JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - I

PHYSICS

Q.1 A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is -

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) $\frac{8}{5}$ (4) $\frac{3}{5}$

Ans. **[3]**

Sol. $M = \int_0^R (\rho_0 r)(2\pi r dr) = 2\pi\rho_0 \frac{R^3}{3}$

$I = MR^2 + \int_0^R 2\pi\rho_0 r^4 dr = MR^2 + \frac{2\pi\rho_0 R^5}{5}$

$I = MR^2 + \frac{3MR^2}{5} = \frac{8MR^2}{5}$

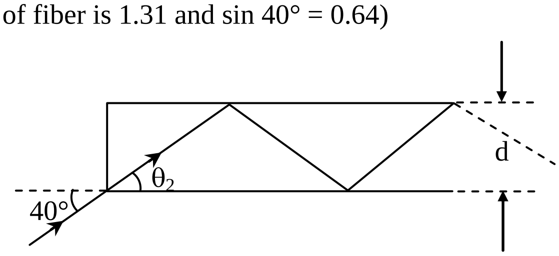
Q.2 A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4Q, the new potential difference between the same two surface is -

(1) – 2V (2) 2V (3) V (4) 4V

Ans. **[3]**

Sol. Shell will produce constant potential. Thus new potential difference = V

Q.3 In figure, the optical fiber is l = 2 m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to – (refractive index of fiber is 1.31 and sin 40° = 0.64)



(1) 66000 (2) 55000 (3) 45000 (4) 57000

Ans. **[4]**

Sol. $1 \sin 40 = 1.31 \sin\theta$

$$\Rightarrow \sin\theta = \frac{0.64}{1.31} \simeq \frac{1}{2} \Rightarrow \theta = 30^{o}$$

$$x = \frac{D}{1/\sqrt{3}} = 20\sqrt{3} \times 10^{-6}\ \text{m}$$

$$\text{No. of reflections} = \frac{20}{20\sqrt{3}\times10^{-6}} \simeq 57000$$

Q.4 An alternating voltage V(t) = 220 sin 100πt volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is -

(1) 5 ms (2) 7.2 ms (3) 3.3 ms (4) 2.2 ms

Ans. **[3]**

Sol. $\Delta\phi = \frac{\pi}{3} = (100\pi)\ \Delta t$

$$\Rightarrow \Delta t = \frac{10^3}{300}\ \text{ms} = 3.3\ \text{ms}$$

Q.5 Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is -

(1) $n = 2 \rightarrow n = 5$ (2) $n = 2 \rightarrow n = 3$ (3) $n = 1 \rightarrow n = 4$ (4) $n = 2 \rightarrow n = 4$

Ans. **[4]**

Sol. for H $\Delta E = Rhc\left[1-\frac{1}{4}\right] = \frac{3Rhc}{4}$

for He $(1\rightarrow4)$ $\Delta E = Rhc\left(1-\frac{1}{16}\right)4 = \frac{15}{4}Rhc$

$(2\rightarrow4)$ $\Delta E = Rhc\left(\frac{1}{4}-\frac{1}{16}\right)4 = \frac{3}{4}Rhc$

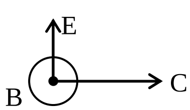
Q.6 A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\ Vm^{-1}$ along y-direction. Its corresponding magnetic field component, B would be -

(1) 2×10^{-8} T along y-direction (2) 6×10^{-8} T along z-direction

(3) 2×10^{-8} T along z-direction (4) 6×10^{-8} T along x-direction

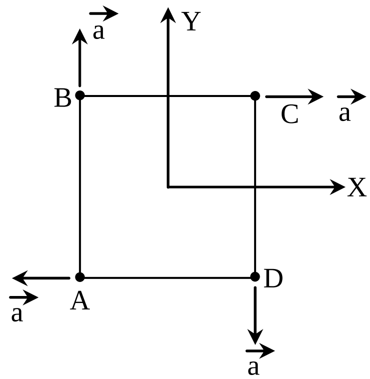
Ans. **[3]**

Sol.



$$B = \frac{E}{C} = \frac{6}{3\times10^8} = 2 \times 10^{-8}\ (+z)$$

Q.7 Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is -

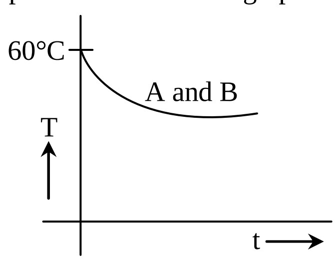


(1) Zero (2) $a(\hat{i}+\hat{j})$ (3) $\frac{a}{5}(\hat{i}-\hat{j})$ (4) $\frac{a}{5}(\hat{i}+\hat{j})$

Ans. **[3]**

Sol. $\vec{a}_c = \dfrac{-ma\hat{i}+2ma\hat{j}+3ma\hat{i}-4ma\hat{j}}{10m} = \dfrac{a}{5}[\hat{i}-\hat{j}]$

Q.8 Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically ? (assume the emissivity of both the beakers to be the same)

(1)


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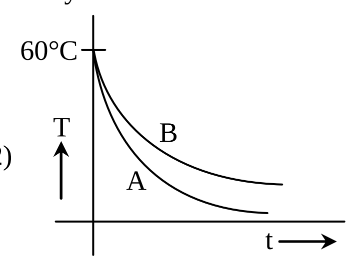

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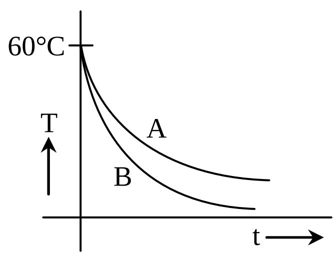

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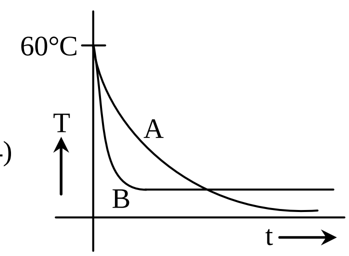


Ans. **[2]**

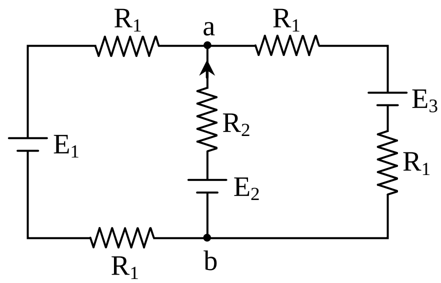
Sol. $-\dfrac{dT}{dt} = \dfrac{e\sigma A}{ms} 4T_0^3 (T - T_0) \propto \dfrac{1}{\rho S}$

For A $\rho s = 800 \times 2000 = 16 \times 10^5$

B $\rho s = 10^3 \times 4000 = 40 \times 10^5$

$(\rho s)_B > (\rho s)_A \Rightarrow \left(-\dfrac{dT}{dt}\right)_B < \left(-\dfrac{dT}{dt}\right)_A$

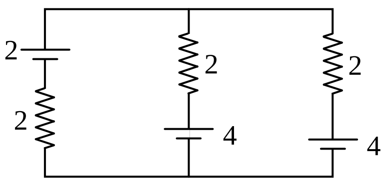
Q.9 For the circuit shown, with R_1 = 1.0 Ω, R_2 = 2.0 Ω, E_1 = 2V and E_2 = E_3 = 4V, the potential difference between the points 'a' and 'b' is approximately (in V) -



(1) 3.7 (2) 2.7 (3) 2.3 (4) 3.3

Ans. **[4]**

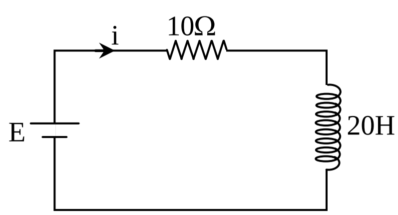
Sol.



$$\frac{\frac{2}{2}+\frac{4}{2}+\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}} = \Delta V$$

$$\Delta V = \frac{5}{3/2} = 3.3 \text{ volt}$$

Q.10 A 20 Henry inductor and coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rated of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is -



(1) $\frac{1}{2}\ell n\, 2$ (2) $\ell n\, 2$ (3) $2\, \ell n\, 2$ (4) $\frac{2}{\ell n\, 2}$

Ans. **[3]**

Sol. $i^2R = \left(L\frac{di}{dt}\right)i$

$\Rightarrow iR = L\frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{i}{\tau}$

$e^{-t/\tau} = 1 - e^{-t/\tau}$

$\Rightarrow 2e^{-t/\tau} = 1 \Rightarrow t = \tau\ell n2$

$\tau = \frac{\tau}{R} = \frac{20}{10} = 2$

$t = 2\ell n2$

Q.11 Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ, of the final particle, is given by -

(1) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (2) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$ (3) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (4) $\lambda = \sqrt{\lambda_1 \lambda_2}$

Ans. **[2]**

Sol. $P_1^2 + P_2^2 = P^2$

$\Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Q.12 In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is -

(1) $A\,T^{-3}\,M\,L^{3/2}$ (2) $A^{-1}\,T\,M\,L^3$ (3) $A\,T^2\,M^{-1}\,L^{-1}$ (4) $A^2\,T^3\,M^{-1}\,L^{-2}$

Ans. **[4]**

Sol. $\epsilon_0 = M^{-1}\,A^2\,L^{-3}\,T^4$

$\mu_0 = MLT^{-2}A^{-2}$

$\sqrt{\frac{\epsilon_0}{\mu_0}} = [M^{-2}A^4L^{-4}T^6]^{1/2} = M^{-1}A^2L^{-2}T^3$

Q.13 In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be -

(1) 9 (2) 2 (3) 18 (4) 4

Ans. **[4]**

Sol. $a_1 : a_2 = 1 : 3$

$\frac{A_{max}}{A_{min}} = \frac{1+3}{3-1} = \frac{4}{2}$

$\frac{I_{max}}{I_{min}} = \left(\frac{4}{2}\right)^2 = 4 : 1$

Q.14 A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire ?

(1) $5.2 \times 10^6\ Nm^{-2}$ (2) $6.2 \times 10^6\ Nm^{-2}$ (3) $4.8 \times 10^6\ Nm^{-2}$ (4) $3.1 \times 10^6\ Nm^{-2}$

Ans. **[4]**

Sol. Stress $= \frac{Mg}{\pi r^2} = \frac{4(3\cdot1\pi)}{\pi 4\times10^{-6}}$

$= 3\cdot1 \times 10^6\,\frac{N}{m^2}$

Q.15 A 200Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be -

(1) 100 Ω (2) 500 Ω (3) 400 Ω (4) 300 Ω

Ans. **[2]**

Sol. Red → Green ⇒ 200Ω → 500Ω

Q.16 Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of - (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1 mPa s)

(1) 10^4 (2) 10^3 (3) 10^2 (4) 10^6

Ans. **[1]**

Sol. $R_e = \dfrac{dv\rho}{\eta}$

$$= \frac{(10^{-1})\left(\frac{10^{-1}}{60}\right)\left(\frac{1}{\pi 25\times10^{-4}}\right)(10^3)}{10^{-3}} \simeq 21\cdot2\times10^3$$

Order = 10^4

Q.17 The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is – (take g = 10 m/s^2)

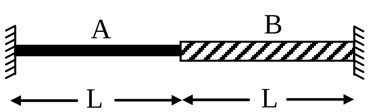
(1) $\tan^{-1}(5.0)$ (2) $\tan^{-1}(0.5)$ (3) $\tan^{-1}(0.2)$ (4) $\tan^{-1}(2.0)$

Ans. **[2]**

Sol. $\tan\theta = \dfrac{qE}{Mg} = \dfrac{5\times10^{-6}\times2\times10^3}{2\times10^{-3}\times10} = \dfrac{1}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{2}\right)$

Q.18 A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is -



(1) 3 : 5 (2) 4 : 9 (3) 1 : 2 (4) 1 : 4

Ans. **[3]**

Sol. $f = \dfrac{N}{2\ell}\sqrt{\dfrac{T}{\rho\pi r^2}} \Rightarrow N \propto r$

$N_1 : N_2 = r : 2r = 1 : 2$

Q.19 An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 2 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be -

(1) 40 cm from the convergent lens, twice the size of the object

(2) 20 cm from the convergent mirror, twice the size of the object

(3) 40 cm from the convergent mirror, same size as the object

(4) 20 cm from the convergent mirror, same size as the object

Ans. **[4]**

Sol.

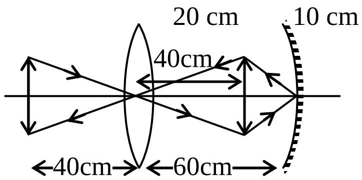


Image will form at object it self of same size and inverted

NTA option is [4]

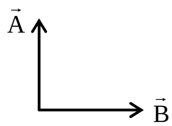
Career point option [bonus]

Q.20 A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is -

(1) $\frac{B\pi r^2 I}{N}$ (2) Zero (3) $B\pi r^2 I\ N$ (4) $\frac{Br^2 I}{\pi N}$

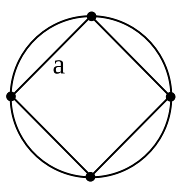
Ans. **[3]**

Sol.



$\tau = NI\pi r^2 B\sin 90^o = B\pi r^2 IN$

Q.21 Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others gravitational field in a circular orbit circumscribing the square ?



(1) $1.16\sqrt{\frac{GM}{a}}$ (2) $1.21\sqrt{\frac{GM}{a}}$ (3) $1.35\sqrt{\frac{GM}{a}}$ (4) $1.41\sqrt{\frac{GM}{a}}$

Ans. **[1]**

Sol. $\frac{Gm^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) = \frac{MV^2}{a/\sqrt{2}}$

$\Rightarrow V = \sqrt{\frac{Gm}{a}}\sqrt{1+\frac{1}{2\sqrt{2}}} = 1\cdot 16\sqrt{\frac{Gm}{a}}$

Q.22 A thermally insulated vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to – (Latent heat of vaporization of water = 2.10×10^6 Jkg^{-1} and Latent heat of fusion of water = 3.36×10^5 J kg^{-1})

(1) 130 g (2) 35 g (3) 150 g (4) 20 g

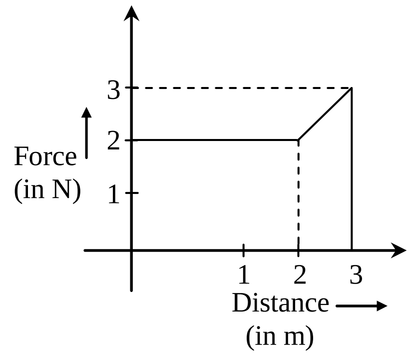
Ans. [4]

Sol. $m(2{\cdot}10 \times 10^6) = (150 - m)(3{\cdot}36 \times 10^5)$

$\frac{21m}{3{\cdot}36} + m = 150$

$\Rightarrow m = \frac{3{\cdot}36 \times 150}{24{\cdot}36} = 20g$

Q.23 A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is -



(1) 2.5 J (2) 6.5 J (3) 4 J (4) 5 J

Ans. [2]

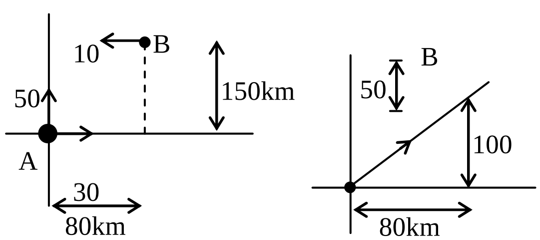
Sol. $\frac{1}{2}mV^2 = 4 + 2 + 0.5 = 6.5$ J

Q.24 Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in -

(1) 4.2 hrs (2) 2.2 hrs (3) 2.6 hrs (4) 3.2 hrs

Ans. [3]

Sol.



$\vec{V}_{AB} = 40\hat{i} + 50\hat{j}$

$\vec{r}_{BA} = 150\hat{j}$

$t_{min} = \frac{|\vec{V}_{AB} \bullet \vec{r}_{BA}|}{|\vec{V}_{AB}|^2} = 2.6$ hrs

Q.25 A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to -

(1) 10^3 Nm^{-2} (2) 10^4 Nm^{-2} (3) 10^6 Nm^{-2} (4) 10^8 Nm^{-2}

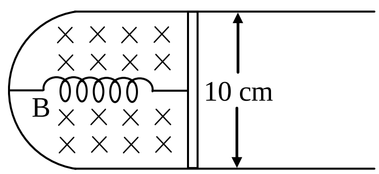
Ans. [3]

Sol. $\frac{1}{2}Y\left(\frac{\Delta\ell}{\ell}\right)^2 A\ell = \frac{1}{2}mV^2$

$\Rightarrow Y = \frac{mV^2\ell}{A(\Delta\ell)^2}$

$Y = \frac{(0.02)(20)^2(0.42)}{\pi(9\times10^{-6})(0.2)^2} \simeq 10^6$

Q.26 A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to -



(1) 50000 (2) 1000 (3) 10000 (4) 5000

Ans. [4]

Sol. $-KX - \frac{v\ell^2B^2}{R} = ma$

$A = A_0e^{-bt/2m}$

$t = \frac{2mR}{B^2\ell^2} = \frac{2(50\times10^{-3})(10)}{(0.1)^2(0.1)^2} = 10^4$

$t = 2\pi\sqrt{m/K} = 2sec \Rightarrow f = 0.5Hz$

N = 5000

Q.27 If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of -

(1) 10^8 N/m^2 (2) 10^4 N/m^2 (3) 10^{16} N/m^2 (4) 10^3 N/m^2

Ans. [4]

Sol. $P = \frac{(2mV)N}{A} = \frac{2(10^{-26})(10^4)10^{22}}{1} = 2\frac{N}{m^2}$

NTA answer is [4]

Career point option [Bonus]

Q.28 Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF ?

(given $\in_0 = 8.86 \times 10^{-12} C^2/Nm^2$)

(1) 6.2 (2) 3.8 (3) 4.5 (4) 8.5

Ans. **[4]**

Sol. $C = \frac{A \in_0 K}{d}$ & $V = Ed$

$\Rightarrow C = \frac{A \in_0 KE}{V}$

$K = \frac{CV}{A \in_0 E} = \frac{(15\times10^{-12})(500)}{10^{-4}(8.86\times10^{-12})(10^6)}$

$= 8.5$

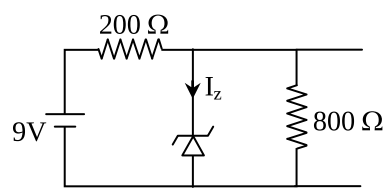
Q.29 The wavelength of the carrier waves in a modern optical fiber communication network is close to -

(1) 1500 nm (2) 600 nm (3) 900 nm (4) 2400 nm

Ans. **[1]**

Sol. $\lambda \simeq 1550nm$ (most widely used wavelength in optical fiber system)

Q.30 The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_z through the Zener is -

(1) 7 mA (2) 10 mA (3) 17 mA (4) 15 mA

Ans. **[2]**

Sol. $i_z = \frac{3.4}{200} - \frac{5.6}{800} = 10$ mA

JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - I

CHEMISTRY

Q.1 The lanthanide ion that would show colour is -

(1) La^{3+} (2) Gd^{3+} (3) Sm^{3+} (4) Lu^{3+}

Ans. **[3]**

Sol. Sm^{+3} = [Xe] $4f^5$

Due to presence of unpaired e^- it is yellow in colour.

Q.2 Given that $E^{\ominus}_{O_2/H_2O} = +1.23$ V; $E^{\ominus}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05$ V

$E^{\ominus}_{Br_2/Br^-} = +1.09$ V; $E^{\ominus}_{Au^{3+}/Au} = +1.4$ V

The strongest oxidizing agent is -

(1) $S_2O_8^{2-}$ (2) O_2 (3) Au^{3+} (4) Br_2

Ans. **[1]**

Sol. Strongest oxidising agent $\propto$ lowest position in electrochemical series (ECS)

So Ans. = $S_2O_8^{2-}$

Q.3 The size of the iso-electronic species Cl^-, Ar and Ca^{2+} is affected by -

(1) nuclear charge

(2) azimuthal quantum number of valence shell

(3) electron-electron interaction in the outer orbitals

(4) principal quantum number of valence shell

Ans. **[1]**

Sol. Due to Zeff. or different nuclear charge.

Q.4 Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is -

(1) A_2BO_4 (2) A_4B_2O (3) AB_2O_4 (4) A_2B_2O

Ans. **[3]**

Sol.

A	:	B	:	O
$\frac{1}{2}$(O.V.)	:	CCP	:	all T.V.
$\frac{4}{2}$	:	4	:	8
2	:	4	:	8
1	:	2	:	4

AB_2O_4

Q.5 The correct order of hydration enthalpies of alkali metal ions is -

(1) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$ (2) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(3) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$ (4) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

Ans. **[1]**

Sol. $Li^{\oplus}$ having minimum radius so maximum hydration and maximum hydration radii.

Q.6 Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively -

(1) H_3BO_3 and B_2O_3 (2) B_2O_3 and H_3BO_3

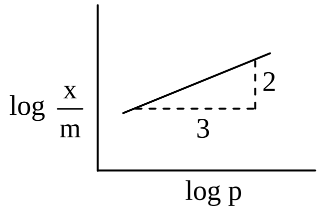
(3) B_2O_3 and $[BH_4]^-$ (4) HBO_2 and H_3BO_3

Ans. **[2]**

Sol. $B_2H_6 + O_2 \rightarrow \underset{\text{(Boric anhydride)}}{B_2O_3 + H_2O}$

$B_2H_6 + H_2O \rightarrow \underset{\text{(Boric acid)}}{H_3BO_3 + H_2}$

Q.7 Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\log\frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to -

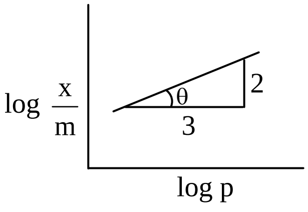


(1) $p^{3/2}$ (2) $p^{2/3}$ (3) p^3 (4) p^2

Ans. **[2]**

Sol. $\frac{x}{m} = Kp^{1/n}$

$\log\frac{x}{m} = \frac{1}{n}\log p + \log K$



Slope = $\tan\theta = \frac{2}{3} = \frac{1}{n}$

$\frac{x}{m} \propto p^{2/3}$

Q.8 The quantum number of four electrons are given below :

I. $n = 4, l = 2, m_l = -2, m_s = -\frac{1}{2}$

II. $n = 3, l = 2, m_l = 1, m_s = +\frac{1}{2}$

III. $n = 4, l = 1, m_l = 0, m_s = +\frac{1}{2}$

IV. $n = 3, l = 1, m_l = 1, m_s = -\frac{1}{2}$

The correct order of their increasing energies will be -

(1) I < III < II < IV (2) IV < II < III < I (3) I < II < III < IV (4) IV < III < II < I

Ans. **[2]**

Sol. Higher the value of $(n + l)$

Higher will be energy of orbital. If $(n + l)$ are equal, then higher the value n higher will be energy

I. $n + l = 6$ $n = 4$

II. $n + l = 5$ $n = 3$

III. $n + l = 5$ $n = 4$

IV. $n + l = 4$ $n = 3$

IV < II < III < I

Q.9 In order to oxidise a mixture of one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is -

(1) 2 (2) 1 (3) 1.5 (4) 3

Ans. **[1]**

Sol. $FeC_2O_4 \longrightarrow Fe^{+3} + CO_2$

$Fe_2(C_2O_4)_3 \longrightarrow CO_2$

$Fe_2(SO_4)_3 \longrightarrow$ No oxidation

$FeSO_4 \longrightarrow Fe^{+3}$

$g_m E (KMnO_4) = g_m E (FeC_2O_4) + g_m E [Fe_2(C_2O_4)_3] + g_m E (FeSO_4)$

moles × V.F. = moles × V.F. + moles × V.F. + mole × V.F.

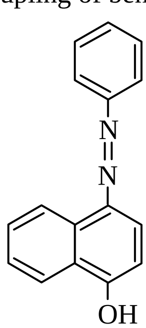
$x \times 5 = (1 \times 3) + (1 \times 6) + (1 \times 1)$

$5x = 10$ $(MnO_4^- \longrightarrow Mn^{+2})$

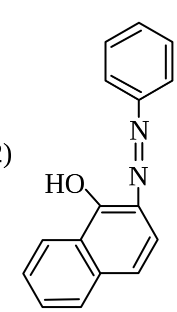
$x = 2$

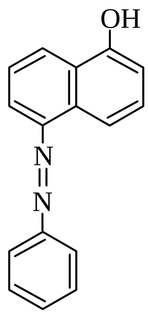
Q.10 Coupling of benzene diazonium chloride with 1-naphthol in alkaline medium will give -

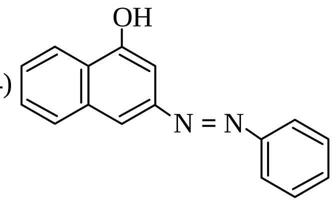
(1)



(2)

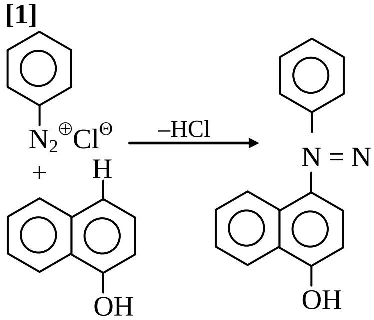


(3) 

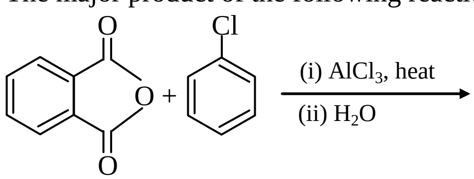
(4) 

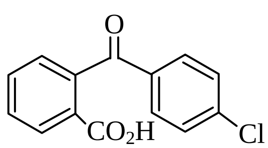
Ans. **[1]**

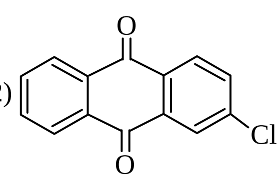
Sol.

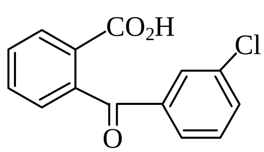


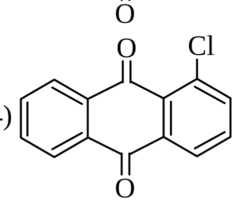
Q.11 The major product of the following reaction is -



(1) 

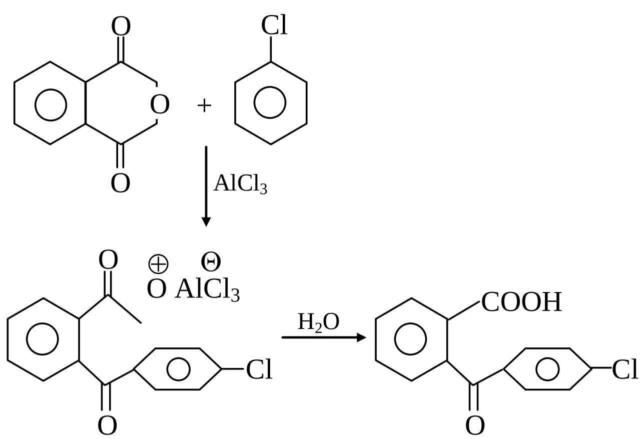
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.12 Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas ? (Assume non-expansion work is zero)

(1) Adiabatic process : $\Delta U = -w$
(2) Cyclic process : $q = -w$
(3) Isothermal process : $q = -w$
(4) Isochoric process : $\Delta U = q$

Ans. **[1]**

Sol. Ist law of thermodynamics

$\Delta E = q + w$

1. Adiabatic process

$q = 0, (\Delta E = w)$ or $(\Delta U = w)$

2. Cyclic process

$\Delta E = 0, (q = -w)$

3. Isothermal process

$\Delta E = nC_V\Delta T, (\Delta T = 0)$

$\Delta E = 0, (q = -w)$

4. Isochoric process $\Delta V = 0$

$w = P\Delta V = 0$

$\Delta E = q$

Q.13 The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are -

(1) 500 mmHg, 0.5, 0.5
(2) 450 mmHg, 0.5, 0.5
(3) 500 mmHg, 0.4, 0.6
(4) 450 mmHg, 0.4, 0.6

Ans. **[3]**

Sol. $P_A^o = 400$ mmHg

$P_B^o = 600$ mmHg

$x_B = 0.5, P_S = ?, y_A = ?, y_B = ?$

$P_S = P_A^o x_A + P_B^o x_B$

$= 400 (0.5) + 600 (0.5)$

$= 200 + 300 = 500$

$$y_A = \frac{P_A}{P_S} = \frac{200}{500} = \frac{2}{5} = 0.4$$

$y_B = 0.6$

Q.14 The IUPAC name of the following compound is -

```
    CH3 OH
     |   |
H3C–CH–CH–CH2–COOH
```

(1) 4,4-Dimethyl-3-hydroxybutanoic acid
(2) 2-Methyl-3-hydroxypentan-5-oic acid
(3) 4-Methyl-3-hydroxypentanoic acid
(4) 3-Hydroxy-4-methylpentanoic acid

Ans. **[4]**

Sol.

```
    CH3 OH
     |   |
CH3–CH–CH–CH2–COOH
 5   4  3   2    1
```

Q.15 With respect to an ore, Ellingham diagram helps to predict the feasibility of its -

(1) Electrolysis (2) Thermal reduction

(3) Zone refining (4) Vapour phase refining

Ans. **[2]**

Sol. Thermal reduction

Reduction by (C) and (CO)

Q.16 The correct order of the spin-only magnetic moment of metal ions in the following low spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$, and $[Cr(NH_3)_6]^{2+}$ is -

(1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$ (2) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(3) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$ (4) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

Ans. **[1]**

Sol. All are low spin complex and no. of unpaired e^- will decide the magnetic moment.

Q.17 In the following compounds, the decreasing order of basic strength will be -

(1) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$ (2) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

(3) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$ (4) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

Ans. **[4]**

Sol. Due to +I effect

Q.18 **Assertion :** Ozone is destroyed by CFCs in the upper stratosphere.

Reason : Ozone holes increase the amount of UV radiation reaching the earth.

(1) Assertion is false, but the reason is correct

(2) Assertion and reason are incorrect

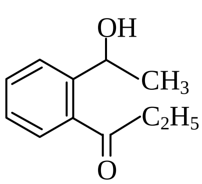
. (3) Assertion and reason are both correct, and the reason is the correct explanation for the assertion

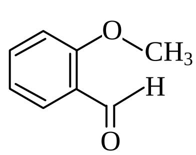
(4) Assertion and reason are correct, but the reason in not the explanation for the assertion

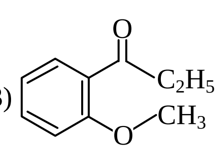
Ans. **[4]**

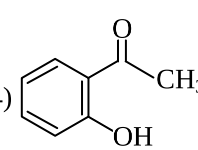
Sol. Ozone layer is depleted due to CFCS

Q.19 An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is -

(1) 

(2) 

(3) 

(4) 

Ans. **[1]**

Sol. No phenolic group

No Aldehyde group

Q.20 For the reaction $2A + B \rightarrow C$, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is :

[A] (mol L^{-1})	[B] (mol L^{-1})	Initial Rate (mol $L^{-1}s^{-1}$)
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

(1) Rate = k[A] [B]
(2) Rate = k $[A][B]^2$
(3) Rate = k $[A]^2[B]^2$
(4) Rate = k $[A]^2[B]$

Ans. **[2]**

Sol. $r = k[A]^p[B]^q$

$$\frac{r_2}{r_1} = \left[\frac{A_2}{A_1}\right]^p \left[\frac{B_2}{B_1}\right]^q$$

$2^1 = 2^p$ (p = 1)

$$\frac{r_3}{r_2} = \left[\frac{A_3}{A_2}\right]^1 \left[\frac{B_3}{B_2}\right]^q$$

$$\frac{0.720}{0.090} = 2(2)^q$$

$$\frac{720}{90 \times 2} = 2^q = 4 = 2^2$$

$q = 2$

$r = k[A]^1[B]^2$

Q.21 If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relation between S and K_{sp} is correct ?

(1) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$
(2) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$
(3) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$
(4) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$

Ans. **[2]**

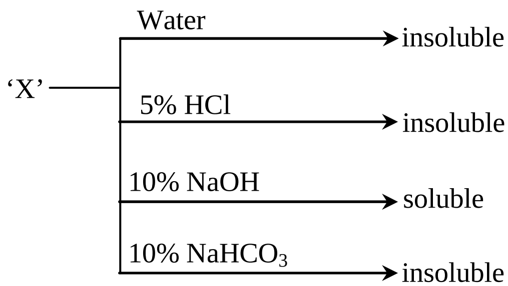
Sol. $Zr_3(PO_4)_4$

$K_{sp} = (3S)^3(4S)^4$

$Ksp = 6912S^7$

$$S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$$

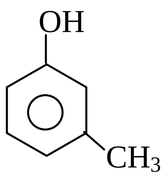
Q.22 An organic compound 'X' showing the following solubility profile is –



(1) Benzamide (2) Oleic acid (3) m-Cresol (4) o-Toluidine

Ans. **[3]**

Sol.



Phenolic group is responsible

Q.23 For silver, $C_p(JK^{-1}\ mol^{-1}) = 23 + 0.01\ T$. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to -

(1) 13 kJ (2) 16 kJ (3) 62 kJ (4) 21 kJ

Ans. **[3]**

Sol. $C_p = 23 + 0.01\ T$

$n = 3$ moles

$T_1 = 300$ K

$T_2 = 1000$ K

$\Delta H = nC_p\ \Delta T$

$$\Delta H = n \int_{T_1}^{T_2} C_p\ \Delta T$$

$$= 3 \int_{300}^{1000} (23 + 0.01T)\, dT$$

$$= 3 \int_{300}^{100} \left(23T + \frac{0.01T^2}{2}\right)$$

= 61.95 kJ/mole

= 62 kJ/mole

Q.24 Which is wrong with respect to our responsibility as a human being to protect our environment ?

(1) Restricting the use of vehicles (2) Avoiding the use of floodlighted facilities

(3) Setting up compost tin in gardens (4) Using plastic bage

Ans. **[4]**

Sol. Plastic bags

Q.25 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is -
(molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1})

(1) 100 ppm (2) 1,000 ppm (3) 5,000 ppm (4) 10,000 ppm

Ans. **[4]**

Sol. ppm of $CaCO_3$ = ?

$$\text{degree of hardness} = \frac{\text{weight of hardness causing salt}}{\text{Mw}} \times 100$$

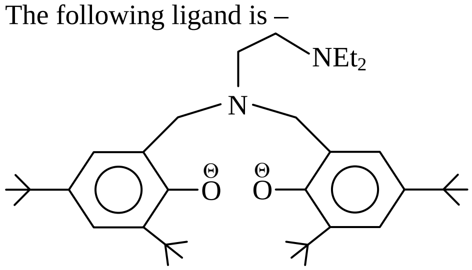
1 ppm = 1 part $CaCO_3$ eq in 10^6 parts water

$$\text{ppm of } CaCO_3 = \frac{\left(\frac{0.73}{146} + \frac{0.81}{162}\right) \times 100}{100} \times 10^6$$

$= 10^4$ ppm

= 10,000 ppm

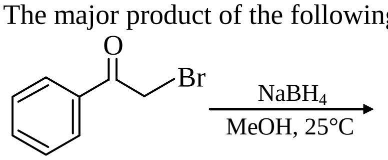
Q.26 The following ligand is –

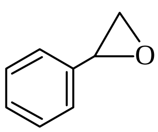


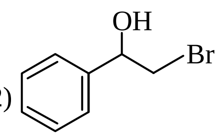
(1) bidentate (2) tridentate (3) tetradentate (4) hexadentate

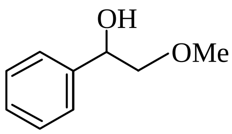
Ans. **[3]**

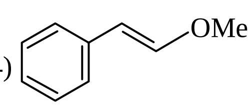
Q.27 The major product of the following reaction is –



(1) 

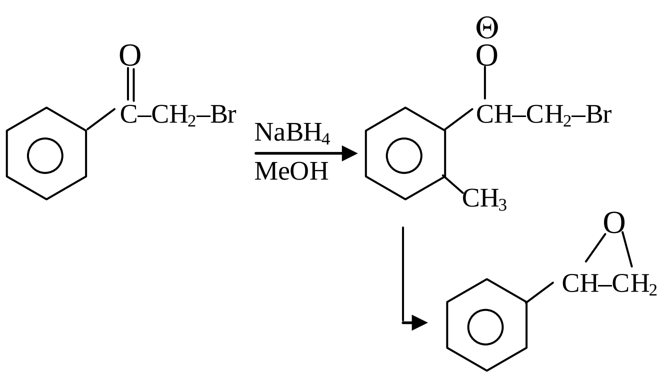
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.28 Maltose on treatment with dilute HCl gives -

(1) D-Glucose (2) D-Fructose

(3) D-Glucose and D-Fructose (4) D-Galactose

Ans. **[1]**

Sol. Dimer of α-D glucose

↓ Acidic hydrolysis

α-D glucose

Q.29 Which of the following amines can be prepared by Gabriel phthalimide reaction ?

(1) triethylamine (2) t-butylamine (3) neo-pentylamine (4) n-butylamine

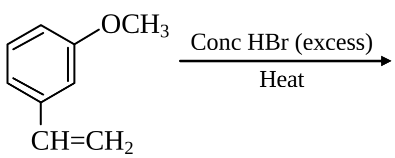
Ans. **[4]**

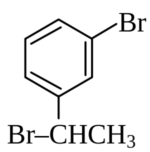
Sol. n – Butyl amine

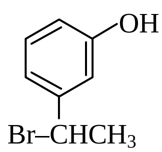
$CH_3–CH_2–CH_2–CH_2–NH_2$

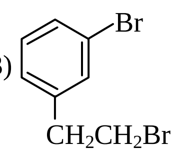
only unhindered 1º Amines are products of this reaction

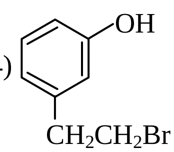
Q.30 The major product of the following reaction is -



(1) 

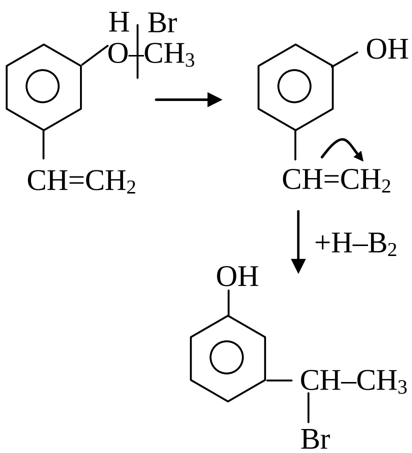
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - I

MATHEMATICS

Q.1 $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}\,dx$ is equal to – (where c is a constant of integration)

(1) 2x + sin x + 2 sin 2x + c (2) 2x + sin x + sin 2x + c

(3) x + 2 sin x + 2 sin 2x + c (4) x + 2 sin x + sin 2x + c

Ans. [4]

Sol. $I = \int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\,dx$

$I = \int \frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{\sin x}\,dx$

$I = \int \frac{\sin(3x)+\sin(2x)}{\sin x}\,dx$

$I = \int \frac{(3\sin x - 4\sin^3 x) + 2\sin x\cos x}{(\sin x)}\,dx$

$I = \int (3 - 4\sin^2 x + 2\cos x)\,dx$

$I = \int (3 - 2(1-\cos 2x) + 2\cos x)\,dx$

$I = \int (3 - 2 + 2\cos 2x + 2\cos x)\,dx$

$I = \int (1 + 2\cos 2x + 2\cos x)\,dx$

I = x + sin 2x + 2sin x + c

Q.2 The equation of a plane containing the line of intersection of the planes 2x – y – 4 = 0 and y + 2z – 4 = 0 and passing through the point (1, 1, 0) is -

(1) x – 3y – 2z = – 2 (2) x – y – z = 0 (3) x + 3y + z = 4 (4) 2x – z = 2

Ans. [2]

Sol. Required equation of plane

$(2x - y - 4) + \lambda\,(y + 2z - 4) = 0$

Required plane passes through (1, 1, 0)

$(2 - 1 - 4) + \lambda\,(1 + 0 - 4) = 0$

$\lambda = -1$

$(2x - y - 4) - (y + 2z - 4) = 0$

$2x - 2y - 2z = 0$

$x - y - z = 0$

Q.3 Let y = y(x) be the solution of the differential equation, $(x^2 + 1)^2 \dfrac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that y(0) = 0. If $\sqrt{a}\;y(1) = \dfrac{\pi}{32}$, then the value of 'a' is -

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{16}$ (4) 1

Ans. **[3]**

Sol. $(x^2 + 1)^2 \dfrac{dy}{dz} + 2x\,(x^2 + 1)\,y = 1$

$\dfrac{dy}{dx} + \left(\dfrac{2x}{x^2+1}\right)y = \dfrac{1}{(x^2+1)^2}$

I.F. = $e^{\int \frac{2x}{1+x^2}dx} = (1 + x^2)$

Solution of diff. eq.

$y\,(1 + x^2) \times \int \dfrac{1}{(1+x^2)^2} \times (1 + x^2)\,dx$

$y\,(1 + x^2) = \tan^{-1}(x) + c$

Given that y(0) = 0

c = 0

$y(1 + x^2) = \tan^{-1}(x)$

Put x = 1

$y \cdot (2) = \dfrac{\pi}{4}$

$y = \dfrac{\pi}{8}$

$\dfrac{1}{4} \cdot y = \dfrac{\pi}{32}$

i.e. $\sqrt{a} = \dfrac{1}{4}$

$a = \dfrac{1}{16}$

Q.4 If $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to -

(1) $2f(x)$ (2) $2f(x^2)$ (3) $(f(x))^2$ (4) $-2f(x)$

Ans. **[1]**

Sol. $f(x) = \log_e \left(\frac{1-x}{1+x}\right)$ |x| + 1

$$f\left(\frac{2x}{1+x^2}\right) = \log_e \left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \log_e \left(\frac{(1-x)^2}{(1+x)^2}\right)$$

$$= 2\log_e \left(\frac{1-x}{1+x}\right) = 2f(x)$$

Q.5 If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is -

(1) $\log_e 1$ (2) $\log_e 3$ (3) $\log_e e$ (4) $\log_e 2$

Ans. **[1]**

Sol. $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ $g(x) = \log_e x$

$$I = \int_{-\pi/4}^{\pi/4} g(f(x)dx$$

$$I = \int_{-\pi/4}^{\pi/4} \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx \quad \text{....(i)}$$

Applying

$$I = \int_{-\pi/4}^{\pi/4} \log_e\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx \quad \text{....(ii)}$$

Equation (i) + (ii)

$$2I = \int_{-\pi/4}^{\pi/4} \log_e(1)\,dx$$

2I = 0 I = 0

Q.6 The sum of the squares of the length of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is -

(1) 160 (2) 320 (3) 210 (4) 105

Ans. **[3]**

Sol. Equation of circle $x^2 + y^2 = 16$

Equation of chord $x + y = N$ $n \in N$

Length of chord = $2\sqrt{16 - \frac{n^2}{2}}$

$(\text{Length of chord})^2 = 2(32 - n^2)$

Possible value of n = 1, 2, 3, 4, 5

Sum of squares of the length of chords

= 2(31) + 2(28) + 2(23) + 2(16) + 2(7)

= 210

Q.7 The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to -

(1) 4 (2) 10 (3) 9 (4) 12

Ans. **[2]**

Sol. $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$ $(x > 0)$

let $\sqrt{x} = t$

$|t - 2| + t(t - 4) + 2 = 0$

$t \geq 2$	$t < 2$
$t - 2 + t^2 - 4t + 2 = 0$	$2 - t + t^2 - 4t + 2 = 0$
$t^2 - 3t = 0$	$t^2 - 5t + 4 = 0$
$t = 0$ (Reject)	$t = 4$ (Reject)
$t = 3$ $\sqrt{x} = 3$	$t = 1$ $\sqrt{x} = 1$
$x = 9$	$x = 1$

Sum of all colution = 9 + 1 = 10

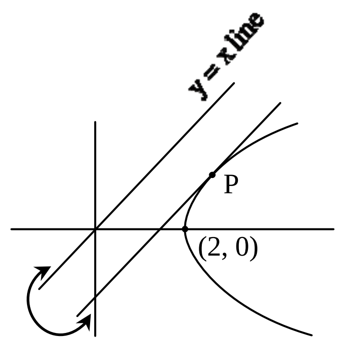
Q.8 The shortest distance between the line y = x and the curve $y^2 = x - 2$ is -

(1) $\frac{7}{4\sqrt{2}}$ (2) 2 (3) $\frac{7}{8}$ (4) $\frac{11}{4\sqrt{2}}$

Ans. **[1]**

Sol. $y = x$ line

$y^2 = x - 2$ parabola



for point P $\left(\frac{dy}{dx}\right)_P = 1$

$\left(\frac{1}{2y}\right)_P = 1$

$(y_{costiate}) = \frac{1}{2}$

$(x_{coordinate}) = \frac{9}{4}$

Point P $\left(\frac{9}{4}, \frac{1}{2}\right)$

Shortest distance = length of perpendicular from P on line x – y = 0

$$= \frac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$$

Q.9 The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$, $(x > 1)$ is equal to -

(1) 24 (2) 26 (3) 29 (4) 32

Ans. **[1]**

Sol. $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$

$= 2\,[{}^6C_0\, x^6\, (x^3 - 1)^0 + {}^6C_2\, x^4\, (x^3 - 1) + {}^6C_4\, x^2\, (x^3 - 1)^2 + {}^6C_6\, x^0\, (x^3 - 1)^3]$

Sum of coefficient at all even power

$= 2\,[{}^6C_0 + {}^6C_2\, \{-1\} + {}^6C_4\, x^2\, \{1 + 1\} + {}^6C_6\, \{{}^{-3}C_1 - {}^3C_3\}]$

$= 2\,[1 + 15\,(-1) + 15\,(2) + (-4)]$

$= 2\,[1 - 15 + 30 - 4]$

$= 2 \times 12 = 24$

Q.10 Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) $P(A|B) = P(B) - P(A)$ (2) $P(A|B) \geq P(A)$ (3) $P(A|B) = 1$ (4) $P(A|B) \leq P(A)$

Ans. **[2]**

Sol. $A \subset B$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

P(B) lies between (0, 1]

So that

$P(A/B) \geq P(A)$

Q.11 The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is -

(1) 3221 (2) 3303 (3) 3203 (4) 3121

Ans. **[4]**

Sol. H.C.F. $(91, n) > 1$

where $100 < n < 200$

sum of all positive values of 'n'

$= 7\,[15 + 16 + \ldots.. + 28] + 13\,[8 + \ldots.. + 15] - (13 \times 14)$

$= 7 \times \frac{14}{2}[15 + 28] + 13 \times \frac{8}{2}[8 + 15] - (13 \times 14)$

$= (49 \times 43) + (52 \times 23) - 182$

$= 2107 + 1196 - 182$

$= 3121$

Q.12 A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in -

(1) 1^{st}, 2^{nd} and 4^{th} quadrants (2) 1^{st} and 2^{nd} quadrants (3) 4^{th} quadrants (4) 1^{st} quadrants

Ans. **[2]**

Sol. 3x + 5y = 15 equation of straight line

$x = y$	$x = -y$
$8x = 15$	$-2x = 15$
$x = 15/8$	$x = -15/2$
$y = 15/8$	$y = 15/2$
$P\left(\frac{15}{8}, \frac{15}{8}\right)$	$Q\left(-\frac{15}{2}, \frac{15}{2}\right)$
I Quadrant	II Quadrant

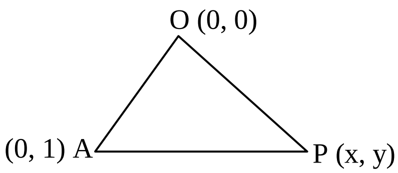
Q.13 Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is -

(1) $8x^2 + 9y^2 - 9y = 18$ (2) $9x^2 + 8y^2 - 8y = 16$

(3) $9x^2 - 8y^2 + 8y = 16$ (4) $8x^2 - 8y^2 + 9y = 18$

Ans. **[2]**

Sol.



Given that

OA + AP + OP = 4

$1 + \sqrt{x^2 + (y-1)^2} + \sqrt{x^2 + y^2} = 4$

$(\sqrt{x^2 + y^2})^2 = (3 - \sqrt{x^2 + (y-1)^2})^2$

$x^2 + y^2 = 9 + x^2 + y^2 - 2y + 1 - 6\sqrt{x^2 + (y-1)^2}$

$3\sqrt{x^2 + (y-1)^2} = (5 - y)$

$9(x^2 + (y-1)^2) = (5 - y)^2$

$9x^2 + 8y^2 - 8y = 16$

Q.14 If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$, $x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to -

(1) $x - \frac{\pi}{6}$ (2) $\frac{\pi}{3} - x$ (3) $\frac{\pi}{6} - x$ (4) $2x - \frac{\pi}{3}$

Ans. **[1]**

Sol. $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2 \quad x \in \left(0, \frac{\pi}{2}\right)$

$2y = \left(\cot^{-1}\left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x}\right)\right)^2$

$$2y = \left(\frac{\pi}{2} - \tan^{-1}\left(\tan\left(\frac{\pi}{3} + x\right)\right)\right)^2$$

$0 < x < \frac{\pi}{6}$

$\left(\frac{\pi}{3} < x + \frac{\pi}{3} < \frac{\pi}{2}\right)$

$2y = \left(\frac{\pi}{2} - \frac{\pi}{3} - x\right)^2$

$2y = \left(\frac{\pi}{6} - x\right)^2$

Differentiation w.r. to x

$2 \cdot \frac{dy}{dx} = 2\left(\frac{\pi}{6} - x\right)(-1)$

$\frac{dy}{dx} = x - \frac{\pi}{6}$

at $x = \frac{\pi}{6}$

f×u is undefined

(non-diff)

$\frac{\pi}{6} < x < \frac{\pi}{2}$

$\frac{\pi}{2} < x + \frac{\pi}{3} < \frac{5\pi}{6}$

$2y = \left(\frac{\pi}{2} - \left(\frac{\pi}{3} + x - \pi\right)\right)^2$

$2y = \left(\frac{7\pi}{6} - x\right)^2$

Differentiation w.r. to x

$2 \cdot \frac{dy}{dx} = 2\left(\frac{7\pi}{6} - x\right)(-1)$

$\frac{dy}{dx} = x - \frac{7\pi}{6}$

Q.15 All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is -

(1) 180 (2) 175 (3) 160 (4) 162

Ans. **[1]**

Sol. Total number of possible numbers

$= \left({}^4C_3 \times \frac{3!}{2!}\right)\left(\frac{6!}{4!\,2!}\right)$

$= 180$

Q.16 If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to -

(1) $\frac{2}{17}$ (2) $\frac{64}{17}$ (3) $\frac{128}{17}$ (4) $\frac{4}{17}$

Ans. **[1]**

Sol. Equation of ellipse $4x^2 + y^2 = 8$

$\frac{dy}{dx} = -\frac{4x}{y}$

tangent at (1, 2) and (a, b) are perpendicular

$\left(-\frac{y}{2}\right)\left(-\frac{4a}{b}\right) = -1$

$b = -8a$ ….(i)

(a, b) lies on ellipse

$4a^2 + b^2 = 8$ (from eq. (i))

$4a^2 + 64a^2 = 8$

$a^2 = \frac{8}{68} = \frac{2}{17}$

Q.17 $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals -

(1) 4 (2) $2\sqrt{2}$ (3) $4\sqrt{2}$ (4) $\sqrt{2}$

Ans. **[3]**

Sol. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$

$\lim_{x\to 0} \left(\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}\right)\left(\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}-\sqrt{1+\cos x}}\right)$

$\lim_{x\to 0} \left(\frac{\sin^2 x}{1-\cos x}\right)(\sqrt{2}+\sqrt{1+\cos x})$

$\lim_{x\to 0} \frac{\sin^2 x}{2\sin^2\left(\frac{x}{2}\right)}(\sqrt{2}+\sqrt{1+\cos x})$

$\lim_{x\to 0} \frac{1}{2}\left(\frac{\sin x}{x}\right)^2 \times x^2 \times \left(\frac{x/2}{\sin x/2}\right)^2 \times \frac{1}{\left(\frac{x^2}{4}\right)} \times (\sqrt{2}+\sqrt{1+\cos x})$

$= \frac{1}{2} \times 4 \times 2\sqrt{2} = 4\sqrt{2}$

Q.18 If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to -

(1) $\frac{63}{52}$ (2) $\frac{63}{16}$ (3) $\frac{33}{52}$ (4) $\frac{21}{16}$

Ans. **[2]**

Sol. $\cos(\alpha+\beta) = \frac{3}{5}$ $\tan(\alpha+\beta) = \frac{4}{3}$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\tan(\alpha-\beta) = \frac{5}{12}$

$\tan(2\alpha) = \tan((\alpha+\beta)+(\alpha-\beta))$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{48+15}{36-20} = \frac{63}{16}$

Q.19 The sum of the series $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \dots + 62\cdot{}^{20}C_{20}$ is equal to -

(1) 2^{23} (2) 2^{25} (3) 2^{26} (4) 2^{24}

Ans. **[2]**

Sol. $S = 2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + \ldots\ldots 59\cdot{}^{20}C_{19} + 62\cdot{}^{20}C_{20}$ .... (i)

$S = 62\cdot{}^{20}C_{20} + 59\cdot{}^{20}C_{19} + \ldots\ldots + 5\cdot{}^{20}C_1 + 2\cdot{}^{20}C_0$ .... (ii)

Add equation (i) & (ii)

$2S = 64\,[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots\ldots + {}^{20}C_{20}]$

$2S = 64 \cdot 2^{20}$

$S = 64 \cdot 2^{19}$

$S = 2^6 \cdot 2^{19} = 2^{25}$

Q.20 The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is -

(1) greater than 3 but less then 4

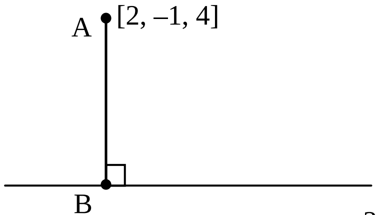
(2) greater than 2 but less than 3

(3) greater than 4

(4) less than 2

Ans. **[1]**

Sol.



$[10\lambda - 3, -7\lambda + 2, \lambda]$ $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{3}{1} = \lambda$

Drs of AB = $(10\lambda - 5, -7\lambda + 3, \lambda - 4)$

Drs of given line = (10, –7, 1)

$10(10\lambda - 5) - 7(-7\lambda + 3) + (\lambda - 4) = 0$

$100\lambda - 50 + 49\lambda - 21 + \lambda - 4 = 0$

$150\lambda = 75$

$\lambda = \frac{1}{2}$

Point B $\left(2, -\frac{3}{2}, \frac{1}{2}\right)$

Distance length AB $= \frac{5}{\sqrt{2}} = 3.53$

Q.21 Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is -

(1) increasing on (0, 2)

(2) increasing on (0, 1) and decreasing on (1, 2)

(3) decreasing on (0, 2)

(4) decreasing on (0, 1) and increasing on (1, 2)

Ans. **[4]**

Sol. Given that $f''(x) > 0$ $\quad x \in (0, 2)$

i.e. $f'(x)$ is increasing function

$\phi(x) = f(x) + f(2 - x)$

$\phi'(x) = f'(x) - f'(2 - x)$

$\phi(x)$ is increasing

i.e. $\phi'(x) > 0$

$f'(x) - f'(2 - x) > 0$

$f'(x) > f'(2 - x)$

$x > 2 - x$

$2x > 2$

$x > 1$

i.e. $x \in (1, 2)$

$\phi(x)$ is decreasing

i.e. $\phi'(x) < 0$

$f'(x) - f'(2 - x) < 0$

$f'(x) < f'(2 - x)$

$x < 2 - x$

$x > 1$

i.e. $x \in (0, 1)$

Q.22 The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is -

(1) $\frac{1}{2}$ (2) 0 (3) 2 (4) –1

Ans. **[1]**

Sol. $\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$

$1(1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$

$1 - c^2 - c^2 - c^3 - c^3 - c^2 = 0$

$2c^3 + 3c^2 - 1 = 0$

$c = -1$

$c = \frac{1}{2}$

Maximum value of $c = \frac{1}{2}$

Q.23 The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is -

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$ (3) $3\sqrt{6}$ (4) $\sqrt{6}$

Ans. **[2]**

Sol. Vector $\perp$ to given to vectors is

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

projection of $\vec{b}$ in direction of $\vec{a}$ $= \frac{|\vec{a}.\vec{b}|}{|\vec{a}|}$

$$= \frac{|(\hat{i}-2\hat{j}+\hat{k})\cdot(2\hat{i}+3\hat{j}+\hat{k})|}{\sqrt{6}}$$

$$= \frac{|2-6+1|}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

Q.24 The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is -

(1) 45 (2) 48 (3) 49 (4) 40

Ans. **[2]**

Sol. Let unknown observation are x_1 & x_2

$$\frac{2+4+10+12+14+x_1+x_2}{7} = 8$$

$x_1 + x_2 = 14$ ....(i)

$$\frac{1}{7}\Sigma x_1^2 - (\bar{x})^2 = 16 \text{ (given)}$$

$$\frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x_1^2 + x_2^2) - (8) = 16$$

$$x_1^2 + x_2^2 = 100$$

$$(x_1 + x_2)^2 - 2x_1x_2 = 100$$

$$2x_1x_2 = 96$$

$$x_1x_2 = 48$$

Q.25 The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3\}$ is -

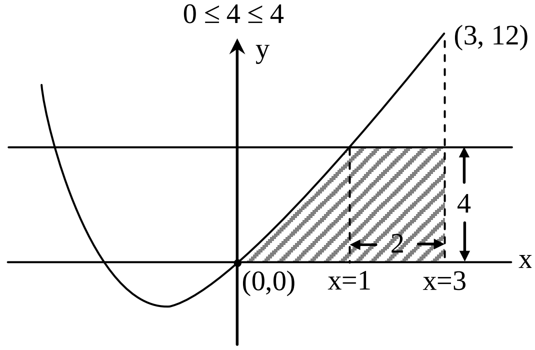
(1) 8 (2) $\frac{53}{6}$ (3) $\frac{26}{3}$ (4) $\frac{59}{6}$

Ans. **[4]**

Sol. $y \le x^2 + 3x$

$0 \le x \le 3$

$0 \le 4 \le 4$



Required region

$$= \int_0^1 [x^2 + 3x]dx + [2 \times 4]$$

$$= \left[\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^1 + 8$$

$$= \frac{2+9+48}{6} = \frac{59}{6}$$

Q.26 If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to -

(1) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (3) $\tan^{-1}\left(\frac{9}{14}\right)$ (4) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Ans. **[2]**

Sol. $\alpha = \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\alpha - \beta = \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{3}\right)$

$$= \tan^{-1}\left(\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right) = \tan^{-1}\left(\frac{1}{1 + \frac{4}{9}}\right) = \tan^{-1}\left(\frac{9}{13}\right)$$

$$= \sin^{-1}\left(\frac{9}{\sqrt{(13)^2 + 9^2}}\right) = \sin^{-1}\left(\frac{9}{\sqrt{250}}\right)$$

$$= \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$$

Q.27 If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then -

(1) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$ (2) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$

(3) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$ (4) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

Ans. **[3]**

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$f'(x) = 36x\,(x^2 + x - 2)$

$f'(x) = 36x\,(x + 2)\,(x - 1)$

Sign of $f'(x)$

⊖ ⊕ ⊖ ⊕

–2 0 1

↑ ↑ ↑

Minina Maxima Minima

$S_1 = \{-2, 1\}$

$S_2 = \{0\}$

Q.28 Let A = $\begin{pmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{pmatrix}$, ($a \in R$) such that $A^{32} = \begin{pmatrix}0 & -1 \\ 1 & 0\end{pmatrix}$. Then a value of α is -

(1) $\frac{\pi}{64}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{16}$

Ans. **[1]**

Sol. $A = \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$A^2 = \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$A^2 = \begin{bmatrix}\cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha\end{bmatrix}$

Similarly

$A^{32} = \begin{bmatrix}\cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha\end{bmatrix} = \begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}$

$32\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{64}$

Q.29 If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is -

(1) 3 (2) 5 (3) 4 (4) 2

Ans. **[3]**

Sol. $x^2 - 2x + 2 = 0$

$(x - 1)^2 = -1$

$x = 1 + i \ \& \ 1 - i$

Let $\alpha = 1 + i$

$\beta = 1 - i$

$\frac{\alpha}{\beta} = \frac{1+i}{1-i} = \frac{(1+i)^2}{2} = \frac{1+i^2+2i}{2}$

$\frac{\alpha}{\beta} = i$

$\left(\frac{\alpha}{\beta}\right)^n = (i)^n = 1$

least value of n = 4

Q.30 The contrapositive of the statement ''If you are born in India, then you are a citizen of India'', is -

(1) If you are a citizen of India, then you are born in India

(2) If you are born in India, then you are not a citizen of India

(3) if you are not a citizen of India, then you are not born in India

(4) If you are not born in India, then you are not a citizen of India

Ans. **[3]**

Sol. Contrapositive of given statement is. If you are not a citizen of India, then you are not born in India.