JEE Main Online Exam 2019

Questions & Solutions

10th January 2019 | Shift - I

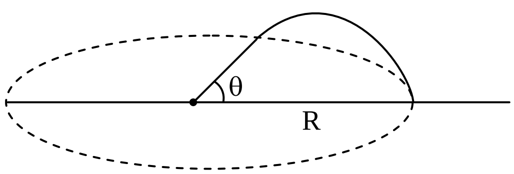
PHYSICS

Q.1 Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is -

(1) 1 : 16 (2) 1 : 8 (3) 1 : 2 (4) 1 : 4

Ans. **[1]**

Sol.



Area covered = πR^2

When R is maximum area covered is also maximum

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R_{max} = \frac{u^2}{g} \; (\theta = 45°)$$

Area covered $\propto u^4$

$$\text{Ratio} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Q.2 A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 $WK^{-1} m^{-1}$, the energy flux through it in the steady state is -

(1) 200 Wm^{-2} (2) 65 Wm^{-2} (3) 120 Wm^{-2} (4) 90 Wm^{-2}

Ans. **[4]**

Sol. Heat current $H = \frac{kA\Delta T}{L}$

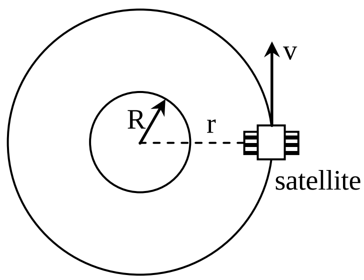
$$\text{Heat flux} = \frac{H}{A} = \frac{k\Delta T}{L} = \frac{0.1(900)}{1} = 90 \; Wm^{-2}$$

Q.3 A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is -

(1) mv^2 (2) $\frac{1}{2}mv^2$ (3) $\frac{3}{2}mv^2$ (4) $2\ mv^2$

Ans. **[1]**

Sol.



KE of revolving particle = $\frac{1}{2}mv^2$

Potential energy = – 2KE

= $-mv^2$

for escape out, total mechanical energy of particle should become zero.

KE + PE = 0

$KE - mv^2 = 0$

$KE = mv^2$

Q.4 A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower is LOS (Line of Sight) mode ? (Given : radius of earth = 6.4×10^6 m).

(1) 40 km (2) 65 km (3) 48 km (4) 80 km

Ans. **[2]**

Sol. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R} = \sqrt{2R}\,(\sqrt{h_T} + \sqrt{h_R})$

$d = \sqrt{2 \times 6400 \times 10^3}\,(\sqrt{140} + \sqrt{40}) = 65$ km

Q.5 A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is -

(1) 19/18 (2) 20/19 (3) 21/20 (4) 18/17

Ans. **[1]**

Sol. $f_2 = f_0\left(\dfrac{v}{v-17}\right)$

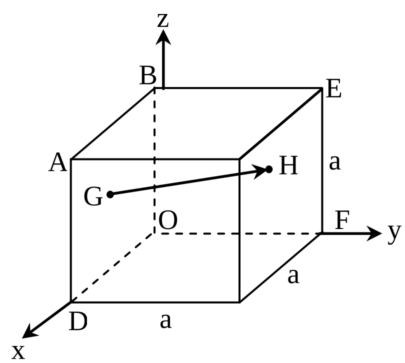
$f_1 = f_0\left(\dfrac{v}{v-34}\right)$

$\dfrac{f_2}{f_1} = \dfrac{v-34}{v-17}$

$= \dfrac{340-34}{340-17} = \dfrac{306}{323} = \dfrac{18}{19}$

$\dfrac{f_1}{f_2} = \dfrac{19}{18}$

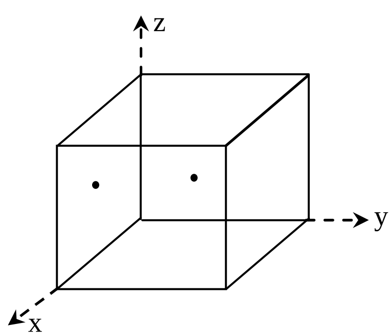
Q.6 In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be -



(1) $\frac{1}{2}a(\hat{k}-\hat{i})$ (2) $\frac{1}{2}a(\hat{j}-\hat{i})$ (3) $\frac{1}{2}a(\hat{j}-\hat{k})$ (4) $\frac{1}{2}a(\hat{i}-\hat{k})$

Ans. **[2]**

Sol.



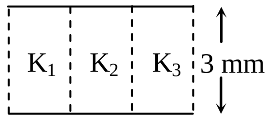
Let side of cube is a

coordinates of point 1 $\left(\frac{a}{2},0,\frac{a}{2}\right)$, $\vec{r}_1 = \frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}$

coordinates of point 2 $\left(0,\frac{a}{2},\frac{a}{2}\right)$, $\vec{r}_2 = \frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}$

$\vec{r}_2 = \vec{r}_1 = \frac{a}{2}\hat{j}-\frac{a}{2}\hat{i}$

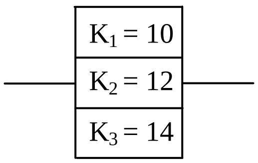
Q.7 A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1 = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be -



(1) 12 (2) 36 (3) 14 (4) 4

Ans. **[1]**

Sol.



$$C_{eq} = \frac{\varepsilon_0}{d}\left(\frac{A}{3}\times 10 + \frac{A}{3}\times 12 + \frac{A}{3}\times 14\right)$$

If whole slab is replaced by single dielectric K then $C = \frac{K \in_0 A}{d}$

Now $\frac{K \in_0 A}{d} = \frac{\varepsilon_0 A}{d}\left[\frac{10}{3}+\frac{12}{3}+\frac{14}{3}\right]$

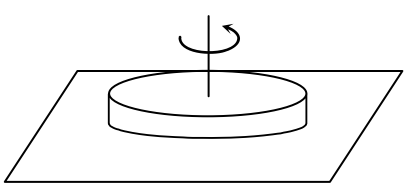
K = 12

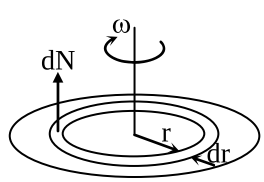
Q.8 To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is -

(1) μFR/2 (2) μFR/3 (3) μFR/6 (4) $\frac{2}{3}$ μFR

Ans. **[4]**

Sol.





$$dN = \frac{F}{\pi R^2}\times 2\pi r dr$$

$$dN = \frac{2F r dr}{R^2}$$

$$f_k = \mu dN$$

$$f_k = \frac{2Fr}{R^2}\mu dr$$

$d\tau_{f_k} = f_k \times r$

$$d\tau_k = \frac{2F\mu}{R^2} r^2 dr$$

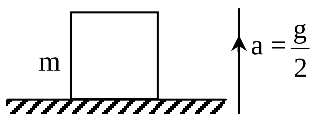
$$\tau_k = \int_0^R \frac{2F\mu}{R^2} r^2 dr$$

$$= \frac{2F\mu}{R^2} \frac{R^3}{3} = \frac{2F\mu R}{3}$$

τ_{ext} will counter balance the torque by friction (τ_k)

$$\therefore\ \tau_{ext} = \frac{2F\mu R}{3}$$

Q.9 A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction on block in time is -



(1) $\frac{mg^2t^2}{8}$ (2) $\frac{3mg^2t^2}{8}$ (3) $-\frac{mg^2t^2}{8}$ (4) 0

Ans. **[2]**

Sol. $N = m\left(g + \frac{g}{2}\right) = \frac{3mg}{2}$

$$S = \frac{1}{2} \times \frac{g}{2} t^2$$

$$\text{Work done} = \frac{3mg}{2} \times \frac{g}{4} t^2 = \frac{3mg^2}{8} t^2$$

Q.10 Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is -

(1) 2.9 cm (2) 5.1 cm (3) 4 cm (4) 1.7 cm

Ans. **[2]**

Sol. $10^{-4} = Ay$

$$10^{-4} = 10^{-4} \times \sqrt{2gh}$$

$$h = \frac{1}{2g} = \frac{100}{2 \times 9.8} = 5.1 \text{ cm}$$

Q.11 A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\hat{i}$ (cos ωt) where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is -

(1) 0.014 J (2) 0.028 J (3) 0.01 J (4) 0.007 J

Ans. **[1]**

Sol. $M = 10^{-2}\hat{i}$

$B = 1\cos(0.125t)\ \hat{i}$

at t = 1 sec

$B = 1\cos(0.125)\ \hat{i}$

$= \cos\left(\frac{125}{1000}\right)\hat{i} = \cos\left(\frac{1}{8}\right)\hat{i}$

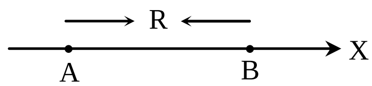
$= 1$

$W_{ext} = [-MB\cos 180° - (-MB\cos 0°)]$

$= MB + MB = 2MB$

$= 2 \times 10^{-2} \times \cos\left(\frac{1}{8}\right) = 0.019$

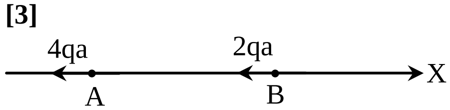
Q.12 Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure. The distance from A at which both of them produce the same potential is -



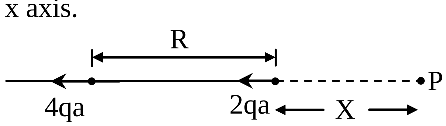
(1) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (2) $\frac{R}{\sqrt{2}+1}$ (3) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (4) $\frac{R}{\sqrt{2}-1}$

Ans. **[3]**

Sol.



On the x axis in left of A potential due to A and B is positive. But potential due to A is higher than that of B, between A & B sign of potential due to A & B is opposite. So potential can be same only in right of B on x axis.



$-\frac{k2qa}{x^2} = -\frac{k4qa}{(R+x)^2}$

$\frac{R+x}{x} = \sqrt{2}$

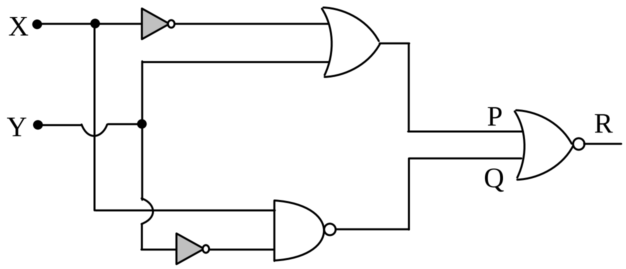
$R + x = x\sqrt{2}$

$x = \frac{R}{\sqrt{2}-1}$

distance from A = $R + \frac{R}{\sqrt{2}-1} \Rightarrow \frac{\sqrt{2}R}{\sqrt{2}-1}$

NTA has given the answer (1) but answer should be (3).

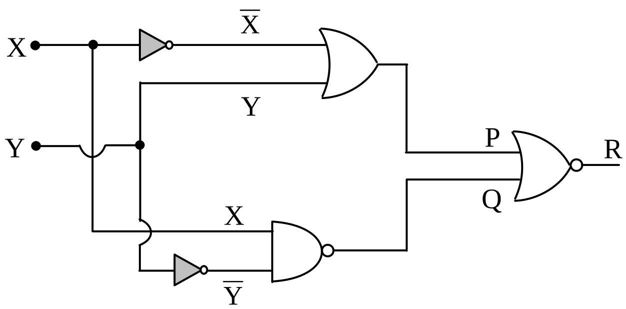
Q.13 To get output 'I' at R, for the given logic gate circuit the input values must be -



(1) X = 0, Y = 0 (2) X = 1, Y = 0 (3) X = 0, Y = 1 (4) X = 1, Y = 1

Ans. **[2]**

Sol.



Truth Table

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X}+Y$	$\overline{X.\overline{Y}}$	R
0	0	1	1	1	1	0
1	0	0	1	0	0	1
0	1	1	0	1	1	0
1	1	0	0	1	1	0

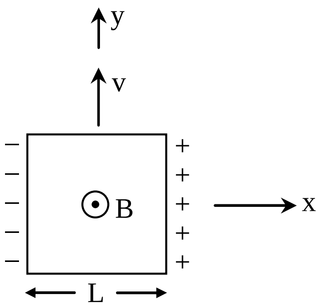
$\therefore$ Ans. X = 1
Y = 0

Q.14 A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is -

(1) 2 mV (2) 12 mV (3) 6 mV (4) 1 mV

Ans. **[2]**

Sol.



Due to magnetic force on electron charge separation get develop as shown in figure

At steady state

eVB = eE

E = VB

Potential difference between two faces = EL = VBL

$= 0.1 \times 6 \times 2 \times 10^{-2}$

= 12 mV

Q.15 In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are -

(1) 400 nm, 500 nm (2) 625 nm, 500 nm (3) 380 nm, 500 nm (4) 380 nm, 525 nm

Ans. **[2]**

Sol. $d \sin\theta = n\lambda$

In question $\theta = \frac{1}{40}$ which is very small

$\therefore \sin\theta = \theta = \frac{1}{40}$

$d \times \theta = n\lambda$

$n = \frac{d\theta}{\lambda} = \frac{0.1\text{mm}}{40\lambda}$

$n = \frac{2500}{\lambda}$ nm

When $\lambda = 380$

$n_1 = \frac{2500}{350} = 6.578$

for $\lambda = 740$ nm

$n_2 = \frac{2500}{740} = 3.378$

$\therefore$ n = 4, 5, 6

for n = 4, λ = 625 nm

for n = 5, λ = 500 nm

Q.16 The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is -

(1) 40 (2) 640 (3) 16 (4) 410

Ans. **[1]**

Sol. $[\text{Density}] = [ML^{-3}]$

$128\,[\text{kg}]\,[\text{m}]^{-3} = n\frac{[50\text{kg}]}{1000}\left[\frac{1}{4}\text{m}\right]^{-3}$

n = 40

Q.17 Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to -

(1) 200 (2) 150 (3) 400 (4) 360

Ans. **[1]**

Sol. $A_0 = 1600$ cps

at t = 8 second

$100 = \frac{1600}{2^n}$

$2^n = 16 = 2^4$

$n = 4 = \frac{8}{T_{1/2}}$

$T_{1/2} = 2$ second

In t = 6 sec number of half life's = 3

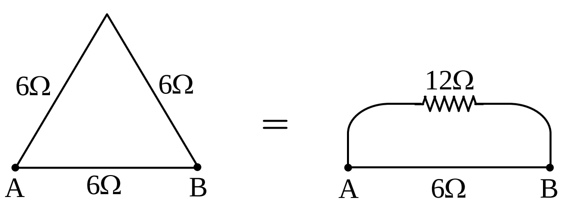
$\therefore\ A = \frac{1600}{2^3} = 200$ cps

Q.18 A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is -

(1) 12 Ω (2) 2 Ω (3) 4 Ω (4) 8 Ω

Ans. **[3]**

Sol. A uniform wire resistance is 18Ω. When it is bended into equilateral triangle then length of each side is same and its resistance is $\frac{18}{3} = 6\Omega$.



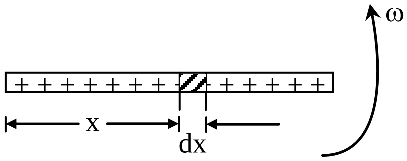
$R_{eqAB} = \frac{12 \times 6}{12 + 6} = \frac{72}{18} = 4\Omega$

Q.19 An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is -

(1) $\frac{\pi}{3} n \rho l^3$ (2) $\frac{\pi}{4} n \rho l^3$ (3) $n \rho l^3$ (4) $\pi n \rho l^3$

Ans. **[2]**

Sol.



Charge of element = dq = ρdx = $\frac{\rho_0 x}{l}$ dx

Magnetic moment of this charge element = $\frac{dqvr}{2}$

$$dM = \frac{\rho_0 x dx}{l} \times \omega x \times \frac{x}{2}$$

$$dM = \frac{\rho_0 \omega x^3 dx}{2l}$$

$$M = \int_0^{\ell} \frac{\rho_0 \omega x^3 dx}{2l} = \frac{\rho_0 \omega}{2l} \int_0^{\ell} x^3 dx$$

$$= \frac{\rho\omega}{2l} \frac{l^4}{4} = \frac{\rho_0 \omega l^3}{8} \text{ (Put } \omega = 2\pi n)$$

$$= \frac{2\pi n \rho_0 \omega l^3}{8} = \frac{\pi}{4} n \rho l^3$$

Q.20 A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as -

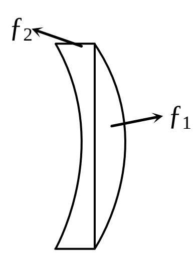
(1) $3\mu_2 - 2\mu_1 = 1$ (2) $\mu_1 + \mu_2 = 3$ (3) $2\mu_1 - \mu_2 = 1$ (4) $2\mu_2 - \mu_1 = 1$

Ans. **[3]**

Sol. $f_1 = 2f_2$

$$\frac{1}{f_1} = \left(\frac{\mu_1 - 1}{1}\right)\frac{1}{R}$$

$$\frac{1}{f_2} = \left(\frac{\mu_2 - 1}{1}\right)\left(-\frac{1}{R}\right)$$



$$f_2 = -\frac{R}{\mu_2 - 1}; \quad f_1 = \frac{R}{\mu_2 - 1}$$

$f_1 = 2f_2$

$$\frac{R}{\mu_1 - 1} = \frac{2R}{\mu_2 - 1}$$

$\mu_2 - 1 = 2\mu_1 - 2$

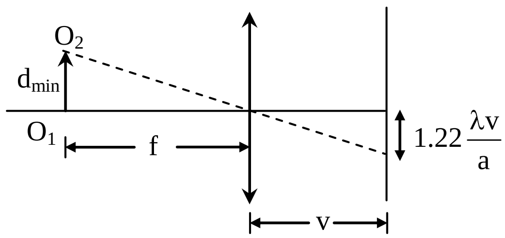
$1 = 2\mu_1 - \mu_2$

Q.21 In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to -

(1) 25 keV (2) 500 keV (3) 100 keV (4) 1 keV

Ans. **[1]**

Sol.



$\frac{d}{f} = \frac{1.22}{v}\frac{\lambda v}{a}$

$d = 1.22\frac{\lambda}{a}$

d_{min} is of order of wavelength of light = 7.5×10^{-12} = 0.075 Å

$7.5 \times 10^{-12} = \frac{h}{\sqrt{2mKE}}$

$0.075 = \frac{12.26}{\sqrt{KE}}$

$KE = \left(\frac{12.26}{0.075}\right)^2$

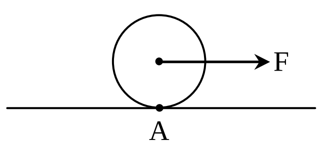
$= 25$ keV

Q.22 A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is -

(1) $\frac{F}{2mR}$ (2) $\frac{2F}{3mR}$ (3) $\frac{3F}{2mR}$ (4) $\frac{F}{3mR}$

Ans. **[2]**

Sol.



$\tau_{abtA} = I_A\alpha$

$F \times R = \left(\frac{1}{2}MR^2 + MR^2\right)\alpha$

$\alpha = \frac{2F}{3mR}$

Note : In this question value of force and its point of application is not given so its answer can not be given. We are giving its answer by assuming that F is applied at centre of cylinder.

Q.23 A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to -

(1) 16.6 cm (2) 10.0 cm (3) 20.0 cm (4) 33.3 cm

Ans. **[3]**

Sol. Speed of wave = $\sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{\frac{5\times10^{-3}}{1}}} = \sqrt{\frac{8\times10^{3}}{5}}$ = 40 m/s

$v = F\lambda$

$\lambda = \frac{40}{100}$ metre

Distance between two consecutive node = $\frac{\lambda}{2} = \frac{20}{100}$ metre = 20 cm

Q.24 A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is -

(1) 0.4 mA (2) 20 mA (3) 63 mA (4) 100 mA

Ans. **[2]**

Sol. From colour code table

$R = 50 \times 10^{2} = 500\ \Omega$

Power = 2 watt

$2 = i^2R$

$2 = i^2 \times 5000$

$\frac{0.4}{1000} = i^2$

$i = \frac{2}{100}$ Amp = 20 mA

Q.25 If the magnetic field of a plane electromagnetic wave is given by (the speed of light = 3×10^8 m/s) $B = 100 \times 10^{-6} \sin\left[2\pi\times2\times10^{15}\left(t-\frac{x}{c}\right)\right]$ then the maximum electric field associated with it is -

(1) 4.5×10^4 N/C (2) 4×10^4 N/C (3) 6×10^4 N/C (4) 3×10^4 N/C

Ans. **[4]**

Sol. $B = 100 \times 10^{-6}\ \sin\left[2\pi\times2\times10^{15}\left(t-\frac{x}{c}\right)\right]$

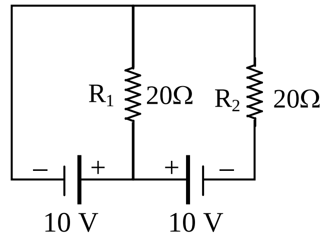
Maximum magnetic field = $B_0 = 100 \times 10^{-6}$ Tesla

$E_0 = cB_0$

$E_0 = 3 \times 10^8 \times 100 \times 10^{-6}$

$= 3 \times 10^4$ N/C

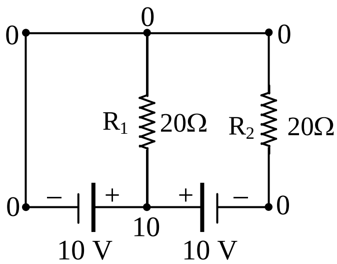
Q.26 In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are -



(1) 0.5, 0 (2) 0, 1 (3) 1, 2 (4) 2, 2

Ans. **[1]**

Sol.



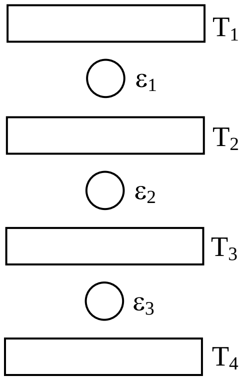
Potential difference across R_1 = 10 – 0 = 10 volt

i through $R_1 = \frac{10}{20}$ = 0.5 Amp

Potential difference across R_2 = 0 – 0 = 0 volt

i through R_2 = 0

Q.27 Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if -



(1) $T_2 = (T_1^3T_4)^{1/4}$; $T_3 = (T_1T_4^3)^{1/4}$

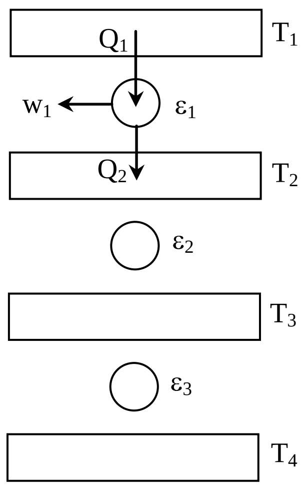
(2) $T_2 = (T_1T_4)^{1/2}$; $T_3 = (T_1^2T_4)^{1/3}$

(3) $T_2 = (T_1T_4^2)^{1/3}$; $T_3 = (T_1^2T_4)^{1/3}$

(4) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

Ans. **[4]**

Sol.



efficiency of carnot engines = $\eta = 1 - \frac{T_2}{T_1}$

$\eta_1 = 1 - \frac{T_2}{T_1}$

$\eta_2 = 1 - \frac{T_3}{T_2}$

$\eta_3 = 1 - \frac{T_4}{T_3}$

As $\eta_1 = \eta_2$

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$T_2 = \sqrt{T_1T_3}$$

As $\eta_2 = \eta_3$

$$1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$T_3 = \sqrt{T_2T_4}$$

As $\eta_1 = \eta_3$

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_4}{T_3} \text{ Hence } T_2T_3 = T_1T_4$$

$$\frac{T_1T_4}{T_3} = \sqrt{T_1T_3} \Rightarrow \frac{T_1^2T_4^2}{T_3^2} = T_1T_3$$

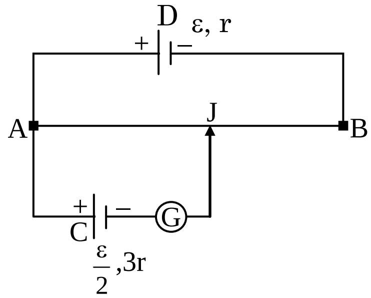
$$T_3 = (T_1T_4^2)^{1/3}$$

$$(T_1T_4^2)^{1/3} = (T_2T_4)^{1/2}$$

$$T_1^{2/3}T_4^{4/3} = T_2T_4$$

$$T_2 = (T_1^2T_4)^{1/3}$$

Q.28 A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is –



(1) $\frac{11}{12}L$ (2) $\frac{13}{24}L$ (3) $\frac{5}{12}L$ (4) $\frac{11}{24}L$

Ans. **[2]**

Sol. y = Potential gradient = $\frac{1R}{L}$

$$= \frac{\varepsilon}{12r + r} = \frac{12r}{L}$$

$$= \frac{12}{13}\frac{\varepsilon}{L}$$

At balanced condition emf of cell = $y\ell$

$$\frac{\varepsilon}{2} = \frac{12}{13}\frac{\varepsilon}{L} \times \ell$$

$$\ell = \frac{13}{24}L$$

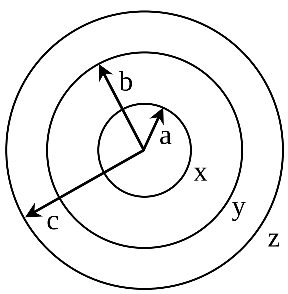
Q.29 A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be -

(1) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$ (2) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

(3) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$ (4) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

Ans. **[4]**

Sol.



$x + y + z = Q$

$\frac{x}{4\pi a^2} = \frac{y}{4\pi b^2} = \frac{z}{4\pi c^2}$ (surface charge density for all are same)

$x : y : z = a^2 : b^2 : c^2$

$x = \frac{a^2 Q}{a^2 + b^2 + c^2}$; $y = \frac{b^2 Q}{a^2 + b^2 + c^2}$; $z = \frac{c^2 Q}{a^2 + b^2 + c^2}$

V at r < a = $\frac{kx}{a} + \frac{ky}{b} + \frac{kz}{c}$

$= \frac{1}{4\pi \in_0}\left[\frac{Qa}{a^2+b^2+c^2} + \frac{Qb}{a^2+b^2+c^2} + \frac{Qc}{a^2+b^2+c^2}\right]$

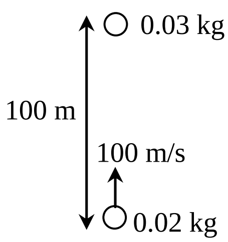
$= \frac{Q}{4\pi \in_0}\left[\frac{a+b+c}{a^2+b^2+c^2}\right]$

Q.30 A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is - ($g = 10\ ms^{-2}$)

(1) 30 m (2) 40 m (3) 20 m (4) 10 m

Ans. **[2]**

Sol.



Bullet will collide with piece of wood at t = 1 second

at t = 1 second

Velocity of piece of wood = u = 0 + 10 × 1 = 10 m m/s

Velocity of bullet = v = 100 – 10 × 1 = 90 m/s↑

from momentum conservation ($p_i = p_f$)

$p_i = 90 \times 0.02 - 0.03 \times 10 = 1.8 - 0.3 = 1.5$

$p_f = (0.03 + 0.02)\, v_f = 0.05\, v_f$

$\therefore\ v_f = \frac{1.5}{0.05} = 30$ m/s

Height from the point of collision H = $\frac{30 \times 30}{2 \times 10} = 45$ m

At time of collision system is h distance below top of tower where h = $\frac{1}{2} \times 10 \times 1^2 = 5$

$\therefore$ Height above tower = 45 – 5 = 40 metre

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th January 2019 | Shift - I

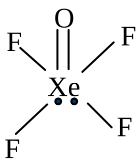
CHEMISTRY

Q.1 The type of hybridisation and number of lone pair (s) of electrons of Xe in $XeOF_4$, respectively, are:

(1) sp^3d and 2 (2) sp^3d^2 and 2 (3) sp^3d and 1 (4) sp^3d^2 and 1

Ans. **[4]**

Sol.



number of b.p = 5

number of l.p = 1

sp^3d^2 hybridisation

Q.2 Which of the following is not an example of heterogenous catalytic reaction ?

(1) Ostwald's process (2) Combustion of coal

(3) Hydrogenation of vegetable oils (4) Haber's process

Ans. **[2]**

Sol. Combustion of coal donot use heterogeneous catalyst

Ostwald process	V_2O_5 (s)
Hydrogenation of vegetable oils	Ni (s)
Haber process	Fe/ Mo(s)

Q.3 Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light ?

$CH_3 - CH_2 - CH = CH_2$

δ γ β α

(E)

(1) β – hydrogen (2) α – hydrogen (3) γ – hydrogen (4) δ– hydrogen

Ans. **[3]**

Sol.

$$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH_2} \;(E) \xrightarrow{Br_2/h\nu} CH_3 - \dot{C}H - CH = CH_2 \longrightarrow \underset{\delta}{CH_3} - \underset{\gamma}{\overset{Br}{\overset{|}{CH}}} - \underset{\beta}{CH} = \underset{\alpha}{CH_2}$$

resonance stabilize free radical

Q.4 The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are :

(1) 3 and 2 (2) 2 and 1 (3) 2 and 0 (4) 3 and 1

Ans. **[4]**

Sol. Total number of isotopes of hydrgen are

$_1H^1$, $_1H^2$ $_1H^3$,

(P) (D) (T)

and only $_1H^3$ or T is an radioactive isotope

Q.5 A process has $\Delta H = 200$ J mol^{-1} and $\Delta S = 40$ JK^{-1} mol^{-1}. Out of the values given below, choose the minimum temperature above which the process will be spontaneous:

(1) 4 K (2) 20 K (3) 5 K (4) 12 K

Ans. **[3]**

Sol. For a process to be spontaneous , $\Delta G < O$

& $\Delta G = \Delta H - T\Delta S$

at equillibrium

$\Delta G = 0$, $\therefore \Delta H = T_e\Delta S$

$$\therefore \quad T_e = \frac{\Delta H}{\Delta S}$$

$$= \frac{200}{40} = 5 \text{ k}$$

$\therefore$ If $T > T_e$, $\Delta G < 0$ and the process will be spontaneous

Q.6 A mixture of 100 m mol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are : (Molar mass of Ca $(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1} , respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

(1) 13.6g, 0.28 mol L^{-1} (2) 13.6g, 0.14 mol L^{-1}

(3) 1.9g, 0.28 mol L^{-1} (4) 1.9g, 0.14 mol L^{-1}

Ans. **[3]**

Sol. $Ca(OH)_2 + Na_2SO_4 \rightarrow CaSO_4 + 2NaOH$

100 m mole 14 m mole

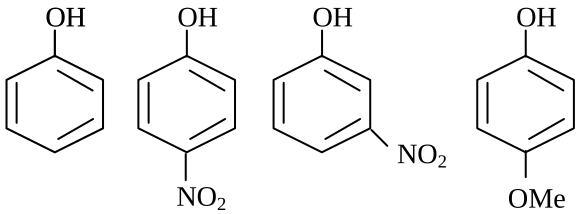
86 m mole – 14 m mole 28 m mole

$\therefore$ W $CaSO_4 = 14 \times 10^{-3} \times 136$

$= 1.9$ gm

$$[OH^-] = \frac{28}{100} = 0.28 \text{ M}$$

Q.7 The increasing order of the pKa values of the following compounds is :

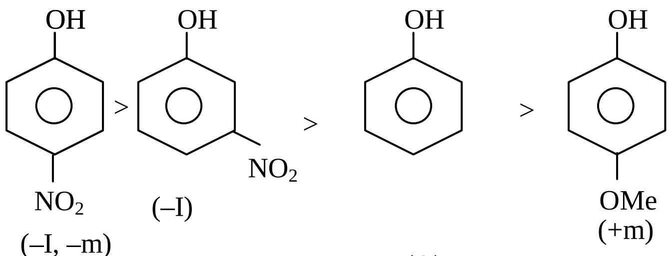


A B C D

(1) B < C < D < A (2) B < C < A < D (3) D < A < C < B (4) C < B < A < D

Ans. **[2]**

Sol.



(B) (C) (A) (D)

pKa order = B < C < A < D

Q.8 Hall- Heroult's process is given by :

(1) $Cu^{2+}(aq) + H_2(g) \rightarrow Cu(s) + 2H^+(aq)$ (2) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(3) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$ (4) $ZnO + C \xrightarrow{\text{Coke,1673K}} Zn + CO$

Ans. **[3]**

Sol. Hall heroult's process it given by

$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Q.9 The electronegativity of aluminium is similar to :

(1) Beryllium (2) Carbon (3) Boron (4) Lithium

Ans. **[1]**

Sol. EN of Al = 1.5

EN of Be = 1.5

Q.10 The metal used for marking X–ray tube window is :

(1) Ca (2) Na (3) Mg (4) Be

Ans. **[4]**

Sol. 'Be' metal is used in x ray tube window it is transparent to x rays

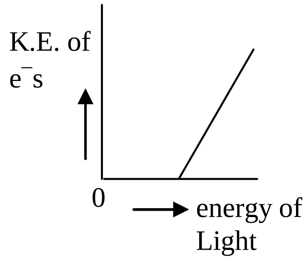
Q.11 Which premitive unit cell has unequal edge lengths ($a \neq b \neq c$) and all axial angles different from 90º?

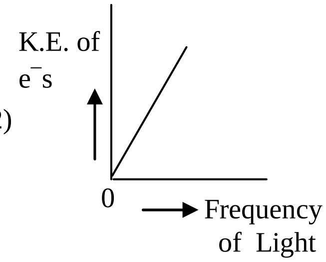
(1) Hexagonal (2) Tetragonal (3) Triclinic (4) Monoclinic

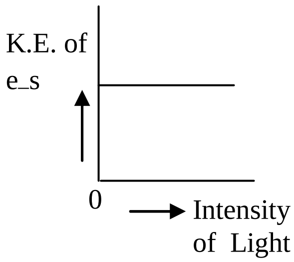
Ans. **[3]**

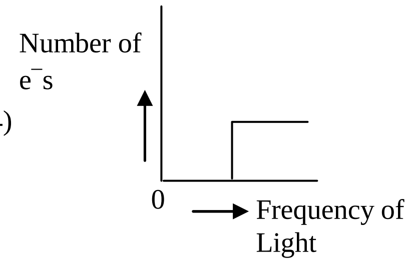
Sol. Triclinic primitive cell has unequal edge lengths & all axial angles different from 90º

Q.12 Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface ?

(1) 

(2) 

(3) 

(4) 

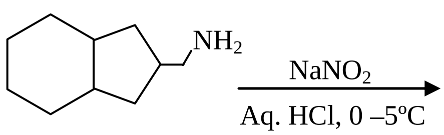
Ans. **[2]**

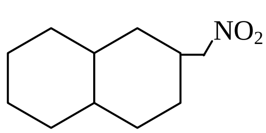
Sol. K.E. = $h\nu - h\nu_0$

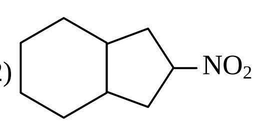
$y = me - c$

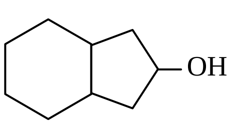
intercept is negative so option (2) is incorrect

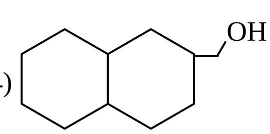
Q.13 The major product formed in the reaction given below will be :



(1) 

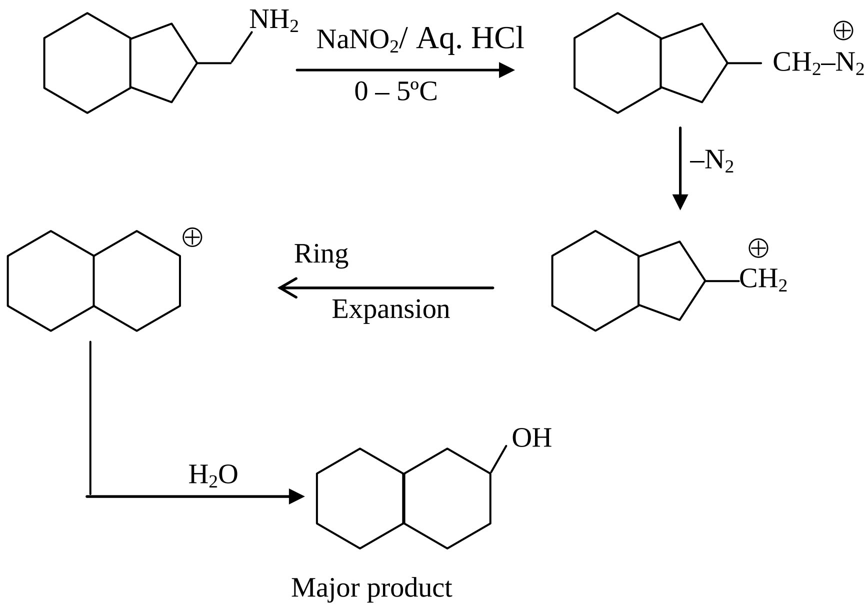
(2) 

(3) 

(4) 

Ans. [Question dropped]

Sol.



Q.14 The values of K_p/K_c for the following reactions at 300 K are, respectively : (At 300 K, RT = 24.62 dm^3 atm mol^{-1})

$N_2(g) + O_2(g) \rightleftharpoons 2\,NO(g)$

$N_2O_4(g) \rightleftharpoons 2\,NO(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2\,NH_3(g)$

(1) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2} 1.65 × 10^{-3} dm^{-6} atm^{-2} mol^2

(2) 1,4.1 × 10^{-2} dm^{-3} atm^{-1} mol, 606 dm^6 atm^2 mol^{-2}

(3) 1,24.62 dm^3 atm mol^{-1} , 1.65 × 10^{-3} dm^{-6} atm $^{-2}$ mol^2

(4) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}

Ans. **[3]**

Sol. $\frac{K_P}{Kc}\ (RT)^{\Delta n_g}$

for reaction I, $\Delta n_g = 0$

$\therefore K_P = Kc \Rightarrow \frac{K_P}{Kc} = 1$

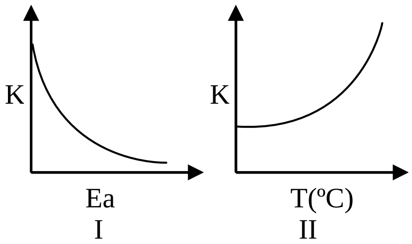
for reaction II, $\Delta n_g = 1$

$\therefore \frac{K_P}{Kc} = (RT)^1 = 24.62\ dm^3\ atm\ \ mol^{-1}$

for reaction III , $\Delta n_g = (-2)$

$\therefore \frac{K_P}{Kc} = (RT)^{-2} = 1.649 \times 10^{-3}\ dm^{-6}\ atm^{-2}\ mol^2$

Q.15 Consider the given plots for a reaction obeying Arrhenius equation (0ºC < T < 300ºC) : (K and E_a are rate constant and activation energy, respectively)



Choose the correct option :

(1) I is right but II is wrong
(2) Both I and II are correct
(3) Both I and II are wrong
(4) I is wrong but II is right

Ans. **[2]**

Sol. $K = Ae^{-E_a/RT}$

On increasing activation energy K decreases & on increasing temperature K increases.

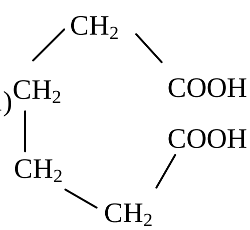
Q.16 If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ?

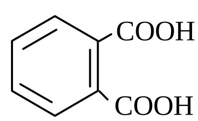
(1) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)
(2) DCM and H_2O will make turbid/ colloidal mixture
(3) DCM and H_2O would stay as lower and upper layer respectively in the S.F.
(4) DCM and H_2O will be miscible clearly

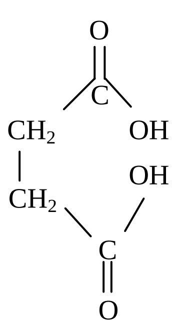
Ans. **[3]**

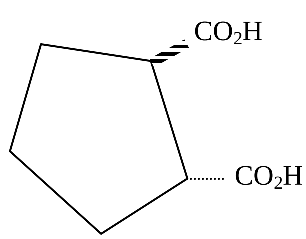
Sol. DCM has higher density and it is immiscible with water it will form lower layer and water will form upper layer in separating funnel

Q.17 Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

(1) 

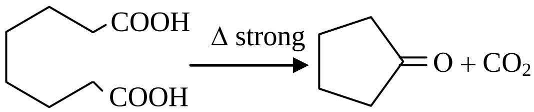
(2) 

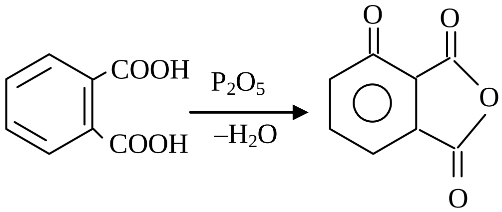
(3) 

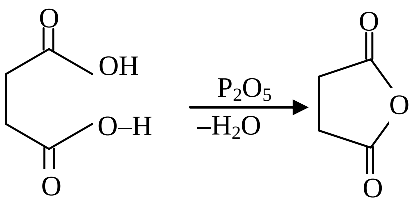
(4) 

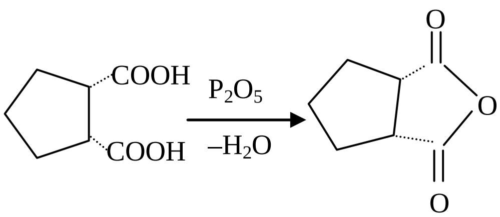
Ans. **[1]**

Sol.

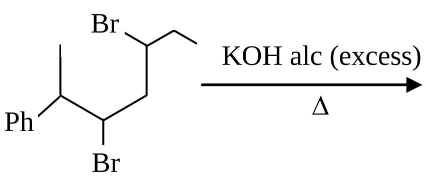
(1) 

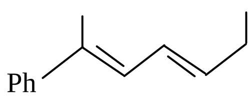
(2) 

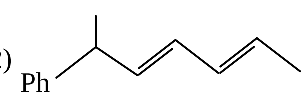
(3) 

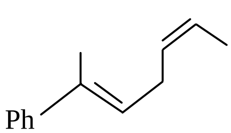
(4) 

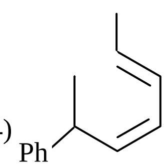
Q.18 The major product of the following reaction is



(1) 

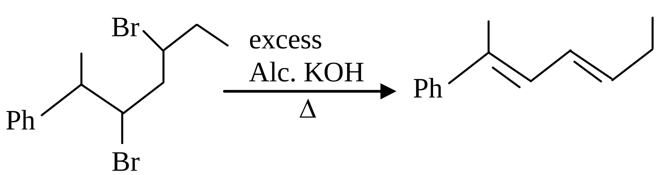
(2) 

(3) 

(4) 

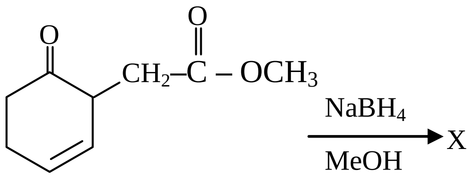
Ans. **[1]**

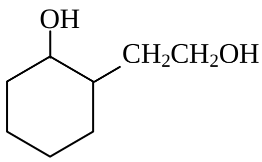
Sol.

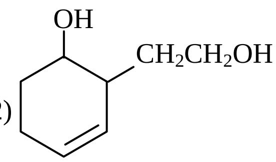


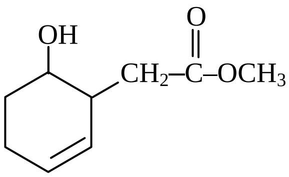
This is example of dehydrohalogenation by E_2 mechanism

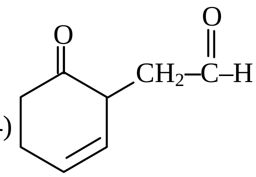
Q.19 The major product 'X' formed in the following reaction is :



(1) 

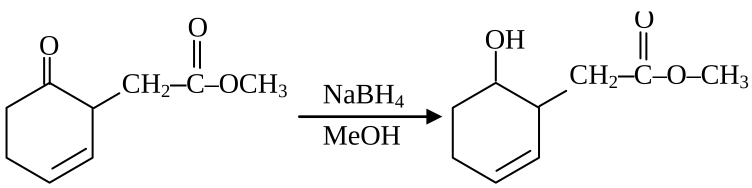
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



$NaBH_4$ is metal hydride it can reducte carbonyl group ($>C = O$) in alchol but it do not reduces ester and C = C

Q.20 Two pi and half sigma bonds are present in :

(1) O_2 (2) N_2^+ (3) O_2^+ (4) N_2

Ans. **[2]**

Sol. N_2^+ $(13e^-) = \sigma\ 1s^2, \sigma^*\ 1s^2, \sigma\ 2s^2, \sigma^*\ 2S^2,\ \ \pi\ 2p_x^{\ 2} = \pi\ 2p_y^{\ 2}\ \sigma\ 2\ p_z^1$

$N_2^+ \Rightarrow BO = 2.5 \Rightarrow [\pi \text{ -bond} = 2\ ,\ \sigma \text{ bond} = \frac{1}{2}]$

Similarly

$N_2 \Rightarrow BO = 3 \Rightarrow [\pi \text{ -bond} = 2\ ,\ \sigma \text{ bond} = 1]$

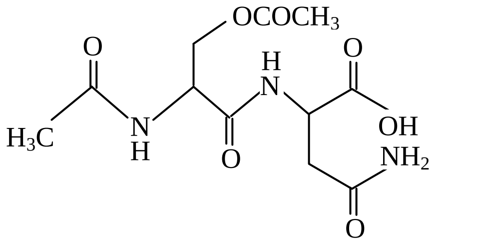
$O_2^+ \Rightarrow BO = 2.5 \Rightarrow [\pi \text{ -bond} = 1.5\ ,\ \sigma \text{ bond} = 1]$

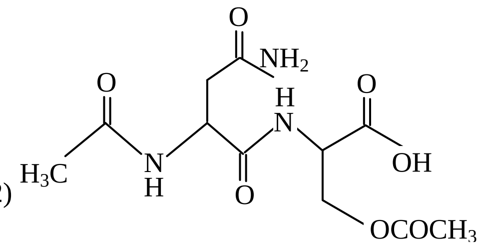
$O_2 \Rightarrow BO = 2 \Rightarrow [\pi \text{ -bond} = 1\ ,\ \sigma \text{ bond} = 1]$

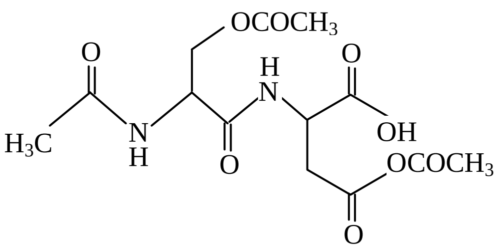
Q.21 The correct structure of product 'P' in the following reactions is :

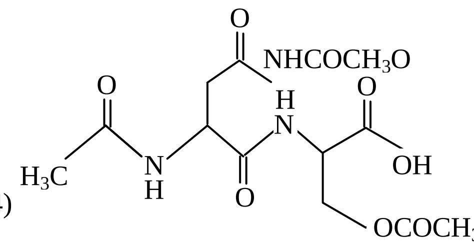
Asn – Ser + $(CH_3CO)_2O \xrightarrow{NEt_3}$ P

(excess)

(1) 

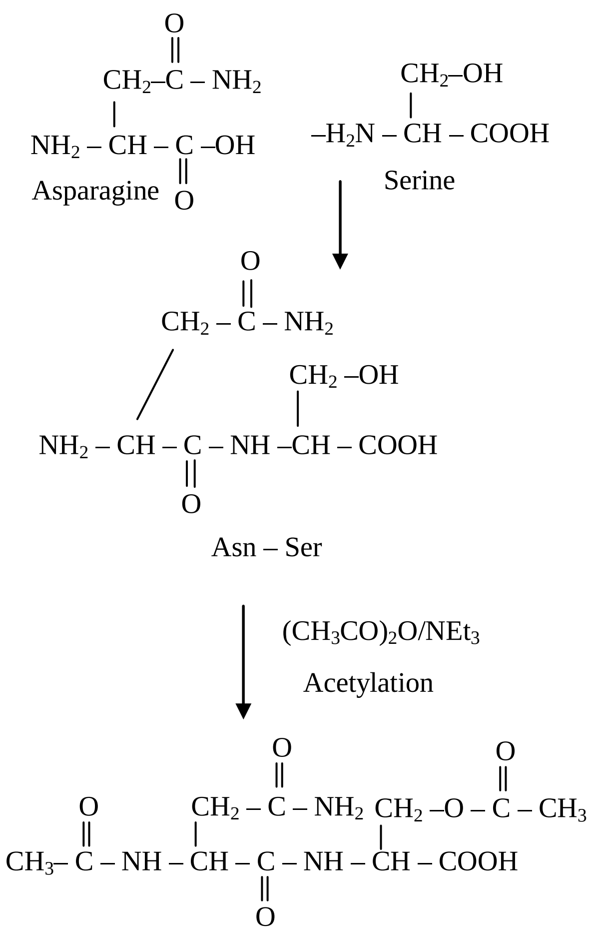
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Q.22 Wilkinson catalyst is : (Et = C_2H_5)

(1) $[(Ph_3P)_3\ RhCl]$

(2) $[(Et_3P)_3RhCl]$

(3) $[(Et_3P)_3IrCl]$

(4) $[(Ph_3P)_3IrCl]$

Ans. **[1]**

Sol. Wilkinson catalyst is $[(Ph_3P)_3\ RhCl]$

Q.23 The chemical nature of hydrogen peroxide is :

(1) Oxidising agent in acidic medium, but not in basic medium

(2) Oxidising and reducing agent in both acidic and basic medium

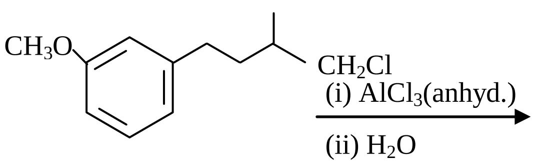
(3) Reducing agent in basic medium, but not in acidic medium

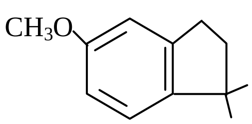
(4) Oxidising and reducing agent in acidic medium, but not in basic medium.

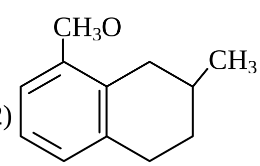
Ans. **[2]**

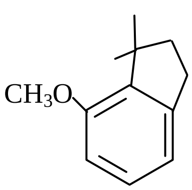
Sol. H_2O_2 act as O.A and R.A in both acidic and basic medium

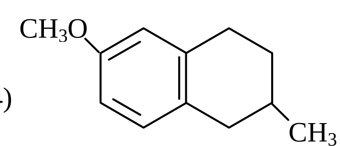
Q.24 The major product of the following reaction is :



(1) 

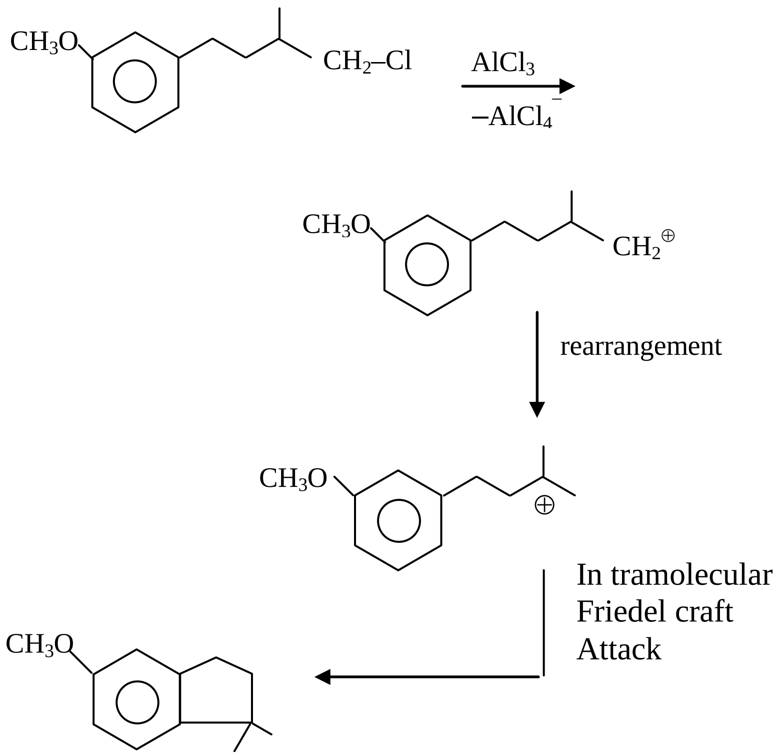
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.25 The effect of lanthanoid contraction in the lanthanoid series of elements by and large means :

(1) increase in atomic radii and decrease in ionic radii

(2) increase in both atomic and ionic radii

(3) decrease in both atomic and ionic radii

(4) decrease in atomic radii and increase in ionic radii

Ans. **[3]**

Sol. Due to lanthanoid contraction both atomic radii and ionic radius decreases gradualy in the lanthanoid series

Q.26 Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is :

(1) B is more polluted than A.

(2) A is suitable for drinking, whereas B is not .

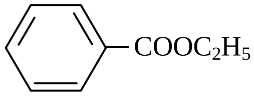
(3) Both A and B are suitable for dirnking

(4) A is more polluted than B.

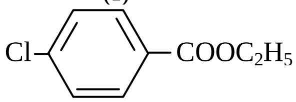
Ans. **[1]**

Sol. Dirnkable water would has BOD value of less than 5ppm where as highly polluted water could have a BOD valve of 17 ppM or more so B is more polluted than A.

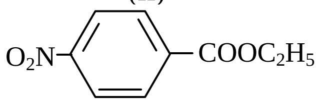
Q.27 The decreasing order of ease of alkaline hydrolysis for the following esters in



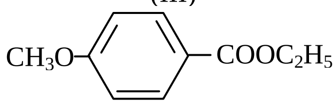
(I)



(II)



(III)



(IV)

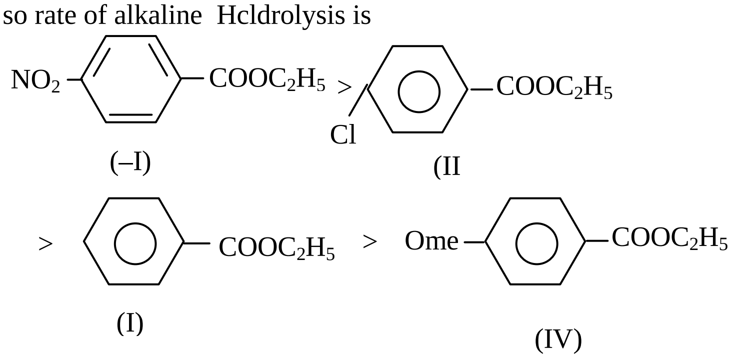
(1) III > II > IV > I (2)IV > II > III > I (3) III > II > I > IV (4) II > III > I > IV

Ans. **[3]**

Sol. Rate of alkaline Hydrolysis of ester occur through nucleophillic substitution by addition elimination in which attack of nucleophile is RDS

Rate of SNAE ∝ the charge on Alyl carbon

so rate of alkaline Hcldrolysis is



Q.28 Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vaapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively . The composition of the vapor in equilibrium with a solution containing 40 mole percent of A at this temperature is :

(1) $x_A = 0.76; x_B = 0.24$ (2) $x_A = 0.28; x_B = 0.72$

(3) $x_A = 0.4; x_B = 0.6$ (4) $x_A = 0.37; x_B = 0.63$

Ans. **[2]**

Sol. $Y_A = \frac{P_A}{P_T} = \frac{P_A^O X_A}{P_A^O X_A + P_B^O X_B}$

$= \frac{7\times10^3\times0.4}{7\times10^3\times0.4+12\times10^3\times0.6}$

$= 0.28$

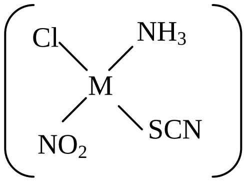
$\therefore\ y_B = 1 - 0.28 = 0.72$

Q.29 The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is :

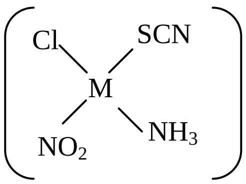
(1) 16 (2) 8 (3) 12 (4) 4

Ans. **[3]**

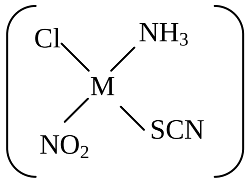
Sol.



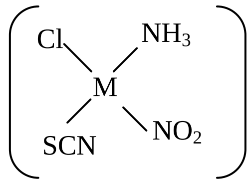
for above complex three G.I. are possible



(A)

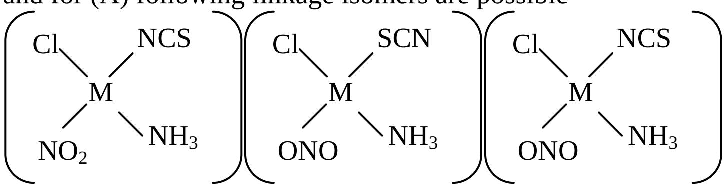


(B)

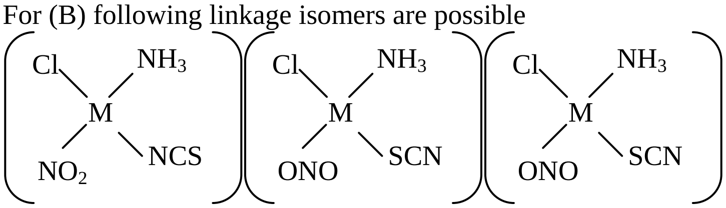


(C)

and for (A) following linkage isomers are possible -



For (B) following linkage isomers are possible



Similarly for (C) also three linkage isomers are present

Q.30 Consider the following reduction processes :

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; $E^o = -0.76$ V

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; $E^o = -2.87$ V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; $E^o = -2.36$ V

$Ni^{2} + 2e^- \rightarrow Ni(s)$; $E^o = -0.25$ V

The reducing power of the metals increases in the order :

(1) Ca < Mg < Zn < Ni (2) Ni < Zn < Mg < Ca

(3) Zn < Mg < Ni < Ca (4) Ca < Zn < Mg < Ni

Ans. **[2]**

Sol. Higher is the oxidation potential better is the tendency to get oxidised and better is the reducing power

$\therefore$ Ni < Zn < Mg < Ca is the correct reducing power of the metals

CAREER POINT

JEE Main Online Exam 2019

Questions & solutions

10th January 2019 | Shift - I

MATHEMATICS

Q.1 In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 42 (2) 102 (3) 1 (4) 38

Ans. **[4]**

Sol. Maths = {2, 4, 6, 8, …… , 138, 140} $\Rightarrow$ n(M) = 70

Physics = {3, 6, 9, …… , 138} $\Rightarrow$ n(P) = 46

Maths & physics = {6, 12, 18, …… , 138} $\Rightarrow$ $n(M \cap P) = 23$

Chem. = {5, 10, 15, 20, …… , 140} $\Rightarrow$ n(C) = 28

Maths & Chem = {10, 20, …… , 140} $\Rightarrow$ $n(M \cap C) = 14$

Phy & Chem = {15, 30, …… , 135} $\Rightarrow$ $n(P \cap C) = 9$

All three subject = {30, 60, ……120} = 4 $\Rightarrow$ $n(P \cap C \cap M) = 4 = n(\cup) - n(P \cup C \cup M)$

Number of students without any subject opted = 140 – (70) – 46 – 28 + 23 + 14 + 9 – 4 = 0 = 42 – 4 = 38

Q.2 Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k}$, and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is -

(1) (1, 5, 1) (2) (1, 3, 1) (3) $\left(-\frac{1}{2}, 4, 0\right)$ (4) $\left(\frac{1}{2}, 4, -2\right)$

Ans. **[3]**

Sol. $\vec{a}.\vec{c} = 0 \Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow \lambda_3 = -2\lambda_1 - 1$

$\vec{b} = 2\vec{a} \Rightarrow 3 - \lambda_2 = 2\lambda_1$

$\Rightarrow \lambda_2 = 3 - 2\lambda_1$

Let $\lambda_1 = 1 ; \lambda_2 = 1 ; \lambda_3 = -3$

$\lambda_1 = -\frac{1}{2} ; \lambda_2 = 4 ; \lambda_3 = 0$

$\lambda_1 = \frac{1}{2} ; \lambda_2 = 2 ; \lambda_3 = -2$

Q.3 Consider the statement : "P(n) : $n^2 - n + 41$ is prime". Then which one of the following is true ?

(1) P(5) is false but P(3) is true (2) Both P(3) and P(5) are true

(3) P(3) is false but P(5) is true (4) Both P(3) and P(5) are false

Ans. **[2]**

Sol. $P(n) = n^2 - n + 41$

$P(5) = 25 - 5 + 41 = 61$

$P(3) = 9 - 3 + 41 = 47$

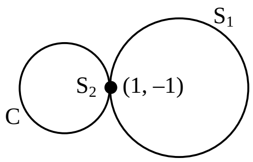
Both true

Q.4 If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, – 1), then the radius of C is -

(1) 5 (2) $2\sqrt{5}$ (3) 4 (4) $\sqrt{57}$

Ans. **[1]**

Sol.



Circle C $S_1 + \lambda S_2 = O$

$x^2 + y^2 + 4x - 6y - 12 + \lambda((x-1)^2 + (y+1)^2) = 0$

it passes through (4, 0)

$16 + 0 + 16 - 0 - 12 + \lambda(9 + 1) = 0$

$\lambda = -2$

$x^2 + y^2 + 4x - 6y - 12 - 2(x^2 + y^2 - 2x + 2y + 2) = 0$

$x^2 + y^2 - 8x + 10y + 16 = 0$

$r = \sqrt{16+25-16} = 5$

Q.5 If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which on of the following is a valid choice for the ordered triad (a, b, c) ?

(1) (1, 1, 3) (2) (1, 1, 0) (3) $\left(\frac{1}{2}, 2, 0\right)$ (4) $\left(\frac{1}{2}, 2, 3\right)$

Ans. **[1] or [1, 2, 3 or 4]**

Sol. $y^2 = 8ax$ Normal $y = mx - 4am - 2am^3$ …..(1)

$y^2 = 4b(x - c)$ Normal $y = m'(x - c) - 2bm' - 2bm'^3$

$y = m'x - m'c - 2bm' - 2bm'^3$ …..(2)

For common normal ; compare eq.(1) & eq.(2)

$$\frac{1}{1} = \frac{m}{m'} = \frac{4am + 2am^3}{m'c + 2bm' + bm'^3}$$

$m = m'$; $m'c + 2bm' + bm'^3 = 4am + 2am^3$

$m(m^2(2a - b) + 4a - 2b - c) = 0$

Case(I) m = 0

x-axis is common normal of both curve

All option are correct

(1), (2), (3), (4)

Case (II) $m \neq 0$

$$m^2 = \frac{2b+c-4a}{2a-b} > 0$$

$$= \frac{c}{2a-b} - 2 > 0$$

Only option (1), (1, 1, 3) satisfy it

Q.6 The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point -

(1) (1, 1, –1) (2) (1, 1, 1) (3) (–1, –1, –1) (4) (–1, –1, 1)

Ans. **[2]**

Sol. Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(-3-4) - \hat{j}(9-2) + \hat{k}(6+1)$

$$= -7\hat{i} - 7\hat{j} + 7\hat{k}$$

D.r.s. of normal of plane = 1, 1, – 1

Eq. of plane $1(x-4) + 1(y+1) - 1(z-2) = 0$

$x + y - z - 1 = 0$

Option (2) point (1, 1, 1) satisfy it

Q.7 Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta} d\theta$ is equal to -

(where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

Ans. **[3]**

Sol. put $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$$I = \int \frac{(t^n - t)^{1/n}}{t^{n+1}} dt = \int \frac{\left(1-\frac{1}{t^{n-1}}\right)^{1/n} dt}{t^n}$$

$$= \int x^{1/n} \frac{dx}{(n-1)} \qquad \text{put } 1 - \frac{1}{t^{n-1}} = x$$

$$= \frac{1}{(n-1)}\frac{x^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C \qquad \Rightarrow \quad (n-1)t^{-(n-1)-1}dt = dx \Rightarrow (n-1)\frac{dt}{t^n} = dx$$

$$= \frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$$

Q.8 The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is -

(1) 6 : 7 (2) 10 : 3 (3) 4 : 9 (4) 5 : 8

Ans. **[3]**

Sol. $\mu = 5$; variance = 9.20

$\mu = \frac{1+3+8+x_1+x_2}{5} = 5 \Rightarrow x_1 + x_2 = 13$

$9.20 = \frac{1}{n}\Sigma x_i^2 - \mu^2 \Rightarrow \Sigma x_i^2 = (9.20 + 25)5$

$1 + 9 + 64 + x_1^2 + x_2^2 = 171$

$x_1^2 + x_2^2 = 97 \Rightarrow x_1^2 + (13 - x_1)^2 = 97$

$(x_1 - 9)(x_1 - 4) = 0$

$x_1 = 9$; $x_2 = 4$

$x_1 = 4$; $x_2 = 9$

$x_1 : x_2 = 4 : 9$ or $9 : 4$

Q.9 If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals -

(1) 8 (2) 21 (3) 18 (4) 5

Ans. **[1]**

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha + 3 + 3 - 2 - 9 - \alpha = 0$

$\Rightarrow \alpha = 5$

Here $P_1 + P_3 = 2P_2$

For ∞ solution

$\frac{5+\beta}{2} = 9 \Rightarrow \beta = 13 \qquad \therefore \beta - \alpha = 8$

Q.10 If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to -

(1) $\frac{7}{4}$ (2) $\frac{5}{4}$ (3) $\frac{3}{4}$ (4) $\frac{3}{2}$

Ans. **[1]**

Sol. Sum of two sides > third side

$5 + 5r > 5r^2 \Rightarrow \quad r^2 - r - 1 < 0$

$$\left(r - \frac{1}{2}\right)^2 - \frac{5}{4} < 0$$

$$\left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$$

$$\frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2} \approx 1.618$$

$$\frac{7}{4} > \frac{1+\sqrt{5}}{2}$$

Q.11 An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ……, 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is -

(1) $\frac{19}{36}$ (2) $\frac{15}{72}$ (3) $\frac{13}{36}$ (4) $\frac{19}{72}$

Ans. **[4]**

Sol. Sum of number of dice is 7

(1, 6) , (2, 5) , (3,4)

(6, 1) , (5, 2) , (4,3)

Sum of number of dice is 8

(2, 6) , (3, 5) , (4, 4), (6, 2), (5, 3)

$P(\overset{\text{sum}}{7} \text{ or } 8) = \frac{11}{36}$

(Head + dice sum 7 or 8) (tail 1 + 7 or 8 appear)

$P(A) = \frac{1}{2}\left(\frac{11}{36}\right) + \frac{1}{2}\left(\frac{2}{9}\right)$

$= \frac{11}{72} + \frac{1}{9} = \frac{19}{72}$

Q.12 Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3),\ x \in R$. Then f(2) equals -

(1) 30 (2) – 2 (3) – 4 (4) 8

Ans. **[2]**

Sol. $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3),\ x \in R$

$f'(x) = 3x^2 + 2xf'(1) + f''(2) \Rightarrow f'(1) = 3 + 2f'(1) + f''(2)$

$\Rightarrow f'(1) + f''(2) + 3 = 0$

$f''(x) = 6x + 2f'(1) \Rightarrow f''(2) = 12 + 2f'(1)$

$\Rightarrow -f'(1) - 3 = 12 + 2f'(1)$

$\Rightarrow f'(1) = -5\ ;\ f''(2) = 2$

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

$f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

Q.13 The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is -

(1) $x - y + 9 = 0$ (2) $x - y - 3 = 0$ (3) $x - y + 1 = 0$ (4) $x - y + 7 = 0$

Ans. **[3]**

Sol. $\frac{x^2}{5} - \frac{y^2}{4} = 1$ $x - y = 2$

$y = mx \pm \sqrt{a^2m^2 - b^2}$ $m = 1$

$y = x \pm \sqrt{5-4}$

$x - y \pm 1 = 0$

Q.14 If $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals -

(1) $\frac{1}{3} + e^6$ (2) $\frac{1}{3}$ (3) $\frac{1}{3} + e^3$ (4) $-\frac{4}{3}$

Ans. **[1]**

Sol. $\frac{dy}{dx} + 3\sec^2 xy = \sec^2 x$ $x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

$IF = e^{\int 3\sec^2 x dx} = e^{3\tan x}$

$ye^{3\tan x} = \int e^{3\tan x} \sec^2 x dx$

$ye^{3\tan x} = \int e^t \frac{dt}{3}$ put $3\tan x = t \Rightarrow 3\sec^2 x dx = dt$

$ye^{3\tan x} = \frac{e^t}{3} + C = \frac{e^{3\tan x}}{3} + C$

$y\left(\frac{\pi}{4}\right) = \frac{4}{3} \Rightarrow \frac{4}{3}e^3 = \frac{e^3}{3} + C \Rightarrow C = e^3$

$y\left(\frac{-\pi}{4}\right)e^{-3} = \frac{e^{-3}}{3} + e^3 \Rightarrow y\left(\frac{-\pi}{4}\right) = \frac{1}{3} + e^6$

Q.15 A point P moves on the line 2x – 3y + 4 = 0. If Q(1, 4) and R (3, – 2) are fixed points, then the locus of the centroid of ΔPQR is a line -

(1) parallel to y-axis (2) with slope $\frac{2}{3}$ (3) parallel to x-axis (4) with slope $\frac{3}{2}$

Ans. **[2]**

Sol. $P\left(h, \frac{2h+4}{3}\right)$

Q (1, 4) Q(3, –2)

Centroid $G(\alpha, \beta) \equiv \left(\frac{h+4}{3}, \frac{2h+4+12-6}{9}\right)$

$\alpha = \frac{h+4}{3}$; $\beta = \frac{2h+10}{9}$; $\beta = \frac{2(3\alpha-4)+10}{9}$; $9\beta = 6\alpha + 2$

Locus α = x ; β = y ; $y = \frac{2}{3}x + \frac{2}{9}$; Slope = $\frac{2}{3}$

Q.16 The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is -

(1) 1356 (2) 1256 (3) 1365 (4) 1465

Ans. **[1]**

Sol. Remainder is 2 {16, 23, ……., 93} 12 terms

Remainder is 5 {12, 19, ……., 96} 13 terms

Sum = 6(16 + 93) + $\frac{13}{2}(12+96)$

= 1356

Q.17 If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$ then k is equal to

(1) 100 (2) 200 (3) 50 (4) 400

Ans. **[1]**

Sol. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{1}{(21)^3}\left(\frac{20.21}{2}\right)^2$

$\frac{100}{21} = \frac{k}{21}$

∴ k = 100

Q.18 Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, c ≠ 5. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is -

(1) 12 (2) 18 (3) 10 (4) 11

Ans. **[4]**

Sol. $f(x) = (c-5)x^2 - 2cx + (c-4) = 0$, c ≠ 5 f(0) f(2) < 0

$\Rightarrow$ (c – 4)(c – 24) < 0

$\Rightarrow$ c $\in$ (4, 24) …….(i)

f(2) f(3) < 0

$\Rightarrow$ (c – 24)(4c – 49) < 0

$\Rightarrow \frac{49}{4} < c < 24$ …….(ii)

Eq.(i) $\cap$ (ii)

$\frac{49}{4} < c < 24$

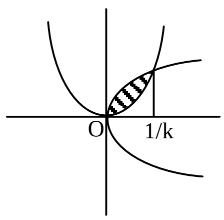
c$\in$ {13, 14, ……, 23} 11 elements

Q.19 If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, (k > 0), is 1 square unit. Then k is -

(1) $\sqrt{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{3}}$

Ans. **[4]**

Sol.



$y = kx^2$

$x = ky^2 \Rightarrow x = k(kx^2)^2 \Rightarrow x = k^3x^4 \Rightarrow x = 0$

$x = \frac{1}{k}$

$$\text{Area} = \int_0^{1/k} \sqrt{\frac{x}{k}} - kx^2 dx$$

$$= \frac{1}{\sqrt{k}}\frac{x^{3/2}}{3/2} - k\frac{x^3}{3}\Big|_0^{1/k} = 1 \Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

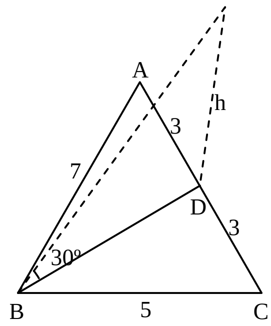
$k^2 = \frac{1}{3} \Rightarrow k = \frac{1}{\sqrt{3}}$

Q.20 Consider a triangular plot ABC with sides AB = 7m, BC = 5m and CA = 6m. A vertical lamp-post at the mid point D of AC subtends an angle 30º at B. The height (in m) of the lamp-post is -

(1) $2\sqrt{21}$ (2) $\frac{3}{2}\sqrt{21}$ (3) $7\sqrt{3}$ (4) $\frac{2}{3}\sqrt{21}$

Ans. **[4]**

Sol.



Median $m_B = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$

$= \frac{1}{2}\sqrt{2(49) + 2(25) - 36} = \sqrt{28}$

$\tan 30^o = \frac{h}{BD} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{\sqrt{28}}{\sqrt{3}} = \frac{2\sqrt{21}}{3}$ cm

Q.21 The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}$, $(x > 0)$, is -

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{5}{4}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{5}}{2}$

Ans. **[4]**

Sol. $(t, \sqrt{t})$ $\left(\frac{3}{2}, 0\right)$

$(\text{distance})^2 = \left(\frac{3}{2} - t\right)^2 + (\sqrt{t})^2 = (t - 1)^2 + \frac{5}{4}$

square of minimum (distance) $= \frac{5}{4}$ at (1, 1)

minimum distance $= \frac{\sqrt{5}}{2}$

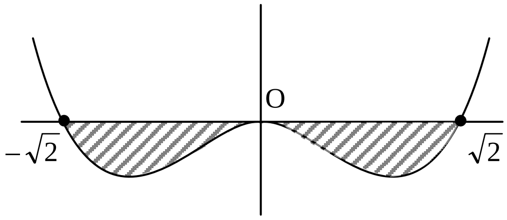
Q.22 Let $I = \int_a^b (x^4 - 2x^2)dx$. If I is minimum then the ordered pair (a, b) is -

(1) $(\sqrt{2}, -\sqrt{2})$ (2) $(0, \sqrt{2})$ (3) $(-\sqrt{2}, \sqrt{2})$ (4) $(-\sqrt{2}, 0)$

Ans. **[3]**

Sol. $I = \int_a^b (x^4 - 2x^2)dx$

$f(x) = x^4 - 2x^2 = x^2 (x - \sqrt{2})(x + \sqrt{2})$



$-\sqrt{2} \le x \le \sqrt{2}$ integration will be greatest negative

so for minimum value of I $\left.\begin{matrix} a = -\sqrt{2} \\ b = \sqrt{2} \end{matrix}\right)$

Q.23 For each $t \in R$, let [t] be the greatest integer less than or equal to t.

Then $\lim\limits_{x \to 1+} \dfrac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|.[1-x]}$

(1) equals –1 (2) equals 1 (3) equals 0 (4) does not exist

Ans. **[3]**

Sol. $\lim\limits_{x \to 1+} \dfrac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|.[1-x]}$

x = 1 + h

$= \lim\limits_{h \to 0} \dfrac{(1-1-h+\sin h)\sin\left(\frac{\pi}{2}(-1)\right)}{h(-1)}$

$= \lim\limits_{h \to 0} \left(\dfrac{\sin h - h}{-h}\right)(-1) = \lim\limits_{h \to 0} \dfrac{\sin h}{h} -1 = 1 - 1 = 0$

Q.24 Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1}$ then -

(1) Im(z) = 0 (2) $|z| = \sqrt{\dfrac{5}{2}}$ (3) $|z| = \dfrac{1}{2}\sqrt{\dfrac{17}{2}}$ (4) Re(z) = 0

Ans. **[Bonus]**

Sol. $3|z_1| = 4|z_2|$

$\Rightarrow 9|z_1|^2 = 16|z_2|^2$

$\Rightarrow 9z_1\bar{z}_1 = 16z_2\bar{z}_2$

$\Rightarrow \dfrac{3z_1}{2z_2} = \dfrac{8}{3}\left(\dfrac{\bar{z}_2}{\bar{z}_1}\right)$ let $\dfrac{3z_1}{2z_2} = re^{i\theta}$

$$re^{i\theta} = \frac{8}{3}\overline{\left(\frac{1}{\left(\frac{2}{3}re^{i\theta}\right)}\right)}$$

$$\Rightarrow re^{i\theta} = \frac{4}{r}e^{+i\theta}$$

$$\Rightarrow r^2 = 4 \Rightarrow r = 2$$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = re^{i\theta} + \frac{1}{re^{i\theta}}$$

$$z = 2e^{i\theta} + \frac{1}{2}e^{-i\theta}$$

$$z = \frac{5}{2}\cos\theta + i\frac{3}{2}\sin\theta$$

$$|z| = \sqrt{\frac{25}{4} + \frac{9}{4}} = \sqrt{\frac{17}{2}}$$

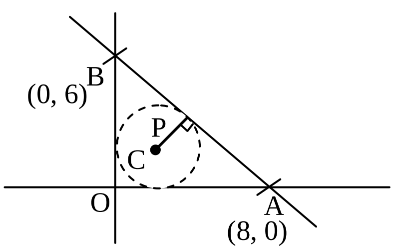
$\therefore$ No option are matched

Q.25 If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is -

(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

Ans. **[2]**

Sol.



Circle C(r, r), radius r, $3x + 4y - 24 = 0$ tangent

$p = r$

$$\left|\frac{3r + 4r - 24}{5}\right| = r$$

$$|7r - 24| = 5r$$

$7r - 24 = \pm 5r$; $r = 6$ (not possible) ; $r = 2$ centre (2, 2)

Q.26 The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is -

(1) $\frac{5\pi}{4}$ (2) $\frac{\pi}{2}$ (3) π (4) $\frac{3\pi}{8}$

Ans. **[2]**

Sol. $1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \cos^4 2\theta - \cos^2 2\theta + \frac{1}{4} = 0$

$\Rightarrow \left(\cos^2 2\theta - \frac{1}{2}\right)^2 = 0$

$\cos^2 2\theta = \frac{1}{2} = \cos^2 \frac{\pi}{4}$

$2\theta = n\pi + \frac{\pi}{4}$

$\theta = \frac{n\pi}{2} + \frac{\pi}{8} \qquad \theta \in \left(0, \frac{\pi}{2}\right)$

$n = 0 \qquad \theta = \frac{\pi}{8}$ ✓

$n = 1 \qquad \theta = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$ ×

$\theta = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}$ ✓

Sum of angles = $\frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

Q.27 Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane x – 4y + 3z = 1 is -

(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $-\frac{1}{4}$

Ans. **[3]**

Sol. $\overrightarrow{AB} = (3-1+3\mu)\hat{i} + (2-\mu+1)\hat{j} + (6-2-5\mu)\hat{k}$

$= (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

$\overrightarrow{AB} \perp$ normal of plane

$(2 + 3\mu)(1) + (3 - \mu)(-4) + (4 - 5\mu)(3) = 0$

$2 - 8\mu = 0 \Rightarrow \mu = \frac{1}{4}$

Q.28 Let $d \in R$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is -

(1) – 7 (2) $2(\sqrt{2}+2)$ (3) – 5 (4) $2(\sqrt{2}+1)$

Ans. **[3]**

Sol. $R_3 \to R_3 + R_1 - 2R_2$

$$\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & 2+\sin\theta & d \\ 1 & 0 & 0 \end{vmatrix}$$

$= d^2 + 4d + 4 - \sin^2\theta$

$\det(A) = (d+2)^2 - \sin^2\theta$

min of $\det(A) = 8 = (d+2)^2 - 1$

$(d+2)^2 = 9 \quad \Rightarrow \quad d + 2 = \pm 3$

(+), $d = 1$, ; (–) $d = -5$

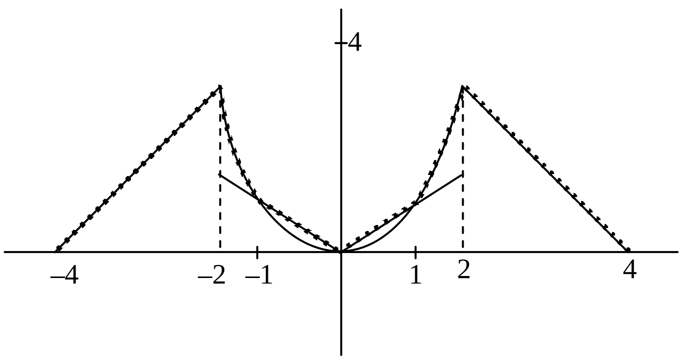
Q.29 Let $f(x) = \begin{cases} \max\{|x|, x^2\} & |x| \le 2 \\ 8 - 2|x| & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (– 4, 4) at which f is not differentiable. Then S

(1) equals {– 2, – 1, 1, 2}
(2) equals {– 2, – 1, 0, 1, 2}
(3) equals {– 2, 2}
(4) is an empty set

Ans. **[2]**

Sol.



not differentiable at $x = \pm 2, \pm 1, 0$

Q.30 If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is -

(1) $2\sqrt{2}$
(2) $4\sqrt{2}$
(3) $\frac{1}{8}$
(4) $\frac{1}{4}$

Ans. **[4]**

Sol. $(1 + x^{\log_2 x})^5$

$T_3 = {}^5C_2 \left(x^{\log_2 x}\right)^2 = 2560$

$x^{\log_2 x} = 16$

$\log_2 x \log_2 x = \log_2 16 = 4$

$\log_2 x = \pm 2$

$\Rightarrow \quad x = 2^2 = 4 x = 2^{-2} = \frac{1}{4}$