JEE Main Online Exam 2019

Questions & Solutions

12^{th} April 2019 | Shift - I

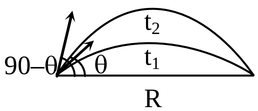
PHYSICS

Q.1 A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is -

(1) 2R/g (2) R/2g (3) R/g (4) R/4g

Ans. **[1]**

Sol.



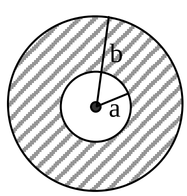
For θ & 90 – θ angle of projection, range will be same

Time of flight for θ : $t_1 = \dfrac{2u\sin\theta}{g}$

Time of flight for 90 – θ : $t_2 = \dfrac{2u\sin(90-\theta)}{g} = \dfrac{2u\cos\theta}{g}$

$\Rightarrow t_1t_2 = \dfrac{4u^2\sin\theta\cos\theta}{g^2} = \dfrac{2u^2}{g}\left(\dfrac{\sin 2\theta}{g}\right) = \dfrac{2}{g}\left(\dfrac{u^2\sin 2\theta}{g}\right) = \dfrac{2R}{g}$

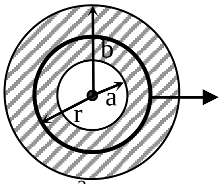
Q.2 A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\dfrac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is:



(1) $\dfrac{a+b}{3}$ (2) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$ (3) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$ (4) $\dfrac{a+b}{2}$

Ans. **[2]**

Sol.



$dm = \left(\frac{\sigma_0}{r}\right)(2\pi r dr) = 2\pi\sigma_0 dr$

$I = mk^2$: k = radius of gyration

$$\int_a^b (dm)r^2 = k^2 \int_a^b dm$$

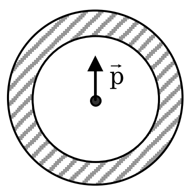
$$\Rightarrow \int_a^b (2\pi\sigma_0 dr)r^2 = k^2 \int_a^b 2\pi\sigma_0 dr$$

$$\Rightarrow 2\pi\sigma_0\left[\frac{b^3 - a^3}{3}\right] = k^2 2\pi\sigma_0 (b-a)$$

$$\Rightarrow \frac{(b-a)(b^2+a^2+ab)}{3} = k^2(b-a)$$

$$k = \sqrt{\frac{a^2+b^2+ab}{3}}$$

Q.3 Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case;



(1) surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

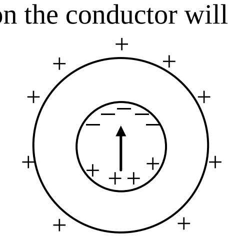
(2) surface charge density on the outer surface depends on $|\vec{p}|$

(3) surface charge density on the inner surface of the shell is zero everywhere

(4) electric field outside the shell is the same as that of a point charge at the centre of the shell

Ans. **[4]**

Sol. The charge distribution at equilibrium on the conductor will be like :



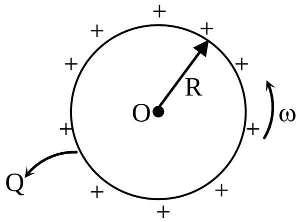
Net charge on the outer surface = Q
Total charge on the inner surface = 0
So for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on the outer surface only.

Q.4 A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

(1) 7×10^{-6} C (2) 4×10^{-5} C (3) 2×10^{-6} C (4) 3×10^{-5} C

Ans. **[4]**

Sol.



$R = 10$ cm $= 10^{-1}$ m

$\omega = 40\pi$ rad/s

$B_0 = 3.8 \times 10^{-9}$T

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q}{\left[\frac{2\pi}{\omega}\right]}$$

$$B_0 = \frac{\mu_0 I}{2R} = \frac{\mu_0\left[\frac{Q}{\left(\frac{2\pi}{\omega}\right)}\right]}{2R}$$

$$\Rightarrow B_0 = \frac{\mu_0 Q\omega}{4\pi R}$$

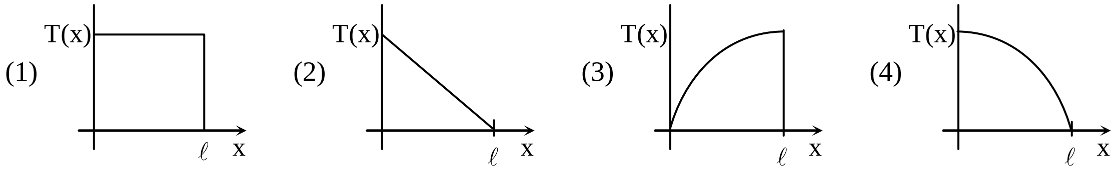
$$\Rightarrow Q = \frac{B_0 4\pi R}{\mu_0 \omega}$$

$$= \frac{(3.8\times10^{-9})(4\pi\times10^{-1})}{(4\pi\times10^{-7})(40\pi)}$$

$$= \frac{380\times10^{-2}\times10^{-9}\times10^{-1}}{40\times3.14\times10^{-7}}$$

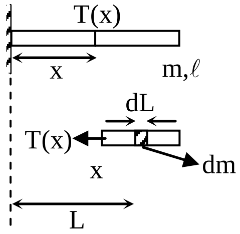
$$= \frac{380}{125.6}\times 10^{-12+7} = 3\times10^{-5}\text{ C}$$

Q.5 A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?



Ans. **[4]**

Sol.



$$T(x) = \int_x^{\ell} (dm)\omega^2 L$$

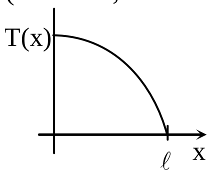
$$= \int_x^{\ell} \left[\frac{m}{\ell} dL\right]\omega^2 L$$

$$= \frac{m}{\ell}\omega^2\left[\frac{L^2}{2}\right]_x^{\ell}$$

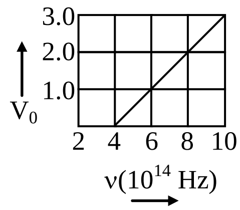
$$T(x) = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

$T(x) = A - Bx^2$

(Parabola, mouth down)



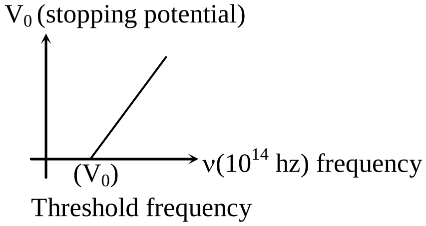
Q.6 The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be: (Given: Planck's constant (h) = 6.63×10^{-34} Js, electron charges e = 1.6×10^{-19} C)



(1) 1.95 eV (2) 2.12 eV (3) 1.82 eV (4) 1.66 eV

Ans. **[4]**

Sol.

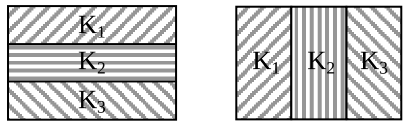


Work function = $\phi_0 = h\nu_0$

$$= \frac{(6.63\times10^{-34})(4\times10^{14})}{1.6\times10^{-19}} \text{ eV} = 1.66 \text{ eV}$$

Q.7 Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig.I, and the second one is filled as shown in fig II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)):



(I) (II)

(1) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{K_1K_2K_3}$

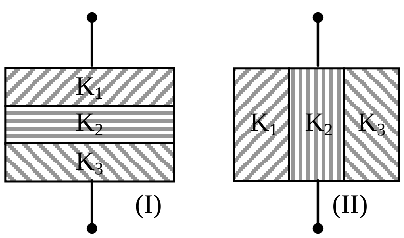
(2) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

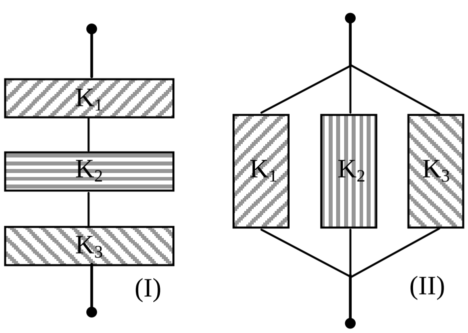
(3) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{9K_1K_2K_3}$

(4) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

Ans. **[4]**

Sol.





$$\frac{1}{C_1} = \frac{1}{k_1\left(\frac{\varepsilon_0 A}{d/3}\right)} + \frac{1}{k_2\left(\frac{\varepsilon_0 A}{d/3}\right)} + \frac{1}{k_3\left(\frac{\varepsilon_0 A}{d/3}\right)}$$

$$\frac{1}{C_1} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]$$

$$C_1 = \frac{3\varepsilon_0 A}{d}\frac{k_1k_2k_3}{(k_1k_2+k_2k_3+k_3k_1)}$$

$$C_2 = k_1\frac{\varepsilon_0 A/3}{d} + k_2\frac{\varepsilon_0 A/3}{d} + k_3\frac{\varepsilon_0 A/3}{d} = \frac{\varepsilon_0 A}{3d}[k_1+k_2+k_3]$$

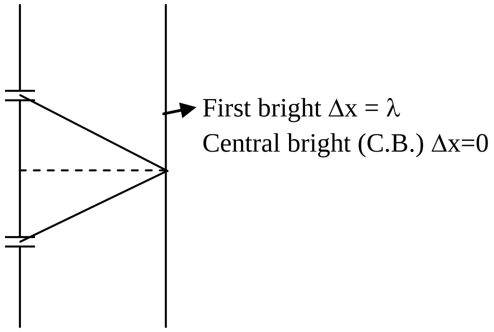
$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_1V^2}{\frac{1}{2}C_2V^2} = \frac{9k_1k_2k_3}{(k_1+k_2+k_3)(k_1k_2+k_2k_3+k_3k_1)}$$

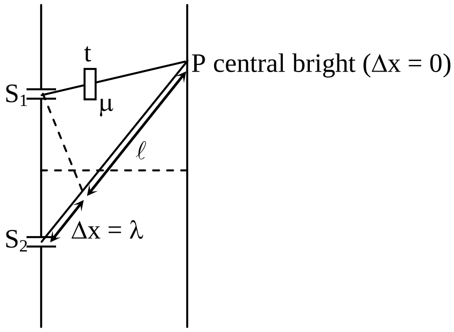
Q.8 In a double slit experiment, when a thin film of thickness t having refractive index μ. is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1) $\frac{\lambda}{2(\mu-1)}$ (2) $\frac{\lambda}{(2\mu-1)}$ (3) $\frac{2\lambda}{(\mu-1)}$ (4) $\frac{\lambda}{(\mu-1)}$

Ans. **[4]**

Sol. Normal YDSE without slab





For central bright at the position of first bright

⇒ $S_2P - S_1P = 0$

⇒ $(\lambda + \ell) - (\ell - t + \mu t) = 0$

optical path length

⇒ $\lambda + \ell - \ell + t - \mu t = 0$

⇒ $\lambda = t(\mu - 1)$

⇒ $\boxed{t = \frac{\lambda}{\mu - 1}}$

Q.9 The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1) 0.12 μm (2) 0.38 μm (3) 0.24 μm (4) 0.48 μm

Ans. **[3]**

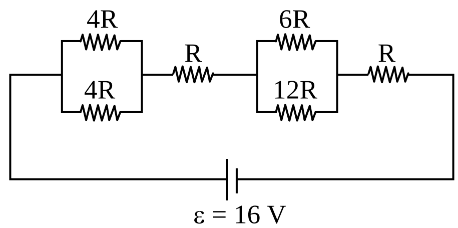
Sol. Numerical aperture of the microscope is given as

$$NA = \frac{0.61\lambda}{d}$$

d = minimum separation between two points to be seen as distinct

$$\Rightarrow d = \frac{0.61\lambda}{NA} = \frac{(0.61)(5000\times10^{-10})}{1.25} = 2.4\times10^{-7}\ m = 0.24\ \mu m$$

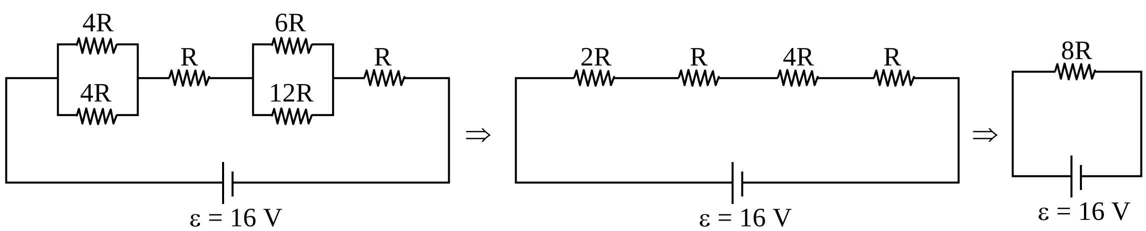
Q.10 The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is:



(1) 16 Ω (2) 1 Ω (3) 8 Ω (4) 6 Ω

Ans. **[3]**

Sol.



$$P = \frac{V^2}{R}$$

$$\Rightarrow 4 = \frac{16\times16}{8R}$$

$$\Rightarrow \boxed{R = 8\Omega}$$

Q.11 The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$) :

(1) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ ms^{-1}$

(2) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(4) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\ ms^{-1}$

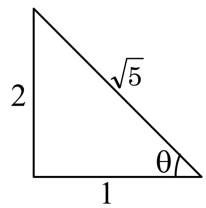
Ans. **[1]**

Sol. $y = x\tan\theta\left(1-\frac{x}{R}\right)$ .....(i)

Given eq^n of trajectory : $y = 2x - 9x^2 = 2x\left(1-\frac{9x}{2}\right) = 2x\left(1-\frac{x}{\left(\frac{2}{9}\right)}\right)$ .... (2)

Comparing equation (1) & (2)

$\tan\theta = 2$ & $R = \frac{2}{9}$



$\cos\theta = \frac{1}{\sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\Rightarrow \frac{u^2\sin 2\theta}{g} = \frac{2}{9}$

$\Rightarrow \frac{u^2\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]}{10} = \frac{2}{9}$

$\Rightarrow u^2\left[\frac{4}{5}\right] = \frac{2}{9}\times 10$

$\Rightarrow u^2 = \frac{2\times 10\times 5}{9\times 4}$

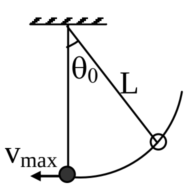
$\Rightarrow u = \frac{10}{6} = \frac{5}{3}$ m/s

Q.12 A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his center of mass moves by a distance $\ell(\ell<<L)$, is close to;

(1) $mg\ell\,(1+\theta_0^2)$ (2) $mg\ell$ (3) $mg\ell\left(1+\frac{\theta_0^2}{2}\right)$ (4) $mg\ell(1-\theta_0^2)$

Ans. **[1]**

Sol.



The force acting on the man at the lowest point

$\Rightarrow F = mg + \frac{mv_{max}^2}{L}$

$= mg + \frac{m}{L}(V_{max})^2$

$= mg + \frac{m}{L}[A\omega]^2$

$[T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \omega = \sqrt{\frac{g}{L}}\,]$

$= mg + \frac{m}{L}[(\theta_0 L)\left(\frac{\sqrt{g}}{L}\right)]^2$

$= mg + mg\,\theta_0^2$

$= mg\,(1 + \theta_0^2)$

Work done = (F) (displacement)

$= [mg(1 + \theta_0^2)][\ell]$

$= mg\ell(1 + \theta_0^2)$

Q.13 A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V= 0 at infinity)

(1) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{-\vec{p}}{4\pi \in_0 d^3}$

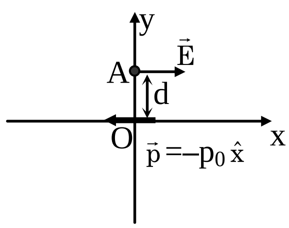
(2) $0, \frac{-\vec{p}}{4\pi \in_0 d^3}$

(3) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{\vec{p}}{4\pi \in_0 d^3}$

(4) $0, \frac{-\vec{p}}{4\pi \in_0 d^3}$

Ans. **[2]**

Sol.



A is an equatorial point w.r.t the dipole :

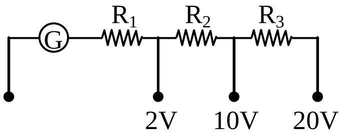
Electric field at A $= \frac{-K\vec{p}}{r^3}$

$= -\left\{\frac{1}{4\pi\varepsilon_0}\frac{(-p_0\hat{x})}{d^3}\right\}$

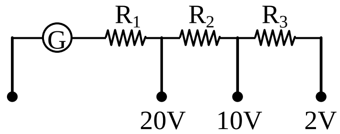
$= \frac{p_0}{4\pi\varepsilon_0 d^3}\hat{x}$

Electric potential at A = 0

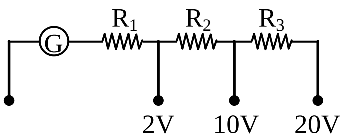
Q.14 A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivitv of 20 μA/division. It is to be converted to a voltmeter with three ranges of 0-2V, 0-10 V and 0-20 V. The appropriate circuit to do so is

(1) 

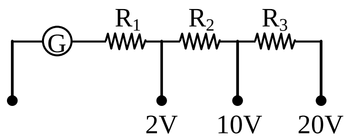
$R_1 = 1900\Omega$
$R_2 = 8000\Omega$
$R_3 = 10000\Omega$

(2) 

$R_1 = 19900\Omega$
$R_2 = 9900\Omega$
$R_3 = 1900\Omega$

(3) 

$R_1 = 1900\Omega$
$R_2 = 9900\Omega$
$R_3 = 19900\Omega$

(4) 

$R_1 = 2000\Omega$
$R_2 = 8000\Omega$
$R_3 = 10000\Omega$

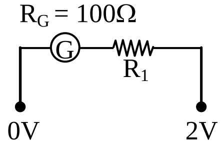
Ans. **[1]**

Sol. sensitivity = 20 μA/div

Total division = 50

$\Rightarrow$ maximum current through galvanometer can be = $I_{max} = (50)(20\ \mu A) = 10^{-3}$ A

$R_G = 100\Omega$



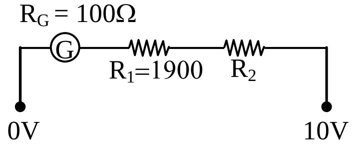
$$I_{max} = \frac{2}{100 + R_1} = 10^{-3}$$

$$\Rightarrow \frac{2}{10^{-3}} = 100 + R_1$$

$\Rightarrow$ $R_1 = 2000 - 100$

$\Rightarrow$ $R_1 = 1900\ \Omega$

$R_G = 100\Omega$



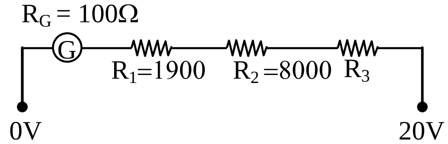
$$I_{max} = \frac{10}{R_G + R_1 + R_2}$$

$$\Rightarrow 10^{-3} = \frac{10}{100 + 1900 + R_2}$$

$$\Rightarrow R_2 + 2000 = \frac{10}{10^{-3}}$$

$\Rightarrow$ $R_2 = 10000 - 2000 = 8000\Omega$

$R_G = 100\Omega$

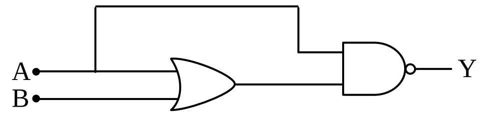
$\Rightarrow$ 

$$I_{max} = \frac{20}{100 + 1900 + 8000 + R_3}$$

$10000 + R_3 = 20000$

$R_3 = 10000\Omega$

Q.15 The truth table for the circuit given in the fig. is:



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

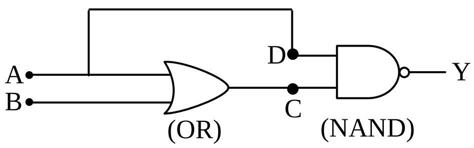
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Ans. **[2]**

Sol.

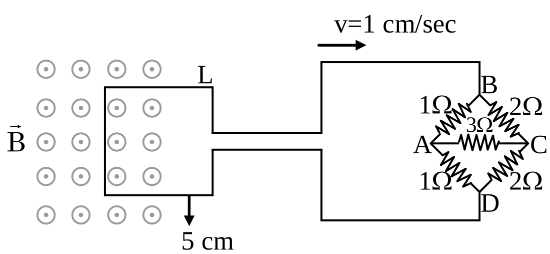


Effectively D $\equiv$ A & C is output of 'OR' gate

Y is output of 'NAND' gate

A	B	C	D	Y
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

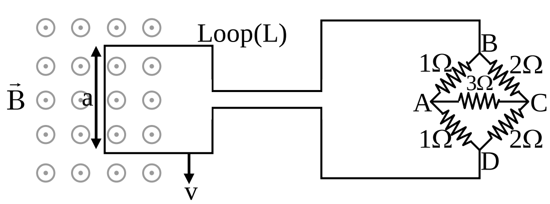
Q.16 The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to :



(1) 115 μA (2) 150 μA (3) 170 μA (4) 60 μA

Ans. **[3]**

Sol.



$V = 1\ cm/s = 10^{-2}\ m/s$

$R_{loop} = 1.7\ \Omega$

$A = 5\ cm = 5 \times 10^{-2}$ m equivalent circuit : $R_{total} = R_{loop} + R_{wheatstone}$

$R_{eq} = \frac{(4)(2)}{4+2} = \frac{8}{6} = \frac{4}{3} = 1.3\ \Omega$

$R_{Total} = 1.7 + 1.3$

$\Rightarrow R_{Total} = 3\Omega$

Induced emf = $VB\ell$

$\Rightarrow$ current = $I = \frac{(VB\ell)}{R_{Total}}$

$= \frac{(10^{-2})(1)(5\times10^{-2})}{3}$

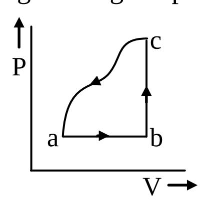
$= \frac{5}{3}\times10^{-4}$

$= 1.67 \times10^{-4}$

$= 167 \times10^{-6}$ A

$\approx 170\ \mu A$

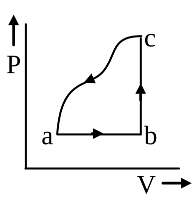
Q.17 A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is:



(1) 140 J (2) 130 J (3) 100 J (4) 120 J

Ans. **[2]**

Sol.



$\Delta U_{ca} = -180J$

$\Delta U_{ac} = +180J$ (state function)

$\Delta Q = \Delta U + \Delta W$

$\Delta Q_{a\to c} = \Delta U_{a\to c} + \Delta W_{a\to c}$

$\Delta Q_{a\to b} + \Delta Q_{b\to c} = 180 + \Delta W_{a\to c}$

$250 + 60 = 180 + \Delta W_{a\to c}$

$\Delta W_{a\to c} = 310 - 180 = 130\ J$

Q.18 An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation $\hat{s}$, is :

(1) $\hat{s} = \frac{-4\hat{k} + 3\hat{j}}{5}$ (2) $\hat{s} = \frac{4\hat{j} - 3\hat{k}}{5}$ (3) $\hat{s} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$ (4) $\hat{s} = \frac{3\hat{i} - 4\hat{j}}{5}$

Ans. **[3]**

Sol. $\vec{E} = E_0\hat{n}[\omega t + (6y - 8z)]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - (8z - 6y)]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - \left(\frac{8}{10}\hat{k} - \frac{6}{10}\hat{j}\right).10]$

$\Rightarrow \vec{E} = E_0\hat{n}[\omega t - \hat{s}k]$

$\hat{s}$ = direction of propagation

$\hat{s} = \left(\frac{8\hat{k} - 6\hat{j}}{10}\right)$

$= \left(\frac{4\hat{k} - 3\hat{j}}{5}\right)$

$= \frac{-3\hat{j} + 4\hat{k}}{5}$

Q.19 At 40º C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40ºC to 20ºC it regains its original length of 0.2 m. The value of M is close to :

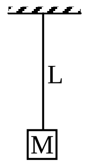
(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; g= 10 ms^{-2})

(1) 1.5 kg (2) 0.5 kg (3) 9 kg (4) 0.9 kg

Ans. **[3]**

Sol. r = 1 mm = 10^{-3} m

L_0 = 0.2 m



$\Delta L = L_0 \times \Delta T$

$Y = \frac{(F/A)}{(\Delta L/L_0)}$

$\Rightarrow \frac{\Delta L}{L_0} = \frac{F}{AY}$

$\Rightarrow \Delta L = \frac{FL_0}{AY}$

$\Rightarrow$ $L_0\,\alpha\,\Delta T = \dfrac{FL_0}{AY}$

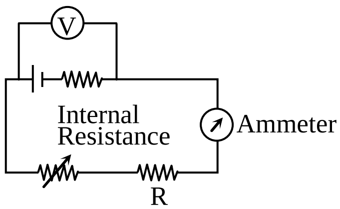
$\Rightarrow$ $\alpha\Delta T = \dfrac{Mg}{AY}$

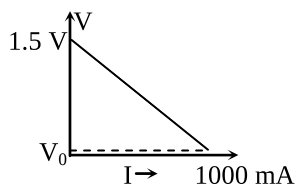
$\Rightarrow$ $M = \dfrac{\alpha\Delta TAY}{g}$

$= \dfrac{(10^{-5})(20)(\pi\times10^{-6})(10^{11})}{(10)}$

$= 2\pi$ kg
$= 2 \times 3.14$ kg
$= 6.28$ kg (closest to 9)

Q.20 To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained;





If V_0 is almost zero, identify the correct statement :

(1) The value of the resistance R is 1.5 Ω

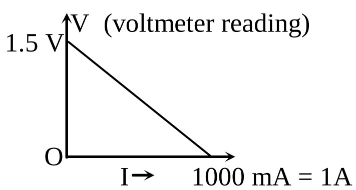
(2) The emf of the battery is l.5 V and its internal resistance is 1.5 Ω

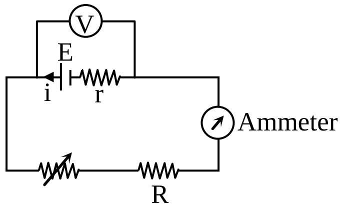
(3) The emf of the battery is l.5 V and the value of R is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

Ans. **[2]**

Sol.





When voltmeter reading is zero

$\Rightarrow$ $E - ir = 0$

$\Rightarrow\ E - \left(\dfrac{E}{R+r}\right)r = 0$

$\Rightarrow\ 1 - \dfrac{r}{R+r} = 0$

$\Rightarrow\ R + r - r = 0$

$\Rightarrow$ R = 0 : when voltmeter reading is zero

$\Rightarrow$ $R_{eq} = r$ (for circuit)

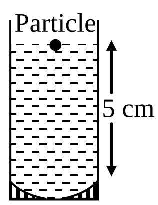
$i = \dfrac{E}{r}$

$\Rightarrow\ 1000\ \text{mA} = \dfrac{1.5}{r}$ (E = 1.5V from graph)

$\Rightarrow\ 1 = \dfrac{1.5}{r}$

$\Rightarrow\ r = 1.5\ \Omega$

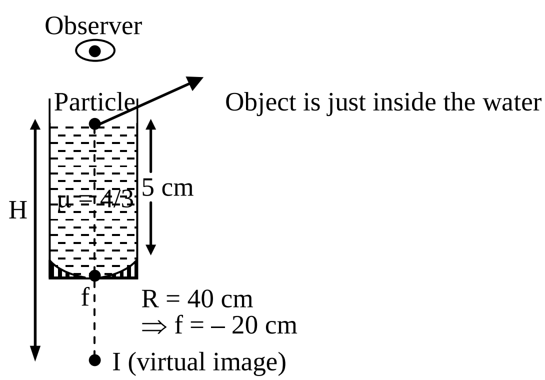
Q.21 A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is dose to: (Refractive index of water = 1.33)



(1) 11.7 cm (2) 6.7 cm (3) 13.4 cm (4) 8.8 cm

Ans. **[4]**

Sol.



$\dfrac{1}{V} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \dfrac{1}{V} + \dfrac{1}{(-5)} = \dfrac{1}{-20}$

$\Rightarrow \dfrac{1}{V} = \dfrac{1}{5} - \dfrac{1}{20}$

$\Rightarrow \dfrac{1}{V} = \dfrac{4-1}{20} = \dfrac{3}{20}$

$\Rightarrow V = \frac{20}{3}$ cm

$H = 5 + \frac{20}{3} = \frac{35}{3}$ cm

$$H_{apperent} = \frac{H}{\mu} = \frac{\left(\frac{35}{3}\right)}{\left(\frac{4}{3}\right)} = \frac{35}{3} \times \frac{3}{4} = \frac{35}{4} = 8.8 \text{ cm}$$

Q.22 Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume ? (R = 8.3 J/mol K)

(1) 21.6 J/mol K (2) 19.7 J/mol K (3) 15.7 J/mol K (4) 17.4 J/mol K

Ans. **[4]**

Sol. $$(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$= \frac{(2)\left[\frac{fR}{2}\right]_{mono} + 3\left[\frac{fR}{2}\right]_{diaatomic}}{2+3}$$

$$= \frac{2\left(\frac{3R}{2}\right) + 3\left(\frac{5R}{2}\right)}{5}$$

$$= \frac{21R}{10}$$

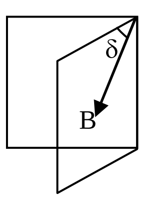
$$= \frac{21 \times 8.3}{10} = 17.4 \text{ J/mol-K}$$

Q.23 A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30º. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(1) 1.8 (2) 2.2 (3) 0.7 (4) 3.6

Ans. **[3]**

Sol.



δ = angle of dip

$\tau = \vec{m} \times \vec{B}$

$\Rightarrow \tau = -mB\sin\theta$

$\Rightarrow \tau = -mB\theta$ (small angular displacement)

$\Rightarrow I\alpha = -m(B\cos\delta)\theta$

$\Rightarrow \alpha = -\left(\frac{mB\cos\delta}{I}\right)\theta$

$\omega^2 = \frac{mB\cos\delta}{I}$

$$T = 2\pi\sqrt{\frac{I}{mB\cos\delta}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\delta_2}{B_1\cos\delta_1}}$$

$$\frac{\left(\frac{60}{30}\right)}{\left(\frac{60}{40}\right)} = \sqrt{\frac{B_2\cos\delta_2}{B_1\cos\delta_1}}$$

$$\Rightarrow \left(\frac{4}{3}\right)^2 = \frac{B_2\cos\delta_2}{B_1\cos\delta_1}$$

$$\Rightarrow \frac{16}{9}\frac{\cos\delta_1}{\cos\delta_2} = \frac{B_2}{B_1}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{9}{16}\times\frac{\sqrt{3}\sqrt{2}}{2\times 1}$$

$$= \frac{9\times\sqrt{6}}{32}$$

$$= \frac{9\times 2.44}{32}$$

$$= \frac{22}{32} = 0.7$$

Q.24 Which of the following combinations has the dimension of electrical resistance ($\in_0$ is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

(1) $\sqrt{\frac{\in_0}{\mu_0}}$ (2) $\frac{\in_0}{\mu_0}$ (3) $\frac{\mu_0}{\in_0}$ (4) $\sqrt{\frac{\mu_0}{\in_0}}$

Ans. [4]

Sol. $[\varepsilon_0] = (M^{-1} L^{-3} T^4 A^2)$

$[\mu_0] = (M L T^{-2} A^{-2})$

$[R] = (M L^2 T^{-3} A^{-2})$

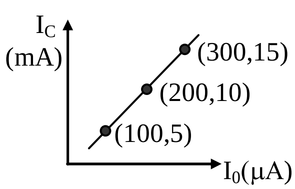
$R = \sqrt{\frac{\mu_0}{\in_0}}$

Q.25 The transfer characteristic curve of a transistor, having input and output resistance 100Ω and 100 kΩ respectively is shown in the figure. The voltage and power gain, are respectively:

(1) $5\times10^4, 5\times10^5$ (2) $5\times10^4, 5\times10^6$ (3) $5\times10^4, 2.5\times10^6$ (4) $2.5\times10^4, 2.5\times10^6$

Ans. [3]

Sol.



$$B = \frac{I_C}{I_B} = \frac{5\times10^{-3}}{100\times10^{-9}} = 50$$

Voltage gain = $A_V = \dfrac{V_{output}}{V_{input}}$

$= \dfrac{I_{output}\ R_0}{I_{input}\ R_{in}}$

$= \left(\dfrac{I_c}{I_b}\right)\left(\dfrac{R_0}{R_{in}}\right)$

$= (\beta)\left(\dfrac{R_0}{R_{in}}\right)$

$= (50)\left[\dfrac{100\times10^3}{100}\right]$

$= 5 \times 10^4$

Power gain = $\beta^2 = \left(\dfrac{R_0}{R_i}\right)$

$= (50)^2 \times 10^3$

$= 25 \times 10^2 \times 10^3$

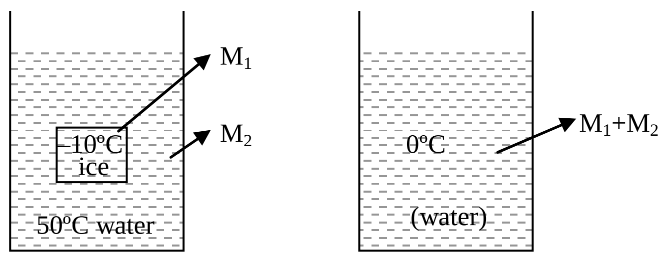
$= 2.5 \times 10^6$

Q.26 When M_1 gram of ice at –10°C (specific heat = 0.5 cal g^{-1}°C^{-1}) is added to M_2, gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is :

(1) $\dfrac{50M_2}{M_1} - 5$ (2) $\dfrac{50M_2}{M_1}$ (3) $\dfrac{5M_2}{M_1} - 5$ (4) $\dfrac{5M_1}{M_2} - 50$

Ans. **[1]**

Sol.



Using energy conservation

$E_{relesed\ by\ water} = E_{used\ by\ ice}$

$\Rightarrow M_2S_W(\Delta T)_{water} = M_1S_{ice}(\Delta T)_{ice} + M_1L_{fusion}$

$\Rightarrow M_2(1)(50) = M_1\left(\dfrac{1}{2}\right)(10) + M_1L_{fusion}$

$\Rightarrow 50M_2 - 5M_1 = M_1L_{fusion}$

$\Rightarrow L_f = \dfrac{50M_2 + 5M_1}{M_1}$

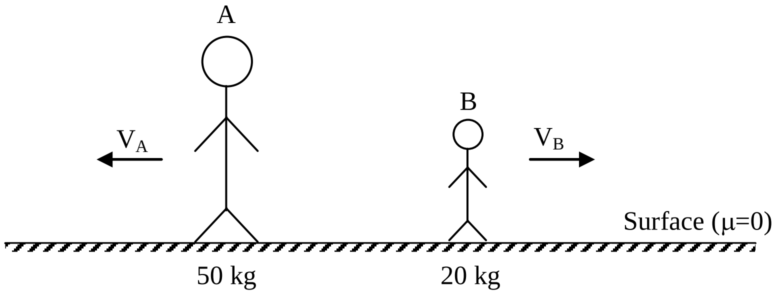
$\Rightarrow L_f = \dfrac{50M_2}{M_1} - 5$

Q.27 A man (mass = 50 kg) and his son (mass =20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.28 ms^{-1} (2) 0.47 ms^{-1} (3) 0.20 ms^{-1} (4) 0.14 ms^{-1}

Ans. **[3]**

Sol.



$\vec{V}_{B/A} = 0.7\,m/s$

$\Rightarrow \vec{V}_{B/S} - \vec{V}_{A/S} = 0.7$

$\Rightarrow V_B - (-V_A) = 0.7$

$\Rightarrow V_B + V_A = 0.7$

$\Rightarrow V_A + V_B = 0.7$

Momentum conservation $(\vec{F}_{ext} = 0)$

$P_C = P_f$

$\Rightarrow 0 = 20(+V_B) + 50(-V_A)$

$\Rightarrow 2V_B = 5V_A$

$\Rightarrow V_B = \frac{5V_A}{2}$

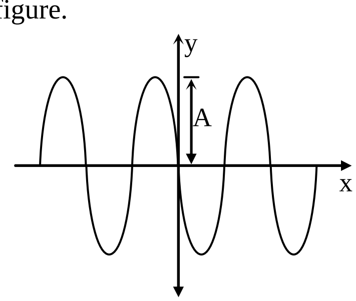
$\Rightarrow V_A + \frac{5V_A}{2} = 0.7$

$\Rightarrow \frac{7V_A}{2} = 0.7$

$\Rightarrow V_A = 0.7\left(\frac{2}{7}\right)$

$\Rightarrow V_A = 0.2$ m/s

Q.28 A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A\sin(kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure.



For this wave, the phase ϕ is :

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) $-\frac{\pi}{2}$

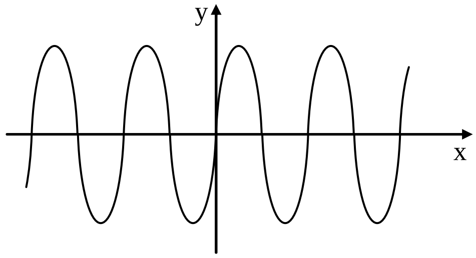
Ans. **[2]**

Sol.

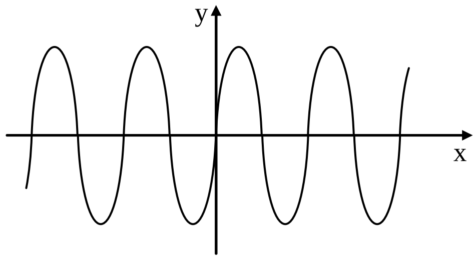
$Y = A \sin (kx - \omega t + \phi)$

At $t = 0$

$Y = A \sin (kx + \phi)$



Graph of : $y = A \sin(kx)$



Graph of : $y = -A \sin(kx)$

$-A \sin(kx) = A \sin (kx + \phi)$

$\Rightarrow A \sin(kx + \pi) = A \sin (kx + \phi)$

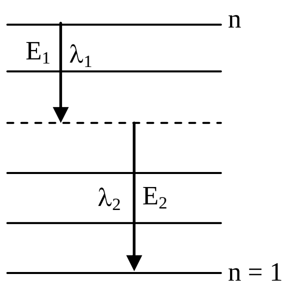
$\Rightarrow \phi = \pi$

Q.29 An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \dfrac{1240\,\text{eV}}{\lambda(\text{in nm})}$.

(1) n = 4 (2) n = 6 (3) n = 5 (4) n = 7

Ans. **[3]**

Sol.



$E = E_1 + E_2$

$$\Rightarrow 13.6\,(Z^2)\left[\frac{1}{1^2} - \frac{1}{n^2}\right] = \frac{1240}{108.5} + \frac{1240}{30.4}$$

$$\Rightarrow 13.6 \times 4 \left[1 - \frac{1}{n^2}\right] = 11.43 + 40.79$$

$$\Rightarrow 1 - \frac{1}{n^2} = \frac{52.22}{54.4}$$

$$\Rightarrow \frac{1}{n^2} = 1 - \frac{52.22}{54.4}$$

$\Rightarrow \dfrac{1}{n^2} = \dfrac{2.18}{54.4}$

$\Rightarrow n^2 = \dfrac{54.4}{2.18}$

$\Rightarrow n^2 = 25$

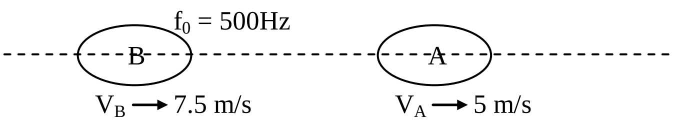
$\Rightarrow n = 5$

Q.30 A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to: (Speed of sound in water =1500 ms^{-1})

(1) 507 Hz (2) 504 Hz (3) 499 Hz (4) 502 Hz

Ans. **[4]**

Sol.



v = speed of sound in water = 1500 m/s

frequency received by A = $f' = \left[\dfrac{V - V_A}{V - V_B}\right] f_0 = \left[\dfrac{1500 - 5}{1500 - 7.5}\right] f_0$

frequency received by B = $f'' = \left[\dfrac{V + V_B}{V + V_A}\right] f' = \left[\dfrac{1500 + 7.5}{1500 + 5}\right]\left[\dfrac{1500 - 5}{1500 - 7.5}\right] f$

$f'' = \left(\dfrac{1500 + 7.5}{1500 - 7.5}\right)\left(\dfrac{1500 - 5}{1500 + 5}\right)(500)$

$= 502$ Hz

JEE Main Online Exam 2019

Questions & Solutions

12th April 2019 | Shift - I

CHEMISTRY

Q.1 The basic structural unit of feldspar, zeolites, mica, and asbestos is:

(1) $-(SiR_2-O)_n-$ (R=Me) (2) $(SiO_4)^{4-}$ (3) $(SiO_3)^{2-}$ (4) SiO_2

Ans. **[2]**

Sol. Feldspar, zeolites, mica and asbestos are silicates which contains SiO_4^{4-} basic unit

Q.2 Which of the following is a thermosetting polymer?

(1) Bakelite (2) NyIon6 (3) PVC (4) Buna-N

Ans. **[1]**

Sol. Bakelite is a thermosetting polymer, thermosetting polymers are also called as 3D-polymer or cross link polymer.

Q.3 The correct set of species responsible for the photochemica1 smog is:

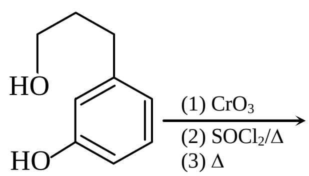
(1) NO, NO_2, O_3 and hydrocarbons (2) N_2, NO_2 and hydrocarbons

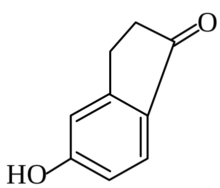
(3) CO_2, NO_2, SO_2 and hydrocarbons (4) N_2, O_2, O_3 and hydrocarbons

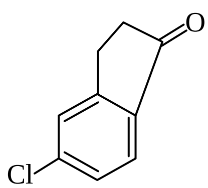
Ans. **[1]**

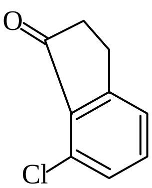
Sol. NO, NO_2, O_3, hydrocarbon are the factors which causes smog.

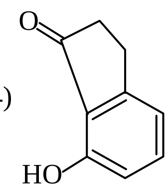
Q.4 The major product of the following reaction is :



(1) 

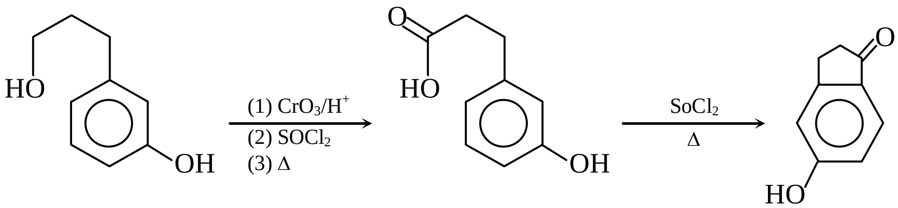
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.5 The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg^{-1}) of the aqueous solution is :

(1) 13.88×10^{-1} (2) 13.88×10^{-3} (3) 13.88 (4) 13.88×10^{-2}

Ans. **[3]**

Sol. $x_A = 0.8, x_B = 0.2$

$$x_B = \frac{mM_A}{1000 + mM_A} \quad \text{or} \quad m = \frac{1000}{M_A} \times \frac{x_B}{1 - x_B}$$

$$m = \frac{1000}{16} \times \frac{0.2}{0.8}$$

m = 13.88

Q.6 Glucose and Galactose are having identical configuration in all the positions except position.

(1) C–5 (2) C–3 (3) C–2 (4) C–4

Ans. **[4]**

Sol. Glucose and galactose differs in configuration on C_4.

Q.7 The correct sequence of thermal stability of the following carbonates is :

(1) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$ (2) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(3) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$ (4) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

Ans. **[2]**

Sol. In 2nd group carbonates thermal stability increases down the group due to increase in lattice energy.

Q.8 What is the molar solubility of $A1(OH)_3$ in 0.2 M NaOH solution ? Given that, solubility product of $A1(OH)_3$ $= 2.4 \times 10^{-24}$:

(1) 3×10^{-22} (2) 3×10^{-19} (3) 12×10^{-21} (4) 12×10^{-22}

Ans. **[1]**

Sol. $Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

Equ. 1-5 5 35 + 0.2

$NaOH \longrightarrow Na^+ + OH^-$

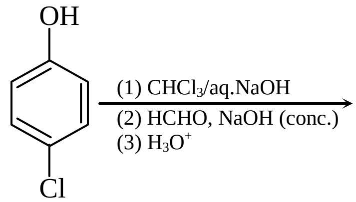
– 0.2 0.2

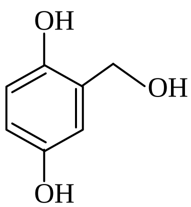
$S(35 + 0.2)^3 = Ksp$ ∵ 0.2 >> 5

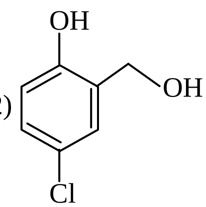
$S \times 0.008 = 2.4 \times 10^{-24}$

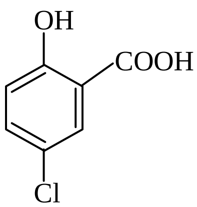
$S = 3 \times 10^{-22}$

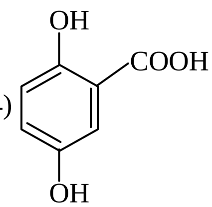
Q.9 The major products of the following reaction are:



(1)  and Formic acid

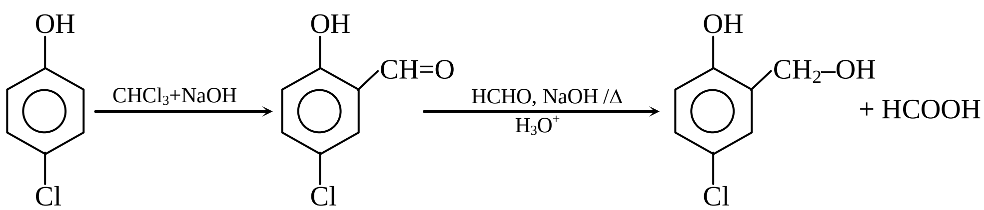
(2)  and Formic acid

(3)  and Methanol

(4)  and Methanol

Ans. **[2]**

Sol.



Q.10 Which of the following statements is not true about RNA?

(1) It controls the synthesis of protein
(2) IL is present in the nucleus of the cell
(3) It usually does not replicate
(4) It has always double stranded α-helix structure

Ans. **[4]**

Sol. RNA is single stranded helix structure

Q.11 Peptization is a:

(1) process of converting soluble particles to form colloidal solution
(2) process of converting precipitate into colloidal solution
(3) process of bringing colloidal molecule into solution
(4) process of converting a colloidal solution into precipitate

Ans. **[2]**

Sol. fact peptization is a process of converting precipitate into colloidal solution.

Q.12 Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200 °C. If specific heat of $I_2(s)$ and l_2 (vap) are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal g^{-1} is :

(1) 2.85
(2) 22.8
(3) 11.4
(4) 5.7

Ans. **[2]**

Sol. $I_{2(s)} \rightleftharpoons I_{2(g)}, \Delta H_r = 24$

$$\Delta H_{r_1 T_2} - \Delta H_{r_1 T_1} = \Delta C_p (T_2 - T_2)$$

$$\Delta H_{r_1 250} - 24 = (0.031 - 0.055) \times 50$$

$$\Delta H_{r_1 250} = 24 - 1.2 = 22.8$$

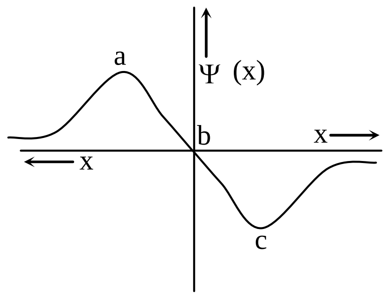
Q.13 The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively. Are :

(1) washer woman and concentration
(2) fisher woman and concentration
(3) wisher man and reduction
(4) fisher man and reduction

Ans. **[1]**

Sol. froth floatation is used for concentration for sulphide ore

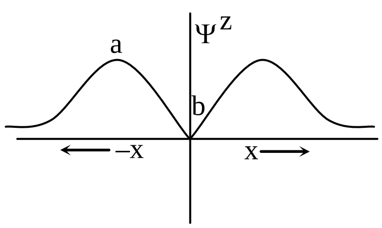
Q.14 The electrons are more likely to be found :



(1) in the region a arid c
(2) only in the region c
(3) in the region a and b
(4) only in the region a

Ans. **[1]**

Sol.



At a & c, probability of finding electron is maximum

Q.15 An organic compound 'A' is oxidizod with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate. Based on above observation, the element present in the given compound is:

(1) Fluorine
(2) Phosphorus
(3) Nitrogen
(4) Sulphur

Ans. **[2]**

Sol. This is qualitative test for phosphorous

$$P + Na_2O_3 \longrightarrow Na_3PO_4 \xrightarrow{3HNO_3} H_3PO_4 + 3NaNO_3$$

$$\underset{\text{Ammonium molybdate}}{H_2PO_4 + 12(NH_4)MoO_4} + 21HNO_3 \longrightarrow \underset{\text{yellow ppt}}{(NH_4)_3PO_4.12MoO_3} + 21\ NH_4NO_3 + 12\ H_2O$$

Q.16 But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give:

(1) one molecule of CH_3CHO and one molecule of CH_3COOH

(2) 2 molecules of CH_3COOH

(3) $CH_3-CH(OH)-CH(OH)-CH_3$

(4) 2 molecules of CH_3CHO

Ans. **[2]**

Sol. $CH_3-CH=CH-CH_3 \xrightarrow[H^+/\Delta]{KMNO_4} 2CH_3COOH$

Q.17 An ideal gas is allowed to expand form 1 L to 10 L against a constant external pressure of I bar. The work done in kJ is:

(1) +10.0 (2) – 9.0 (3) – 2.0 (4) –0.9

Ans. **[4]**

Sol. $w = -P_{ext}(V_2 - V_1)$

$= -1(10-1)$

$= -9$ L–bar

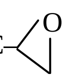
$= -9 \times 100$ J (1L – bar = 100)

$= -0.9$ kJ

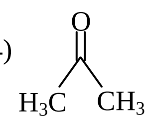
Q.18 The major product of the following addition reaction is –

$H_3C-CH=CH_2 \xrightarrow{Cl_2/H_2O}$

(1) $H_3C-CH(OH)-CH_2(Cl)$

(2) H_3C- 

(3) $CH_3-CH(Cl)-CH_2(OH)$

(4) 

Ans. **[1]**

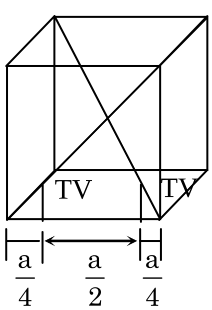
Sol. $H_3C-CH=CH_2 \xrightarrow{Cl_2/HOH} H_3C-CH(OH)-CH_2(Cl)$

Q.19 An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is:

(1) a (2) $\frac{3}{2}a$ (3) $\frac{a}{2}$ (4) $\sqrt{2}a$

Ans. **[3]**

Sol.



Minimum distance between two tetrahedral voids is $\frac{a}{2}$

Q.20 In the following reaction; $xA \rightarrow yB$

$$\text{Log10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(1) n-Butane and Iso-butane (2) C_2H_4 and C_4H_8

(3) C_2H_4 and C_6H_6 (4) N_2O_4 and NO_2

Ans. **[2]**

Sol. $x_A \longrightarrow y_B$

$$-\frac{1}{x}\frac{\Delta A}{dt} = +\frac{1}{y}\frac{\Delta B}{dt}$$

$$-\frac{\Delta A}{dt} = +\frac{x}{y}\frac{\Delta B}{dt} \qquad \text{\_\_\_\_(1)}$$

$$\because \log\left[-\frac{dA}{dt}\right] = \log\left[+\frac{dB}{dt}\right] + \log 2$$

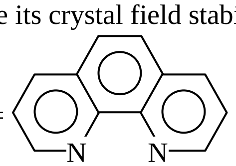
$$-\frac{dA}{dt} = +\frac{2}{1}\frac{dB}{dt} \qquad \text{\_\_\_\_(2)}$$

Compare (1) & (2)

x = 2, y = 1

$2C_2H_4 \rightleftharpoons C_4H_5$

Q.21 The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is :

(Phen = 

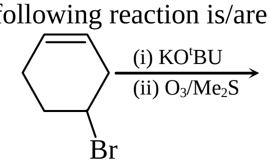
and ignore pairing energy)

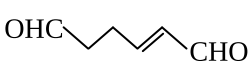
(1) $[Fe(phen)_3]^{2+}$ (2) $[Zn(phen)_3]^{2+}$ (3) $[Co(phen)_3]^{2+}$ (4) $[Ni('phen)_3]^{2+}$

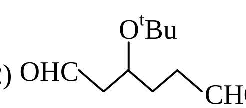
Ans. **[1]**

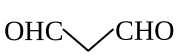
Sol. In the complexes of Fe^{2+}, CO^{2+}, Ni^{2+} upon oxidation, complexes of Fe^{3+}, CO^{2+} & Ni^{3+} stabilizes whereas in case of removing 3^{rd} electron from Zn becomes difficult.

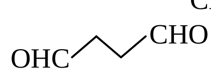
Q.22 The major product(s) obtained in the following reaction is/are :



(1) 

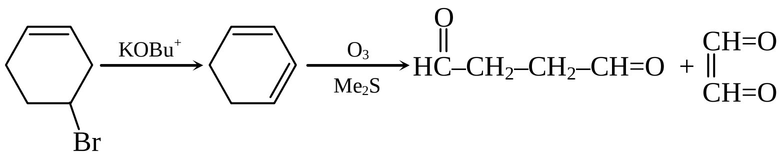
(2) 

(3) 

(4) 

and OHC–CHO

Ans. **[4]**

Sol. 

Q.23 The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are:

(1) 16, 6 and 3 (2) 15, 6 and 12 (3) 16, 5 and 2 (4) 15, 5 and 3

Ans. **[4]**

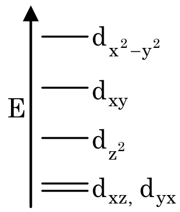
Sol. $z = 15\ (3s^2\ 3p^2)$

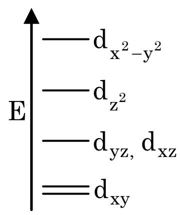
hν. No. = 15

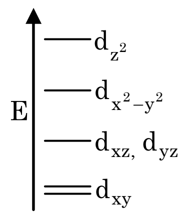
Valence e^{-}s = 5

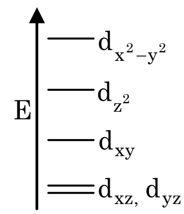
Valence = 3

Q.24 Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

(1)


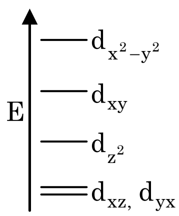
(2)


(3)


(4)


Ans. **[1]**

Sol. In this case, this leads to square planar complex



Q.25 5 moles of AB_2 weigh 125×10^{-5} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of A (M_A) and molar mass of B (M_B) in kg mol are:

(1) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$ (2) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

(3) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$ (4) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$

Ans. **[3]**

Sol. $5 = \dfrac{125}{M_A + 2M_B}$

$M_A + 2M_B = 25$ ….(i)

$10 = \dfrac{300}{2M_A + 2M_B}$

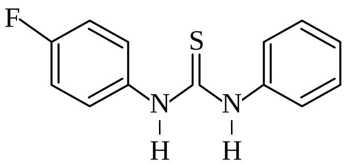
$2M_A + 2M_B = 30$ ….(2)

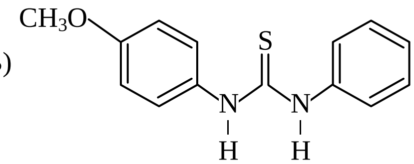
Solving (1) & (2) ;

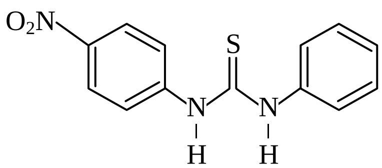
$M_A = 5$ g/mol $= 5 \times 10^{-3}$ kg/mol

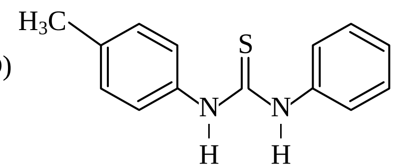
$M_B = 10$ g/mol $= 10 \times 10^{-3}$ kg/mol

Q.26 The increasing order of the pK_b of the following compound is:

(A) 

(B) 

(C) 

(D) 

(1) (C) < (A) < (D) < (B)
(2) (B) < (D) < (C) < (A)
(3) (B) < (D) < (A) < (C)
(4) (A) < (C) < (D) < (B)

Ans. **[3]**

Sol.

A	<	C	<	D	<	B
↓		↓		↓		↓
–M		–I		+M		+M

Q.27 An example of a disproportionation reaction is :

(1) $2NaBr + Cl_2 \longrightarrow 2NaCl + Br_2$
(2) $2KMnO_4 \longrightarrow 2KMnO_4 + MnO_2 + O_2$
(3) $2CuBr \longrightarrow CuBr_2 + Cu$
(4) $2MnO_4^- + 10I^- + 16H^+ \longrightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

Ans. **[3]**

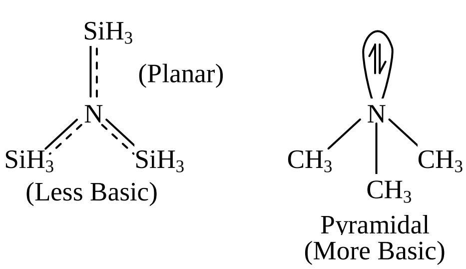
Sol. $2\overset{+1}{Cu}Br \longrightarrow \overset{+2}{Cu}Br_2 + Cu^0$

Q.28 The correct statement among the following is :

(1) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$
(2) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$
(3) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$
(4) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

Ans. **[4]**

Sol.



Q.29 Given

$CO^{3+} + e^{-} \rightarrow CO^{2+}$; E° = + 1.81 V

$Pb^{4+} + 2e^{-} \rightarrow Pb^{2+}$; E° = + 1.67 V

$Ce^{4+} + e^{-} \rightarrow Ce^{3+}$; E° = + 1.61 V

$Bi^{3+} + 3e^{-} \rightarrow Bi$; E° = + 0.20 V

Oxidizing power of the species will increase in the order:

(1) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$
(2) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$
(3) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$
(4) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Ans. **[4]**

Sol. Higher reduction potential, higher oxidizing power.
$Bi^{3+} < Ce^{4} < Pb^{4+} < Co^{3+}$

Q.30 The metal that gives hydrogen gas upon treatment with both acid as well as base is:

(1) mercury (2) zinc (3) iron (4) magnesium

Ans. **[2]**

Sol. $Zn + \underset{(dil)}{NaOH} \longrightarrow H_2\uparrow$

$Zn + \underset{(dil)}{2HCl} \longrightarrow H_2\uparrow$

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

12th April 2019 | Shift - I

MATHEMATICS

Q.1 The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e \frac{|x^3+1|}{x^2} + C$ (2) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$ (3) $\log_e \left|\frac{x^3+1}{x}\right| + C$ (4) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|} + C$

Ans. **[3]**

Sol. $\int \frac{2x^3 - 1}{x^4 + x} dx$

$\Rightarrow \int \frac{(4x^3+1)-(2x^3+2)}{x^4+x} dx$

$\Rightarrow \int \frac{4x^3+1}{x^4+x} dx - 2\int \frac{1}{x} dx$

$x^4 + x = t \Rightarrow (4x^3 + 1)\, dx = dt$

$\Rightarrow \int \frac{dt}{t} - 2\int \frac{1}{x} dx$

$\Rightarrow \ell n|t| - 2\ell nx + C$

$\Rightarrow \ell n \left|\frac{x^4+x}{x^2}\right| + C \Rightarrow \ell n \left|\frac{x^3+1}{x}\right| + C$

Q.2 For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$ If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{7\pi}{12}$ (2) $\tan\frac{11\pi}{12}$ (3) $\tan\frac{\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

Ans. **[2]**

Sol. $\phi(x) = [(hof)og](x)$

$hof(x) = \frac{1-x}{1+x}$

$(hof)g(x) = \frac{1-\tan x}{1+\tan x}$

$$\phi\left(\frac{\pi}{3}\right) = \frac{1-\tan\frac{\pi}{3}}{1+\tan\left(\frac{\pi}{3}\right)}$$

$$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4}-\frac{\pi}{3}\right) = \tan\left(\frac{11\pi}{12}\right)$$

Q.3 Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio :

(1) 14 : 13 (2) 13 : 11 (3) 5 : 4 (4) 2 : 1

Ans. **[3]**

Sol.

$$\left.\begin{matrix} C_1 : y^2 = 12x \\ C_2 : x^2 - \frac{y^2}{8} = 1 \end{matrix}\right]\text{common tangent}$$

$$\left.\begin{matrix} \text{Tangent of } C_1 : y = mx + \frac{3}{m} \\ C_2 : y = mx \pm \sqrt{1(m^2) - 8} \end{matrix}\right] \Rightarrow \text{Both same}$$

$$\left(\frac{3}{m}\right)^2 = m^2 - 8$$

$9 = m^2 (m^2-8)$

let $m^2 = t$

$\Rightarrow t (t-8) = 9$

$\Rightarrow t^2 - 8t - 9 = 0$

$\Rightarrow (t-9) (t+1) = 0$

$m^2 = 9$

$\Rightarrow m = -3, 3$

common tangent $y = 3x + 1$

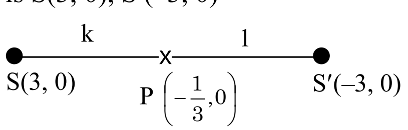
$y = -3x - 1$

P intersection of tangents $P\left(-\frac{1}{3}, 0\right)$

foci of hyperbola $a = 1, b = 2\sqrt{2}$

$8 = 1 (e^2-1) \Rightarrow e = 3$

is S(3, 0), S′(–3, 0)



$$-\frac{1}{3} = \frac{-3k+3}{k+1}$$

$\Rightarrow k = 5 : 4$

Q.4 If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then m·n is equal to

(1) – 1 (2) 1 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

Ans. **[1]**

Sol. $I = \int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx$

$I = \int_0^{\pi/2} \frac{\cos x}{1+\cos x} = \int_0^{\pi/2} \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 x/2\right) dx$

$I = \left[x - \frac{2}{2}\tan(x/2)\right]_0^{\pi/2}$

$\Rightarrow \left(\frac{\pi}{2} - 1\right) = \frac{(\pi - 2)}{2}$

$m = \frac{1}{2}, n = -2$

$m \cdot n = -1$

Q.5 If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to :

(1) $\frac{7}{116}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{21}{346}$

Ans. **[3]**

Sol. $\alpha + \beta = \frac{25}{375}$

$\alpha\beta = \frac{-2}{375}$

α & $\beta \in (-1, 1)$, then

$\lim_{n\to\infty} \sum_{r=1}^{n} (\alpha^r + \beta^r)$

$\Rightarrow \alpha + \alpha^2$ ……….infinite + $\beta + \beta^2$........ infinite

$\Rightarrow \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$

$\Rightarrow \frac{\alpha(1-\beta) + \beta(1-\alpha)}{(1-\alpha)(1-\beta)} = \frac{1}{12}$

Q.6 The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 5 (2) 3 (3) 7 (4) 4

Ans. **[1]**

Sol. $1 + \sin^4 x = \cos^2 3x$; $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$

$\Rightarrow$ L.H.S. ≥ 1

R.H.S. ≤ 1

Both satisfy when

L.H.S. = R.H.S. = 1

$\sin^4 x = 0$; $\cos^2 3x = 1$

$x = -2\pi, -\pi, 0, \pi, 2\pi$ total 5 solution

Q.7 The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

(1) a circle of radius 1.

(2) the line through the origin with slope – 1

(3) a circle of radius $\frac{1}{2.}$

(4) the line through the origin with slope 1.

Ans. **[4]**

Sol. $|z - i| = |z - 1|$

$z = x + iy$

$|x + i(y - 1)| = |(x - 1) + iy|$

$\Rightarrow x^2 + (y - 1)^2 = (x - 1)^2 + y^2$

$\Rightarrow y = x$ straight line having slope (+)ve 1,

pass origin (0, 0)

Q.8 If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx,}, \frac{d^2y}{dx^2,}\right)$ at x = 0 is equal to :

(1) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (3) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

Ans. **[1]**

Sol. $e^y + xy = e$

at $x = 0$; $e^y + 0 = e$

$y = 1$ $y(0) = 1$

diff. w.r.t. x

$e^y.\ y_1 + xy_1 + y = 0$

$e^{(1)}.y_1 + 0 + 1 = 0$

$y_1 = -1/e$ at $x = 0$

diff again w.r.t. to x

$e^y.y_2 + y_1\, e^y.y_1 + xy_2 + y_1 + y_1 = 0$

at $x = 0$

$$\Rightarrow e^{(1)}(y_2) + \left(\frac{-1}{e}\right)^2 e^1 + 0.y_2 + 2\left(\frac{-1}{e}\right) = 0$$

$y_2 = 1/e^2 \quad x = 0$

$$\left(\frac{-1}{e}, \frac{1}{e^2}\right)$$

Q.9 The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) $2^{20} - 1$ (2) 2^{20} (3) 2^{21} (4) $2^{20} + 1$

Ans. **[2]**

Sol. total 31 $\Rightarrow \underbrace{\text{21 distinct \& 10 identical}}_{10}$

$S = {}^{21}C_{10} + {}^{21}C_9(1) + {}^{21}C_8(1) + {}^{21}C_7 + {}^{21}C_6 \ldots\ldots {}^{21}C_0(1)$

$S = {}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} \ldots\ldots {}^{21}C_{21}$

$2S = 2^{21}$

$S = 2^{20}$

Q.10 For x $\in$R, let [x] denote the greatest integer $\le$ x, then the sum of the series

$$\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \ldots.. + \left[-\frac{1}{3} - \frac{99}{100}\right] \text{ is :}$$

(1) – 153 (2) – 135 (3) – 133 (4) – 131

Ans. **[3]**

Sol.
$$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \ldots..\left[-\frac{1}{3} - \frac{66}{100}\right]}_{-1(67\text{ times})} + \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + \left[-\frac{1}{3} - \frac{68}{100}\right] + \ldots..\left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33\text{ times})}$$

$= -67 - 66 = -133$

Q.11 If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(5, 3\sqrt{6})$ (2) $(4, 3\sqrt{3})$ (3) $(4, 3\sqrt{2})$ (4) $(3, 3\sqrt{3})$

Ans. **[2]**

Sol. $f(x)\uparrow : f'(x) \ge 0$; & $f(x_1) \le f(x_2)$, $x\in[0, 3]$

$\Rightarrow f(0) \le f(3)$

$0 \le 3\sqrt{3k - 9}$

$k \ge 3$

$$f'(x) = \frac{x(k-2x)}{2\sqrt{kx - x^2}} + \sqrt{kx - x^2}$$

$$= \frac{x(k-2x) + 2(kx - x^2)}{2\sqrt{kx - x^2}} \ge 0$$

$$= \frac{3kx - 4x^2}{2\sqrt{kx - x^2}} \ge 0$$

$\&\ kx - x^2 \ge 0$

$3kx - 4x^2 \geq 0$

$x(3k - 4x) \geq 0 \quad x \in [0,3]$

$x(4x - 3x) \leq 0$

$k \geq 4$

$k = 4 \Rightarrow$ minimum value of k

$m = 4$

$f(3) = M = 3\sqrt{4(3) - 9} = 3\sqrt{3}$

$M = 3\sqrt{3}$

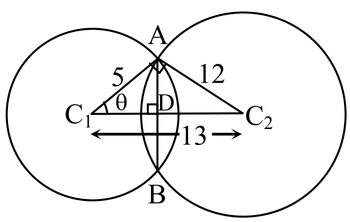
$(m, M) = (4, 3\sqrt{3})$

Q.12 If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{13}{5}$ (2) $\frac{60}{13}$ (3) $\frac{120}{13}$ (4) $\frac{13}{2}$

Ans. **[3]**

Sol.



ΔAC_1C_2

$\tan\theta = \frac{12}{5} \Rightarrow \sin\theta = \frac{12}{13}$ …..(i)

ΔACD:

$\sin\theta = \frac{AB/2}{5}$ …..(ii)

(i) & (ii)

$\Rightarrow \frac{AB}{2.5} = \frac{12}{13}$

$AB = \frac{120}{13}$

Q.13 The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is :

(1) 126 (2) – 84 (3) – 126 (4) 84

Ans. **[4]**

Sol. coefficient of x^{18}

$(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$\Rightarrow (1 + x)(1 - x)(1 - x)^9(1 + x + x^2)^9$

$\Rightarrow (1 - x^2)(1 - x^3)^9$

$\Rightarrow (1 - x^2)\ [{}^9C_r\,(-1)^r\,(x^{3r})]$

$\Rightarrow {}^9C_r\,(-1)^r\,x^{3r} - {}^9C_r\,(-1)^r\,x^{3r+2}$

for x^{18}

$3r = 18$	$3r+2=18$
$r = 6$	r not possible

then coefficient of x^{18} is ${}^9C_6\,(-1)^6 = 84$

Q.14 Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$, If value of y is 1 when x = 1, then the value of x for which y = 2, is :

(1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$ (3) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (4) $\frac{5}{2} + \frac{1}{\sqrt{e}}$

Ans. **[1]**

Sol. $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int 1/y^2 dy} = e^{-1/y}$

sol. $xe^{-1/y} = \int e^{-1/y}\,(1/y^3)dy$

$-\frac{1}{y} = t$

$\Rightarrow \frac{1}{y^2}dy = dt$

$xe^t = \int e^t\,(-t).dt$

$x.e^t = -\left[t.e^t - e^t\right] + c$

$x = -t + 1 + c.e^{-t}$

$x = \frac{1}{y} + 1 + c.\,e^{1/y}$

give $y(1) = 1$

$1 = 1 + 1 + c.e^{1/1}$

$c = -e^{-1}$

$x = \frac{1}{y} + 1 + (-)e^{-1}.\,e^{1/y}$

at $y = 2$

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

Q.15 If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to :

(1) $\frac{8}{3}$ (2) 6 (3) $\frac{10}{3}$ (4) $-\frac{2}{3}$

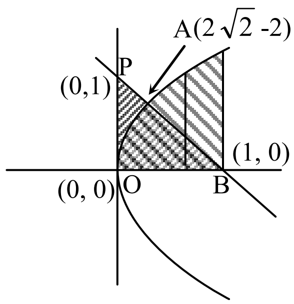
Ans. **[2]**

Sol. $C_1 : y^2 \le 4x$

$C_2 : x + y \le 1$

$x \ge 0$

$y \ge 0$



$\Rightarrow y^2 = 4x \; ; \; y^2 = 4(1-y)$

$y^2 + 4y + 4 = 0$

$y = 2\sqrt{2} - 2 \, , \, -2\sqrt{2} - 2$

Area : shaded region of curve OAB

A = Area of Δ_{OBP} – Area of region OAP

$\Delta_{OBP} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of OAP $= \int_0^{2\sqrt{2}-2} \frac{y^2}{4} dy + \int_{2\sqrt{2}-2}^{1} (1-y)\,dy$

$= \frac{1}{12}\left[y^3\right]_0^{2\sqrt{2}-2} + \left[y - \frac{y^2}{2}\right]_{2\sqrt{2}-2}^{1}$

$= \frac{1}{12}[(2\sqrt{2}-2)^3] + \left[\left(1-\frac{1}{2}\right) - \left\{(2\sqrt{2}-2) - \frac{(2\sqrt{2}-2)^2}{2}\right\}\right]$

$= \frac{23}{6} - \frac{8}{3}\sqrt{2}$

$A = \frac{1}{2} - \frac{23}{6} + \frac{8\sqrt{2}}{3}$

$a = \frac{8}{3}, \; b = -\frac{20}{6}$

$\therefore a - b = 6$

Q.16 If the data $x_1, x_2, \ldots\ldots x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is :

(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. **[2]**

Sol. $x_1 + x_2 + x_3 + x_4 = 44$

$x_5 + x_6 \ldots\ldots x_{10} = 96$

$\sum x_i = 140$

$\bar{x} = \frac{140}{40} = 14$

$\sum \bar{x}^2 = 2000$

$$SD = \sqrt{\frac{\sum \bar{x}^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{2000}{10} - \left(\frac{140}{10}\right)^2} = \sqrt{200 - 196} = 2$$

Q.17 If A is a symmetric matrix and B is a skew-symmetric matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Ans. **[4]**

Sol. A is symmetric $A^T = A$

B is skew 8 symmetry $B^T = -B$

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ … (i)

Transpose

$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$ ….(ii)

From (i) + (ii)

$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$

From (i) – (ii)

$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

then $AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Q.18 If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, 2x + y = 4 and the tangent to the ellipse at P passes through Q(4,4) then PQ is equal to :

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{\sqrt{221}}{2}$ (3) $\frac{\sqrt{157}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

Ans. **[4]**

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Normal at P : $(2\cos\theta, \sqrt{3}\sin\theta)$ is $2x\sec\theta - \sqrt{3}\,\text{cosec}\,\theta\, y = 4 = 3 = 1$

Normal at P: $2x\sec\theta - \sqrt{3}\, y\,\text{cosec}\theta = 1$

Slope of normal = $\frac{-2\sec\theta}{-\sqrt{3}\,\text{cosec}\theta} = \frac{2}{\sqrt{3}} \frac{\sin\theta}{\cos\theta}$

Normal parallel to 2x + y = 4

then $\frac{2}{\sqrt{3}}\tan\theta = -2;\ \tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

Point P $\left(-1, \frac{3}{2}\right)$, Q(4, 4)

$PQ = \sqrt{(4+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$

Q.19 Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :

(1) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (3) $4(2\hat{i} + 2\hat{j} + \hat{k})$ (4) $4(2\hat{i} + 2\hat{j} - \hat{k})$

Ans. **[1]**

Sol. $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$

$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$

A vector $\vec{r}$ perpendicular to $(\vec{a} + \vec{b})$ & $(\vec{a} - \vec{b})$ & magnitude 12

$\vec{r} = 12.\hat{n}$

$\therefore\ \hat{n} = \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|}$

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 8[2\hat{i} - 2\hat{j} - \hat{k}]$

$|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = 8.3$

$\hat{n} = \frac{(2\hat{i} - 2\hat{j} - \hat{k})}{3}$

$\vec{r} = 4.\ (2\hat{i} - 2\hat{j} - \hat{k})$

Q.20 If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det(A) + 1 = 0, is :

(1) 2 (2) – 1 (3) 0 (4) 1

Ans. **[4]**

Sol. If B is inverse of A then

AB = I

det(AB) = det(I)

det(A).det(B) = 1

Given det (A) = –1

then det (B) = –1

$$\begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = -1$$

$5(-2-3) - 2\alpha[0-\alpha] + 1[-2\alpha] = -1$

$2\alpha^2 - 2\alpha - 25 = -1$

$2\alpha^2 - 2\alpha - 24 = 0$

$(\alpha - 4)(\alpha + 3) = 0$; $\alpha = 4, -3$

Sum of value of $\alpha = 4 - 3 = 1$

Q.21 A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec, then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

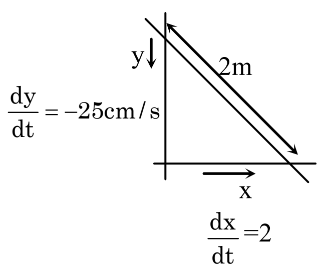
(1) $\frac{25}{3}$ (2) 25 (3) $25\sqrt{3}$ (4) $\frac{25}{\sqrt{3}}$

Ans. **[4]**

Sol. $x^2 + y^2 = 4$

Diff. w.r.t. t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



At y = 1; x = $\sqrt{3}$

then

$$\boxed{\frac{dx}{dt} = \frac{25}{\sqrt{3}} \text{cm/sec}}$$

Q.22 The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in :

(1) first, second and fourth quadrants
(2) first, third and fourth quadrants
(3) second and third quadrants only
(4) third and fourth quadrants only

Ans. **[4]**

Sol. $y = \sin x \sin(x + 2) - \sin^2(x + 1)$

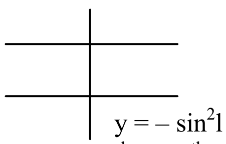
$$y = \frac{1}{2}[2\sin x \sin(x+2)] - \frac{1}{2}[2\sin^2(x+1)]$$

$$y = \frac{1}{2}[\cos(2) - \cos(2x+2)] - \frac{1}{2}[1 - \cos 2(x+1)]$$

$$y = \frac{1}{2}[\cos 2 - 1)]$$

$$y = (-)\frac{1}{2}(2)\sin^2 1$$

$$y = -\sin^2 1$$



Graph III[rd] & IV[th]

Q.23 If the truth value of the statement $p \to (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively :

(1) T, F, T
(2) F, T, T
(3) T, T, F
(4) T, F, F

Ans. **[3]**

Sol. $p \to (\sim q \vee r)$ is false
It is true when
$p \to T$ & $(\sim q \vee r) = \text{false}$
It will true : $\sim q$ false & r false
$\sim q \to F \mid r \to F$
$\Rightarrow q \to T$
Truth value of p, q, r $\Rightarrow$ T, T, F

Q.24 Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 17
(2) 1
(3) 137
(4) 121

Ans. **[3]**

Sol. Given : np = 8 ; nqp = 4

$$q = \frac{1}{2} \Rightarrow p = \frac{1}{2} \Rightarrow n = 16$$

$P(x \le 2) = P(x = 0) + P(x = 1) + (x = 2)$

$$= {}^{16}C_0 \left(\frac{1}{2}\right)^{16}\left(\frac{1}{2}\right)^{0} + {}^{16}C_1 \left(\frac{1}{2}\right)^{15}\left(\frac{1}{2}\right)^{1} + {}^{16}C_2 \left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{16}[1 + 16 + 120] = \frac{137}{2^{16}}$$

$\therefore k = 137$

Q.25 Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) – 320 (2) – 380 (3) – 410 (4) – 260

Ans. **[1]**

Sol. S_n = Sum of n terms of an A.P.

$S_4 = 16 = a + 3d$ …..(i)

$S_6 = -48 = a + 5d$ …..(ii)

From (i) & (ii)

$d = -32 \quad a = 112$

$S_{10} = \frac{10}{2}[2.(112) + (10-1)(-32)] = 5[-64]$

$S_{10} = -320$

Q.26 Let f : R → R be a continuously differentiable function such that f(2) = 6 and f(2) = $\frac{1}{48}$. If $\int_6^{f(x)} 4t^3\,dt = (x-2)g(x)$, then $\lim\limits_{x\to 2} g(x)$ is equal to :

(1) 18 (2) 36 (3) 12 (4) 24

Ans. **[1]**

Sol. $\int_6^{f(x)} 4t^3.dt\ (x-2)g(x)$; f(2) = 6 ; f'(2) = $\frac{1}{48}$

$g(x) = \frac{\int_6^{f(x)} 4t^3 dt}{(x-2)}$

$\lim\limits_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{(x-2)}$ at x = 2 $\frac{0}{0}$ form

L-hospital rule

$\lim\limits_{x\to 2} \frac{4[f(x)]^3.f'(x)}{1}$

At x = 2, $4[f(2)]^3.f'(2)$

$\Rightarrow 4(6)^3\left(\frac{1}{48}\right) = 18$

Q.27 The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

(1) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$ (2) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$ (3) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$ (4) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Ans. **[4]**

Sol. $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\left[\frac{12}{13}\sqrt{1-\frac{9}{25}} - \frac{3}{5}\sqrt{1-\frac{144}{169}}\right]$

$= \sin^{-1}\left(\frac{33}{65}\right)$

$= \cos^{-1}\left(\frac{56}{65}\right)$

$= \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Q.28 If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

(1) $-\frac{1}{\sqrt{3}}$ (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

Ans. **[3]**

Sol. Volume of parallelopiped

$$\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = 1[1 - 0] - \lambda[-\lambda^2] + 1[-\lambda]$$

$V = |1 + \lambda^3 - \lambda|$

$\frac{dV}{d\lambda} = 3\lambda^2 - 1 \Rightarrow \frac{dV}{d\lambda} = 0 \Rightarrow \lambda = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

$\frac{d^2V}{d\lambda^2} = 6\lambda$ for minimum

$\frac{d^2V}{d\lambda^2} > 0$ at $\lambda = \frac{1}{\sqrt{3}}$

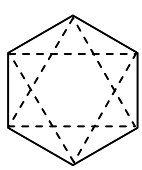
Q.29 If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\frac{1}{10}$ (2) $\frac{3}{10}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

Ans. **[1]**

Sol. Number of total triangle = ${}^6C_3$

Equilateral $\Delta = 2$



Prob. $= \frac{2}{{}^6C_3} = \frac{1}{10}$

Q.30 If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane 2x + 3y – z+13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to :

(1) $2\sqrt{7}$ (2) 14 (3) $2\sqrt{14}$ (4) $\sqrt{14}$

Ans. **[3]**

Sol. $\frac{x-2}{3}=\frac{y+1}{g}=\frac{z-1}{-1}=\lambda$

General point

$(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ = intersect plane

2x + 3y – z + 13 = 0 at P then

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$\lambda = -1$

P(– 1, – 3, 2)

Line intersect plane : 3x + y + 4z = 16 at Q then

$3(3\lambda + 2) + 2\lambda - 1 + 4(-\lambda + 1) = 16 \Rightarrow \lambda = 1$

Q(5, 1, 0) then PQ = $\sqrt{(5+1)^2+(1+3)^2+(0-2)^2}$

PQ = $2\sqrt{14}$