

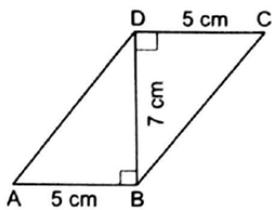
## 10. Area

### Exercise 10A

#### 1. Question

In the adjoining figure, show that ABCD is a parallelogram.

Calculate the area of llgm ABCD.



#### Answer

In the given figure consider  $\triangle ABD$  and  $\triangle BCD$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times BD$$

$$= \frac{1}{2} \times 5 \times 7 = \frac{35}{2} \text{ ----- 1}$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times DC \times DB$$

$$= \frac{1}{2} \times 5 \times 7 = \frac{35}{2} \text{ ----- 2}$$

From 1 and 2 we can tell that area of two triangle that is  $\triangle ABD$  and  $\triangle BCD$  are equal

Since the diagonal BD divides ABCD into two triangles of equal area and opp sides  $AB = DC$

$\therefore$  ABCD is a parallelogram

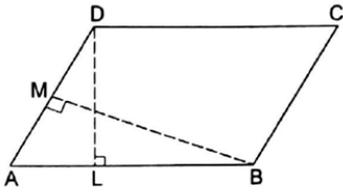
$\therefore$  Area of parallelogram ABCD = Area of  $\triangle ABD$  + Area of  $\triangle BCD$

$$= \left( \frac{35}{2} + \frac{35}{2} \right) = \frac{70}{2} \text{ cm}^2 = 35 \text{ cm}^2$$

$\therefore$  Area of parallelogram ABCD =  $35 \text{ cm}^2$

#### 2. Question

In a parallelogram ABCD, it is being given that  $AB = 10 \text{ cm}$  and the altitudes corresponding to the sides AB and AD are  $DL = 6 \text{ cm}$  and  $BM = 8 \text{ cm}$ , respectively. Find AD.



### Answer

Given

$$AB = 10 \text{ cm}$$

$$DL = 6 \text{ cm}$$

$$BM = 8 \text{ cm}$$

$$AD = ? \text{ (To find)}$$

Here, Area of parallelogram = base  $\times$  height

In the given figure if we consider AB as base Area =  $AB \times DL$

If we consider DM as base Area =  $AD \times BM$

$$\therefore \text{Area} = AB \times DL = AD \times BM$$

$$\Rightarrow 10 \times 6 = AD \times 8$$

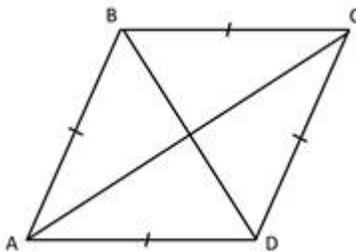
$$\Rightarrow 60 = 8 \times AD$$

$$\Rightarrow AD = \frac{60}{8} = 7.5 \text{ cm}$$

### 3. Question

Find the area of a rhombus, the lengths of whose diagonals are 16 cm and 24 cm respectively.

### Answer



Here, Let ABCD be Rhombus with diagonals AC and BD

Here let  $AC = 24$  and  $BD = 16$

We know that, in a Rhombus, diagonals are perpendicular bisectors to each other

$\therefore$  if we consider  $\triangle ABC$  AC is base and OB is height

Similarly, in  $\triangle ADC$  AC is base and OD is height

Now, Area of Rhombus = Area of  $\triangle ABC$  + Area of  $\triangle ADC$

$$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$$

$$= \frac{1}{2} \times 24 \times \frac{BD}{2} + \frac{1}{2} \times 24 \times \frac{BD}{2} \text{ (Since AC and BC are perpendicular bisectors } \therefore OB = OD = \frac{BD}{2})$$

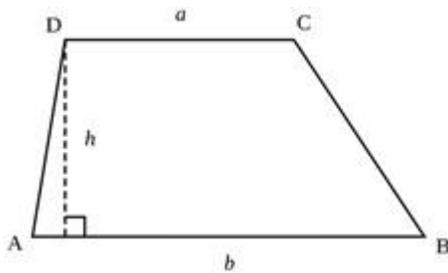
$$= \frac{1}{2} \times 24 \times \frac{16}{2} + \frac{1}{2} \times 24 \times \frac{16}{2} = 96 + 96 = 192 \text{ cm}^2$$

$\therefore$  Area of Rhombus ABCD is  $192\text{cm}^2$

#### 4. Question

Find the area of a trapezium whose parallel sides are 9 cm and 6 cm respectively and the distance between these sides is 8 cm.

**Answer**



Given

$$AB = a = 9 \text{ cm}$$

$$DC = b = 6 \text{ cm}$$

$$\text{Height (h)} = 8 \text{ cm}$$

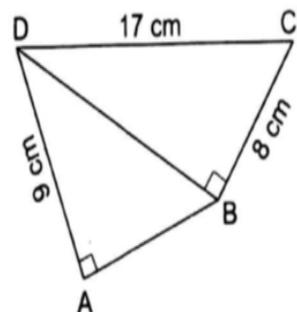
We know that area of trapezium is  $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$$\text{Therefore, Area of trapezium ABCD} = \frac{1}{2} \times (AB + DC) \times h = \frac{1}{2} \times (9 + 6) \times 8 = 60 \text{ cm}^2$$

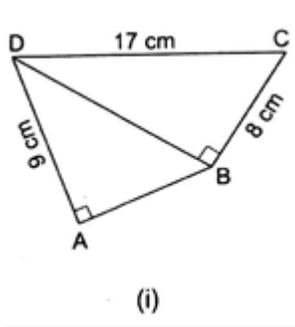
$\therefore$  Area of Trapezium ABCD =  $60\text{cm}^2$

#### 5A. Question

Calculate the area of quad. ABCD, given in Fig. (i).



## Answer



Given

$$AD = 9 \text{ cm}$$

$$BC = 8 \text{ cm}$$

$$DC = 17 \text{ cm}$$

Here Area of Quad ABCD = Area of  $\triangle ABD$  + Area of  $\triangle BCD$

$$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$$

By Pythagoras theorem in  $\triangle BCD$

$$DC^2 = BD^2 + BC^2$$

$$17^2 = BD^2 + 8^2$$

$$BD^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$\therefore BD = 15 \text{ cm}$$

Similarly in  $\triangle ABD$  using Pythagoras theorem

$$BD^2 = AD^2 + AB^2$$

$$15^2 = 9^2 + AB^2$$

$$AB^2 = 15^2 - 9^2 = 225 - 81 = 144$$

$$\therefore AB = 12 \text{ cm}$$

Now, Area of Quad ABCD = Area of  $\triangle ABD$  + Area of  $\triangle BCD$

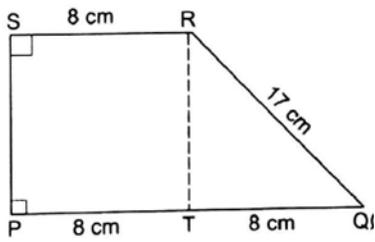
$$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$$

$$= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 8 \times 15 = 54 + 60 = 114 \text{ cm}^2$$

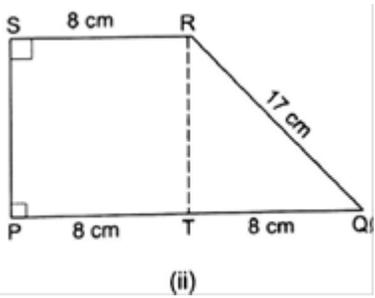
$$\therefore \text{Area of Quadrilateral ABCD} = 114 \text{ cm}^2$$

## 5B. Question

Calculate the area of trap. PQRS, given in Fig. (ii).



**Answer**



Given :- Right trapezium

$$RS = 8 \text{ cm}$$

$$PT = 8 \text{ cm}$$

$$TQ = 8 \text{ cm}$$

$$RQ = 17 \text{ cm}$$

$$\text{Here } PQ = PT + TQ = 8 + 8 = 16$$

We know that area of trapezium is  $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$$\text{That is } \frac{1}{2} \times (AB + DC) \times RT$$

Consider  $\triangle TQR$

By Pythagoras theorem

$$RQ^2 = TQ^2 + RT^2$$

$$17^2 = 8^2 + RT^2$$

$$RT^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$\therefore RT = 15 \text{ cm}$$

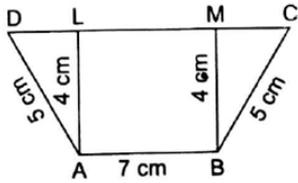
$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (RS + PQ) \times RT$$

$$= \frac{1}{2} \times (8 + 16) \times 15 = 180 \text{ cm}^2$$

∴ Area of trapezium PQRS =  $180\text{cm}^2$

## 6. Question

In the adjoining figure, ABCD is a trapezium in which  $AB \parallel DC$ ;  $AB = 7\text{ cm}$ ;  $AD = BC = 5\text{ cm}$  and the distance between AB and DC is  $4\text{ cm}$ . Find the length of DC and hence, find the area of trap. ABCD.



## Answer

Given

$$AB = 7\text{ cm}$$

$$AD = BC = 5\text{ cm}$$

$$AL = BM = 4\text{ cm (height)}$$

$$DC = ?$$

Here in the given figure  $AB = LM$

$$\therefore LM = 7\text{ cm} \text{ -----1}$$

Now Consider  $\triangle ALD$

By Pythagoras theorem

$$AD^2 = AL^2 + DL^2$$

$$5^2 = 4^2 + DL^2$$

$$DL^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\therefore DL = 3\text{ cm} \text{ -----2}$$

Similarly in  $\triangle BMC$

By Pythagoras theorem

$$BC^2 = BM^2 + MC^2$$

$$5^2 = 4^2 + MC^2$$

$$MC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\therefore MC = 3\text{ cm} \text{ --3}$$

∴ from 1 2 and 3

$$DC = DL + LM + MC = 3 + 7 + 3 = 13\text{ cm}$$

We know that area of trapezium is  $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

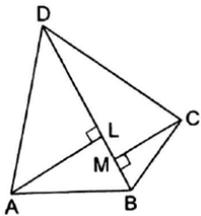
$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (AB + DC) \times AL$$

$$= \frac{1}{2} \times (7 + 13) \times 4 = 40 \text{ cm}^2$$

$$\therefore \text{Area of trapezium ABCD} = 180 \text{ cm}^2$$

### 7. Question

BD is one of the diagonals of a quad. ABCD. If  $AL \perp BD$  and  $CM \perp BD$ , show that  $\text{ar}(\text{quad. ABCD}) = \frac{1}{2} \times BD \times (AL + CM)$ .



### Answer

Given :

$AL \perp BD$  and  $CM \perp BD$

To prove :  $\text{ar}(\text{quad. ABCD}) = \frac{1}{2} \times BD \times (AL + CM)$

Proof:

$$\text{Area of } \triangle ABD = \frac{1}{2} \times BD \times AL$$

$$\text{Area of } \triangle CBD = \frac{1}{2} \times BD \times CM$$

Now area of Quad ABCD = Area of  $\triangle ABD$  + Area of  $\triangle CBD$

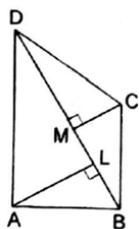
$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \frac{1}{2} \times BD \times (AL + CM)$$

Hence proved

### 8. Question

In the adjoining figure, ABCD is a quadrilateral in which diag.  $BD = 14$  cm. If  $AL \perp BD$  and  $CM \perp BD$  such that  $AL = 8$  cm and  $CM = 6$  cm, find the area of quad. ABCD.



### Answer

Given

$AL \perp BD$  and  $CM \perp BD$

$BD = 14$  cm

$AL = 8$  cm

$CM = 6$  cm

Here,

$$\text{Area of } \triangle ABD = \frac{1}{2} \times BD \times AL$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times BD \times CM$$

Now area of Quad ABCD = Area of  $\triangle ABD$  + Area of  $\triangle BCD$

$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

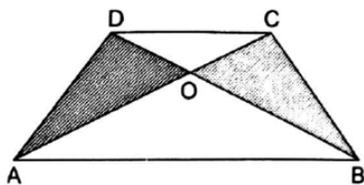
$$= \frac{1}{2} \times BD \times (AL + CM)$$

$$\therefore \text{Area of quad ABCD} = \frac{1}{2} \times BD \times (AL + CM) = \frac{1}{2} \times 14 \times (8 + 6) = 98\text{cm}^2$$

$$\therefore \text{Area of quad ABCD} = 98\text{cm}^2$$

### 9. Question

In the adjoining figure, ABCD is a trapezium in which  $AB \parallel DC$  and its diagonals AC and BD intersect at O. Prove that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$ .



### Answer

Given

AB  $\parallel$  DC

To prove that:  $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$

Here in the given figure Consider  $\triangle ABD$  and  $\triangle ABC$ ,

we find that they have same base AB and lie between two parallel lines AB and CD

According to the theorem: triangles on the same base and between same parallel lines have equal areas.

$\therefore$  Area of  $\triangle ABD = \text{Area of } \triangle BCA$

Now,

Area of  $\triangle AOD = \text{Area of } \triangle ABD - \text{Area of } \triangle AOB$  ---1

Area of  $\triangle COB = \text{Area of } \triangle BCA - \text{Area of } \triangle AOB$  ---2

$\therefore$  From 1 and 2

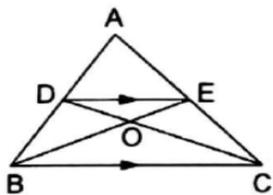
We can conclude that  $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$  (Since Area of  $\triangle AOB$  is common)

Hence proved

### 10. Question

In the adjoining figure,  $DE \parallel BC$ . Prove that

- (i)  $\text{ar}(\triangle ACD) = \text{ar}(\triangle ABE)$ ,
- (ii)  $\text{ar}(\triangle OCE) = \text{ar}(\triangle OBD)$ .



### Answer

Given

AB  $\parallel$  DC

To prove that : (i)  $\text{area}(\triangle ACD) = \text{area}(\triangle ABE)$

(ii)  $\text{area}(\triangle OCE) = \text{area}(\triangle OBD)$

(i)

Here in the given figure Consider  $\triangle BDE$  and  $\triangle ECD$ ,

we find that they have same base DE and lie between two parallel lines BC and DE

According to the theorem: triangles on the same base and between same parallel lines have equal

areas.

$$\therefore \text{Area of } \triangle BDE = \text{Area of } \triangle ECD$$

Now,

$$\text{Area of } \triangle ACD = \text{Area of } \triangle ECD + \text{Area of } \triangle ADE \text{ ---1}$$

$$\text{Area of } \triangle ABE = \text{Area of } \triangle BDE + \text{Area of } \triangle ADE \text{ ---2}$$

$\therefore$  From 1 and 2

We can conclude that  $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$  (Since Area of  $\triangle ADE$  is common)

Hence proved

(ii)

Here in the given figure Consider  $\triangle BCD$  and  $\triangle BCE$ ,

we find that they have same base BC and lie between two parallel lines BC and DE

According to the theorem : triangles on the same base and between same parallel lines have equal areas.

$$\therefore \text{Area of } \triangle BCD = \text{Area of } \triangle BCE$$

Now,

$$\text{Area of } \triangle OBD = \text{Area of } \triangle BCD - \text{Area of } \triangle BOC \text{ ---1}$$

$$\text{Area of } \triangle OCE = \text{Area of } \triangle BCE - \text{Area of } \triangle BOC \text{ ---2}$$

$\therefore$  From 1 and 2

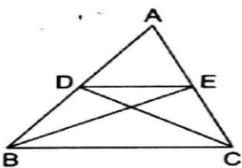
We can conclude that  $\text{area}(\triangle OCE) = \text{area}(\triangle OBD)$  (Since Area of  $\triangle BOC$  is common)

Hence proved

### 11. Question

In the adjoining figure, D and E are points on the sides AB and AC of  $\triangle ABC$  such that  $\text{ar}(\triangle BCE) = \text{ar}(\triangle BCD)$ .

Show that  $DE \parallel BC$ .



**Answer**

Given

A triangle ABC in which points D and E lie on AB and AC of  $\Delta ABC$  such that  $\text{ar}(\Delta BCE) = \text{ar}(\Delta BCD)$ .

To prove:  $DE \parallel BC$

Proof:

Here, from the figure we know that  $\Delta BCE$  and  $\Delta BCD$  lie on same base BC and

It is given that  $\text{area}(\Delta BCE) = \text{area}(\Delta BCD)$

Since two triangle have same base and same area they should equal altitude(height)

That means they lie between two parallel lines

That is  $DE \parallel BC$

$\therefore DE \parallel BC$

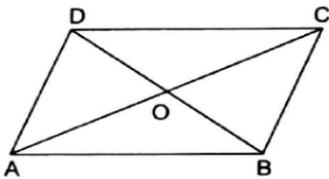
Hence proved

## 12. Question

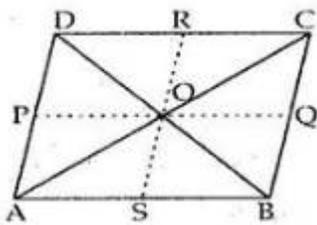
In the adjoining figure, O is any point inside a parallelogram ABCD. Prove that

(i)  $\text{ar}(\Delta OAB) + \text{ar}(\Delta OCD) = \frac{1}{2} \text{ar}(\text{llgm ABCD}),$

(ii)  $\text{ar}(\Delta OAD) + \text{ar}(\Delta OBC) = \frac{1}{2} \text{ar}(\text{llgm ABCD}).$



## Answer



Given : A parallelogram ABCD with a point 'O' inside it.

To prove : (i)  $\text{area}(\Delta OAB) + \text{area}(\Delta OCD) = \frac{1}{2} \text{area}(\text{llgm ABCD}),$

(ii)  $\text{area}(\Delta OAD) + \text{area}(\Delta OBC) = \frac{1}{2} \text{area}(\text{llgm ABCD}).$

Construction : Draw  $PQ \parallel AB$  and  $RS \parallel AD$

Proof:

(i)

$\triangle AOB$  and parallelogram  $ABQP$  have same base  $AB$  and lie between parallel lines  $AB$  and  $PQ$ .

According to theorem: If a triangle and parallelogram are on the same base and between the same parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$$\therefore \text{area}(\triangle AOB) = \frac{1}{2} \text{area}(\text{llgm } ABQP) \text{ ---1}$$

$$\text{Similarly, we can prove that } \text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{llgm } PQCD) \text{ ---2}$$

$\therefore$  Adding -1 and -2 we get,

$$\text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{llgm } ABQP) + \frac{1}{2} \text{area}(\text{llgm } PQCD)$$

$$\text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} [\text{area}(\text{llgm } ABQP) + \text{area}(\text{llgm } PQCD)] = \frac{1}{2} \text{area}(\text{llgm } ABCD)$$

$$\therefore \text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{llgm } ABCD)$$

Hence proved

(ii)

$\triangle OAD$  and parallelogram  $ASRD$  have same base  $AD$  and lie between parallel lines  $AD$  and  $RS$ .

According to theorem: If a triangle and parallelogram are on the same base and between the same parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$$\therefore \text{area}(\triangle OAD) = \frac{1}{2} \text{area}(\text{llgm } ASRD) \text{ ---1}$$

$$\text{Similarly, we can prove that } \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{llgm } BCRS) \text{ ---2}$$

$\therefore$  Adding -1 and -2 we get,

$$\text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{llgm } ASRD) + \frac{1}{2} \text{area}(\text{llgm } BCRS)$$

$$\text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} [\text{area}(\text{llgm } ASRD) + \text{area}(\text{llgm } BCRS)] = \frac{1}{2} \text{area}(\text{llgm } ABCD)$$

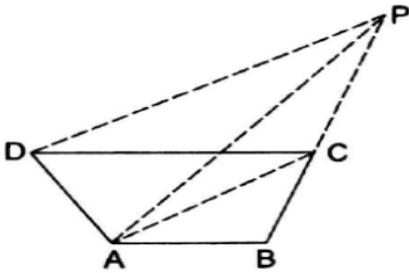
$$\therefore \text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{llgm } ABCD)$$

Hence proved

### 13. Question

In the adjoining figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P.

Prove that  $\text{ar}(\Delta ABP) = (\text{quad. ABCD})$ .



**Answer**

Given : ABCD is a quadrilateral in which a line through D drawn parallel to AC which meets BC produced in P.

To prove: area of  $(\Delta ABP) = \text{area of (quad ABCD)}$

Proof:

Here, in the given figure

$\Delta ACD$  and  $\Delta ACP$  have same base and lie between same parallel line AC and DP.

According to the theorem : triangles on the same base and between same parallel lines have equal areas.

$$\therefore \text{area of } (\Delta ACD) = \text{area of } (\Delta ACP) \text{ -----1}$$

Now, add area of  $(\Delta ABC)$  on both side of (1)

$$\therefore \text{area of } (\Delta ACD) + (\Delta ABC) = \text{area of } (\Delta ACP) + (\Delta ABC)$$

$$\text{Area of (quad ABCD)} = \text{area of } (\Delta ABP)$$

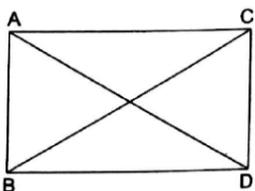
$$\therefore \text{area of } (\Delta ABP) = \text{Area of (quad ABCD)}$$

Hence proved

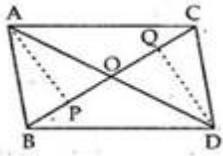
**14. Question**

In the adjoining figure,  $\Delta ABC$  and  $\Delta DBC$  are on the same base BC with A and D on opposite sides of BC such that  $\text{ar}(\Delta ABC) = \text{ar}(\Delta DBC)$ .

Show that BC bisects AD.



**Answer**



Given :  $\Delta ABC$  and  $\Delta DBC$  having same base BC and  $\text{area}(\Delta ABC) = \text{area}(\Delta DBC)$ .

To prove:  $OA = OD$

Construction : Draw  $AP \perp BC$  and  $DQ \perp BC$

Proof :

Here area of  $\Delta ABC = \frac{1}{2} \times BC \times AP$  and area of  $\Delta DBC = \frac{1}{2} \times BC \times DQ$

since,  $\text{area}(\Delta ABC) = \text{area}(\Delta DBC)$

$$\therefore \frac{1}{2} \times BC \times AP = \frac{1}{2} \times BC \times DQ$$

$$\therefore AP = DQ \text{ ----- 1}$$

Now in  $\Delta AOP$  and  $\Delta QOD$ , we have

$$\angle APO = \angle DQO = 90^\circ \text{ and}$$

$$\angle AOP = \angle DOQ \text{ [Vertically opposite angles]}$$

$$AP = DQ \text{ [from 1]}$$

Thus by AAS congruency

$$\Delta AOP \cong \Delta QOD \text{ [AAS]}$$

Thus By corresponding parts of congruent triangles law [C.P.C.T]

$$\therefore OA = OD \text{ [C.P.C.T]}$$

Hence BC bisects AD

Hence proved

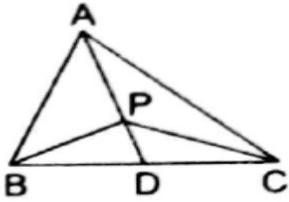
### 15. Question

In the adjoining figure, AD is one of the medians of a  $\Delta ABC$  and P is a point on AD.

Prove that

$$(i) \text{ar}(\Delta BDP) = \text{ar}(\Delta CDP)$$

$$(ii) \text{ar}(\Delta ABP) = \text{ar}(\Delta ACP)$$



### Answer

Given : A  $\Delta ABC$  in which AD is the median and P is a point on AD

To prove: (i)  $\text{ar}(\Delta BDP) = \text{ar}(\Delta CDP)$ ,

(ii)  $\text{ar}(\Delta ABP) = \text{ar}(\Delta ACP)$ .

(i)

In  $\Delta BPC$ , PD is the median. Since median of a triangle divides the triangles into two equal areas

So,  $\text{area}(\Delta BDP) = \text{area}(\Delta CDP)$ -----1

Hence proved

(ii)

In  $\Delta ABC$  AD is the median

So,  $\text{area}(\Delta ABD) = \text{area}(\Delta ADC)$  ----2 and

$\text{area}(\Delta BDP) = \text{area}(\Delta CDP)$  [from 1]

Now subtracting  $\text{area}(\Delta BDP)$  from ---2 , we have

$\text{area}(\Delta ABD) - \text{area}(\Delta BDP) = \text{area}(\Delta ADC) - \text{area}(\Delta BDP)$

$\text{area}(\Delta ABD) - \text{area}(\Delta BDP) = \text{area}(\Delta ADC) - \text{area}(\Delta CDP)$  [since  $\text{area}(\Delta BDP) = \text{area}(\Delta CDP)$  from -1]

$\therefore \text{area}(\Delta ABP) = \text{area}(\Delta ACP)$

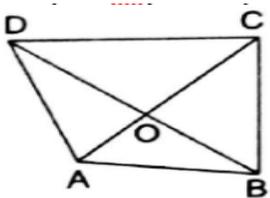
Hence proved.

### 16. Question

In the adjoining figure, the diagonals AC and BD of a quadrilateral ABCD intersect at O.

If  $BO = OD$ , prove that

$\text{Ar}(\Delta ABC) = \text{ar}(\Delta ADC)$ .



### Answer

Given : A quadrilateral ABCD with diagonals AC and BD and  $BO = OD$

To prove: Area of  $(\Delta ABC) = \text{area of } (\Delta ADC)$

Proof :  $BO = OD$  [given]

Here  $AO$  is the median of  $\Delta ABD$

$$\therefore \text{Area of } (\Delta AOD) = \text{Area of } (\Delta AOB) \text{ ----- 1}$$

And  $OC$  is the median of  $\Delta BCD$

$$\therefore \text{Area of } (\Delta COD) = \text{Area of } (\Delta BOC) \text{ ----- 2}$$

Now by adding -1 and -2 we get

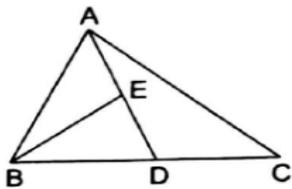
$$\text{Area of } (\Delta AOD) + \text{Area of } (\Delta COD) = \text{Area of } (\Delta AOB) + \text{Area of } (\Delta BOC)$$

$$\therefore \text{Area of } (\Delta ABC) = \text{Area of } (\Delta ADC)$$

Hence proved

### 17. Question

$ABC$  is a triangle in which  $D$  is the midpoint of  $BC$  and  $E$  is the midpoint of  $AD$ . Prove that  $\text{ar}(\Delta BED) = \frac{1}{4} \text{ar}(\Delta ABC)$ .



### Answer

Given : A  $\Delta ABC$  in which  $AD$  is the median and  $E$  is the midpoint on line  $AD$

$$\text{To prove: } \text{area}(\Delta BED) = \frac{1}{4} \text{area}(\Delta ABC)$$

Proof : here in  $\Delta ABC$   $AD$  is the median

$$\therefore \text{Area of } (\Delta ABD) = \text{Area of } (\Delta ADC)$$

$$\text{Hence Area of } (\Delta ABD) = \frac{1}{2} [\text{Area of } (\Delta ABC)] \text{ ----- 1}$$

Now in  $\Delta ABD$   $E$  is the midpoint of  $AD$  and  $BE$  is the median

$$\therefore \text{Area of } (\Delta BDE) = \text{Area of } (\Delta ABE)$$

$$\text{Hence Area of } (\Delta BED) = \frac{1}{2} [\text{Area of } (\Delta ABD)] \text{ ----- 2}$$

Substituting (1) in (2), we get

$$\text{Hence Area of } (\Delta BED) = \frac{1}{2} \left[ \frac{1}{2} \text{ Area of } (\Delta ABC) \right]$$

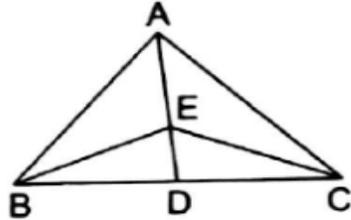
$$\therefore \text{area}(\Delta BED) = \frac{1}{4} \text{area}(\Delta ABC)$$

Hence proved

### 18. Question

The vertex A of  $\Delta ABC$  is joined to a point D on the side BC. The midpoint of AD is E. Prove that

$$\text{ar}(\Delta BEC) = \frac{1}{2} \text{ar}(\Delta ABC).$$



### Answer

Given : A  $\Delta ABC$  in which AD is a line where D is a point on BC and E is the midpoint of AD

$$\text{To prove: } \text{ar}(\Delta BEC) = \frac{1}{2} \text{ar}(\Delta ABC)$$

Proof: In  $\Delta ABD$  E is the midpoint of side AD

$$\therefore \text{Area of } (\Delta BDE) = \text{Area of } (\Delta ABE)$$

$$\text{Hence Area of } (\Delta BDE) = \frac{1}{2} [\text{Area of } (\Delta ABD)] \quad -1$$

Now, consider  $\Delta ACD$  in which E is the midpoint of side AD

$$\therefore \text{Area of } (\Delta ECD) = \text{Area of } (\Delta AEC)$$

$$\text{Hence Area of } (\Delta ECD) = \frac{1}{2} [\text{Area of } (\Delta ACD)] \quad -2$$

Now, adding -1 and -2, we get

$$\text{Area of } (\Delta BDE) + \text{Area of } (\Delta ECD) = \frac{1}{2} [\text{Area of } (\Delta ABD)] + \frac{1}{2} [\text{Area of } (\Delta ACD)]$$

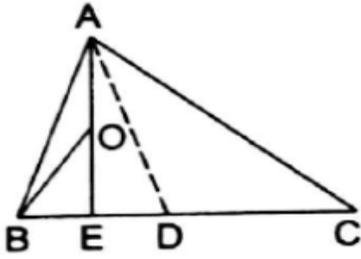
$$\therefore \text{area}(\Delta BEC) = \frac{1}{2} [\text{area}(\Delta ABD) + \text{area}(\Delta ACD)]$$

$$\therefore \text{Area}(\Delta BEC) = \frac{1}{2} \text{Area}(\Delta ABC)$$

Hence proved

### 19. Question

D is the midpoint of side BC of  $\Delta ABC$  and E is the midpoint of BD. If O is the midpoint of AE, prove that  $\text{ar}(\Delta BOE) = \frac{1}{8} \text{ar}(\Delta ABC)$ .



**Answer**

Given : D is the midpoint of side BC of  $\Delta ABC$  and E is the midpoint of BD and O is the midpoint of AE

To prove :  $\text{ar}(\Delta BOE) = \frac{1}{8} \text{ar}(\Delta ABC)$

Proof : Consider  $\Delta ABC$  here D is the midpoint of BC

$$\therefore \text{Area of } (\Delta ABD) = \text{Area of } (\Delta ACD)$$

$$\therefore \text{Area}(\Delta ABD) = \frac{1}{2} \text{Area}(\Delta ABC) \text{---1}$$

Now, consider  $\Delta ABD$  here E is the midpoint of BD

$$\therefore \text{Area of } (\Delta ABE) = \text{Area of } (\Delta AED)$$

$$\therefore \text{Area}(\Delta ABE) = \frac{1}{2} \text{Area}(\Delta ABD) \text{---2}$$

Substituting -1 in -2 , we get

$$\therefore \text{Area}(\Delta ABE) = \frac{1}{2} \left( \frac{1}{2} \text{Area}(\Delta ABC) \right)$$

$$\text{Area}(\Delta ABE) = \frac{1}{4} \text{Area}(\Delta ABC) \text{---3}$$

Now consider  $\Delta ABE$  here O is the midpoint of AE

$$\therefore \text{Area of } (\Delta BOE) = \text{Area of } (\Delta AOB)$$

$$\therefore \text{Area}(\Delta BOE) = \frac{1}{2} \text{Area}(\Delta ABE) \text{---4}$$

Now, substitute -3 in -4 , we get

$$\text{Area}(\Delta BOE) = \frac{1}{2} \left( \frac{1}{4} \text{Area}(\Delta ABC) \right)$$

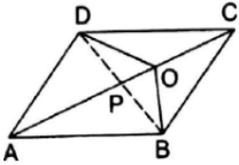
$$\therefore \text{area}(\Delta BOE) = \frac{1}{8} \text{area}(\Delta ABC)$$

Hence proved

### 20. Question

In the adjoining figure, ABCD is a parallelogram and O is any point on the diagonal AC.

Show that  $\text{ar}(\triangle AOB) = \text{ar}(\triangle AOD)$ .



### Answer

Given : A parallelogram ABCD in which AC is the diagonal and O is some point on the diagonal AC

To prove:  $\text{area}(\triangle AOB) = \text{area}(\triangle AOD)$

Construction : Draw a diagonal BD and mark the intersection as P

Proof:

We know that in a parallelogram diagonals bisect each other, hence P is the midpoint of  $\triangle ABD$

$$\therefore \text{Area of } (\triangle APB) = \text{Area of } (\triangle APD) \text{---1}$$

Now consider  $\triangle BOD$  here OP is the median, since P is the midpoint of BD

$$\therefore \text{Area of } (\triangle OPB) = \text{Area of } (\triangle OPD) \text{---2}$$

Adding -1 and -2 we get

$$\text{Area of } (\triangle APB) + \text{Area of } (\triangle OPB) = \text{Area of } (\triangle APD) + \text{Area of } (\triangle OPD)$$

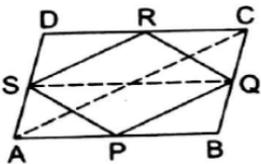
$$\therefore \text{Area of } (\triangle AOB) = \text{Area of } (\triangle AOD)$$

Hence proved

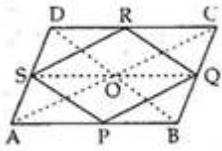
### 21. Question

P,Q,R,S are respectively the midpoints of the sides AB, BC, CD and DA of  $\parallel$  gm ABCD. Show that PQRS is a parallelogram and also show that

$$\text{Ar}(\parallel\text{gm PQRS}) = \frac{1}{2} \times \text{ar}(\parallel\text{gm ABCD}).$$



### Answer



Given : ABCD is a parallelogram and P,Q,R,S are the midpoints of AB,BC,CD,AD respectively

To prove: (i) PQRS is a parallelogram

(ii)  $\text{Area}(\text{llgm PQRS}) = \frac{1}{2} \times \text{area}(\text{llgm ABCD})$

Construction : Join AC ,BD,SQ

Proof:

(i)

As S and R are midpoints of AD and CD respectively, in  $\Delta ACD$

$SR \parallel AC$  [By midpoint theorem] ----- (1)

Similarly in  $\Delta ABC$  , P and Q are midpoints of AB and BC respectively

$PQ \parallel AC$  [By midpoint theorem] ----- (2)

From (1) and (2)

$SR \parallel AC \parallel PQ$

$\therefore SR \parallel PQ$  ----- (3)

Again in  $\Delta ACD$  as S and P are midpoints of AD and CB respectively

$SP \parallel BD$  [By midpoint theorem] ----- (4)

Similarly in  $\Delta ABC$  , R and Q are midpoints of CD and BC respectively

$RQ \parallel BD$  [By midpoint theorem] ----- (5)

From (4) and (5)

$SP \parallel BD \parallel RQ$

$\therefore SP \parallel RQ$  ----- (6)

From (3) and (6)

We can say that opposite sides are Parallel in PQRS

Hence we can conclude that PQRS is a parallelogram.

(ii)

Here ABCD is a parallelogram

S and Q are midpoints of AD and BC respectively

∴ SQ || AB

∴ SQAB is a parallelogram

$$\text{Now, area}(\Delta SQP) = \frac{1}{2} \times \text{area of (SQAB)} \text{ ----- 1}$$

[Since  $\Delta SQP$  and ||gm SQAB have same base and lie between same parallel lines]

Similarly

S and Q are midpoints of AD and BC respectively

∴ SQ || CD

∴ SQCD is a parallelogram

$$\text{Now, area}(\Delta SQR) = \frac{1}{2} \times \text{area of (SQCD)} \text{ ----- 2}$$

[Since  $\Delta SQR$  and ||<sup>gm</sup> SQCD have same base and lie between same parallel lines]

Adding (1) and (2) we get

$$\text{area}(\Delta SQP) + \text{area}(\Delta SQR) = \frac{1}{2} \times \text{area of (SQAB)} + \frac{1}{2} \times \text{area of (SQCD)}$$

$$\therefore \text{area(PQRS)} = \frac{1}{2} (\text{area of (SQAB)} + \text{area of (SQCD)})$$

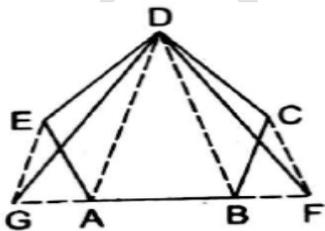
$$\therefore \text{Area}(\text{||gm PQRS}) = \frac{1}{2} \times \text{area}(\text{||gm ABCD})$$

Hence proved

## 22. Question

The given figure shows a pentagon ABCDE, EG, drawn parallel to DA, meets BA produced at G, and CF, drawn parallel to DB, meets AB produced at F.

Show that  $\text{ar}(\text{pentagon ABCDE}) = \text{ar}(\Delta DGF)$ .



## Answer

Given : ABCDE is a pentagon EG is drawn parallel to DA which meets BA produced at G and CF is drawn parallel to DB which meets AB produced at F

To prove:  $\text{area}(\text{pentagon ABCDE}) = \text{area}(\Delta DGF)$

Proof:

Consider quadrilateral ADEG. Here,

$$\text{area}(\triangle AED) = \text{area}(\triangle ADG) \text{ ----- (1)}$$

[since two triangles are on same base AD and lie between parallel line i.e, AD||EG]

Similarly now, Consider quadrilateral BDCF. Here,

$$\text{area}(\triangle BCD) = \text{area}(\triangle BDF) \text{ ----- (2)}$$

[since two triangles are on same base AD and lie between parallel line i.e, AD||EG]

Adding Eq (1) and (2) we get

$$\text{area}(\triangle AED) + \text{area}(\triangle BCD) = \text{area}(\triangle ADG) + \text{area}(\triangle BDF) \text{ ----- (3)}$$

Now add  $\text{area}(\triangle ABD)$  on both sides of Eq (3), we get

$$\therefore \text{area}(\triangle AED) + \text{area}(\triangle BCD) + \text{area}(\triangle ABD) = \text{area}(\triangle ADG) + \text{area}(\triangle BDF) + \text{area}(\triangle ABD)$$

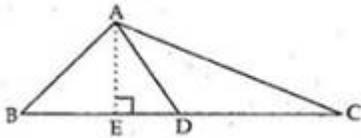
$$\therefore \text{area}(\text{pentagon } ABCDE) = \text{area}(\triangle DGF)$$

Hence proved

### 23. Question

Prove that a median divides a triangle into two triangles of equal area.

**Answer**



Given : A  $\triangle ABC$  with D as median

To prove : Median D divides a triangle into two triangles of equal areas.

Constructions: Drop a perpendicular AE onto BC

Proof: Consider  $\triangle ABD$

$$\text{area}(\triangle ABD) = \frac{1}{2} \times BD \times AE$$

Now , Consider  $\triangle ACD$

$$\text{area}(\triangle ACD) = \frac{1}{2} \times CD \times AE$$

since D is the median

$$BD = CD$$

$$\therefore \frac{1}{2} \times BD \times AE = \frac{1}{2} \times CD \times AE$$

Hence , area( $\Delta ABD$ ) = area( $\Delta ACD$ )

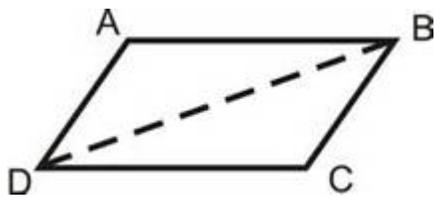
$\therefore$  we can say that Median D divides a triangle into two triangles of equal areas.

Hence proved

#### 24. Question

Show that a diagonal divides a parallelogram into two triangles of equal area.

**Answer**



Given: A parallelogram ABCD with a diagonal BD

To prove: area( $\Delta ABD$ ) = area( $\Delta BCD$ )

Proof:

We know that in a parallelogram opposite sides are equal, that is

$$AD = BC \text{ and } AB = CD$$

Now, consider  $\Delta ABD$  and  $\Delta BCD$

$$\text{Here } AD = BC$$

$$AB = CD$$

$$BD = BD \text{ (common)}$$

Hence by SSS congruency

$$\Delta ABD \cong \Delta BCD$$

By this we can conclude that both the triangles are equal

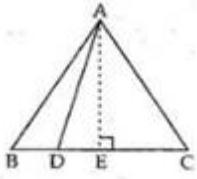
$$\therefore \text{area}(\Delta ABD) = \text{area}(\Delta BCD)$$

Hence proved

#### 25. Question

The base BC of  $\Delta ABC$  is divided at D such  $BD = \frac{1}{2} DC$ . Prove that  $\text{ar}(\Delta ABD) = \frac{1}{3} \times \text{ar}(\Delta ABC)$ .

**Answer**



Given: A  $\Delta ABC$  with a point D on BC such that  $BD = \frac{1}{2} DC$

To prove:  $\text{area}(\Delta ABD) = \frac{1}{3} \times \text{area}(\Delta ABC)$

Construction: Drop a perpendicular onto BC

Proof:  $\text{area}(\Delta ABC) = \frac{1}{2} \times BC \times AE$  -----(1)

and,  $\text{area}(\Delta ABD) = \frac{1}{2} \times BD \times AE$  ----- (2)

given that  $BD = \frac{1}{2} DC$  ----- (3)

so,  $BC = BD + DC = BD + 2BD = 3BD$  [from 2]

$\therefore BD = \frac{1}{3} (BC)$

Sub BD in (1), we get

$\text{area}(\Delta ABD) = \frac{1}{2} \times \left(\frac{1}{3} (BC)\right) \times AE$

$\text{area}(\Delta ABD) = \frac{1}{3} \times \left(\frac{1}{2} BC \times AE\right)$

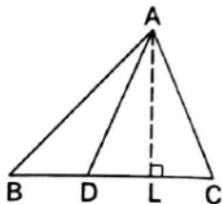
$\therefore \text{area}(\Delta ABD) = \frac{1}{3} \times \text{area}(\Delta ABC)$  [from 1]

Hence proved

## 26. Question

In the adjoining figure, the points D divides the

Side BC of  $\Delta ABC$  in the ratio  $m:n$ . prove that  $\text{area}(\Delta ABD) : \text{area}(\Delta ABC) = m:n$



## Answer

Given : A  $\Delta ABC$  in which a point D divides the Side BC in the ratio m:n.

To prove:  $\text{area}(\Delta ABD) : \text{area}(\Delta ABC) = m : n$

Construction : Drop a perpendicular AL on BC

Proof:

$$\text{area}(\Delta ABD) = \frac{1}{2} \times BD \times AL \text{ ----- (1)}$$

$$\text{and, } \text{area}(\Delta ADC) = \frac{1}{2} \times DC \times AL \text{ ----- (2)}$$

$$BD : DC = m : n$$

$$\frac{BD}{DC} = \frac{m}{n}$$

$$\therefore BD = \frac{m}{n} \times DC \text{ -----(3)}$$

sub Eq (3) in eq (1)

$$\text{area}(\Delta ABD) = \frac{1}{2} \times \left(\frac{m}{n} \times DC\right) \times AL$$

$$\text{area}(\Delta ABD) = \frac{m}{n} \times \left(\frac{1}{2} \times DC \times AL\right)$$

$$\text{area}(\Delta ABD) = \frac{m}{n} \times \text{area}(\Delta ADC)$$

$$\therefore \frac{\text{area}(\Delta ABD)}{\text{area}(\Delta ADC)} = \frac{m}{n}$$

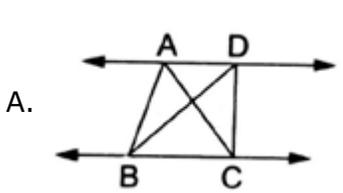
$$\therefore \text{Area}(\Delta ABD) : \text{Area}(\Delta ABC) = m : n$$

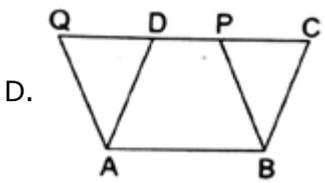
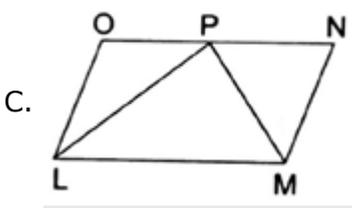
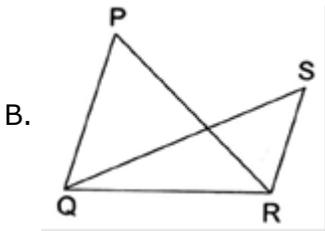
Hence proved

## CCE Questions

### 1. Question

Out of the following given figures which are on the same base but not between the same parallels?





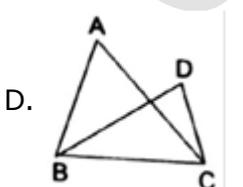
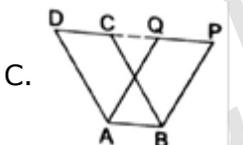
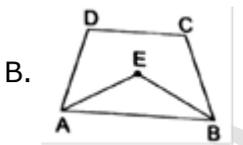
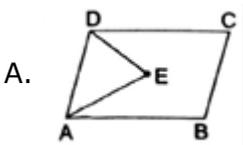
**Answer**

Here,  $\Delta PQR$  and  $\Delta PSR$  are on the same base QR but there is no parallel line to QR.

$\therefore$  Here, Figure in option B is on the same base but not between the same parallels.

**2. Question**

In which of the following figures, you find polygons on the same base and between the same parallels?



**Answer**

Here parallelogram ABCD and parallelogram ABQP lie on the same base AB and lie between the parallel line AB and DP.

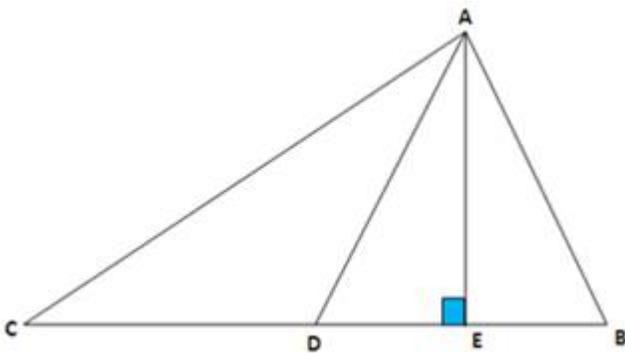
∴ Here, Figure in option C is on the same base and between the same parallels.

### 3. Question

The median of a triangle divides it into two

- A. Triangles of equal area
- B. Congruent triangles
- C. Isosceles triangles
- D. Right triangles

### Answer



In  $\triangle ABC$ , AD is the median

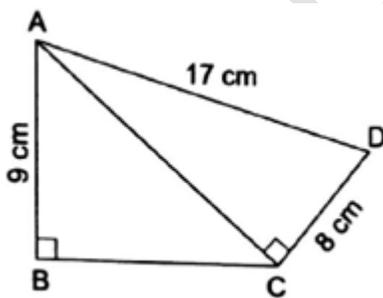
Hence  $BD = DC$  Draw  $AE \perp BC$

Area of  $\triangle ABD =$  Area of  $\triangle ADC$

Thus median of a triangle divides it into two triangles of equal area.

### 4. Question

The area of quadrilateral ABCD in the given figure is



- A.  $57\text{cm}^2$
- B.  $108\text{cm}^2$
- C.  $114\text{cm}^2$
- D.  $195\text{cm}^2$

### Answer

Given:

$$\angle ABC = 90^\circ$$

$$\angle ACD = 90^\circ$$

$$CD = 8\text{cm}$$

$$AB = 9\text{cm}$$

$$AD = 17\text{cm}$$

Consider  $\Delta ACD$

Here, By Pythagoras theorem :  $AD^2 = CD^2 + AC^2$

$$17^2 = 8^2 + AC^2$$

$$\Rightarrow AC^2 = 17^2 - 8^2$$

$$\Rightarrow AC^2 = 289 - 64 = 225$$

$$\Rightarrow AC = 15$$

Now, Consider  $\Delta ABC$

Here, By Pythagoras theorem :  $AC^2 = AB^2 + BC^2$

$$15^2 = 9^2 + BC^2$$

$$\Rightarrow BC^2 = 15^2 - 9^2$$

$$\Rightarrow BC^2 = 225 - 81 = 144$$

$$\Rightarrow BC = 12$$

Here,

$$\text{Area (quad. ABCD)} = \text{Area } (\Delta ABC) + \text{Area } (\Delta ACD)$$

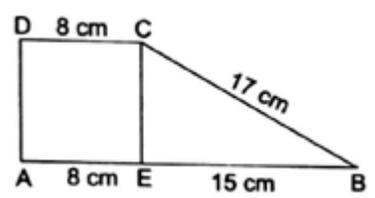
$$\text{Area (quad. ABCD)} = \frac{1}{2} \times AB \times BC + \frac{1}{2} \times AC \times CD$$

$$\text{Area (quad. ABCD)} = \frac{1}{2} \times 9 \times 12 + \frac{1}{2} \times 15 \times 8 = 54 + 60 = 104\text{cm}^2$$

$$\therefore \text{Area (quad. ABCD)} = 114\text{cm}^2$$

### 5. Question

The area of trapezium ABCD in the given figure is



A.  $62\text{cm}^2$

B.  $93\text{cm}^2$

C.  $124\text{cm}^2$

D.  $155\text{cm}^2$

**Answer**

Given:

$$\angle BEC = 90^\circ$$

$$\angle DAE = 90^\circ$$

$$CD = AE = 8\text{cm}$$

$$BE = 15\text{cm}$$

$$BC = 17\text{cm}$$

Consider  $\triangle CEB$

Here, By Pythagoras theorem

$$BC^2 = CE^2 + EB^2$$

$$17^2 = CE^2 + 15^2$$

$$CE^2 = 17^2 - 15^2$$

$$CE^2 = 289 - 225 = 64$$

$$CE = 8$$

Here,

$$\angle AEC = 90^\circ$$

$$CD = CE = 8\text{cm}$$

$\therefore$  AECD is a Square.

$$\therefore \text{Area (Trap. ABCD)} = \text{Area (Sq. AECD)} + \text{Area } (\triangle CEB)$$

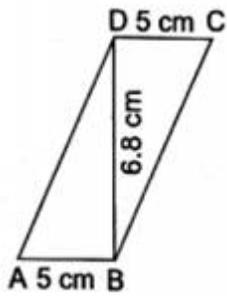
$$\text{Area (Trap. ABCD)} = AE \times EC + \frac{1}{2} \times CE \times EB$$

$$\text{Area (Trap. ABCD)} = 8 \times 8 + \frac{1}{2} \times 8 \times 15 = 64 + 60 = 104\text{cm}^2$$

$$\therefore \text{Area (Trap. ABCD)} = 124\text{cm}^2$$

**6. Question**

In the given figure, ABCD is a llgm in which  $AB = CD = 5\text{cm}$  and  $BD \perp DC$  such that  $BD = 6.8\text{cm}$ . Then, the area of llgm ABCD = ?



- A.  $17\text{cm}^2$
- B.  $25\text{cm}^2$
- C.  $34\text{cm}^2$
- D.  $68\text{cm}^2$

**Answer**

Given:

$$AB = CD = 5\text{cm}$$

$$BD \perp DC$$

$$BD = 6.8\text{cm}$$

Now, consider the parallelogram ABCD

Here, let DC be the base of the parallelogram then BD becomes its altitude (height).

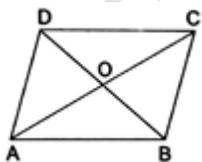
Area of the parallelogram is given by: Base  $\times$  Height

$$\therefore \text{area of llgm ABCD} = CD \times BD = 5 \times 6.8 = 34\text{cm}^2$$

$$\therefore \text{area of llgm ABCD} = 34\text{cm}^2.$$

**7. Question**

In the given figure, ABCD is a llgm in which diagonals AC and BD intersect at O. If  $\text{ar}(\text{llgm ABCD})$  is  $52\text{cm}^2$ , then the  $\text{ar}(\triangle OAB) = ?$



- A.  $26\text{cm}^2$
- B.  $18.5\text{cm}^2$
- C.  $39\text{cm}^2$
- D.  $13\text{cm}^2$

## Answer

Given: ABCD is a llgm in which diagonals AC and BD intersect at O and  $\text{ar}(\text{llgm ABCD})$  is  $52\text{cm}^2$ .

Here,

$$\text{Ar}(\triangle ABD) = \text{ar}(\triangle ABC)$$

( $\because$   $\triangle ABD$  and  $\triangle ABC$  on same base AB and between same parallel lines AB and CD)

Here,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABC) = 1/2 \times \text{ar}(\text{llgm ABCD})$$

( $\because$   $\triangle ABD$  and  $\triangle ABC$  on same base AB and between same parallel lines AB and CD are half the area of the parallelogram)

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ABC) = 1/2 \times 52 = 26\text{cm}^2$$

Now, consider  $\triangle ABC$

Here OB is the median of AC

( $\because$  diagonals bisect each other in parallelogram)

$$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC)$$

( $\because$  median of a triangle divides it into two triangles of equal area)

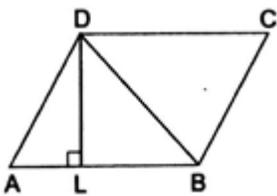
$$\text{ar}(\triangle AOB) = 1/2 \times \text{ar}(\triangle ABC)$$

$$\text{ar}(\triangle AOB) = 1/2 \times 26 = 13\text{cm}^2$$

$$\therefore \text{ar}(\triangle AOB) = 13\text{cm}^2$$

## 8. Question

In the given figure, ABCD is a llgm in which  $DL \perp AB$ . If  $AB = 10\text{cm}$  and  $DL = 4\text{cm}$ , then the  $\text{ar}(\text{llgm ABCD}) = ?$



A.  $40\text{cm}^2$

B.  $80\text{cm}^2$

C.  $20\text{cm}^2$

D.  $196\text{cm}^2$

## Answer

Area of parallelogram is: base  $\times$  height

Here,

$$\text{Base} = AB = 10\text{cm}$$

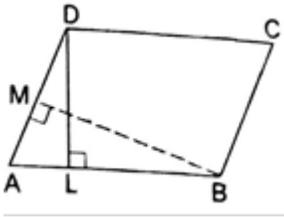
$$\text{Height} = DL = 4\text{cm}$$

$$\therefore \text{ar}(\text{llgm ABCD}) = AB \times DL = 10 \times 4 = 40\text{cm}^2$$

$$\therefore \text{ar}(\text{llgm ABCD}) = 40\text{cm}^2$$

### 9. Question

In llgm ABCD, it is given that  $AB = 10\text{cm}$ ,  $DL \perp AB$  and  $BM \perp AD$  such that  $DL = 6\text{cm}$  and  $BM = 8\text{cm}$ . Then,  $AD = ?$



A. 7.5cm

B. 8cm

C. 12cm

D. 14cm

### Answer

Given:

$$AB = 10\text{cm}$$

$$DL \perp AB$$

$$BM \perp AD$$

$$DL = 6\text{cm}$$

$$BM = 8\text{cm}$$

Now, consider the parallelogram ABCD

Here, let AB be the base of the parallelogram then DL becomes its altitude (height).

Area of the parallelogram is given by:  $\text{Base} \times \text{Height}$

$$\therefore \text{area of llgm ABCD} = AB \times DL = 10 \times 6 = 60\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BM becomes its altitude (height)

$$\therefore \text{area of llgm ABCD} = AD \times BM = 60\text{cm}^2$$

$$AD \times 8 = 60\text{cm}^2$$

$$AD = 60/8 = 7.5\text{cm}$$

∴ length of AD = 7.5cm.

### 10. Question

The lengths of the diagonals of a rhombus are 12cm and 16cm. The area of the rhombus is

- A.  $192\text{cm}^2$
- B.  $96\text{cm}^2$
- C.  $64\text{cm}^2$
- D.  $80\text{cm}^2$

### Answer

Given:

Length of diagonals of rhombus: 12cm and 16cm.

Area of the rhombus is given by:  $\frac{\text{product of diagonals}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{12 \times 16}{2} = 96\text{cm}^2$$

### 11. Question

Two parallel sides of a trapezium are 12cm and 8cm long and the distance between them 6.5cm. The area of the trapezium is

- A.  $74\text{cm}^2$
- B.  $32.5\text{cm}^2$
- C.  $65\text{cm}^2$
- D.  $130\text{cm}^2$

### Answer

Given:

Lengths of parallel sides of trapezium: 12cm and 8cm

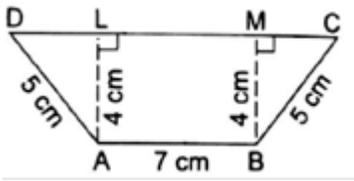
Distance between two parallel lines (height): 6.5cm

Area of the trapezium is given by:  $\frac{(\text{sum of parallel sides}) \times \text{height}}{2}$

$$\therefore \text{Area of the trapezium} = \frac{(12 + 8) \times 6.5}{2} = 65\text{cm}^2$$

### 12. Question

In the given figure ABCD is a trapezium such that  $AL \perp DC$  and  $BM \perp DC$ . If  $AB = 7\text{cm}$ ,  $BC = AD = 5\text{cm}$  and  $AL = BM = 4\text{cm}$ , then  $\text{ar}(\text{trap. ABCD}) = ?$



- A.  $24\text{cm}^2$
- B.  $40\text{cm}^2$
- C.  $55\text{cm}^2$
- D.  $27.5\text{cm}^2$

**Answer**

Given:

$$AL \perp DC$$

$$BM \perp DC$$

$$AB = 7\text{cm}$$

$$BC = AD = 5\text{cm}$$

$$AL = BM = 4\text{cm}$$

Here,

$$MC = DL \text{ and } AB = LM = 7\text{ cm}$$

Consider the  $\Delta BMC$

Here, by Pythagoras theorem

$$BC^2 = BM^2 + MC^2$$

$$5^2 = 4^2 + MC^2$$

$$MC^2 = 25 - 16$$

$$MC^2 = 9$$

$$MC = 3\text{cm}$$

$$\therefore MC = DL = 3\text{cm}$$

$$CD = DL + LM + MC = 3 + 7 + 3 = 13\text{cm}$$

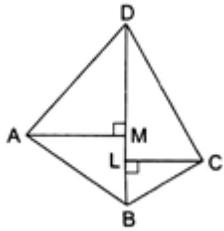
Now,

Area of the trapezium is given by:  $\frac{(\text{sum of parallel sides}) \times \text{height}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{(13 + 7) \times 4}{2} = 40\text{cm}^2$$

### 13. Question

In a quadrilateral ABCD, it is given that  $BD = 16\text{cm}$ . If  $AL \perp BD$  and  $CM \perp BD$  such that  $AL = 9\text{cm}$  and  $CM = 7\text{cm}$ , then  $\text{ar}(\text{quad. ABCD}) = ?$



- A.  $256\text{cm}^2$
- B.  $128\text{cm}^2$
- C.  $64\text{cm}^2$
- D.  $96\text{cm}^2$

### Answer

Given:

$$BD = 16\text{cm}$$

$$AL \perp BD$$

$$CM \perp BD$$

$$AL = 9\text{cm}$$

$$CM = 7\text{cm}$$

Here,

$$\text{Area of quadrilateral ABCD} = \text{area}(\triangle ABD) + \text{area}(\triangle BCD)$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

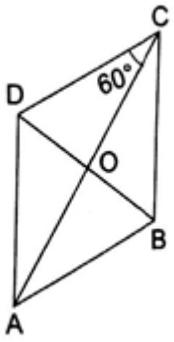
$$\text{area}(\triangle ABD) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BD \times CM = \frac{1}{2} \times 16 \times 7 = 56\text{cm}^2$$

$$\text{area}(\triangle BCD) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times 16 \times 9 = 72\text{cm}^2$$

$$\therefore \text{Area of quadrilateral ABCD} = \text{area}(\triangle ABD) + \text{area}(\triangle BCD) = 56 + 72 = 128\text{cm}^2$$

### 14. Question

ABCD is a rhombus in which  $\angle C = 60^\circ$ . Then,  $AC : BD = ?$



- A. 3:1
- B. 3:2
- C. 3:1
- D. 3:2

**Answer**

Given:  $\angle DCB = 60^\circ$

Let the length of the side be  $x$

Here, consider  $\triangle BCD$

$BC = DC$  (all sides of rhombus are equal)

$\therefore \angle CDB = \angle CBD$  (angles opposite to equal sides are equal)

Now, by angle sum property

$$\angle CDB + \angle CBD + \angle BCD = 180^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$\therefore 2 \times \angle CBD = 120^\circ$$

$$\angle CBD = \frac{120}{2} = 60^\circ$$

$$\therefore \angle CDB = \angle CBD = 60^\circ$$

$\therefore \triangle ADC$  is equilateral triangle

$$\therefore BC = CD = BD = x \text{ cm}$$

In Rhombus diagonals bisect each other.

Consider  $\triangle COD$

By Pythagoras theorem

$$CD^2 = OD^2 + OC^2$$

$$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$$

$$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$$

$$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$$

$$OC = \frac{\sqrt{3} \times x}{2} \text{ cm}$$

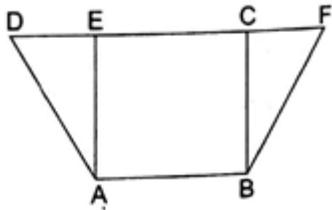
$$\therefore AC = 2 \times OC = 2 \times \frac{\sqrt{3} \times x}{2} = \sqrt{3}x$$

$$AC: BD = \sqrt{3}x : x = \sqrt{3} : 1$$

$$\therefore AC: BD = \sqrt{3} : 1$$

### 15. Question

In the given figure ABCD and ABFE are parallelograms such that ar(quad. EABC) = 17cm<sup>2</sup> and ar(||gm ABCD) = 25cm<sup>2</sup>. Then, ar(ΔBCF) = ?



A. 4cm<sup>2</sup>

B. 4.8cm<sup>2</sup>

C. 6cm<sup>2</sup>

D. 8cm<sup>2</sup>

### Answer

Given: ar(quad. EABC) = 17cm<sup>2</sup> and ar(||gm ABCD) = 25cm<sup>2</sup>

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area} (\text{||gm ABCD}) = \text{Area} (\text{||gm ABFE}) = 25\text{cm}^2$$

Here,

$$\text{Area} (\text{||gm ABFE}) = \text{Area} (\text{quad. EABC}) + \text{Area} (\Delta BCF)$$

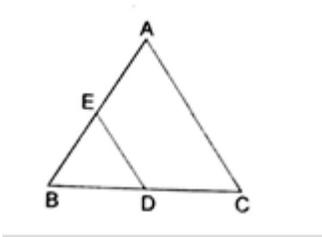
$$25\text{cm}^2 = 17\text{cm}^2 + \text{Area} (\Delta BCF)$$

$$\text{Area} (\Delta BCF) = 25 - 17 = 8\text{cm}^2$$

$$\therefore \text{Area} (\Delta BCF) = 8\text{cm}^2$$

### 16. Question

$\triangle ABC$  and  $\triangle BDE$  are two equilateral triangles such that D is the midpoint of BC. Then,  
 $\text{ar}(\triangle BDE) : \text{ar}(\triangle ABC) = ?$



A. 1:2

B. 1:4

C. 3:2

D. 3:4

### Answer

Given:  $\triangle ABC$  and  $\triangle BDE$  are two equilateral triangles, D is the midpoint of BC.

Consider  $\triangle ABC$

Here, let  $AB = BC = AC = x$  cm (equilateral triangle)

Now, consider  $\triangle BED$

Here,

$$BD = \frac{1}{2} BC$$

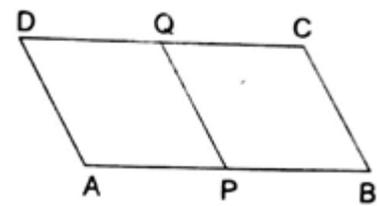
$$\therefore BD = ED = EB = \frac{1}{2} BC = \frac{x}{2} \text{ (equilateral triangle)}$$

Area of the equilateral triangle is given by:  $\frac{\sqrt{3}}{4} a^2$  (a is side length)

$$\therefore \text{ar}(\triangle BDE) : \text{ar}(\triangle ABC) = \frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2 : \frac{\sqrt{3}}{4} x^2 = \frac{1}{4} : 1 = 1:4$$

### 17. Question

In a  $\parallel\text{gm}$  ABCD, if Point P and Q are midpoints of AB and CD respectively and  $\text{ar}(\parallel\text{gm ABCD}) = 16\text{cm}^2$ , then  $\text{ar}(\parallel\text{gm APQD}) = ?$



A.  $8\text{cm}^2$

B.  $12\text{cm}^2$

C.  $6\text{cm}^2$

D.  $9\text{cm}^2$

**Answer**

Given:

P and Q are midpoints of AB and CD respectively

$$\text{ar}(\text{llgm ABCD}) = 16\text{cm}^2$$

Now, consider the (llgm ABCD)

Here,

Q is the midpoint of DC and P is the midpoint of AB.

$\therefore$  By joining P and Q (llgm ABCD) is divided into two equal parallelograms.

That is,  $\text{ar}(\text{llgm ABCD}) = \text{ar}(\text{llgm APQD}) + \text{ar}(\text{llgm PQCB})$

$$\text{ar}(\text{llgm ABCD}) = 2 \times \text{ar}(\text{llgm APQD}) \quad (\because \text{ar}(\text{llgm APQD}) = \text{ar}(\text{llgm PQCB}))$$

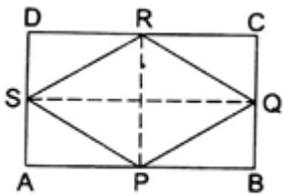
$$2 \times \text{ar}(\text{llgm APQD}) = 16\text{cm}^2 \quad (\because \text{ar}(\text{llgm ABCD}) = 16\text{cm}^2)$$

$$\text{ar}(\text{llgm APQD}) = 16/2 = 8\text{cm}^2$$

$$\therefore \text{ar}(\text{llgm APQD}) = 8\text{cm}^2$$

**18. Question**

The figure formed by joining the midpoints of the adjacent sides of a rectangle of sides 8cm and 6cm is a



A. Rectangle of area  $24\text{cm}^2$

B. Square of area  $24\text{cm}^2$

C. Trapezium of area  $24\text{cm}^2$

D. Rhombus of area  $24\text{cm}^2$

**Answer**

Given: A rectangle with sides 8cm and 6cm.

Consider the Rectangle ABCD

Here  $DR = RD = AP = PB = 8/2 = 4\text{cm}$  ( $\because$  P and R are the midpoints of DC and AB respectively)

and  $AS = SD = BQ = QC = 6/2 = 3\text{cm}$  ( $\because$  S and Q are the midpoints of AD and BC respectively)

Now, consider the  $\Delta RSD$

By Pythagoras theorem

$$SR^2 = SD^2 + DR^2$$

$$SR^2 = 3^2 + 4^2$$

$$SR^2 = 9 + 16$$

$$SR^2 = 25$$

$$SR = 5 \text{ cm}$$

Similarly using Pythagoras theorem in  $\Delta QRC$ ,  $\Delta PBQ$  and  $\Delta APS$

We get  $RQ = QP = PS = 5\text{cm}$

$$\therefore SR = RQ = QP = PS = 5\text{cm}$$

$\therefore$  PQSR is Rhombus of side length 5cm

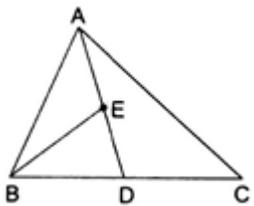
Area of the rhombus is given by:  $\frac{\text{product of diagonals}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{PR \times SQ}{2} = \frac{8 \times 6}{2} = 24\text{cm}^2$$

$$\therefore \text{Area}(PQRS) = 24\text{cm}^2$$

### 19. Question

In  $\Delta ABC$ , if D is the midpoint of BC and E is the midpoint of AD, then  $\text{ar}(\Delta BED) = ?$



A.  $\frac{1}{2} \text{ar}(\Delta ABC)$

B.  $\frac{1}{3} \text{ar}(\Delta ABC)$

C.  $\frac{1}{4} \text{ar}(\Delta ABC)$

D.  $\frac{2}{3} \text{ar}(\Delta ABC)$

## Answer

Given: D is the midpoint of BC and E is the midpoint of AD

Here,

D is the midpoint of BC and AD is the median of  $\Delta ABC$

Area ( $\Delta ABD$ ) = Area ( $\Delta ADC$ ) ( $\because$  median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area} (\Delta ABD) = \text{Area} (\Delta ADC) = \frac{1}{2} \text{Area} (\Delta ABC)$$

Now, consider  $\Delta ABD$

Here, BE is the median

Area ( $\Delta ABE$ ) = Area ( $\Delta BED$ )

$$\therefore \text{Area} (\Delta ABE) = \text{Area} (\Delta BED) = \frac{1}{2} \text{Area} (\Delta ABD)$$

$$\text{Area} (\Delta BED) = \frac{1}{2} \text{Area} (\Delta ABD)$$

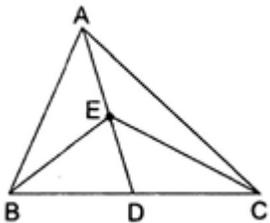
$$\text{Area} (\Delta BED) = \frac{1}{2} \times \left[ \frac{1}{2} \text{Area} (\Delta ABC) \right] (\because \text{Area} (\Delta ABD) = \frac{1}{2} \text{Area} (\Delta ABC))$$

$$\text{Area} (\Delta BED) = \frac{1}{4} \text{Area} (\Delta ABC)$$

$$\therefore \text{Area} (\Delta BED) = \frac{1}{4} \text{Area} (\Delta ABC)$$

## 20. Question

The vertex A of  $\Delta ABC$  is joined to a point D on BC. If E is the midpoint of AD, then  $\text{ar}(\Delta BEC) = ?$



A.  $\frac{1}{2} \text{ar}(\Delta ABC)$

B.  $\frac{1}{3} \text{ar}(\Delta ABC)$

C.  $\frac{1}{4} \text{ar}(\Delta ABC)$

D.  $\frac{1}{6} \text{ar}(\Delta ABC)$

**Answer**

Given:

Here,

D is the midpoint of BC and AD is the median of  $\Delta ABC$

Area ( $\Delta ABD$ ) = Area ( $\Delta ADC$ ) ( $\because$  median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area} (\Delta ABD) = \text{Area} (\Delta ADC) = \frac{1}{2} \text{Area} (\Delta ABC)$$

Now, consider  $\Delta ABD$

Here, BE is the median

$$\text{Area} (\Delta ABE) = \text{Area} (\Delta BED)$$

$$\therefore \text{Area} (\Delta ABE) = \text{Area} (\Delta BED) = \frac{1}{2} \text{Area} (\Delta ABD)$$

$$\text{Area} (\Delta BED) = \frac{1}{2} \text{Area} (\Delta ABD)$$

$$\text{Area} (\Delta BED) = \frac{1}{2} \times \left[ \frac{1}{2} \text{Area} (\Delta ABC) \right] (\because \text{Area} (\Delta ABD) = \frac{1}{2} \text{Area} (\Delta ABC)) -1$$

$$\text{Area} (\Delta BED) = \frac{1}{4} \text{Area} (\Delta ABC)$$

Similarly,

$$\text{Area} (\Delta EDC) = \frac{1}{4} \text{Area} (\Delta ABC) -2$$

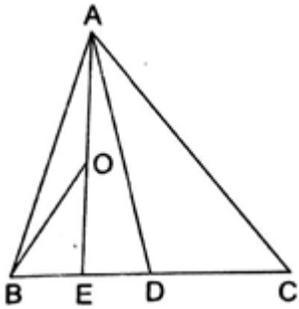
Add -1 and -2

$$\text{Area } (\Delta \text{ BED}) + \text{Area } (\Delta \text{ EDC}) = \frac{1}{4} \text{Area } (\Delta \text{ ABC}) + \frac{1}{4} \text{Area } (\Delta \text{ ABC}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC})$$

$$\therefore \text{Area } (\Delta \text{ BEC}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC})$$

### 21. Question

In  $\Delta \text{ ABC}$ , it is given that D is the midpoint of BC; E is the midpoint of BD and O is the midpoint of AE. Then  $\text{ar}(\Delta \text{ BOE}) = ?$



A.  $\frac{1}{3} \text{ar}(\Delta \text{ ABC})$

B.  $\frac{1}{4} \text{ar}(\Delta \text{ ABC})$

C.  $\frac{1}{6} \text{ar}(\Delta \text{ ABC})$

D.  $\frac{1}{8} \text{ar}(\Delta \text{ ABC})$

### Answer

Given: D is the midpoint of BC; E is the midpoint of BD and O is the midpoint of AE.

Here,

D is the midpoint of BC and AD is the median of  $\Delta \text{ ABC}$

$\text{Area } (\Delta \text{ ABD}) = \text{Area } (\Delta \text{ ADC})$  ( $\because$  median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area } (\Delta \text{ ABD}) = \text{Area } (\Delta \text{ ADC}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC})$$

Now, consider  $\Delta \text{ ABD}$

Here, AE is the median

$\text{Area } (\Delta \text{ ABE}) = \text{Area } (\Delta \text{ BED})$

$$\therefore \text{Area} (\Delta ABE) = \text{Area} (\Delta BED) = \frac{1}{2} \text{Area} (\Delta ABD)$$

$$\text{Area} (\Delta ABE) = \frac{1}{2} \text{Area} (\Delta ABD)$$

$$\text{Area} (\Delta ABE) = \frac{1}{2} \times \left[ \frac{1}{2} \text{Area} (\Delta ABC) \right] (\because \text{Area} (\Delta ABD) = \frac{1}{2} \text{Area} (\Delta ABC) ) -1$$

$$\text{Area} (\Delta ABE) = \frac{1}{4} \text{Area} (\Delta ABC)$$

Consider  $\Delta ABE$

Here, BO is the median

$$\text{Area} (\Delta BOE) = \text{Area} (\Delta BOA)$$

$$\therefore \text{Area} (\Delta BOE) = \text{Area} (\Delta BOA) = \frac{1}{2} \text{Area} (\Delta ABE)$$

$$\text{Area} (\Delta BOE) = \frac{1}{2} \times \left[ \frac{1}{4} \text{Area} (\Delta ABC) \right] (\because \text{Area} (\Delta ABE) = \frac{1}{4} \text{Area} (\Delta ABC) )$$

$$\text{Area} (\Delta BOE) = \frac{1}{8} \text{Area} (\Delta ABC)$$

$$\therefore \text{Area} (\Delta BOE) = \frac{1}{8} \text{Area} (\Delta ABC)$$

## 22. Question

If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the parallelogram is

- A. 1:2
- B. 1:3
- C. 1:4
- D. 3:4

## Answer

Given:

We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\text{Area}(\triangle ABF) = 1/2 \text{ Area}(\text{||gm } ABCD) - 1$$

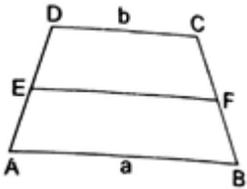
$$\text{Area}(\triangle ABF) : \text{Area}(\text{||gm } ABCD) = 1/2 \text{ Area}(\text{||gm } ABCD) : \text{Area}(\text{||gm } ABCD) \text{ (from } -1 \text{ )}$$

$$\text{Area}(\triangle ABF) : \text{Area}(\text{||gm } ABCD) = 1/2 : 1 = 1:2$$

$$\therefore \text{Area}(\triangle ABF) : \text{Area}(\text{||gm } ABCD) = 1:2$$

### 23. Question

In the given figure ABCD is a trapezium in which  $AB \parallel DC$  such that  $AB = a$  cm and  $DC = b$  cm. If E and F are the midpoints of AD and BC respectively. Then, ar (ABFE) : ar(EFCD) = ?



- A. A:b
- B.  $(a + 3b):(3a + b)$
- C.  $(3a + b):(a + 3b)$
- D.  $(2a + b):(3a + b)$

### Answer

Given: ABCD is a trapezium,  $AB \parallel DC$ ,  $AB = a$  cm and  $DC = b$  cm, E and F are the midpoints of AD and BC.

Since E and F are midpoints of AD and BC, EF will be parallel to both AB and CD.

$$EF = \frac{a+b}{2}$$

Height between EF and DC and height between EF and AB are equal, because E and F are midpoints of AD and BC and  $EF \parallel AB \parallel DC$ .

Let height between EF and DC and height between EF and AB be  $h$  cm.

Area of trapezium =  $1/2 \times (\text{sum of parallel lines}) \times \text{height}$

Now,

$$\text{Area}(\text{Trap. ABFE}) = 1/2 \times \left(a + \frac{a+b}{2}\right) \times h.$$

and

$$\text{Area}(\text{Trap. EFCD}) = 1/2 \times \left(b + \frac{a+b}{2}\right) \times h.$$

$$\text{Area}(\text{Trap. ABFE}) : \text{Area}(\text{Trap. EFCD}) = 1/2 \times \left(a + \frac{a+b}{2}\right) \times h : 1/2 \times \left(b + \frac{a+b}{2}\right) \times h$$

$$\text{Area}(\text{Trap. ABFE}) : \text{Area}(\text{Trap. EFCD}) = \frac{2a+a+b}{2} : \frac{2b+a+b}{2} = 3a + b : a + 3b$$

$$\therefore \text{Area (Trap.ABFE)} : \text{Area (Trap.ABFE)} = 3a + b : a + 3b$$

#### 24. Question

ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD is

- A. a rectangle
- B. allgm
- C. a rhombus
- D. all of these

#### Answer

Given: a quadrilateral whose diagonal AC divides it into two parts, equal in area.

Here,

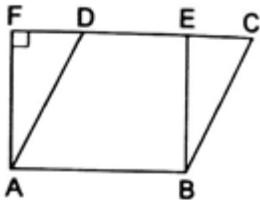
A quadrilateral is any shape having four sides, it is given that diagonal AC of the quadrilateral divides it into two equal parts.

We know that the rectangle, parallelogram and rhombus are all quadrilaterals, in these quadrilaterals if a diagonal is drawn say AC it divides it into equal areas.

$\therefore$  This diagonal divide the quadrilateral into two equal or congruent triangles.

#### 25. Question

In the given figure, a llgm ABCD and a rectangle ABEF are of equal area. Then,



- A. Perimeter of ABCD = perimeter of ABEF
- B. Perimeter of ABCD < perimeter of ABEF
- C. Perimeter of ABCD > perimeter of ABEF
- D. Perimeter of ABCD =  $\frac{1}{2}$  (perimeter of ABEF)

#### Answer

Given: Area (llgm ABCD) = Area (rectangle ABEF)

Consider  $\triangle AFD$

Clearly AD is the hypotenuse

$\therefore AD > AF$

Perimeter of Rectangle ABEF =  $2 \times (AB + AF)$  -1

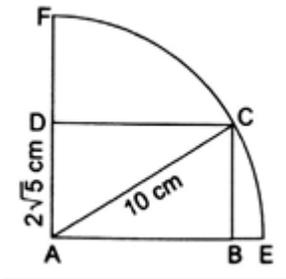
Perimeter of Parallelogram ABCD =  $2 \times (AB + AD)$  -2

On comparing -1 and -2, we can see that

Perimeter of ABCD > perimeter of ABEF ( $\because AD > AF$ )

## 26. Question

In the given figure, ABCD is a rectangle inscribed in a quadrant of a circle of radius 10cm. If AD = 25cm, then area of the rectangle is



A.  $32\text{cm}^2$

B.  $40\text{cm}^2$

C.  $44\text{cm}^2$

D.  $48\text{cm}^2$

## Answer

Given: ABCD is a rectangle inscribed in a quadrant of a circle of radius 10cm and AD = 25cm

Consider  $\Delta ADC$

By Pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$10^2 = (25)^2 + DC^2$$

$$DC^2 = 10^2 - (25)^2$$

$$DC^2 = 100 - 625 = -525$$

$$DC = \sqrt{-525}$$

Area of rectangle = length  $\times$  breadth = DC  $\times$  AD

$$\text{Area of rectangle} = \sqrt{-525} \times 25 = 40\text{cm}^2$$

$$\therefore \text{Area of rectangle} = 40\text{cm}^2$$

## 27. Question

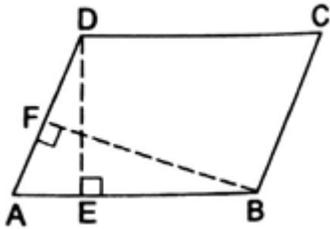
Look at the statements given below:

(I) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

(II) In a llgm ABCD, it is given that AB = 10cm. The altitudes DE on AB and BF on AD being 6cm and 8cm respectively, then AD = 7.5 cm.

(III) Area of a llgm =  $\frac{1}{2}$  x base x altitude.

Which is true?



- A. I only
- B. II only
- C. I and II
- D. II and III

**Answer**

Consider Statement (I) :

Two or more parallelograms on the same base and between the same parallels are equal in area. Rectangle is also a parallelogram.

∴ It is true.

Consider Statement (II) :

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base × Height

$$\therefore \text{Area of llgm ABCD} = AB \times DE = 10 \times 6 = 60\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

$$\therefore \text{area of llgm ABCD} = AD \times BF = 60\text{cm}^2$$

$$AD \times 8 = 60\text{cm}^2$$

$$AD = \frac{60}{8} = 7.5\text{cm}$$

∴ length of AD = 7.5cm.

∴ Statement (II) is correct.

Consider Statement (III)

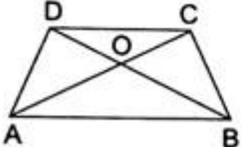
Area of parallelogram is  $\text{base} \times \text{height}$

$\therefore$  Statement (III) is false

$\therefore$  Statement (I) and (II) are true and statement (III) is false

### 28. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	Reason (R)
<p>In a trapezium ABCD we have <math>AB \parallel DC</math> and the diagonals AC and BD intersect at O.</p> <p>Then, <math>\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)</math></p> 	<p>Triangles on the same base and between the same parallels are equal in areas.</p>

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

### Answer

Assertion:

Here,  $\text{Area}(\triangle ABD) = \text{Area}(\triangle ABC)$  ( $\because$  Triangles on same base and between same parallel lines) -1

Subtract  $\text{Area}(\triangle AOB)$  on both sides of -1

$\text{Area}(\triangle ABD) - \text{Area}(\triangle AOB) = \text{Area}(\triangle ABC) - \text{Area}(\triangle AOB)$

$$\text{Area } (\triangle AOD) = \text{Area } (\triangle BOC)$$

$\therefore$  Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

### 29. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	Reason (R)
If ABCD is a rhombus whose one angle is $60^\circ$ , then the ratio of the lengths of its diagonals is 3:1.	Median of a triangle divides it into two triangles of equal area.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

### Answer

Given:  $\angle DCB = 60^\circ$

Let the length of the side be  $x$

Here, consider  $\triangle BCD$

$BC = DC$  (all sides of rhombus are equal)

$\therefore \angle CDB = \angle CBD$  (angles opposite to equal sides are equal)

Now, by angle sum property

$$\angle CDB + \angle CBD + \angle BCD = 180^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$\therefore 2 \times \angle CBD = 120^\circ$$

$$\angle CBD = \frac{120}{2} = 60^\circ$$

$$\therefore \angle CDB = \angle CBD = 60^\circ$$

$\therefore \Delta ADC$  is equilateral triangle

$$\therefore BC = CD = BD = x \text{ cm}$$

In Rhombus diagonals bisect each other.

Consider  $\Delta COD$

By Pythagoras theorem

$$CD^2 = OD^2 + OC^2$$

$$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$$

$$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$$

$$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$$

$$OC = \frac{\sqrt{3} \times x}{2} \text{ cm}$$

$$\therefore AC = 2 \times OC = 2 \times \frac{\sqrt{3} \times x}{2} = \sqrt{3}x$$

$$AC: BD = \sqrt{3}x : x = \sqrt{3} : 1$$

$$\therefore AC: BD = \sqrt{3} : 1$$

$\therefore$  Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

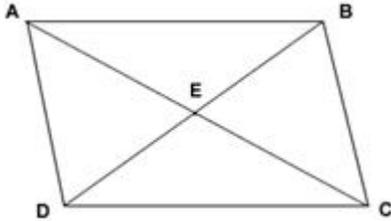
### 30. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	Reason (R)
The diagonals of a $\parallel$ gm divide it into four triangles of equal area.	A diagonal of a $\parallel$ gm divides it into two triangles of equal area.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**



Consider  $\Delta ABD$

We know that diagonals in a parallelogram bisect each other

$\therefore E$  is the midpoint of  $BD$ ,  $AE$  is median of  $\Delta ABD$

$\therefore \text{Area}(\Delta ADE) = \text{Area}(\Delta AEB)$  ( $\because$  Median divides the triangle into two triangles of equal areas)

Similarly we can prove

$$\text{Area}(\Delta ADE) = \text{Area}(\Delta DEC)$$

$$\text{Area}(\Delta DEC) = \text{Area}(\Delta CEB)$$

$$\text{Area}(\Delta CEB) = \text{Area}(\Delta AEB)$$

$\therefore$  Diagonals of a  $\parallel$  gm divide into four triangles of equal area.

$\therefore$  Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

**31. Question**

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	Reason (R)
The area of a trapezium whose parallel sides measure 25cm and 15cm respectively and the distance between them is 6cm, is $120\text{cm}^2$ .	The area of an equilateral triangle of side 8cm is $163\text{cm}^2$ .

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

Area of trapezium =  $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height} = \frac{1}{2} \times (25 + 15) \times 6 = 120\text{cm}^2$

$\therefore$  Area of trapezium =  $120\text{cm}^2$

$\therefore$  Assertion is correct.

Area of an equilateral triangle is given by:  $\frac{\sqrt{3}}{4} \times a^2$  (here 'a' is length of the side)

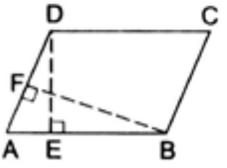
$\therefore$  Area of an equilateral triangle with side length 8 cm =  $\frac{\sqrt{3}}{4} \times 8^2 = 16\sqrt{3}$

$\therefore$  Reason is correct

$\therefore$  Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

**32. Question**

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	Reason (R)
<p>In the given figure, ABCD is a <math>\parallel</math> gm in which <math>DE \perp AB</math> and <math>BF \perp AD</math>. If <math>AB = 16\text{cm}</math>, <math>DE = 8\text{cm}</math> and <math>BF = 10\text{cm}</math>, then AD is 12cm.</p> 	<p>Area of a <math>\parallel</math> gm = base <math>\times</math> height.</p>

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base  $\times$  Height

$$\therefore \text{Area of } \parallel\text{gm ABCD} = AB \times DE = 16 \times 8 = 128\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

$$\therefore \text{area of } \parallel\text{gm ABCD} = AD \times BF = 128\text{cm}^2$$

$$AD \times 10 = 128\text{cm}^2$$

$$AD = \frac{128}{10} = 12.8\text{cm}$$

$$\therefore \text{length of AD} = 12.8\text{cm}$$

$\therefore$  Assertion is false and Reason is true

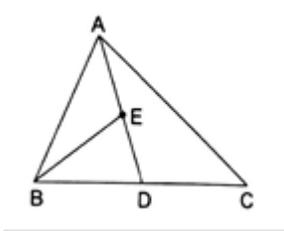
### 33. Question

Which of the following is a false statement?

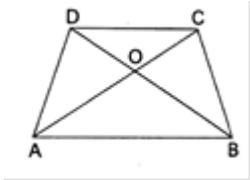
(A) A median of a triangle divides it into two triangles of equal areas.

(B) The diagonals of a llgm divide it into four triangles of equal areas.

(C) In a  $\Delta ABC$ , if E is the midpoint of median AD, then  $\text{ar}(\Delta BED) = \frac{1}{4} \text{ar}(\Delta ABC)$ .



(D) In a trap. ABCD, it is given that  $AB \parallel DC$  and the diagonals AC and BD intersect at O. Then,  $\text{ar}(\Delta AOB) = \text{ar}(\Delta COD)$ .



### Answer

The correct answer is Option (D)

$\Delta ABC$  and  $\Delta BCD$  does not lie between parallel lines and also  $\Delta AOB$  and  $\Delta COD$  are not congruent.

### 34. Question

Which of the following is a false statement?

A) If the diagonals of a rhombus are 18cm and 14cm, then its area is  $126\text{cm}^2$ .

B) Area of a llgm =  $\frac{1}{2}$  x base x corresponding height.

C) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

D) If the area of a ll gm with one side 24cm and corresponding height h cm is  $192\text{cm}^2$ , then  $h = 8\text{cm}$ .

### Answer

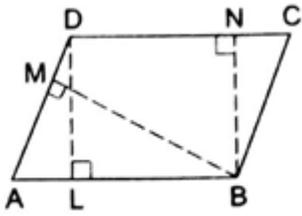
The correct answer is Option (B)

Area of parallelogram = base  $\times$  corresponding height.

## Formative Assessment (Unit Test)

### 1. Question

The area of || gm ABCD is



- A.  $AB \times BM$
- B.  $BC \times BN$
- C.  $DC \times DL$
- D.  $AD \times DL$

**Answer**

Area of the ||gm is  $\text{Base} \times \text{Height}$

Here, height is distance between the Base and its corresponding parallel side.

$$\therefore \text{Area (||gm ABCD)} = \text{Base} \times \text{Height} = DC \times DL$$

( $\because$  Here DC is taken as length and DL is the distance between DC and its corresponding parallel side AB).

**2. Question**

Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

- A. 1:2
- B. 1:1
- C. 2:1
- D. 3:1

**Answer**

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

Consider two ||gms ABCD and PQRS which are on same base and lie between same parallel lines.

$$\therefore \text{ar}(\text{||gm ABCD}) = \text{ar}(\text{||gm PQRS}) - 1$$

$$\therefore \text{ar}(\text{||gm ABCD}) : \text{ar}(\text{||gm PQRS}) = 1:1 (\because \text{eq -1})$$

**3. Question**

ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area. Then, ABCD

- A. Is a rectangle
- B. is a rhombus
- C. is a parallelogram

D. need not be any of (A), (B), (C)

### Answer

Quadrilateral is any closed figure which has four sides.

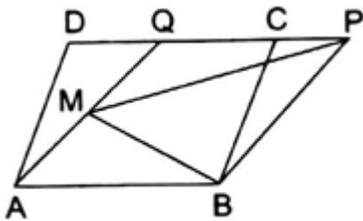
Rhombus, Rectangle, Parallelograms are few Quadrilaterals.

When a Diagonal AC of a quadrilateral divides it into two parts of equal areas, it is not necessary for the figure to be a Rhombus or a Rectangle or a Parallelogram, it can be any Quadrilateral.

### 4. Question

In the given figure, ABCD and ABPQ are two parallelograms and M is a point on AQ and BMP is a triangle.

Then,  $\text{ar}(\triangle BMP) = \frac{1}{2} \text{ar}(\text{||gm ABCD})$ .



A. True

B. False

### Answer

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{ar}(\text{||gm ABCD}) = \text{ar}(\text{||gm ABPQ}) - 1$$

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\therefore \text{ar}(\triangle BMP) = \frac{1}{2} \text{ar}(\text{||gm ABPQ})$$

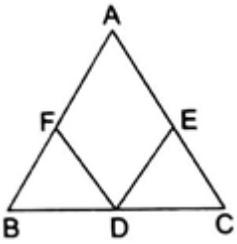
But, from -1

$$\text{ar}(\text{||gm ABCD}) = \text{ar}(\text{||gm ABPQ})$$

$$\therefore \text{ar}(\triangle BMP) = \frac{1}{2} \text{ar}(\text{||gm ABCD})$$

### 5. Question

The midpoints of the sides of a triangle along with any of the vertices as the fourth point makes a parallelogram of area equal to



- A.  $\frac{1}{2}$  (ar  $\Delta ABC$ )
- B.  $\frac{1}{3}$  (ar  $\Delta ABC$ )
- C.  $\frac{1}{4}$  (ar  $\Delta ABC$ )
- D. ar ( $\Delta ABC$ )

**Answer**

Join EF

Here Area ( $\Delta AEF$ ) = Area ( $\Delta BDF$ ) = Area ( $\Delta DEF$ ) = Area ( $\Delta DEC$ ) =  $\frac{1}{4}$  Area ( $\Delta ABC$ ) - 1

Consider any vertex of the triangle.

Let us consider Vertex B

Here, BDEF form a parallelogram.

Area (||gm BDEF) = Area ( $\Delta BDF$ ) + Area ( $\Delta DEF$ )

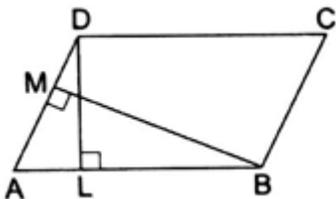
Area (||gm BDEF) =  $\frac{1}{4}$  Area ( $\Delta ABC$ ) +  $\frac{1}{4}$  Area ( $\Delta ABC$ ) =  $\frac{1}{2}$  Area ( $\Delta ABC$ ) (from -1)

$\therefore$  Area (||gm BDEF) =  $\frac{1}{2}$  Area ( $\Delta ABC$ )

Similarly, we can prove for other vertices.

**6. Question**

Let ABCD be a || gm in which  $DL \perp AB$  and  $BM \perp AD$  such that  $AD = 6$  cm,  $BM = 10$  and  $DL = 8$  cm. Find AB.



**Answer**

Given:

$AD = 6$  cm

$DL \perp AB$

$BM \perp AD$

$$DL = 8\text{cm}$$

$$BM = 10\text{cm}$$

Now, consider the parallelogram ABCD

Here, let AD be the base of the parallelogram then BM becomes its altitude (height).

Area of the parallelogram is given by: Base  $\times$  Height

$$\therefore \text{area of llgm ABCD} = AD \times BM = 6 \times 10 = 60\text{cm}^2$$

Now,

Consider AB as base of the parallelogram then DL becomes its altitude (height)

$$\therefore \text{area of llgm ABCD} = AB \times DL = 60\text{cm}^2$$

$$AB \times 8 = 60\text{cm}^2$$

$$AB = \frac{60}{8} = 7.5\text{cm}$$

$$\therefore \text{length of AB} = 7.5\text{cm}.$$

### 7. Question

Find the area of the trapezium whose parallel sides are 14 cm and 10 cm and whose height is 6 cm.

### Answer

Given: Length of parallel sides 14 cm and 10 cm, height is 6cm

We know that area of trapezium is given by:  $\frac{1}{2}$  (sum of parallel sides)  $\times$  height

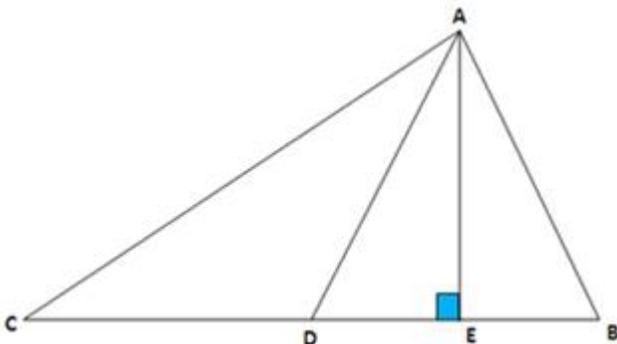
$$\therefore \text{Area of trapezium} = \frac{1}{2} (14 + 10) \times 6 = 72\text{cm}^2$$

$$\therefore \text{Area of trapezium} = 72\text{cm}^2$$

### 8. Question

Show that the median of a triangle divides it into two triangles of equal area.

### Answer



Consider the Figure

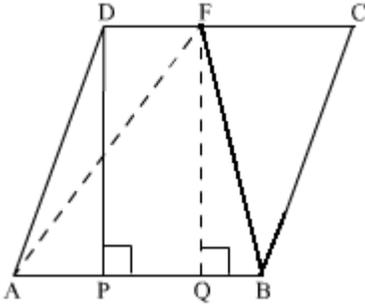
Here,

In  $\triangle ABC$ , AD is the median Hence  $BD = DC$  Draw  $AE \perp BC$  Area of  $\triangle ABD =$  Area of  $\triangle ADC$  Thus median of a triangle divides it into two triangles of equal area.

### 9. Question

Prove that area of a triangle =  $\frac{1}{2}$  X base X altitude.

### Answer



We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

Consider the figure,

Here,

$$\text{Area}(\triangle ABF) = \frac{1}{2} \text{Area}(\text{||gm } ABCD) \text{ (From above statement) } -1$$

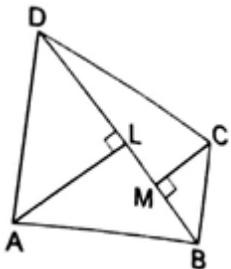
$$\text{Area}(\text{||gm } ABCD) = \text{Base} \times \text{Height} -2$$

Sub -2 in -1

$$\text{Area}(\triangle ABF) = \frac{1}{2} \times \text{Base} \times \text{Height}$$

### 10. Question

In the adjoining figure, ABCD is a quadrilateral in which diagonal  $BD = 14\text{cm}$ . If  $AL \perp BD$  and  $CM \perp BD$  such that  $AL = 8\text{ cm}$  and  $CM = 6\text{ cm}$ , find the area of quad. ABCD.



### Answer

Given:  $BD = 14\text{cm}$ ,  $AL = 8\text{ cm}$ ,  $CM = 6\text{ cm}$  and also,  $AL \perp BD$  and  $CM \perp BD$ .

Here,

$$\text{Area (Quad. ABCD)} = \text{Area} (\triangle ABD) + \text{Area} (\triangle ABC)$$

$$\text{Area } (\Delta ABD) = 1/2 \text{ base} \times \text{height} = 1/2 \times BD \times AL = 1/2 \times 14 \times 8 = 56\text{cm}^2$$

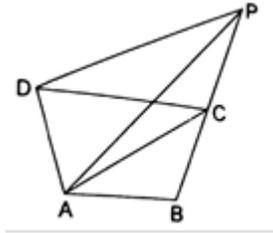
$$\text{Area } (\Delta ABC) = 1/2 \text{ base} \times \text{height} = 1/2 \times BD \times CM = 1/2 \times 14 \times 6 = 42\text{cm}^2$$

$$\therefore \text{Area (Quad.ABCD)} = \text{Area } (\Delta ABD) + \text{Area } (\Delta ABC) = 56 + 42 = 98 \text{ cm}^2$$

$$\therefore \text{Area (Quad.ABCD)} = 98 \text{ cm}^2$$

### 11. Question

In the adjoining figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P. Prove that  $\text{ar}(\Delta ABP) = \text{ar}(\text{quad. ABCD})$ .



### Answer

Given:  $AC \parallel DP$

We know that any two or Triangles having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area } (\Delta ACD) = \text{Area } (\Delta ACP) \text{ ---1}$$

Add Area  $(\Delta ABC)$  on both sides of eq -1

We get,

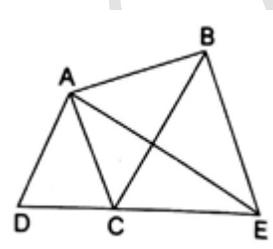
$$\text{Area } (\Delta ACD) + \text{Area } (\Delta ABC) = \text{Area } (\Delta ACP) + \text{Area } (\Delta ABC)$$

That is,

$$\text{Area (quad.ABCD)} = \text{Area } (\Delta ABP)$$

### 12. Question

In the given figure, ABCD is a quadrilateral and  $BE \parallel AC$  and also BE meets DC produced at E. Show that the area of  $\Delta ADE$  is equal to the area of quad. ABCD.



### Answer

Given:  $BE \parallel AC$

We know that any two or more Triangles having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area}(\Delta ACE) = \text{Area}(\Delta ACB) - 1$$

Add Area ( $\Delta ADC$ ) on both sides of eq -1

We get,

$$\text{Area}(\Delta ACE) + \text{Area}(\Delta ADC) = \text{Area}(\Delta ACB) + \text{Area}(\Delta ADC)$$

That is,

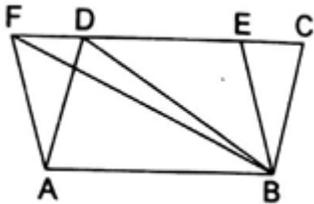
$$\text{Area}(\Delta ADE) = \text{Area}(\text{quad. } ABCD)$$

### 13. Question

In the given figure, area of  $\parallel$  gm ABCD is  $80 \text{ cm}^2$ .

Find (i)  $\text{ar}(\parallel\text{gm } ABEF)$

(ii)  $\text{ar}(\Delta ABD)$  and (iii)  $\text{ar}(\Delta BEF)$ .



### Answer

Given: area of  $\parallel$  gm ABCD is  $80 \text{ cm}^2$

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{ar}(\parallel\text{gm } ABCD) = \text{ar}(\parallel\text{gm } ABEF) - 1$$

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\therefore \text{ar}(\Delta ABD) = 1/2 \times \text{ar}(\parallel\text{gm } ABCD) \text{ and,}$$

$$\text{ar}(\Delta BEF) = 1/2 \times \text{ar}(\parallel\text{gm } ABEF)$$

(i)

$$\text{ar}(\parallel\text{gm } ABCD) = \text{ar}(\parallel\text{gm } ABEF)$$

$$\therefore \text{ar}(\parallel\text{gm } ABEF) = 80\text{cm}^2 (\because \text{ar}(\parallel\text{gm } ABCD) = 80\text{cm}^2)$$

(ii)

$$\text{ar}(\Delta ABD) = 1/2 \times \text{ar}(\parallel\text{gm } ABCD)$$

$$\text{ar}(\Delta ABD) = 1/2 \times 80 = 40\text{cm}^2 (\because \text{ar}(\parallel\text{gm } ABCD) = 80\text{cm}^2)$$

$$\therefore \text{ar}(\triangle ABD) = 40\text{cm}^2$$

(iii)

$$\text{ar}(\triangle BEF) = 1/2 \times \text{ar}(\text{||gm ABEF})$$

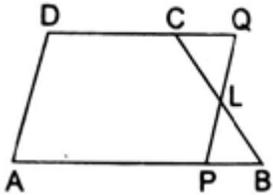
$$\text{ar}(\triangle BEF) = 1/2 \times 80 = 40\text{cm}^2 (\because \text{ar}(\text{||gm ABEF}) = 80\text{cm}^2)$$

$$\therefore \text{ar}(\triangle BEF) = 40\text{cm}^2$$

#### 14. Question

In trapezium ABCD,  $AB \parallel DC$  and L is the midpoint of BC. Through L, a line  $PQ \parallel AD$  has been drawn which meets AB in Point P and DC produced in Q.

Prove that  $\text{ar}(\text{trap. ABCD}) = \text{ar}(\text{||gm APQD})$ .



#### Answer

Given:  $AB \parallel DC$  and L is the midpoint of BC,  $PQ \parallel AD$

Construction: Drop a perpendicular DM from D onto AP

Consider  $\triangle PBL$  and  $\triangle CQL$

Here,

$$\angle LPB = \angle LQC \text{ (Alternate interior angles, } AB \parallel DQ)$$

$$BL = LC \text{ (L is midpoint of BC)}$$

$$\angle PLB = \angle QLC \text{ (vertically opposite angles)}$$

$\therefore$  By AAS congruency

$$\triangle PBL \cong \triangle CQL$$

$$\therefore PB = CQ \text{ (C.P.C.T)}$$

$$\text{Area (||gm APQD)} = \text{base} \times \text{height} = AP \times DM \text{ --1}$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (\text{sum of parallel sides}) \times \text{height} = 1/2 \times (AB + DC) \times DM$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (AB + DC) \times DM = 1/2 \times (AP + PB + DC) \times DM (\because AB = AP + PB)$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (AP + CQ + DC) \times DM (\because PB = CQ)$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (AP + DQ) \times DM (\because DC + CQ = DQ)$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (2 \times AP) \times DM (\because AP = DQ)$$

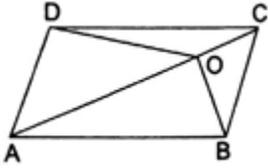
$$\text{Area (Trap.ABCD)} = AP \times DM \text{ --2}$$

From -1 and -2

$$\text{Area (Trap. ABCD)} = \text{Area (||gm APQD)}$$

### 15. Question

In the adjoining figure, ABCD is a || gm and O is a point on the diagonal AC. Prove that  $\text{ar}(\Delta AOB) = \text{ar}(\Delta AOD)$ .



### Answer

Given: ABCD is a || gm and O is a point on the diagonal AC.

Construction: Drop perpendiculars DM and BN onto diagonal AC.

Here,

$DM = BN$  (perpendiculars drawn from opposite vertices of a || gm to the diagonal are equal)

Now,

$$\text{Area } (\Delta AOB) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AO \times BN \text{ -1}$$

$$\text{Area } (\Delta AOD) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AO \times DM \text{ -2}$$

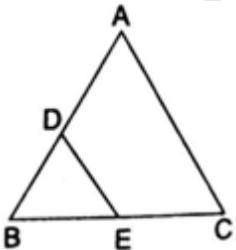
From -1 and -2

$$\text{Area } (\Delta AOB) = \text{Area } (\Delta AOD) \text{ } (\because BN = DM)$$

### 16. Question

$\Delta ABC$  and  $\Delta BDE$  are two equilateral triangles such that D(E) is the midpoint of BC. Then, prove that

$$\text{ar}(\Delta BDE) = \frac{1}{4} \text{ar}(\Delta ABC).$$



### Answer

Given:  $\Delta ABC$  and  $\Delta BDE$  are two equilateral triangles, D is the midpoint of BC.

Consider  $\Delta ABC$

Here, let  $AB = BC = AC = x$  cm (equilateral triangle)

Now, consider  $\Delta BED$

Here,

$$BD = 1/2 BC$$

$$\therefore BD = ED = EB = 1/2 BC = x/2 \text{ (equilateral triangle)}$$

Area of the equilateral triangle is given by:  $\frac{\sqrt{3}}{4} a^2$  (a is side length)

$$\therefore \text{ar}(\triangle BDE) : \text{ar}(\triangle ABC) = \frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2 : \frac{\sqrt{3}}{4} x^2 = \frac{1}{4} : 1 = 1 : 4$$

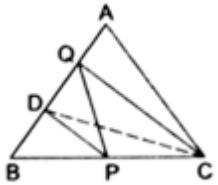
$$\text{That is } \frac{\text{ar}(\triangle BDE)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$

$$\therefore \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

Hence Proved

### 17. Question

In  $\triangle ABC$ , D is the midpoint of AB and P Point is any point on BC. If  $CQ \parallel PD$  meets AB in Q, then prove that  $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$ .



### Answer

Given: D is the midpoint of AB and P Point is any point on BC,  $CQ \parallel PD$

In Quadrilateral DPQC

$$\text{Area}(\triangle DPQ) = \text{Area}(\triangle DPC)$$

Add Area  $(\triangle BDP)$  on both sides

We get,

$$\text{Area}(\triangle DPQ) + \text{Area}(\triangle BDP) = \text{Area}(\triangle DPC) + \text{Area}(\triangle BDP)$$

$$\text{Area}(\triangle BPQ) = \text{Area}(\triangle BCD) - 1$$

D is the midpoint BC, and CD is the median

$$\therefore \text{Area}(\triangle BCD) = \text{Area}(\triangle ACD) = 1/2 \times \text{Area}(\triangle ABC) - 2$$

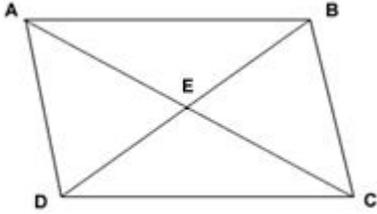
Sub -2 in -1

$$\text{Area}(\triangle BPQ) = 1/2 \times \text{Area}(\triangle ABC) (\because \text{Area}(\triangle BCD) = 1/2 \times \text{Area}(\triangle ABC))$$

### 18. Question

Show that the diagonals of a || gm divide into four triangles of equal area.

**Answer**



Consider  $\Delta ABD$

We know that diagonals in a parallelogram bisect each other

$\therefore$  E is the midpoint of BD, AE is median of  $\Delta ABD$

$\therefore$  Area ( $\Delta ADE$ ) = Area ( $\Delta AEB$ ) ( $\because$  Median divides the triangle into two triangles of equal areas)

Similarly we can prove

$$\text{Area } (\Delta ADE) = \text{Area } (\Delta DEC)$$

$$\text{Area } (\Delta DEC) = \text{Area } (\Delta CEB)$$

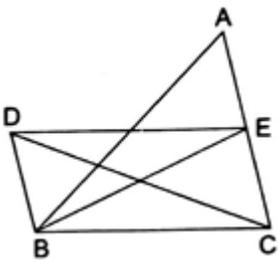
$$\text{Area } (\Delta CEB) = \text{Area } (\Delta AEB)$$

$\therefore$  Diagonals of a || gm divide into four triangles of equal area.

**19. Question**

In the given figure,  $BD \parallel CA$ , E is the midpoint of CA and  $BD = \frac{1}{2} CA$ .

Prove that  $\text{ar}(\Delta ABC) = 2 \times \text{ar}(\Delta DBC)$ .



**Answer**

Given:  $BD \parallel CA$ , E is the midpoint of CA and  $BD = \frac{1}{2} CA$

Consider  $\Delta BCD$  and  $\Delta DEC$

Here,

$$BD = EC \left( \because E \text{ is the midpoint of } AC \text{ that is } CE = \frac{1}{2} CA, BD = \frac{1}{2} CA \right)$$

$$CD = CD \text{ (common)}$$

$$\angle BDC = \angle ECD \text{ (alternate interior angles, } DB \parallel AC)$$

$\therefore$  By SAS congruency

$$\Delta BCD \cong \Delta DEC$$

$$\therefore \text{Area} (\Delta BCD) = \text{Area} (\Delta DEC) \text{ --1}$$

Here,

$$\text{Area} (\Delta BCE) = \text{Area} (\Delta DEC) \text{ (triangles on same base } CE \text{ and between same parallel lines) --2}$$

E is the midpoint of AC, BE is the median of  $\Delta ABC$

$$\therefore \text{Area} (\Delta BCE) = \text{Area} (\Delta ABE) = \frac{1}{2} \times \text{Area} (\Delta ABC)$$

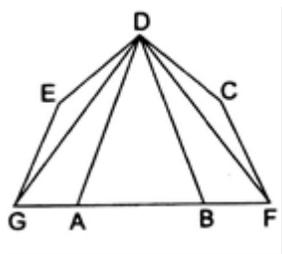
$$\therefore \text{Area} (\Delta DEC) = \frac{1}{2} \times \text{Area} (\Delta ABC) \left( \because \text{Area} (\Delta BCE) = \text{Area} (\Delta DEC) \right)$$

$$\therefore \text{Area} (\Delta BCD) = \frac{1}{2} \times \text{Area} (\Delta ABC) \left( \because \text{Area} (\Delta DEC) = \text{Area} (\Delta BCD) \right)$$

## 20. Question

The given figure shows a pentagon ABCDE in which EG, drawn parallel to DA, meets BA produced at G and CF drawn parallel to DB meets AB produced at F.

Show that  $\text{ar}(\text{pentagon } ABCDE) = \text{ar}(\Delta DGF)$ .



## Answer

Given:  $EG \parallel DA, CF \parallel DB$

Here, in Quadrilateral ADEG

$$\text{Area} (\Delta AED) = \text{Area} (\Delta ADG) \text{ --1}$$

In Quadrilateral CFBD

$$\text{Area} (\Delta CBD) = \text{Area} (\Delta BCF) \text{ --2}$$

Add -1 and -2

$$\text{Area} (\Delta AED) + \text{Area} (\Delta CBD) = \text{Area} (\Delta ADG) + \text{Area} (\Delta BCF) \text{ --3}$$

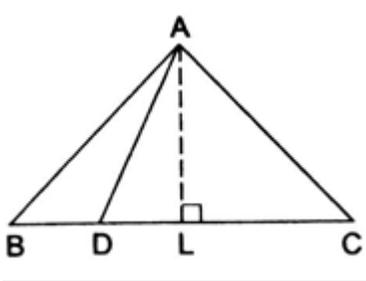
Add Area ( $\Delta ABD$ ) to -3

$$\text{Area } (\Delta AED) + \text{Area } (\Delta CBD) + \text{Area } (\Delta ABD) = \text{Area } (\Delta ADG) + \text{Area } (\Delta BCF) + \text{Area } (\Delta ABD)$$

$$\text{Area (pentagon ABCDE)} = \text{Area } (\Delta DGF)$$

## 21. Question

In the adjoining figure, the point D divides the side BC of  $\Delta ABC$  in the ratio  $m:n$ . Prove that  $\text{ar}(\Delta ABD) : \text{ar}(\Delta ADC) = m:n$ .



## Answer

Given: D divides the side BC of  $\Delta ABC$  in the ratio  $m:n$

$$\text{Area } (\Delta ABD) = \frac{1}{2} \times BD \times AL$$

$$\text{Area } (\Delta ADC) = \frac{1}{2} \times CD \times AL$$

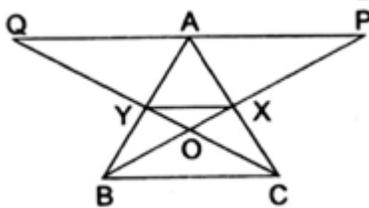
$$\text{Area } (\Delta ABD) : \text{Area } (\Delta ADC) = \frac{1}{2} \times BD \times AL : \frac{1}{2} \times CD \times AL$$

$$\text{Area } (\Delta ABD) : \text{Area } (\Delta ADC) = BD : CD$$

$$\text{Area } (\Delta ABD) : \text{Area } (\Delta ADC) = m : n \quad (\because BD : CD = m : n)$$

## 22. Question

In the give figure, X and Y are the midpoints of AC and AB respectively,  $QP \parallel BC$  and  $CYQ$  and  $BXP$  are straight lines. Prove that  $\text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$ .



## Answer

Given: X and Y are the midpoints of AC and AB respectively,  $QP \parallel BC$  and  $CYQ$  and  $BXP$  are straight lines.

Construction: Join QB and PC

In Quadrilateral BCQP

$\text{Area } (\Delta QBC) = \text{Area } (\Delta BCP)$  (Triangles on same base BC and between same parallel lines are equal in area) -1 and,

$\text{Area } (\text{||gm } ACBQ) = \text{Area } (\text{||gm } ABPC)$  (parallelograms on same base BC and between same parallel lines are equal in area) -2

Subtract  $-1$  from  $-2$

$\text{Area}(\text{||gm ACBQ}) - \text{Area}(\Delta \text{QBC}) = \text{Area}(\text{||gm ABCP}) - \text{Area}(\Delta \text{BCP})$

$\text{Area}(\Delta \text{ACQ}) = \text{Area}(\Delta \text{ABP})$

$\therefore \text{Area}(\Delta \text{ABP}) = \text{Area}(\Delta \text{ACQ})$

CAREER POINT