$u = -\infty$; $v = ?$; $\mu_r = \frac{4}{3}$

$R = +7.8$

$\frac{4}{3v} - 0 = \frac{4/3 - 1}{7.8} = \frac{1}{3 \times 7.8}$

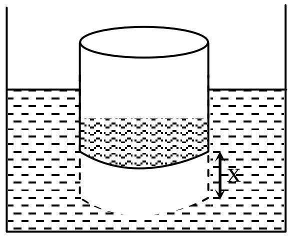
$v = +4 \times 7.8 = 31.2$ mm = 3.1 cm

Q.22 A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to – (density of water = 10^3 kg/m^3).

(1) 2.50 rad s^{-1} (2) 3.75 rad s^{-1} (3) 5.00 rad s^{-1} (4) 1.25 rad s^{-1}

Ans. **[Bonus]**

Sol.



On depressed slightly restoring force $A\rho_{water}\,xg$

Mass of bottle with water in it = $V \times \rho_{water}$

$= 310 \times 10^{-6} \times 10^{3+}$

$= 0.31$ kg

$0.31\,a = A\rho_{water}\,x \times 10$

$$a = \pi\left(\frac{2.5}{100}\right)^2 \times \frac{10\times10}{0.31}x$$

$$= \frac{3.1}{0.31}\times\frac{(2.5)^2\times10^4}{10^4}x$$

$a = 10 \times (2.5)^2 x$

$a = 62.5\,x$

$\omega = \sqrt{62.5}$ rad/s

$\omega = 8$ rad/s

as no answer is matching.

Q.23 A hope and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then -

(1) $T_h = 1.5\,T_c$ (2) $T_h = T_c$ (3) $T_h = 2T_c$ (4) $T_h = 0.5T_c$

Ans. **[2]**

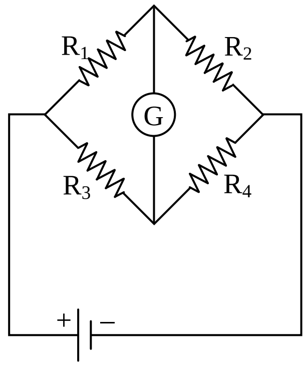
Sol. $M_C = 2M$ & $M_H = M$

Using $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\therefore \frac{T_H}{T_C} = \sqrt{\frac{I_H}{M_H}\frac{M_C}{I_C}} = \sqrt{\frac{mR^2}{\frac{1}{2}mR^2 2M}} = \frac{1}{1}$$

$T_\ell = T_C$

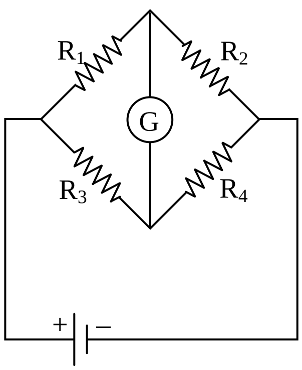
Q.24 The Wheatstone bridge shown in figure, here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be -



(1) Brown, Blue, Brown
(2) Grey, Black, Brown
(3) Red, Green, Brown
(4) Brown, Blue, Black

Ans. **[2]**

Sol.



Color code for R_1 orange, red, brown

$\therefore R_1 = 32 \times 10 = 320\ \Omega$

Balanced condition

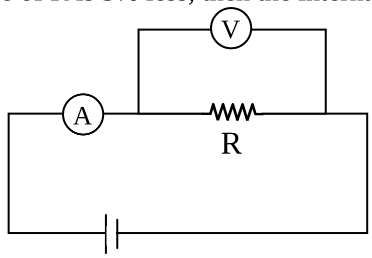
$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$R_3 = 160\ \Omega$

Corresponding color code for R_3 = Brown, Blue and Brown

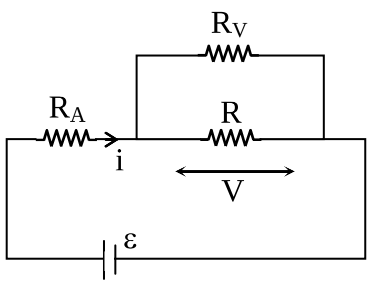
Q.25 The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is -



(1) 570 Ω (2) 600 Ω (3) 350 Ω (4) 35 Ω

Ans. **[1]**

Sol.



$$i = \frac{\varepsilon}{R_A + \frac{R_V R}{R_V + R}} = \frac{\varepsilon (R_V + R)}{R_A R_V + R_A R + R_V R}$$

$$V = \frac{R_V R \varepsilon}{(R_V + R)\left[R_A + \frac{R_V R}{R_V + R}\right]}$$

$$V = \frac{R_V R \varepsilon}{R_A R_V + R_A R + R_V R}$$

$$R_{res} = \frac{V}{i}$$

$$R_{res} = 30 - 30 \times \frac{5}{100} = 30\left[1 - \frac{5}{100}\right] = 28.5$$

$$28.5 = \frac{R_V R}{R_V + R}$$

$28.5\ R_V + 28.5 \times 30 = R_V \times 30$

$30 \times 28.5 = R_V \times 1.5$

$$R_V = \frac{28.5}{1.5} \times 30 = 570\ \Omega$$

Q.26 An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5ºC. (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1) 458 J kg^{-1} K^{-1} (2) 1232 J kg^{-1} K^{-1} (3) 654 J kg^{-1} K^{-1} (4) 916 J kg^{-1} K^{-1}

Ans. **[4]**

Sol. Final temperature = 21.5°C

Total heat gain = 0

$192 \times 5(21.5 - 100) + \frac{128}{1000} \times 394(21.5 - 8.4) + 240 \times 4.2\,(21.5 - 8.4) = 0$

$-5 \times 15072 + 13865.45 = 0$

$S = 916\ J\ kg^{-1}\ K^{-1}$

Q.27 A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(1) 9 J (2) 10 J (3) 12 J (4) 15 J

Ans. **[4]**

Sol. $w = \vec{F}.\vec{d} = (3\hat{i} - 12\hat{j}) \cdot (4\hat{i})$

w = 12 Joule

W.E.T.

$w_{Total} = \Delta k = k_f - k_i$

$12 = k_f - 3J$

$\Rightarrow k_f = 15J$

Q.28 A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is -

(1) $\frac{4\pi}{3}$ (2) $\frac{3}{8}\pi$ (3) $\frac{7}{3}\pi$ (4) $\frac{8\pi}{3}$

Ans. **[4]**

Sol. $v = \omega\sqrt{A^2 - x^2}$

$a = \omega^2 x$

$v = a$ (according to question |velocity| = |acceleration|).

$\omega\sqrt{A^2 - x^2} = \omega^2 x$

$\sqrt{A^2 - x^2} = \omega x$

$A^2 - x^2 = \omega^2 x^2$

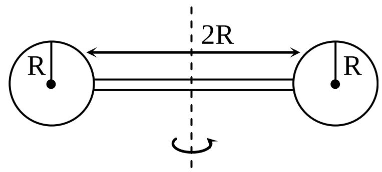
$25 - 16 = \omega^2 \times 16$

$9 = \omega^2 \times 16$

$\omega = \sqrt{\frac{9}{16}} = \frac{3}{4}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3}$ sec

Q.29 Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{17}{15}MR^2$ (2) $\frac{137}{15}MR^2$ (3) $\frac{209}{15}MR^2$ (4) $\frac{152}{15}MR^2$

Ans. **[1]**

Sol. $I = 2I_{sp} + I_{rod}$

$$= 2\left[\frac{2}{5}MR^2 + M(2R)^2\right] + \frac{M(2R)^2}{12}$$

$$= 2\left[\frac{22}{5}MR^2\right] + \frac{MR^2}{3}$$

$$= \frac{(132+5)}{15}MR^2$$

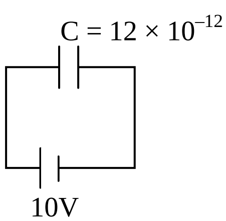
$$I = \frac{137}{15}MR^2$$

Q.30 A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is :

(1) 508 pJ (2) 692 pJ (3) 560 pJ (4) 600 pJ

Ans. **[1]**

Sol.



Internal energy = u

$Q = CV$

$= 12 \times 10^{-12} \times 10$

$= 12 \times 10^{-11}$ J

$u_i = \frac{1}{2} \times 12 \times 10^{-12} \times 100$

$= 600 \times 10–12$

$= 6 \times 10^{-10}$ J

After insertion

$C' = KC = 6.5 \times 12 \times 10^{-12}$

Final energy $u_f = \frac{Q^2}{2C'} = \frac{12\times12\times10^{-11}\times10^{-11}}{2\times6.5\times12\times10^{-12}}$

So energy dissipated = $u_i - u_f$

$\Rightarrow$ 508 pJ

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th January 2019 | Shift - II

CHEMISTRY

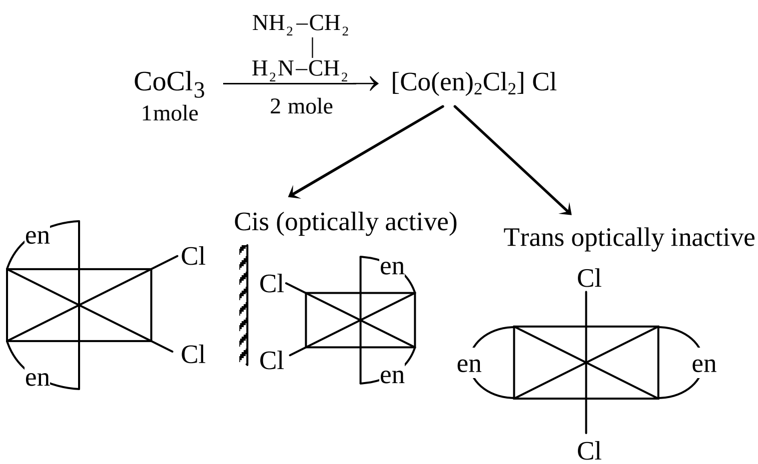
Q.1 A reaction of cobalt (III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) and B (green coloured). A can show optical activity, but B is optically inactive. What type of isomers does A and B represent ?

(1) Ionisation isomers   (2) Linkage isomers

(3) Coordination isomers   (4) Geometrical isomers

Ans. **[4]**

Sol.



Trans

Q.2 The pair that contains two P –H bonds in each of the oxoacids is :

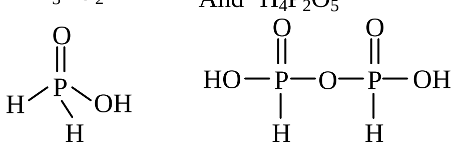
(1) H_3PO_2 and $H_4P_2O_5$ (2) $H_4P_2O_5$ and $H_4P_2O_6$

(3) $H_4P_2O_5$ and H_3PO_3 (4) H_3PO_3 and H_3PO_2

Ans. **[1]**

Sol.

H_3PO_2 And $H_4P_2O_5$



Both contain 2 p–H bonds.

Q.3 In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is :

(1) 10 (2) 2 (3) 1 (4) 5

Ans. **[3]**

Sol. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

The number of e^- involved in producing 10 mole of CO_2 is 10, so number of e^- involved in producing 1 mole of CO_2 will be 1.

Q.4 The reaction that is NOT involved in the ozone layer depletion mechanism in the stratosphere is :

(1) $Cl\dot{O}(g) + O(g) \rightarrow \dot{Cl}(g) + O_2(g)$

(2) $CF_2Cl_2(g) \xrightarrow{uv} \dot{Cl}(g) + \dot{C}F_2Cl(g)$

(3) $CH_4 + 2O_3 \rightarrow 3CH_2 = O + 3\ H_2O$

(4) $HOCl(g) \xrightarrow{hv} \dot{O}H(g) + \dot{Cl}(g)$

Ans. **[3]**

Sol. $Cl\dot{O}(g) + O(g) \longrightarrow \dot{Cl}(g) + O_2(g)$

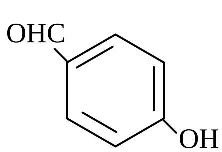
$CF_2Cl_2(g) \xrightarrow{uv} \dot{Cl}(g) + \dot{C}F_2Cl(g)$

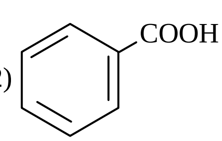
$HOCl(g) \xrightarrow{hv} \dot{O}H(g) + \dot{Cl}(g)$

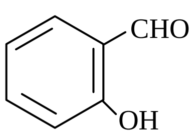
All the above three reaction are involved in the ozone layer deplection mechanism.

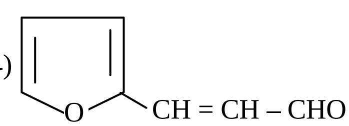
$CH_4 + 2O_3 \longrightarrow 3CH_2 = O + 3H_2O$ [not involved]

Q.5 An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N.. The structure of 'A' is :

(1) 

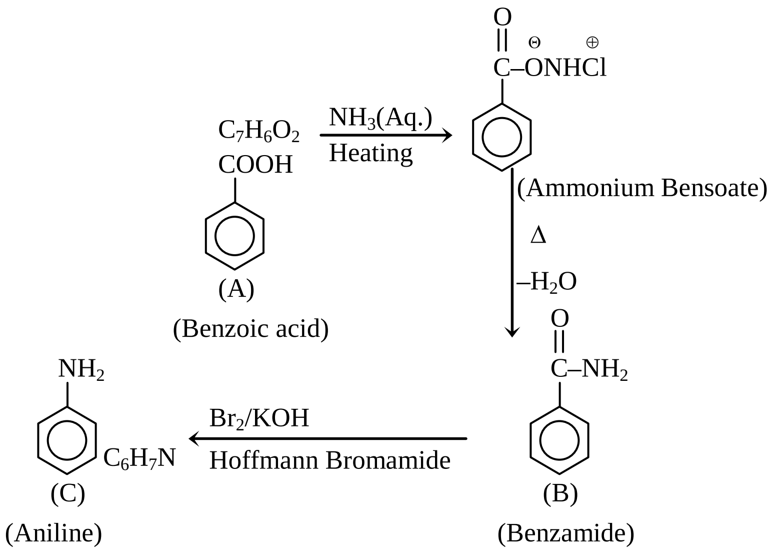
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Q.6 For an elementary chemical reaction, $A_2 \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is

(1) $2K_1[A_2] - K_{-1}[A]^2$ (2) $K_1[A_2] - K_{-1}[A]^2$

(3) $K_1[A_2] + K_{-1}[A]^2$ (4) $2K_1[A_2] - 2K_{-1}[A]^2$

Ans. **[4]**

Sol. $\frac{d[A]}{dt} = 2K_1[A_2] - 2K_{-1}[A]^2$

Q.7 The electrolytes usually used in the electroplating of gold and silver, respectively are:

(1) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$ (2) $[Au(CN)_2]^-$ and $[Ag(Cl_2)]^-$

(3) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$ (4) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$

Ans. **[1]**

Sol. Theoritical.

Q.8 Among the following reactions of hydrogen with halogens, the one that requires a catalyst is

(1) $H_2 + Br_2 \rightarrow 2HBr$ (2) $H_2 + Cl_2 \rightarrow 2HCl$

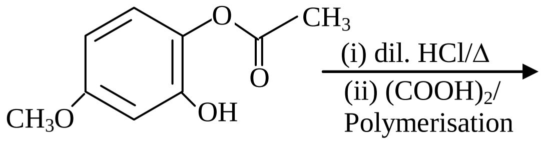
(3) $H_2 + F_2 \rightarrow 2HF$ (4) $H_2 + I_2 \rightarrow 2HI$

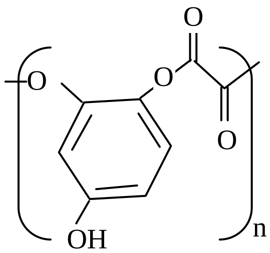
Ans. **[4]**

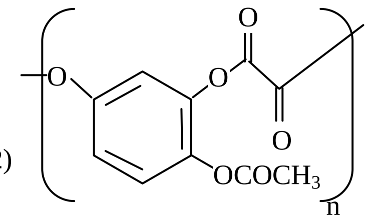
Sol. $H_2 + I_2 \rightleftharpoons 2HI$

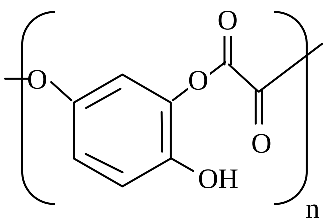
In this reaction catalyst is required because I_2 is a less reactive.

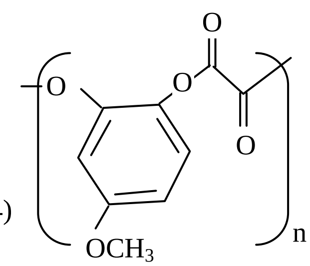
Q.9 The major product of the following reaction is



(1) 

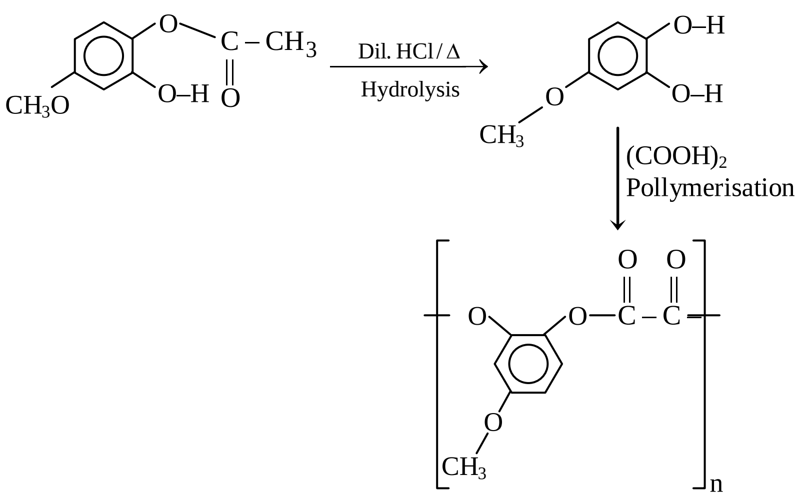
(2) 

(3) 

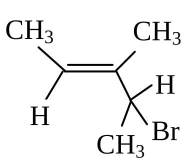
(4) 

Ans. **[4]**

Sol.



Q.10 What is the IUPAC name of the following compound ?



(1) 4–Bromo-3-methylpent-2-ene

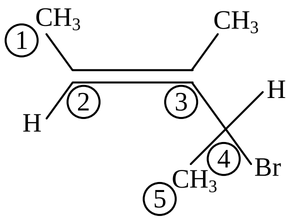
(2) 3–Bromo 1, 2-dimethylbut-1-ene

(3) 3–Bromo-3-methyl-1, 2-dimethylprop-1-ene

(4) 2-Bromo-3-methylpent-3-ene

Ans. **[1]**

Sol.



4 -Bromo- 3 - methyl Pent-2-ene

Q.11 A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms :

(1) hcp lattice – A, $\frac{1}{3}$ Tetrahedral voids-B

(2) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids -A

(3) hcp lattice - A, $\frac{2}{3}$ Tetrahedral voids -B

(4) hcp lattice -B, $\frac{2}{3}$ Tetrahedral voids-A

Ans. **[2]**

Sol. Total effective atoms in HCP unit cell = 6

Total no. of tetrahedral voids = 12

If B is placed at HCP lattice points then $\frac{1}{3}$ of tetrahedral voids will be occupied by A.

So the general formulae becomes A_2B_3.

Q.12 Soldium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of :

(1) ammoniated electrons (2) sodamide

(3) sodium-ammonia complex (4) sodium ion-ammonia complex

Ans. **[1]**

Sol. $Na + (x + y)\, NH_3 \longrightarrow \underset{\text{Ammoniated cation}}{Na^+(NH_3)_x} + \underset{\text{Ammoniated electron}}{e^-(NH_3)_y}$

Ammoniated electron is responsible for blue colour of the solution.

Q.13 Which of the following test cannot be used for identifying amino acids ?

(1) Ninhydrin test (2) Barfoed lest

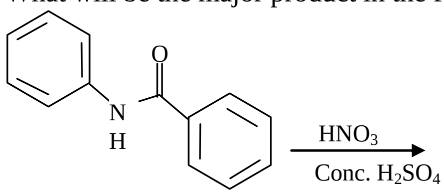
(3) Xanthoproteic test (4) Biuret test

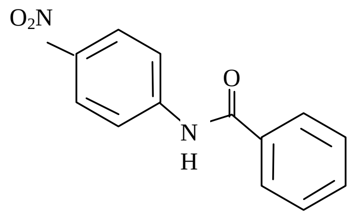
Ans. **Ans.** **[2]**

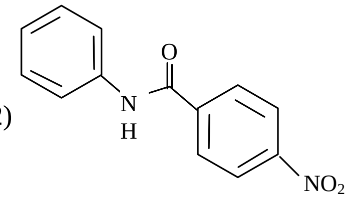
Sol. Ninhydrin, xantho proteic and biuret test is used for test of protein and amino acid but barfoed's test is used to detect the presence of monosaccharide (reducing).

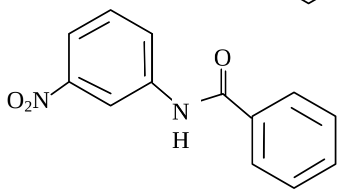
$$\text{Monosaccharide} \xrightarrow[\substack{+ \\ \text{Copper II Acetate} \\ \text{(Barfoed reagetn)}}]{\text{ethonic acid}} \text{Red colour}$$

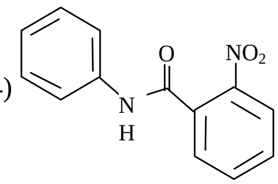
Q.14 What will be the major product in the following mononitration reaction ?



(1) 

(2) 

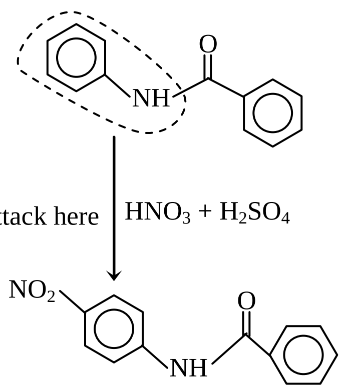
(3) 

(4) 

Ans. **[1]**

Sol.

More
e^- rich so
$E^{\oplus}$ will attack here



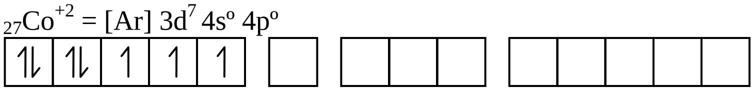
Q.15 The difference in the number of unpaired electrons of a metal ion in its high spin and low-spin octahedral complexes is two. The metal ion is

(1) Mn^{2+} (2) Ni^{2+} (3) Co^{2+} (4) Fe^{2+}

Ans. **[3]**

Sol.

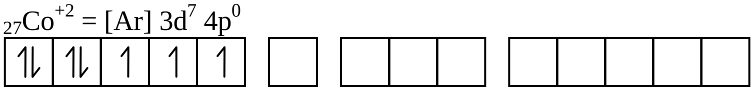
$_{27}Co^{+2}$ = [Ar] $3d^7\ 4s^o\ 4p^o$



3d 4s 4p 4d

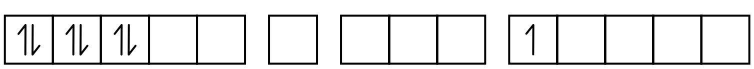
No. of unpaired e^- = 3 (For weak field ligand)

$_{27}Co^{+2}$ = [Ar] $3d^7\ 4p^0$



3d 4s 4p 4d

For strong field ligand



3d 4s 4p 4d

No. of unpaired e^- = 1

Q.16 The ground state energy of hydrogen atom is – 13.6 eV. The energy of second excited state of He^+ ion in eV is :

(1) –6.04 (2) – 54.4 (3) – 27.2 (4) – 3.4

Ans. **[1]**

Sol. $E = -13.6\frac{z^2}{n^2}$ eV

$\therefore E = -13.6 \times \frac{2^2}{3^2} = -6.04$ eV

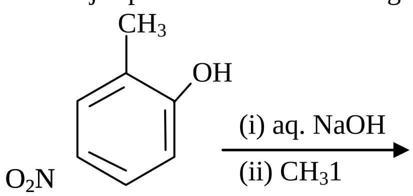
Q.17 The 71st electron of an element X with an atomic number of 71 enters into the orbital;

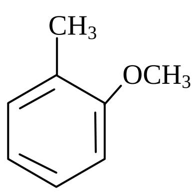
(1) 4f (2) 6s (3) 6p (4) 5d

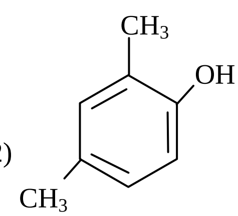
Ans. **[4]**

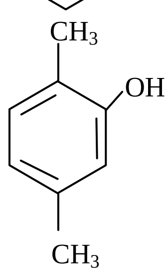
Sol. Electronic configuration of element having atomic number 71 is [Xe] $4f^{14}6s^2\,5d^1$.

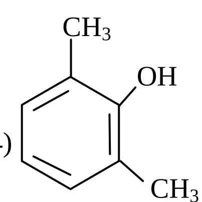
Q.18 The major product of the following reaction



(1) 

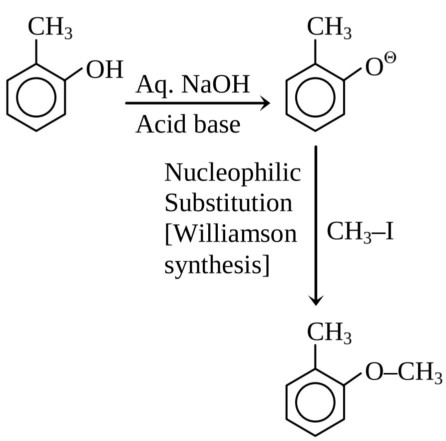
(2) 

(3) 

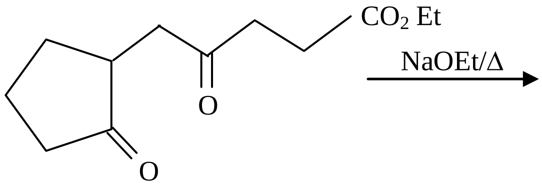
(4) 

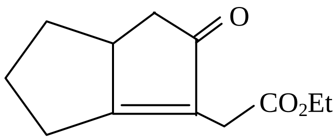
Ans. **[1]**

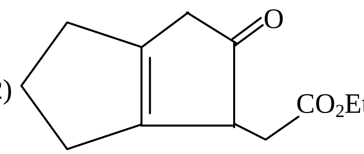
Sol.

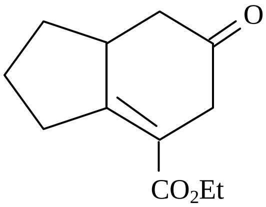


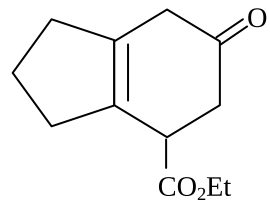
Q.19 The major product obtained in the following reaction is :



(1) 

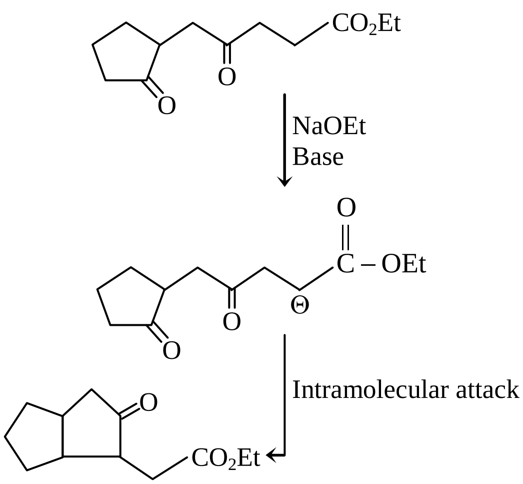
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.20 In the cell Pt|(s)|H_2(g, 1 bar)|HCl(aq)|AgCl|(s)|Ag(s)|Pt(s) the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of ($AgCl/Ag,Cl^-$) electrode is :

$$\left\{\text{Given, } \frac{2.303RT}{F} = 0.06\text{V at } 298\text{K}\right\}$$

(1) 0.94 V (2) 0.40V (3) 0.76 V (4) 0.20 V

Ans. **[4]**

Sol. $H_{2(g)} + 2AgCl_{(s)} \longrightarrow 2Ag_{(s)} + 2Cl^-_{(aq)} + 2H^+_{(aq)}$

$$E_{cell} = E^o_{cell} - \frac{0.06}{2}\log [[H^+]^2 [Cl^-]^2)$$

$$0.92 = 0 + E^o_{AgCl/Ag,Cl^-} - \frac{0.06}{2}\log [(10^{-6})^2 (10^{-6})^2]$$

$$0.92 + \frac{0.06}{2}\log 10^{-24} = E^o_{AgCl/Ag,Cl^-}$$

$$E^o_{AgCl/Ag,Cl^-} = 0.20$$

Q.21 Which is the most suitable reagent for the following transformation ?

$$CH_3-CH=CH-CH_2-\underset{}{\overset{\displaystyle OH \atop |}{CH}}-CH_3 \rightarrow CH_3-CH=CH-CH_2CO_2H$$

(1) alkaline $KMnO_4$ (2) Tollen's neagent (3) $I_2/NaOH$ (4) CrO_2Cl_2/CS_2

Ans. **[3]**

Sol.

$$CH_3–CH=CH–CH_2–\overset{\displaystyle O-H \atop |}{CH}-CH_3$$

$$\Big\downarrow I_2/NaOH$$

$$CH_3–CH=CH–CH_2–\overset{\displaystyle O \atop \|}{C}-CH_3$$

$$\Big\downarrow \begin{matrix} I_2/NaOH \\ \text{Iodoform Reaction} \end{matrix}$$

$$CH_3–CH=CH–CH_2–\underset{\displaystyle \| \atop O}{C}-\overset{\ominus}{O}\overset{\oplus}{N}a + CHI_3$$

$$\Big\downarrow H^{\oplus}$$

$$CH_3–CH=CH–CH_2–COOH$$

Q.22 The process with negative entropy change is
(1) Dissociation of $CaSO_4$ (s) to CaO(s) and SO_3(g)
(2) Dissolution of iodine in water
(3) Synthesis of ammonia from N_2 and H_2
(4) Sublimation of dry ice

Ans. **[3]**

Sol. $N_{2(g)} + H_{2(g)} \rightleftharpoons 2NH_{3(g)}$; $\Delta ng < 0$
entropy decreases in above reaction.

Q.23 Elevation in the boiling point for 1 molar solution of glucose is 2 K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2 K. The relation between K_b and K_f is :

(1) $K_b = K_f$ (2) $K_b = 0.5\ K_f$ (3) $K_b = 1.5\ K_f$ (4) $K_b = 2\ K_f$

Ans. **[4]**

Sol. $\Delta T_b = K_b m$
$2 = K_b \times 1$
$\therefore K_b = 2$
$\Delta T_f = K_f\ m$
$2 = K_f \times 2$
$\therefore K_f = 1$
$\therefore K_b = 2K_f$

Q.24 The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2L of its 0.1 M aqueous solution is :

(1) 17. 1g (2) 34.2g (3) 68.4g (4) 136.8 g

Ans. **[3]**

Sol. $\eta_{C_{12}H_{22}O_{11}} = 0.1 \times 2 = 0.2$

$\omega_{C_{12}H_{22}O_{11}} = 0.2 \times 342 = 68.4$ gm

Q.25 The correct match between item 'I' and item 'II' is :

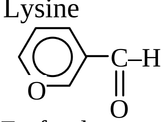
Item 'I' (compound)	Item 'II' (reagent)
(A) Lysine	(P) 1-naphthol
(B) Furfural	(Q) ninhydrin
(C) Benzylalcohol	(R) $KMnO_4$
(D) Styrene	(S) Ceric ammonium nitrate

(1) (A) → (Q); (B) → (R); (C) → (S); (D) →(P)
(2) (A) → (Q); (B) → (P); (C) → (S); (D) →(R)
(3) (A) → (R); (B) → (P); (C) → (Q); (D) →(S)
(4) (A) → (Q); (B) → (P); (C) → (R); (D) →(S)

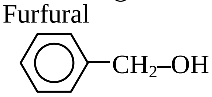
Ans. **[2]**

Sol.

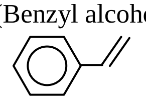
(A) Lysine ⟶ (Q) Ninhydrin

(B)  ⟶ (P) 1 Napthol (Molisch reagent)

Furfural

(C)  ⟶ (S) Cerric Ammonium nitrate

(Benzyl alcohol)

(D)  ⟶ (R) $KMnO_4$ (Unsaturation test)

Styrene

Q.26 An ideal gas undergoes isothermal compression from $5m^3$ to $1\ m^3$ against a constant external pressure of $4\ Nm^{-2}$. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J $mol^{-1}\ K^{-1}$, the temperature of Al increases by :

(1) $\frac{2}{3}$K (2) $\frac{3}{2}$K (3) 1 K (4) 2 K

Ans. **[1]**

Sol. $w = -\int P_{ext}\, dV$

$= -4\,(1-5) = 16$ J

$\Delta U = 0$ for isothermal process (ideal gas)

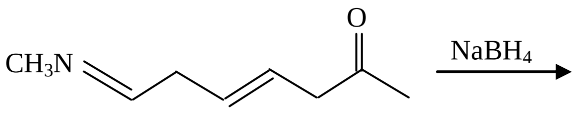
$\therefore q = -w$

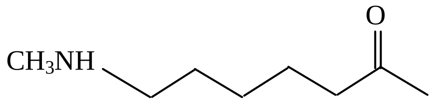
$\therefore q = -16$ J & $|q| = nC_V\Delta T$

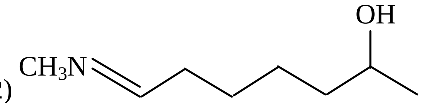
$|-16| = 1 \times 24 \times \Delta T$

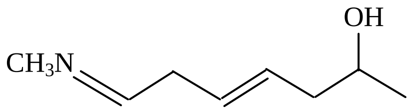
$\therefore \Delta T = \frac{2}{3}$ K

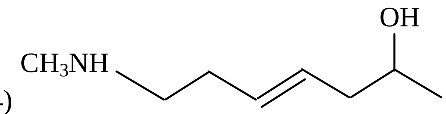
Q.27 The major product of the following reaction is :



(1) 

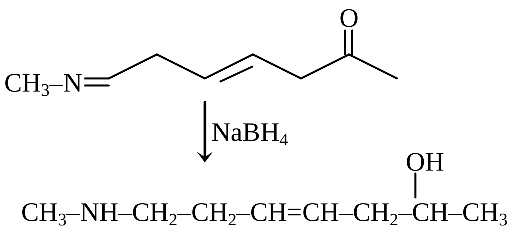
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.

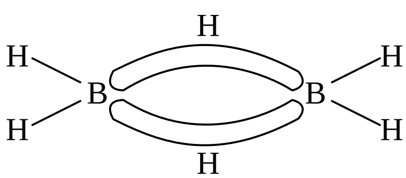


Q.28 The number of 2-centre-2-electron and 3-centre -2-electron bonds in B_2H_6, respectively, are :

(1) 2 and 2 (2) 2 and 1 (3) 2 and 4 (4) 4 and 2

Ans. **[4]**

Sol.



2-centre-2 electron bond = 4

3-centre-2 electron bond = 2

Q.29 Haemoglobin and gold sol are examples of :

(1) positively charged sols

(2) negatively charged sols

(3) Positively and negatively charged sols, respectively

(4) negatively and positively charged sols, respectively

Ans. **[3]**

Sol. Fact

Q.30 5.1 g NH_4SH is introduced in 3.0 L evacuated flask at 327ºC. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases . The K_p of the reaction at 327ºC is (R = 0.082 L atm mol^{-1} K^{-1}, Molar mass of S = 32 g mol^{-1} molar mass of N = 14 g mol^{-1})

(1) 0.242×10^{-4} atm^2
(2) 1×10^{-4} atm^2
(3) 4.9×10^{-3} atm^2
(4) 0.242 atm^2

Ans. **[4]**

Sol. $NH_4SH_{(s)} \rightleftharpoons NH_{3(g)} + H_2C_{s(g)}\ H_2S(g)$

t = 0 $\frac{5.1}{51} = 0.1$ mole

t = t $0.1 - 0.1 \times 0.3$ 0.03 0.03

$\eta_{NH_3} + \eta_{H_2S} = 0.06$

PV = nRT

$P_T \times 3 = 0.06 \times 0.082 \times 600$

$P_T = 0.984$

$\therefore\ P_{NH_3} = P_{H_2S} = 0.492$

$\therefore\ K_P = (0.492)^2 = 0.242$ atm^2

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th January 2019 | Shift - II

MATHEMATICS

Q.1 The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is -

(1) $8\sqrt{2}$ (2) $6\sqrt{3}$ (3) $3\sqrt{2}$ (4) $2\sqrt{11}$

Ans. **[2]**

Sol. Parabola $x^2 = 4y$ ....(i)

Chord $x - \sqrt{2}\,y + 4\sqrt{2} = 0$ ....(ii)

Solving (i) & (2)

$(\sqrt{2})^2 (y-4)^2 = 4y$

$\Rightarrow 2y^2 - 16y + 32 = 4y$

$\Rightarrow 2y^2 - 20y + 32 = 0$

$\Rightarrow y^2 - 10y + 16 = 0$

$\Rightarrow (y-2)(y-8) = 0$

$y = 2$ and $y = 8$

from (2) $x = -2\sqrt{2}$ | $x = 4\sqrt{2}$

$A(-2\sqrt{2}, 2)$ | $B(4\sqrt{2}, 8)$

Length of chord = distance between A and B

$= \sqrt{(6\sqrt{2})^2 + (6)^2}$

$= 6\sqrt{3}$

Q.2 Let N be the set of natural numbers and two functions f and g be defined as f, g : $N \to N$ such that

$$f(n) = \begin{cases} \dfrac{n+1}{2} & ; \text{ if } n \text{ is odd} \\ \dfrac{n}{2} & ; \text{ if } n \text{ is even} \end{cases}$$; and $g(n) = n - (-1)^n$. Then fog is

(1) neither one-one nor onto

(2) onto but not one-one

(3) both one-one and onto

(4) one-one but not onto

Ans. **[2]**

Sol. For f(g(n))

at, n = 1

$g(n) = 1 - (-1)^1$

$= 1 - (-1) = 2$

$f(2) = \frac{2}{2} = 1$

at, n = 2

$g(n) = 2 - (-1)^2$

$= 2 - (1) = 1$

$f(1) = 0$

at, n = 3

$g(n) = 3 - (-1)^3$

$= 3 - (-1) = 4$

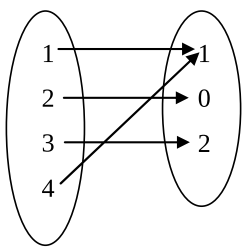
$f(4) = 2$

at, n = 4

$g(n) = 4 - (-1)^4$

$= 4 - (1) = 3$

$f(3) = 1$



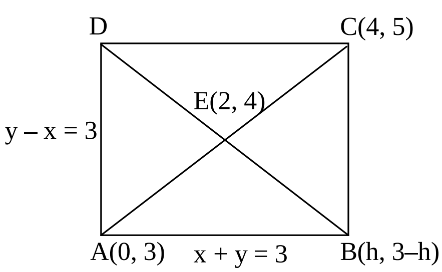
many one onto or onto but not one-one

Q.3 Two sides of a parallelogram are along the lines, x + y = 3 & x – y + 3 = 0. If its diagonals intersect at (2, 4), then one of its vertex is -

(1) (2, 1) (2) (2, 6) (3) (3, 5) (4) (3, 6)

Ans. **[4]**

Sol.



Slope of BC = 1

$\Rightarrow \frac{3-h-5}{h-4} = 1 \Rightarrow h = 1 \Rightarrow B(1, 2)$

Now, E is the mid point of BD so D(3, 6)

Q.4 If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \ldots, x_5$ and –50 is equal to

(1) 582.5 (2) 507.5 (3) 586.5 (4) 509.5

Ans. **[2]**

Sol. S.D. = 3

variance = 9

$$9 = \frac{1}{5}\sum_{i=1}^{5} x_1^2 - (\bar{x})^2$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 5 \times 109$$

variance of x_1, x_2, x_3, x_4, x_5 & –50

$$\text{variance} = \frac{1}{6}\sum_{i=1}^{6} x_1^2 - (\bar{x})^2$$

$$= \frac{1}{6}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 2500\right) - (\bar{x})^2$$

($\because$ $\bar{x}$ = 0 of x_1, x_2, x_3, x_4, x_5, –50)

$$= \frac{1}{6}(5 \times 109 + 2500)$$

= 507.5

Q.5 The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is -

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. **[3]**

Sol. $$\sum_{n=1}^{19}\cot^{-1}\left(1+\frac{2n(n+1)}{2}\right)$$

$$= \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right) = \sum_{n=1}^{19}\tan^{-1}(n+1) - \tan^{-1}(n)$$

$$= \tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + \ldots + \tan^{-1}(20) - \tan^{-1}(19)$$

$$\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right] = \cot(\tan^{-1}20 - \tan^{-1}1)$$

$$= \cot\left(\tan^{-1}\left(\frac{20-1}{1+20.1}\right)\right) = \cot\left(\tan^{-1}\left(\frac{19}{21}\right)\right)$$

$$= \cot\left(\cot^{-1}\left(\frac{21}{19}\right)\right) = \frac{21}{19}$$

Q.6 The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}........\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is -

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$ (3) $\frac{1}{1024}$ (4) $\frac{1}{512}$

Ans. **[4]**

Sol. $\left\{\cos\theta.\cos(2^1\theta).\cos(2^2\theta)......\cos(2^{n-1}\theta) = \frac{\sin 2^n\theta}{2^n\sin\theta}\right\}$

$= \left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\cos\frac{\pi}{2^4} \text{ .... } \cos\frac{\pi}{2^{10}}\right)\sin\frac{\pi}{2^{10}}$

$= \left(\cos\frac{\pi}{2^{10}}.\cos\frac{\pi}{2^9}.\cos\frac{\pi}{2^8}....\cos\frac{\pi}{2^2}\right)\sin\left(\frac{\pi}{2^{10}}\right)$

$= \frac{\sin\left(2^9.\frac{\pi}{2^{10}}\right)}{2^9\sin\left(\frac{\pi}{2^{10}}\right)} \text{ . } \sin\left(\frac{\pi}{2^{10}}\right)$

$= \frac{\sin\left(\frac{\pi}{2}\right)}{2^9} = \frac{1}{512}$

Q.7 If $\sum_{r=0}^{25}\{{}^{50}C_r.{}^{50-r}C_{25-r}\} = K({}^{50}C_{25})$, then K is equal to -

(1) 2^{24} (2) $2^{25} - 1$ (3) 2^{25} (4) $(25)^2$

Ans. **[3]**

Sol. $\sum_{r=0}^{25}{}^{50}C_r.{}^{50-r}C_{25-r}$

$= \sum_{r=0}^{25}\frac{50!}{r!(50-r)!}\times\frac{(50-r)!}{25!(25-r)!}$

$= {}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r = 2^{25}.{}^{50}C_{25}$

$\therefore$ K = 2^{25}

Q.8 Let $a_1, a_2, a_3, ... a_{10}$ be in G.P. with $a_i > 0$ for i = 1, 2, ….., 10 and S be the set of pairs (r, k), r, $k \in N$ (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$. Then the number of elements in S, is -

(1) 10 (2) 4 (3) 2 (4) infinitely many

Ans. **[4]**

Sol. $a_1^r\, a_2^k = a^r(aR)^k = a^{r+k} R^k$

$a_2^r\, a_3^k = (aR)^r (aR^2)^k = a^{r+k} R^{r+2k}$

$a_3^r\, a_1^k = (aR^2)^r (aR^3)^k = a^{r+k} R^{2r+3k}$

$$\Rightarrow \begin{vmatrix} \ln(a^{r+k}R^k) & \ln(a^{r+k}R^{r+2k}) & \ln(a^{r+k}R^{2r+3k}) \\ \ln(a^{r+k}R^{3r+4k}) & \ln(a^{r+k}R^{4r+5k}) & \ln(a^{r+k}R^{5r+6k}) \\ \ln(a^{r+k}R^{6r+7k}) & \ln(a^{r+k}R^{7r+8k}) & \ln(a^{r+k}R^{8r+9k}) \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1,\quad R_3 \to R_3 - R_2$

$$\Rightarrow \begin{vmatrix} \ln(a^{r+k}R^k) & \ln(a^{r+k}R^{r+k}) & \ln(a^{r+k}R^{2r+3k}) \\ \ln\left(\frac{R^{3r+4k}}{R^k}\right) & \ln\left(\frac{R^{4r+5k}}{R^{r+2k}}\right) & \ln\left(\frac{R^{5r+6k}}{R^{2r+3k}}\right) \\ \ln\left(\frac{R^{6r+7k}}{R^{3r+4k}}\right) & \ln\left(\frac{R^{7r+8k}}{R^{4r+5k}}\right) & \ln\left(\frac{R^{8r+9k}}{R^{5r+6k}}\right) \end{vmatrix} = 0$$

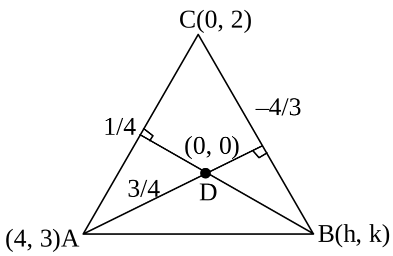
$$\Rightarrow \begin{vmatrix} \ln(a^{r+k}R^k) & \ln(a^{r+k}R^{r+k}) & \ln(a^{r+k}R^{2r+3k}) \\ \ln(R^{3r+3k}) & \ln(R^{3r+3k}) & \ln(R^{3r+3k}) \\ \ln(R^{3r+3k}) & \ln(R^{3r+3k}) & \ln(R^{3r+3k}) \end{vmatrix} = 0$$

Q.9 Two vertices of a triangle are (0, 2) and (4, 3). If its orthocenter is at the origin, then its third vertex lies in which quadrant -

(1) third (2) fourth (3) second (4) first

Ans. **[3]**

Sol.



(BC and AD are perpendicular)

$\Rightarrow \frac{(k-2)}{h} = -\frac{4}{3} \Rightarrow 3k - 6 = -4h \Rightarrow -12h - 6 = -4h \Rightarrow -6 = 8h \Rightarrow h = -\frac{3}{4}$

$\frac{k}{h} = -4 \Rightarrow k = -4h$

$k = -4\left(\frac{-3}{4}\right) \Rightarrow k = 3$

$B\left(\frac{-3}{4}, 3\right)$; IInd Quadrant

Q.10 If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target atleast once is greater than $\frac{5}{6}$, is-

(1) 4 (2) 6 (3) 5 (4) 3

Ans. [3]

Sol. $1-\left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\Rightarrow \frac{1}{6} > \left(\frac{2}{3}\right)^n$

$\Rightarrow 3^n > 2^n \times 6$

$\Rightarrow 3^{n-1} > 2^{n+1}$

$\Rightarrow$ n = 5 (minimum)

Q.11 The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point

(1) $\left(\frac{4}{3}, 2e\right)$ (2) (3, 6e) (3) (2, 3e) (4) $\left(\frac{5}{3}, 2e\right)$

Ans. [1]

Sol. $\frac{dy}{dx} = e^{x^2} + xe^{x^2}.2x$

At x = 1, slope of tangent m = 3e

Equation of tangent :

$y - e = 3e(x - 1)$

$\Rightarrow y = 3ex - 2e$

$\left(\frac{4}{3}, 2e\right)$ lies on it

Q.12 The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is -

(1) $\frac{1}{12}(7\pi-5)$ (2) $\frac{1}{12}(7\pi+5)$ (3) $\frac{3}{10}(4\pi-3)$ (4) $\frac{3}{20}(4\pi-3)$

Ans. [4]

Sol. $\int_{-\pi/2}^{-1} \frac{dx}{2-1} + \int_{-1}^{0} \frac{dx}{3-1} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\pi/2} \frac{dx}{5+0}$

$= (x)_{-\pi/2}^{-1} + \frac{1}{2}(x)_{-1}^{0} + \frac{1}{4}(x)_0^1 + \frac{1}{5}(x)_1^{\pi/2} = \left(-1+\frac{\pi}{2}\right) + \frac{1}{2}(1) + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2}-1\right)$

$= -1 + \frac{\pi}{2} + \frac{1}{2} + \frac{1}{4} + \frac{\pi}{10} - \frac{1}{5} = -\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$

$= \frac{-10+5-4}{20} + \frac{5\pi+\pi}{10} = -\frac{9}{20} + \frac{6\pi}{10} = -\frac{9}{20} + \frac{3\pi}{5} = \frac{3}{20}(4\pi-3)$

Q.13 Let $S = \left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$; $r \neq \pm 1$. Then S represents

(1) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where r > 1

(2) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when 0 < r < 1

(3) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when 0 < r < 1

(4) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when r > 1

Ans. **[4]**

Sol. $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

If $r \in (0, 1)$ then this curve is a hyperbola whose

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1-r}{1+r}} = \sqrt{\frac{2}{1+r}}$

If r > 1 then this curve is an ellipse whose

$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\left(\frac{r-1}{r+1}\right)} = \sqrt{\frac{2}{r+1}}$

Q.14 If $\int_0^x f(t)\,dt = x^2 + \int_x^1 t^2 f(t)\,dt$ then $f'\left(\frac{1}{2}\right)$ is -

(1) $\frac{18}{25}$ (2) $\frac{6}{25}$ (3) $\frac{24}{25}$ (4) $\frac{4}{5}$

Ans. **[3]**

Sol. $f(x) = 2x - x^2 f(x)$

$f(x) = \frac{2x}{1+x^2}$, $f\left(\frac{1}{2}\right) = \frac{1}{1+(1/4)} = \frac{4}{5}$

$f'(x) = \frac{(1+x^2)^2 - 2x(2x)}{(1+x^2)^2} = 2\left[\frac{1-x^2}{(1+x^2)^2}\right]$

$f'\left(\frac{1}{2}\right) = \frac{2\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{2(3/4)}{25/16} = \frac{24}{25}$

Q.15 Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0. Then the minimum value of $\frac{\det(A)}{b}$ is -

(1) $\sqrt{3}$ (2) $-2\sqrt{3}$ (3) $-\sqrt{3}$ (4) $2\sqrt{3}$

Ans. **[4]**

Sol. $\det(A) = 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$

$= 2(b^2 + 2) - b^2 - 1$

$= b^2 + 3$

$\frac{\det(A)}{b} = \frac{b^2+3}{b} = b + \frac{3}{b}$

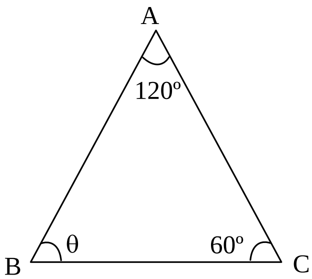
Minimum value = $2\sqrt{3}$

Q.16 With the usual notation, in ΔABC, if $\angle A + \angle B = 120^o$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$, then the ratio $\angle A : \angle B$, is -

(1) 9 : 7 (2) 7 : 1 (3) 5 : 3 (4) 3 : 1

Ans. **[2]**

Sol. $\frac{a}{b} = \frac{\sin A}{\sin B}$



$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sin(120^o - \theta)}{\sin\theta}$

$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta}{\sin\theta}$

$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{1}{2} = \frac{\sqrt{3}}{2}\cot\theta$

$\Rightarrow \frac{3+2\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\cot\theta$

$\Rightarrow \cot\theta = \sqrt{3} + 2$

$\Rightarrow \tan\theta = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2 - \sqrt{3}$

$\theta = 15^o$

$\therefore \angle A = 120^o - 15^o = 105^o$

$\therefore \frac{\angle A}{\angle B} = \frac{105}{15} = \frac{7}{1} = 7 : 1$

Q.17 The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is -

(1) 1 (2) 2 (3) $\frac{15}{8}$ (4) $\frac{4}{9}$

Ans. **[2]**

Sol. Sum of roots $\alpha + \beta = -(3 - \lambda)$

Product of roots $\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (3 - \lambda)^2 - 2(2 - \lambda)$

$= 9 + \lambda^2 - 6\lambda - 4 + 2\lambda$

$= \lambda^2 - 4\lambda + 5$

$\alpha^2 + \beta^2$ is least then $\lambda = -\frac{b}{2a}$

$$\lambda = -\frac{-(-4)}{2} = 2$$

Q.18 On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?

(1) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$ (2) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$ (4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Ans. **[3]**

Sol. $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1} = \lambda$ (any point)

P $(2\lambda + 4, 2\lambda + 5, \lambda + 3)$ lies on plane $x + y + z = 2$

$\therefore 2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$\boxed{\lambda = -2} \Rightarrow P(0, 1, 1)$

Hence option (3) is correct

Q.19 The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$ is 720, is -

(1) 4 (2) $2\sqrt{2}$ (3) 3 (4) $\sqrt{5}$

Ans. **[1]**

Sol. $T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^r = {}^{10}C_r\,\lambda^r\,x^{5-\frac{r}{2}-2r}$

$\Rightarrow 5 - \frac{r}{2} - 2r = 0 \Rightarrow r = 2$

coefficient of $x^2 \Rightarrow {}^{10}C_2\,\lambda^2 = 720 \Rightarrow \lambda^2 = 16 \Rightarrow \lambda = \pm 4 \Rightarrow \lambda = 4$

Q.20 Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I(z) respectively denote the real and imaginary parts of z, then -

(1) R(z) = – 3
(2) R(z) < 0 and I(z) > 0
(3) I(z) = 0
(4) R(z) > 0 and I(z) > 0

Ans. **[3]**

Sol. $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$

$z = \left(e^{i\frac{\pi}{6}}\right)^5 + \left(e^{-i\frac{\pi}{6}}\right)^5$

$z = e^{i\frac{5\pi}{6}} + e^{-i\frac{5\pi}{6}}$

$z = 2\cos\frac{5\pi}{6}$

$z = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

It means Im(z) = 0

Q.21 The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$
$-x + 4y + 7z = 0$
$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is -

(1) two
(2) one
(3) four
(4) three

Ans. **[1]**

Sol. For non trivial solution $\Delta = 0$

$\Rightarrow \begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$

$\Rightarrow 4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{3}{2}$

For $\theta \in (0, \pi)$

$\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$ are satisfy the equation

Number of values of $\theta = 2$

Q.22 The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xy\,dy = 0$ which passes through (1, 1) is

(1) a circle with centre on the y-axis

(2) an ellipse with major axis along the y-axis

(3) a circle with centre on the x-axis

(4) a hyperbola with transverse axis along the x-axis

Ans. **[3]**

Sol. $(x^2 - y^2)dx + 2xydy = 0$

$$\frac{dy}{dx} = \frac{-(x^2 - y^2)}{2xy}$$

Put y = vx

$$\frac{dy}{dx} = v + \frac{xdv}{dx}$$

$$\Rightarrow \quad v + \frac{xdv}{dx} = \frac{-(x^2 - v^2x^2)}{2x(vx)}$$

$$\Rightarrow \quad v + \frac{xdv}{dx} = \frac{-1 + v^2}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-1 + v^2}{2v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\Rightarrow \quad \int \frac{2vdv}{1 + v^2} = -\int \frac{dy}{x}$$

$\Rightarrow$ $\log(1 + v^2) = -\log x + \log C$

$$\Rightarrow \quad 1 + \frac{y^2}{x^2} = \frac{C}{x}$$

$\Rightarrow$ $x^2 + y^2 = Cx$

it passes through (1, 1)

$\therefore$ C = 2

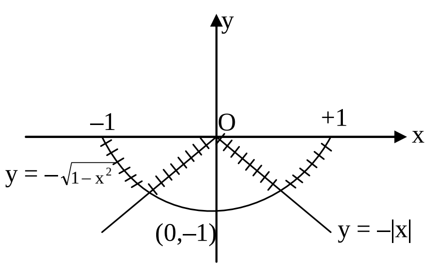
$\therefore$ Curve $x^2 + y^2 - 2x = 0$

Q.23 Let $f : (-1, 1) \to R$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1 - x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly -

(1) one element (2) three elements (3) five elements (4) two elements

Ans. **[2]**

Sol. $x \in (-1, 1)$



three points

Q.24 Consider the following three statements :

P : 5 is a prime number

Q : 7 is a factor of 192

R : L.C.M. of 5 and 7 is 35

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q) \vee (\sim R)$ (2) $P \vee (\sim Q \wedge R)$ (3) $(\sim P) \wedge (\sim Q \wedge R)$ (4) $(\sim P) \vee (Q \wedge R)$

Ans. **[2]**

Sol. P is true

Q is false

R is true

by option

$P \vee (\sim Q \wedge R)$

$= T \vee (T \wedge T)$

$= T$

Q.25 Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$

(1) does not exist (2) exists and equals $\frac{4}{7}$

(3) exists and equals 4 (4) exists and equals 0

Ans. **[3]**

Sol. $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x} (x > 0)$

Given $f(1) \neq 4$, $\lim_{x\to 0+} x + \left(\frac{1}{x}\right) = 7$

$\Rightarrow \frac{dy}{dx} + \frac{3}{4}\frac{y}{x} = 7$ (linear diff. equation)

I.F. $= e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\log|x|} = x^{3/4}$

$yx^{3/4} = \int 7.x^{3/4}dx$

$\Rightarrow yx^{3/4} = 7\frac{x^{7/4}}{(7/4)} + C$

$\Rightarrow f(x) = y = 4x + Cx^{-3/4}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{3/4}$

$\lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = \lim_{x\to 0^+} (4 + C.x^{7/4}) = 4$

Q.26 If $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is -

(1) 4 (2) 3 (3) –3 (4) –4

Ans. **[4]**

Sol. $\vec{\alpha} = \mu\vec{\beta}$

$(\lambda - 2)\,\vec{a} + \vec{b} = \mu\,[(4\lambda - 2)\,\vec{a} + 3\vec{b}\,]$

$\lambda - 2 = \mu(4\lambda - 2)$

$\Rightarrow 1 = 3\mu$

$\Rightarrow \mu = \dfrac{1}{3}$

$\Rightarrow \lambda - 2 = \dfrac{1}{3}(4\lambda - 2)$

$\Rightarrow 3\lambda - 6 = 4\lambda - 2$

$\Rightarrow -\lambda = 4$

Q.27 The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points ?

(1) (2, 1, 3)

(2) (4, –1, 2)

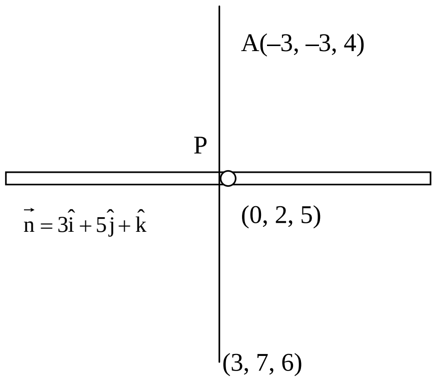
(3) (4, 1, – 2)

(4) (– 2, 3, 5)

Ans. **[3]**

Sol. Mid point of AB (0, 2, 5)

Equation of plane



$3(x - 0) + 5(y - 2) + 1\,(z - 5) = 0$

$\Rightarrow\ 3x + 5y + z = 15$

Hence option (3) satisfy the equation of plane

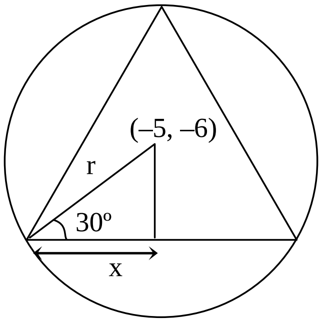
Q.28 If the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq units then c is equal to

(1) 20 (2) 25 (3) –25 (4) 13

Ans. **[2]**

Sol. C(–5, –6)

$r = \sqrt{25 + 36 - c}$



$\cos 30^\circ = \frac{x}{r}$

$\frac{\sqrt{3}}{2} = \frac{x}{r}$

$\Rightarrow x = \frac{\sqrt{3}\,r}{2}$

side of triangle = $\sqrt{3}\,r$

Area = $\frac{\sqrt{3}}{4}(\sqrt{3}\,r)^2$

$\frac{3\sqrt{3}}{4}r^2 = 27\sqrt{3}$

$\Rightarrow r^2 = 36$

$\Rightarrow 25 + 36 - c = 36$

$\Rightarrow c = 25$

Q.29 If $\int x^5 . e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to -

(1) $-2x^3 - 1$ (2) $-2x^3 + 1$ (3) $4x^3 + 1$ (4) $-4x^3 - 1$

Ans. **[4]**

Sol. $\int x^5 . e^{-4x^3} dx = \int x^3 . x^2 e^{-4x^3} dx$

Let $4x^3 = t$

$12x^2\, dx = dt$

$= \int \frac{t}{4} . \frac{1}{12} e^{-t}\, dt = \frac{1}{48} \int \underset{I}{t} . \underset{II}{e^{-t}}\, dt$

$= \frac{1}{48}\left[\int - t . e^{-t} + \int e^{-t} dt\right]$ (integration by part) $= \frac{1}{48}(-te^{-t} - e^{-t})$

$= -\frac{1}{48} e^{-t}(t + 1) = -\frac{1}{48} e^{-4x^3}(4x^3 + 1)$

$\because f(x) = -(4x^3 + 1) = -4x^3 - 1$

Q.30 A helicopter is flying along the curve given by $y - x^{3/2} = 7,\ (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is -

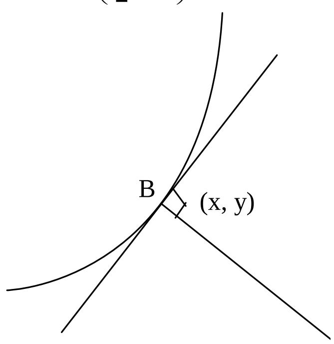
(1) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (2) $\frac{\sqrt{5}}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}\sqrt{\frac{7}{3}}$

Ans. **[1]**

Sol. $y - x^{3/2} = 7\ (x \geq 0)$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$$



$$\frac{3}{2}x^2 = \frac{1}{2} - x$$

$(x + 1)(3x - 1) = 0$

$x = -1$ (rejected)

$$x = \frac{1}{3}$$

$$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$$

$$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3}$$

$$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$$