JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - II

(Memory Based)

PHYSICS

Q.1 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3\times10^8$ m/s) :

(1) 20×10^{-8} N (2) 35×10^{-8} N (3) 10×10^{-8} N (4) 15×10^{-8} N

Ans. **[1]**

Sol. $F = \frac{25}{100}\left(\frac{2I}{c}\right) + \frac{75}{100}\left(\frac{I}{c}\right)$

$= \frac{225}{100}\left(\frac{I}{c}\right)$

$= \frac{125}{100} \times \frac{50}{30\times10^8}$

$= 20.8 \times 10^{-8}$ N

Q.2 The physical sizes of the transmitter and receiver antenna in a communication system are :

(1) inversely proportional to modulation frequency
(2) proportional to carrier frequency
(3) inversely proportional to carrier frequency
(4) independent of both carrier and modulation frequency

Ans. **[3]**

Sol. By theory size of atenna of reciever and transmitter both inverse to carrier frequency

Q.3 A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :

(1) 10^{-3} (2) 10^{-1} (3) 10^{-4} (4) 10^{-2}

Ans. **[1]**

Sol. $\tau = \vec{M} \times \vec{B}$

$C\dot{\theta} = NIAB$

$10^{-6}\ \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175\ B$

$B = 10^{-3}$ T

Q.4 Two coil 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P' a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :

(1) 6.67×10^{-3} Wb (2) 3.67×10^{-4} Wb (3) 6.67×10^{-4} Wb (4) 3.67×10^{-3} Wb

Ans. **[3]**

Sol. $\phi = BA$

$$\phi_P = \frac{\mu_0 i_1 R^2}{2(R^2+x^2)^{3/2}}\,\pi r^2 = 10^{-3}$$

$$\phi_Q = \frac{\mu_0 i_2 r^2}{2(r^2+x^2)^{3/2}}\,\pi R^2$$

$$\frac{\phi_P}{\phi_q} = \frac{i_2}{i_1}\left(\frac{R^2+x^2}{r^2+x^2}\right)^{3/2}$$

$$\phi_P = \frac{2\times 10^{-3}}{3} = 6.67 \times 10^{-4}$$

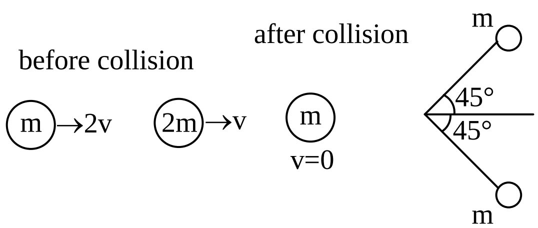
Q.5 A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be :

(1) $\sqrt{2}\,v$ (2) $v/(2\sqrt{2})$ (3) $v/\sqrt{2}$ (4) $2\sqrt{2}\,v$

Ans. **[4]**

Sol.



Momentum conservation in x directon

$2mv + 2mv = mv'\cos 45° + mv'\cos 45°$

$4v = \sqrt{2}\,v'$

$v' = 2\sqrt{2}\,v$

Q.6 A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :

(1) 0.7 (2) 0.5 (3) 0.8 (4) 0.6

Ans. **[4]**

Sol. For case 1

$mg = F_3$

$mg = m'g$

$m = \frac{4}{5} V d_w$

For case 2

$mg = F_{bw} + F_{bo}$

$m = \frac{v}{2} d_w + \frac{v}{2} d_o$

By equation 1 & 2

$\frac{4}{5} Vdw = \frac{V}{2} d_w + \frac{V}{2} d_o$

$\frac{d_0}{d_w} = \frac{6}{10} = 0.6$

Q.7 The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \frac{b^2}{4c}$ (2) $a + \frac{b^2}{c}$ (3) $a + \frac{b^2}{3c}$ (4) $a + \frac{b^2}{2c}$

Ans. **[3]**

Sol. $x = at + bt^2 - ct^3$

$v = a + 2bt - 3\,ct^2$

acceleration $= 2b - 6ct = 0$

$f = \frac{2b}{6c} = \frac{b}{3c}$

so velocity at $t = \frac{b}{3c}$

$v = a + 2b\,\frac{b}{3c} - 3c\,\frac{b^2}{9c^2}$

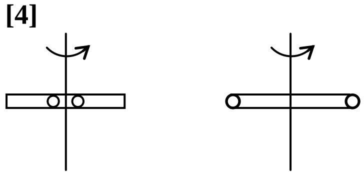
$= a + \frac{b^2}{3c}$

Q.8 A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be :

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+2m}$ (3) $\frac{M\omega_0}{M+m}$ (4) $\frac{M\omega_0}{M+6m}$

Ans. **[4]**

Sol.

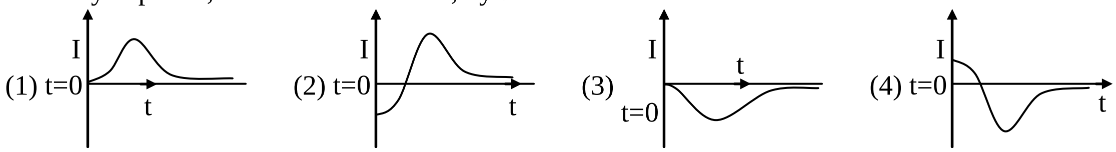


$J_1 = J_2$ (Angular momentum conservation)

$\frac{M\ell^2}{12}\omega_0 = \left(\frac{M\ell^2}{12} + \frac{2m\ell^2}{4}\right)\omega$

$\omega = \frac{M\omega_0}{M+6m}$

Q.9 A very long solenoid of readius R is carrying current $I(t) = kte^{-\alpha t}$ (k > 0), as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by :



Ans. **[2]**

Sol. $\phi = (\mu_0 nKte^{-\alpha t})\, 4\pi R^2$

$e = \dfrac{-d\phi}{dt} = -ce^{-\alpha t}(1 - \alpha t)$

$i_{induced} = \dfrac{-ce^{-\alpha t}(1-\alpha t)}{R}$

at t = 0

$i_{ind} \Rightarrow +V_c$

Q.10 Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of :

(1) 5 s (2) 3 s (3) 2.5 s (4) 2 s

Ans. **[4]**

Sol. $\omega_1 = 0,\ \alpha = 20$

$\omega_2 = \omega_1 + \alpha t$

$\omega_2 = 20\,t$

$\dfrac{1}{2} I\omega^2 = 1200$

$\dfrac{1}{2} 1.5 \times 400\, t^2 = 1200$

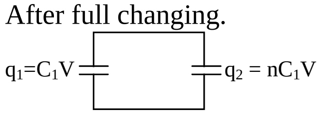
t = 2 sec

Q.11 The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :

(1) $\dfrac{V}{K+n}$ (2) $\dfrac{nV}{K+n}$ (3) $\dfrac{(n+1)V}{(K+n)}$ (4) V

Ans. **[3]**

Sol. After full changing.



$q = CV + nCV$

$= (n+1)\, CV$

Due to insertion of dielectric

$q_1 = KCV_C$

$q_2 = nCV_C$

$V_C = \dfrac{q_{total}}{C_{eff}}$

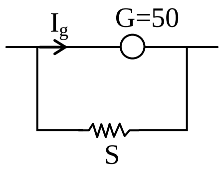
$= \dfrac{(n+1)CV}{KC+nC} = \dfrac{(n+1)}{K+n} V$

Q.12 The resistance of a galvanometer is 50 ohm and the maximum current which can be passed though it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0–0.5 A ?

(1) 0.2 ohm (2) 0.002 ohm (3) 0.02 ohm (4) 0.5 ohm

Ans. **[1]**

Sol.



$I_y = 0.002$ A

S (0.5 – 0.002) = 50 × 0.002

$S = \dfrac{0.1}{0.498} = 0.2$

Q.13 The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\,\hat{i} + (4-20t^2)\,\hat{j}$. What is the magnitude of the acceleration at t = 1 ?

(1) 100 (2) 40 (3) 50 (4) 25

Ans. **[3]**

Sol. $\vec{r}(t) = 15t^2\,\hat{i} + (4-20t^2)\,\hat{j}$

V = 30 t i – 40 tj

acceleration = 30 i – 40 j

$a = \sqrt{30^2 + (-40)^2}$

= 50

Q.14 A massless spring (k = 800 N/m) attached with mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(1) 10^{-3} K (2) 10^{-4} K (3) 10^{-5} K (4) 10^{-1} K

Ans. **[3]**

Sol. $\dfrac{1}{2}Kx^2 = (m\,SdT)_0 + (m\,SdT)_w$

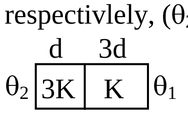
$\dfrac{1}{2}800\left(\dfrac{2}{100}\right)^2 = (0.5\times400+1\times4184)\,dT$

$\dfrac{1600}{4264\times10^4} = dT$

$dT = 3 \times 10^{-5}$

Order 10^{-5} K

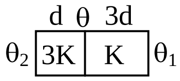
Q.15 Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness d and '3d' respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectivlely, ($\theta_2 > \theta_1$). The temperature at the interface is :



(1) $\dfrac{\theta_2+\theta_1}{2}$ (2) $\dfrac{\theta_1}{3}+\dfrac{2\theta_2}{3}$ (3) $\dfrac{\theta_1}{10}+\dfrac{9\theta_2}{10}$ (4) $\dfrac{\theta_1}{6}+\dfrac{5\theta_2}{6}$

Ans. **[3]**

Sol.



Heat current will be same in both

$H_1 = H_2$

$$\frac{3KA}{d}(\theta_2 - \theta) = \frac{KA}{3d}(\theta - \theta_1)$$

$9\theta_2 - 9\theta = \theta - \theta_1$

$$\theta = \frac{\theta_1 + 9\theta_2}{10}$$

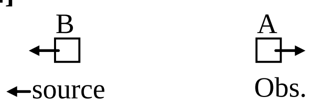
$$= \frac{\theta_1}{10} + \frac{9\theta_2}{10}$$

Q.16 Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B?
(speed of sound in air = 340 ms^{-1})

(1) 2060 Hz (2) 2150 Hz (3) 2300 Hz (4) 2250 Hz

Ans. **[4]**

Sol.



V_B = 20 m/sec V_A = 20 m/sec

$$n' = n\left(\frac{v - v_0}{v + v_s}\right)$$

$$2000 = n\left(\frac{340 - 20}{340 + 20}\right)$$

$$2000 = n\left(\frac{320}{360}\right)$$

$$n = 2000 \times \frac{9}{8} = 2250 \text{ hr}$$

Q.17 Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. coming from a distant object, the limit of resolution of the telescope if close to :

(1) 2.0×10^{-7} rad (2) 4.5×10^{-7} rad (3) 1.5×10^{-7} rad (4) 3.0×10^{-7} rad

Ans. **[4]**

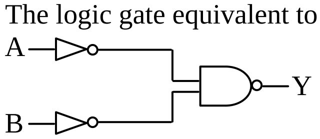
Sol. resolution limit$^+$ = $\frac{1.22\lambda}{d}$

$$= \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$$

$= 2.9 \times 10^{-7}$ rad

$= 3.0 \times 10^{-7}$

Q.18 The logic gate equivalent to the given logic circuit is :



(1) AND (2) OR (3) NOR (4) NAND

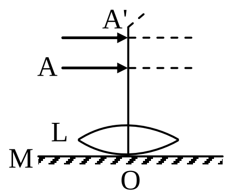
Ans. **[2]**

Sol.

A	B	
0	0	0
0	1	1
1	0	1
1	1	1

So it is OR gate

Q.19 A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_ℓ is put between the lens and the mirror, the pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_ℓ will be :



(1) $\sqrt{3}$ (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) $\sqrt{2}$

Ans. **[2]**

Sol. $\frac{1}{f_1} = \frac{1}{2} \times \frac{2}{18} = \frac{1}{18}$

$\frac{1}{f_1} = \frac{\mu_\ell - 1}{-18}$

$P = \frac{2}{18} - \frac{2}{18}(\mu_\ell - 1)$

$F_m = -\left(\frac{18}{2 - \mu_\ell}\right)$

$2 = 6 - 3\,\mu_\ell$

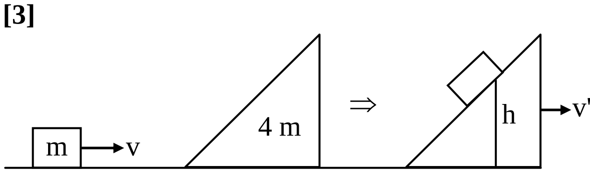
$\mu_\ell = \frac{4}{3}$

Q.20 A wedge of mass M=4 m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{2g}$ (3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{g}$

Ans. **[3]**

Sol.



conservation of momentum

mv = 5 mv'

$v' = \frac{v}{5}$

energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}5mv'^2 + mgh$$

$$\frac{1}{2}v^2 = \frac{5}{2}\left(\frac{v}{5}\right)^2 + gh$$

$$\frac{v^2}{2} - \frac{v^2}{10} = gh$$

$$h = \frac{2v^2}{5g}$$

Q.21 The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is :

(1) 37.03 cm^2 (2) 37.0 cm^2 (3) 37 cm^2 (4) 37.030 cm^2

Ans. **[1]**

Sol. Total area = 7 × 5.29

= 37.03 cm^2

special comment :

Answer should be in two digit after decimal so answer should be (1), NTA give (4)

Q.22 The specific heats, C_P and C_V of a gas of diatomic milecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic moleucles, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :

(1) Both A and B have a vibrational mode each (2) A is rigid but B has a vibrational mode.

(3) A has a vibrational mode but B has none. (4) A has one vibrational mode and B has two.

Ans. **[3]**

Sol. For A :

$$\frac{C_p}{C_V} = \frac{29}{22}$$

$$1 + \frac{2}{f} = \frac{29}{22}$$

$$\frac{2}{f} = \frac{7}{22}$$

$$f = \frac{44}{7} = 6.6$$

3 translation, 2 rotation, remaining vibration

For B :

$$1 + \frac{2}{f} = \frac{30}{21}$$

$$\frac{2}{f} = \frac{9}{21}$$

$$f = \frac{42}{9}$$

No vibrational

Q.23 In a conductor, if the number of conduction electrons per unit volume is $8.5 \times 10^{28}\ m^{-3}$ and mean free time is 25 *f*s (femto second), it's approximate resistivity is :

($m_e = 9.1 \times 10^{-31}$ kg)

(1) $10^{-5}\ \Omega m$ (2) $10^{-7}\ \Omega m$ (3) $10^{-8}\ \Omega m$ (4) $10^{-6}\ \Omega m$

Ans. **[3]**

Sol. $m = ne^2 \tau\rho$

$n = \dfrac{m}{\tau e^2 \rho}$

$= \dfrac{9.1 \times 10^{-31}}{25 \times 10^{-15} \times (1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28}}$

$= 1.67 \times 10^{-8}\ \Omega$ m

Q.24 A He^+ ion is in its first excited state. Its ionization energy is :

(1) 48.36 eV (2) 13.60 eV (3) 54.40 eV (4) 6.04 eV

Ans. **[2]**

Sol. $E = -13.6\ \dfrac{z^2}{n^2}$ eV

Z = 2 , n = 2

E = –13.6 eV

ionisation energy is

= +13.6 eV

Q.25 A metal wire of resistance 3Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :

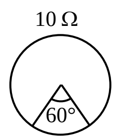
(1) $\dfrac{5}{2}\Omega$ (2) $\dfrac{5}{3}\Omega$ (3) $\dfrac{7}{2}\Omega$ (4) $\dfrac{12}{5}\Omega$

Ans. **[2]**

Sol. R = 3

When length become double

$R = 12\ \Omega$, $\left(R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell^2}{V}\right)$



$R \propto \ell$

$R_{Eff} = \dfrac{10}{6}$

$= \dfrac{5}{3}$

Q.26 Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d, y = +2d, respectively. The magnitude of the electric field E at a point a the x-axis at x = D, with D>>d, will behave as :

(1) $E \propto \dfrac{1}{D}$ (2) $E \propto \dfrac{1}{D^3}$ (3) $E \propto \dfrac{1}{D^4}$ (4) $E \propto \dfrac{1}{D^2}$

Ans. [3]

Sol. $E_P = 2E_1 \cos\theta_1 - 2E_1 \cos\theta_2$

$= \frac{2Kq}{d^2+D^2} \times \frac{D}{(d^2+D^2)^{3/2}} - \frac{2Kq}{(2d)^2+D^2} \times \frac{D}{[(2d)^2+D^2]^{3/2}}$

$= 2K\,qD\,[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}]$ $\qquad d << D$

$= \frac{2KqD}{D^2}\left(1 - \frac{3}{2}\frac{d^2}{D^2} - 1 - \frac{3\times 4d}{2D^2}\right)$

$= \frac{9Kqd^2}{D^4}$

Q.27 A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in oppoiste directions, then the de-Broglie wavelength of 'P' is :

(1) $\lambda_x - \lambda_y$ (2) $\frac{\lambda_x\lambda_y}{|\lambda_x - \lambda_y|}$ (3) $\lambda_x + \lambda_y$ (4) $\frac{\lambda_x\lambda_y}{\lambda_x + \lambda_y}$

Ans. [2]

Sol. Momentum conservation

$P_1 + P_2 = P$

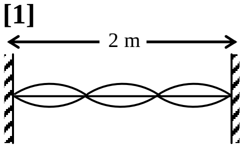
$\frac{h}{\lambda_1} - \frac{h}{\lambda_2} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{\lambda_1\lambda_2}{|\lambda_2 - \lambda_1|}$

Q.28 A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :

(1) 320 m/s, 80 Hz (2) 180 m/s, 120 Hz (3) 320 m/s, 120 Hz (4) 180 m/s, 80 Hz

Ans. [1]

Sol.



$\frac{3V}{2\ell} = 240$

$\frac{V}{2\ell} = 80$ (fundumental frequency)

$\frac{V}{2\times 2} = 80$

V = 320 m/sec (velocity)

Q.29 A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distance x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :

(1) 4 : 3 (2) 3 : 1 (3) 2 : 1 (4) 5 : 3

Ans. [2]

Sol. Magnification = 2

for $x_1 = 3\frac{f}{2}$

$x_2 = \frac{f}{2}$

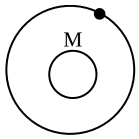
$\frac{x_1}{x_2} = 3 : 1$

Q.30 A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$.

Identify the correct relation between the radius R of the particle's orbit and its period T :

(1) T/R^2 is a constant (2) T/R is a constant (3) T^2/R^3 is a constant (4) TR is a constant

Ans. **[2]**



Sol.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$dm = (4\pi r^2 dr)\,\rho$$

$$\int dm = \int 4\pi r^2 dr.\frac{K}{r^2}$$

$$m = 4\pi Kr$$

$$V = \sqrt{\frac{G4\pi Kr}{r}}$$

$$V = \sqrt{4\pi KG}$$

$$T = \frac{2\pi R}{V}$$

$$\frac{T}{R} \rightarrow \text{const.}$$

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Questions & Solutions

9th April 2019 | Shift - II

CHEMISTRY

Q.1 The maximum number of possible oxidation states of actinoides are shown by :

(1) nobelium (No) and lawrencium (Lr)
(2) neptunium (Np) and lawrencium (Pu)
(3) actinium (Ac) and thorium (Th)
(4) berkelium (Bk) and californium (Cf)

Ans. **[2]**

Sol.

Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
+3		+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
	+4	+4	+4	+4	+4	+4	4	4						
		+5	+5	+5	+5	+5								
			+6	+6	+6	+6								
				+7	+7									

$\therefore$ Np and Pu has maximum number of possible O.S.

Q.2 In the following reaction carbonyl compound + MeOH $\underset{}{\overset{HCl}{\rightleftharpoons}}$ acetal Rate of the reaction is the highest for :

(1) Propanal as substrate and methanol in stoichiometric amount
(2) Propanal as substrate and methanol in excess
(3) Acetone as substrate and methanol in stoichiometric amound
(4) Acetone as substrate and methanol in excess

Ans. **[2]**

Sol. Ketones < Aldehydes $\rightarrow$ Rate of NAR

Only aldehydes are responsible for the function of acetals.

Q.3 Among the following species, the diamagnetic molecule is :

(1) NO (2) O_2 (3) CO (4) B_2

Ans. **[3]**

Sol. CO is diamagnetic in nature as all orbitals are fully filled.

CO MOT diagram = $\sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \sigma 2p_z^2$

$\therefore$ Ans. = 3

Q.4 The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

(II) Oxides of aluminium and gallium are amphoteric.

(III) Oxides of indium and thallium are basic.

(1) (I) and (II) only (2) (I) and (III) only

(3) (II) and (III) only (4) (I), (II) and (III) only

Ans. **[4]**

Sol. B_2O_3 is acidic in nature Al_2O_3 and Ga_2O_3 are amphoteric, oxides of In and $T\ell$ are basic in nature.

$\therefore$ And. 4

Q.5 A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?

(1) 0.10 (2) 0.05 (3) 0.20 (4) 0.15

Ans. **[2]**

Sol. gm E (Ni) = F

Moles × V.F. = 0.1

x × 2 = 0.1

x = 0.05

$N_i^{+2} + 2e^- \rightarrow Ni$

Q.6 Molal depression constant for a solvent is 4.0 K kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2OS_4 is :

(Assume complete dissociation of the electrolyte)

(1) 0.36 K (2) 0.18 K (3) 0.12 K (4) 0.24 K

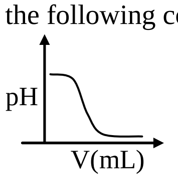
Ans. **[1]**

Sol. $\Delta T_f = ik_fm$

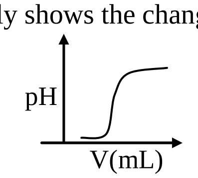
$= 3\,(4)\,(0.03)$

$\Delta T_f = 0.36$ K

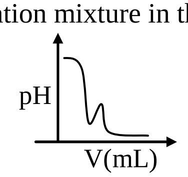
Q.7 In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?



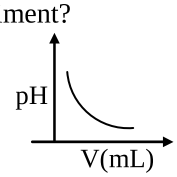
(A)



(B)



(C)

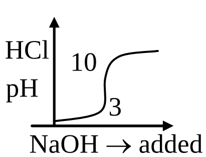


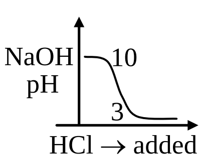
(D)

(1) (C) (2) (B) (3) (A) (4) (D)

Ans. **[3]**

Sol.





Q.8 The correct statements among I to III are :

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes.

(II) Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes.

(III) Valennce bond theory cannot distinguish ligands as weak and strong field ones.

(1) (I), (II) and (III)   (2) (II) and (III) only

(3) (I) and (II) only   (4) (I) and (III) only

Ans. **[4]**

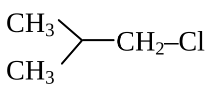
Sol. (1) VBT does not explain colour exhibited by complex of transition metal.

(2) VBT does not distinguish between strong and weak complex.

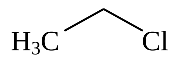
(3) VBT does not give quantitative interpretation of magnetic properties.

$\therefore$ According to question statement I and III are correct $\therefore$ Ans. 4

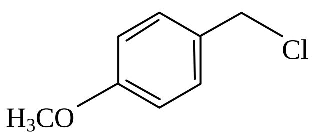
Q.9 Increasing order of reactivity of the following compounds for S_N1 substitution is :



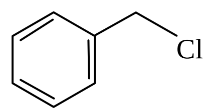
(A)



(B)



(C)



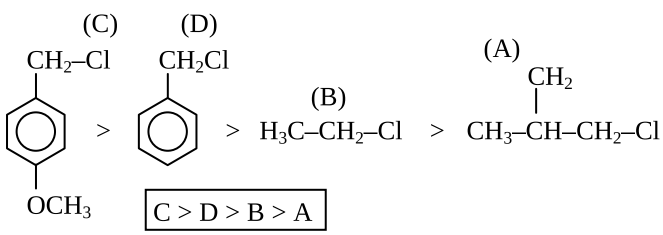
(D)

(1) (A) < (B) < (D) < (C)   (2) (B) < (C) < (D) < (A)

(3) (B) < (C) < (A) < (D)   (4) (B) < (A) < (D) < (C)

Ans. **[4]**

Sol.



C > D > B > A

Official answer is $\rightarrow$ 4 (for NTA)

Fact $\rightarrow$ stability of carbocation $\propto$ Rate of SN^2

Q.10 HF has highest boiling point among hydrogen halides, because it has :

(1) lowest ionic character   (2) lowest dissociation enthalpy

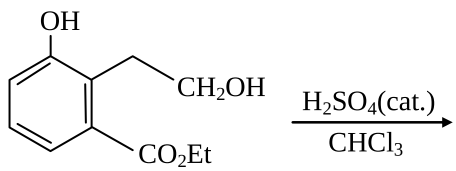
(3) strongest van der Waals intreactions   (4) strongest hydrogen bonding

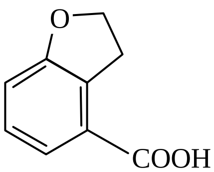
Ans. **[4]**

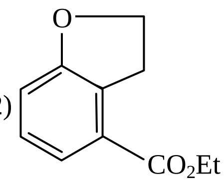
Sol. HF has strong hydrogen bond $\therefore$ it has highest B.P. among hydrogen halides.

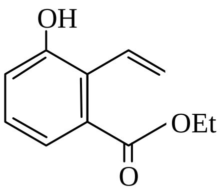
$\therefore$ Ans. 4

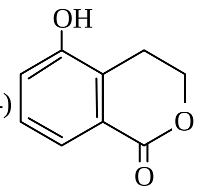
Q.11 The major product of the following reaction if :



(1) 

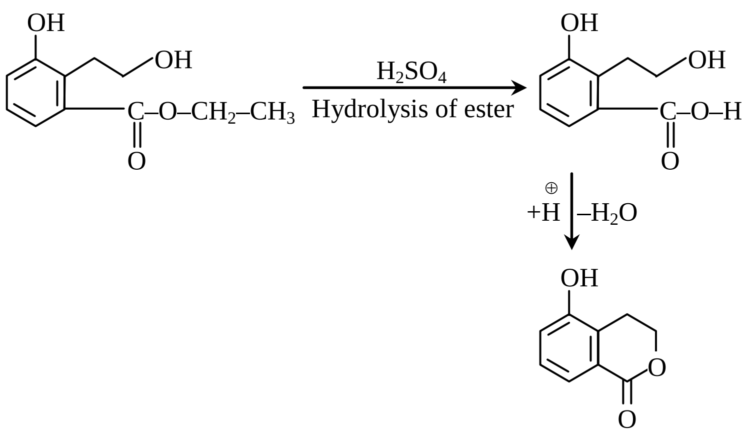
(2) 

(3) 

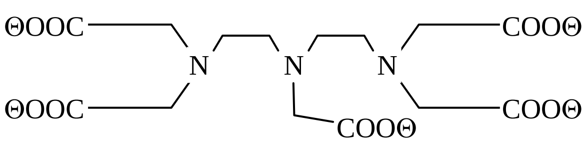
(4) 

Ans. **[4]**

Sol.



Q.12 The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are :



(1) 6 and 6 (2) 8 and 8 (3) 8 and 6 (4) 6 and 8

Ans. **[4]**

Sol. General C.No. of CN^- in transition elements is 6 and in inner transition elements is 8 – 12.
∴ ans. 4

Q.13 Noradrenaline is a / an :

(1) Antacid (2) Antihistamine (3) Antidepressant (4) Neurotransmitter

Ans. **[4]**

Sol. Nonadrenaline is a neurotransmitter.

Q.14 What would be the molality of 20 % (mass/mass) aqueous solution of KI ?
(molar mass of KI = 166 g mol^{-1})
(1) 1.51 (2) 1.35 (3) 1.08 (4) 1.48

Ans. **[1]**

Sol. 20% w/w K.I.
20 gm K.I. is 100 gm solution

Molality = $\frac{\text{gm(solute)}}{\text{mw} \times \text{kg(sovlent)}}$

= $\frac{20 \times 1000}{166 \times 80}$

= 1.506
= 1.51

Q.15 The peptide that gives positive ceric ammonium nitrate and carbylamine tests is :
(1) Gln - Asp (2) Asp - Gln (3) Ser - Lys (4) Lys - Asp

Ans. **[3]**

Sol. Ser-lys
Due to –OH group of serine it gives ⊕ve serric ammonium nitrates tent whereas due to $–NH_2$ group lysine gives ⊕ve. Carbylamine test.

Q.16 10 mL of 1 mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as a cube, what is its edge length?
(1) 2.0 nm (2) 0.1 nm (3) 2.0 pm (4) 1.0 pm

Ans. **[3]**

Sol.

Moles = $\frac{mv_{ml}}{1000} = \frac{10^{-3} \times 10}{1000} = 10^{-5}$ mole

10^{-5} N_A molecules covering area = 0.24 cm^2

1 N_A molecules covering area = $\frac{0.24}{10^{-5} N_A}$ cm^2

$\frac{0.24}{10^{-5} \times 6 \times 10^{23}} = a^2$

$a^2 = 4 \times 10^{-20}$ cm^2
$a = 2 \times 10^{-10}$ cm
$a = 2 \times 10^{-12}$ m
a = 2 pm

Q.17 Which of the following compounds is a constituent of the polymer $\left[HN–\overset{\overset{O}{||}}{C}–NH–CH_2 \right]_N$?
(1) N-Methyl urea (2) Ammonia (3) Formaldehyde (4) Methylamine

Ans. **[3]**

Sol.

Urea + Formaldehyde:

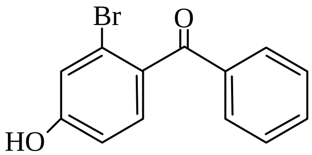
$H_2N–\underset{\underset{O}{||}}{C}–NH_2 + H–\underset{\underset{O}{||}}{C}–H$

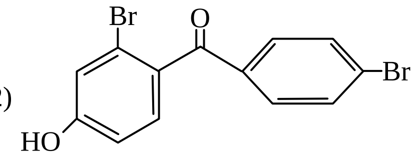
↓

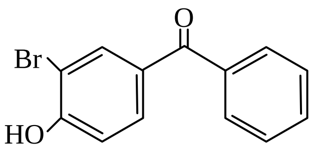
$\left[NH–\underset{\underset{O}{||}}{C}–NH–CH_2 \right]_n$

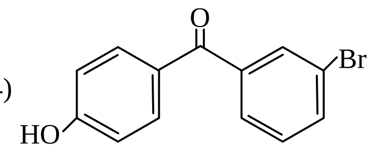
Urea formaldehyde resin

Q.18 p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives :

(1) 

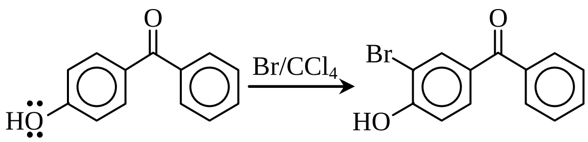
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



(M effect)

+M effect having group is responsible for activation towards ESR.

Q.19 Hinsberg' reagent is :

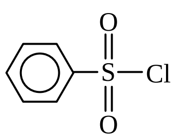
(1) $SOCl_2$ (2) $(COCl)_2$ (3) C_6H_5COCl (4) $C_6H_5SO_2Cl$

Ans. **[4]**

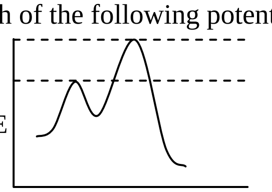
Sol.

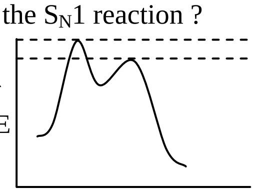
Benzene sulphonyl chloride

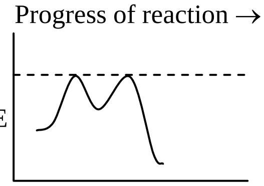
$C_6H_5SO_2C\ell$

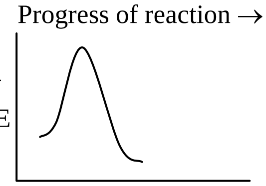


Q.20 Which of the following potential energy (PE) diagrams represents the S_N1 reaction ?

(1) 

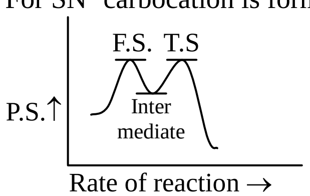
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. For SN^1 carbocation is formed.



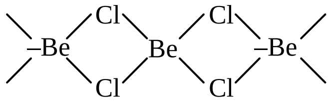
Q.21 The structures of beryllium chloride in the solid state and vapour phase, respectively, are :

(1) chain and chain (2) dimeric and dimeric

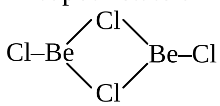
(3) dimeric and chain (4) chain and dimeric

Ans. **[4]**

Sol. Solid state-chain



In vapour state dimeric



Q.22 Assertion : For the extraction of iron, haematite ore is used.

Reason : Haematite is a carbonate ore of iron.

(1) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(2) Both the assertion and reason are correct and the reason is the correct explanation for the assertion.

(3) Only the reason is correct.

(4) Only the assertion is correct.

Ans. **[4]**

Sol. Extration of Fe is done from Haematite ore this is true but reason is wrong as Haematite is Fe_2O_3.

$\therefore$ Ans. 4

Q.23 During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is :

(1) 12 (2) –8 (3) 8 (4) –12

Ans. **[3]**

Sol. Work done on system = +10 kT

Heat escaped = –2kI

$\Delta U = q + w$

$= 10 - 2 = 8$ KJ.

Q.24 At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as $P = \frac{RT}{V - b}$ at T.

(1) Kr (2) Ar (3) Xe (4) Ne

Ans. **[3]**

Sol. $P = \frac{RT}{(V - b)}$

$P(V-b) = RT$

$\left(P + \frac{a}{V^2}\right)(V - b) = RT$

At high pressure.

$P(V - b) = RT$

$PV - Pb = RT$

$\frac{PV}{RT} - \frac{Pb}{RT} = 1$

$Z = 1 + \frac{Pb}{RT}$

$Z > 1, Z \propto b$

Q.25 The one that is not a carbonate ore is :

(1) malachite (2) bauxite (3) calamine (4) siderite

Ans. **[2]**

Sol. Malachite = $CuCO_3 . Cu(OH)_2$

Bauxite = $Al_2O_3 . 2H_2O$ or $AlO_x . (OH)_{3-2x}$ (where $0 < x < 1$)

Calamine = $ZnCO_3$

Siderite = $FeCO_3$

$\therefore$ Ans is Bauxite option-2.

Q.26 The amorphous from of silica is :

(1) kieselguhr (2) tridymite (3) cristobalite (4) quartz

Ans. **[1]**

Sol. Kieselguhr is amorphous form of silica.

$\therefore$ Ans. 1

Q.27 Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect ? (The Bohr radius is represented by a_0).

(1) The magnitude of the potential energy is double that of its kinetic energy on an average.

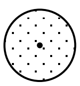
(2) The probability density of finding the electron is maximum at the nucleus.

(3) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus.

(4) The electron can be found at a distance $2a_0$ from the nucleus.

Ans. **[3]**

Sol. (1) T.E. = -K.E. = $\frac{PE}{2}$

(4) 

Is does not have any boundary.

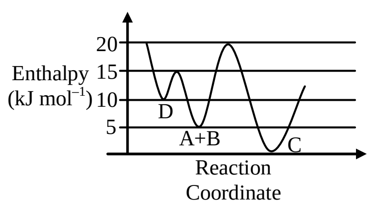
Q.28 The layer of atmosphere between 10 km to 50 km above the sea level is called as :

(1) troposphere (2) thermosphere (3) stratosphere (4) mesosphere

Ans. **[3]**

Sol. Memory based.

Q.29 Consider the given plot of enthalpy of the following reaction between A and B. $A + B \rightarrow C + D$. Identify the incorrect statement.



(1) C is the thermodynamically stable product.

(2) Formation of A and B from C has highest enthalpy of activation.

(3) Activation enthalpy to form C is 5 kJ mol^{-1} less than that to form D.

(4) D is kinetically stable product.

Ans. **[3]**

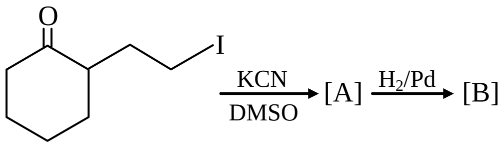
Sol. $E_a = (d \rightarrow c) = 15 - 0 = 15$

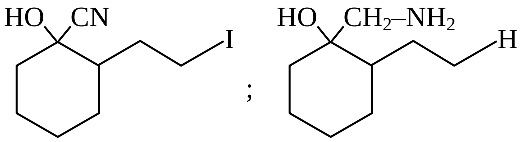
$E_a = (A + B) \rightarrow C = 15$

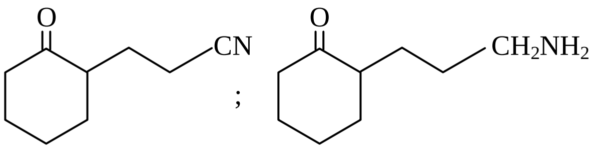
$E_a = (A + B \rightarrow D) = 10$

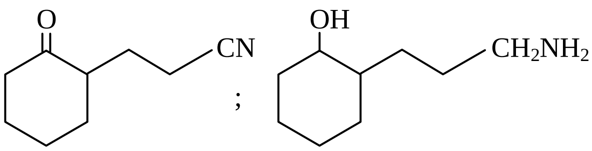
$E_a = C \rightarrow (A + B) = 20$

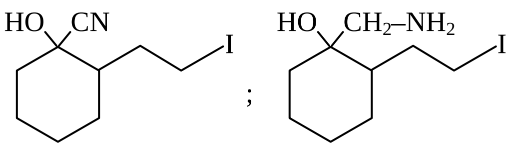
Q.30 The major products A and B for the following reactions are, respectively :



(1) 

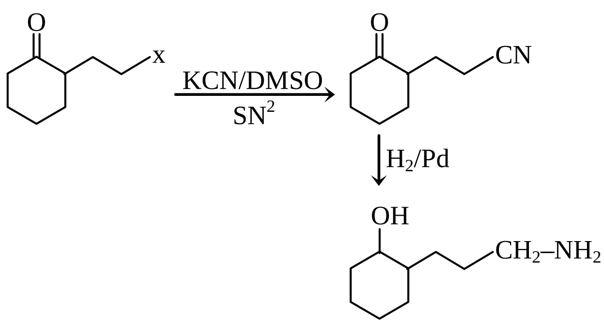
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - II

(Memory Based)

MATHEMATICS

Q.1 If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $-\frac{1}{4}$ (4) –4

Ans. **[1]**

Sol. Given system of equations has non-trival solution

$$\therefore \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow K = \frac{9}{2}$$

so equation are

$$2x + 3y - z = 0 \quad ....(1)$$

$$x + \frac{9}{2}y - 2z = 0 \quad ....(2)$$

$$2x - y + z = 0 \quad ....(3)$$

$(1) - (3) \Rightarrow 4y - 2z = 0$

$$\Rightarrow 2y = z \quad ....(4)$$

$$\Rightarrow \boxed{\frac{y}{z} = \frac{1}{2}}$$

From equation (1) & (4)

$$2x + 3y - 2y = 0$$

$$\Rightarrow 2x + y = 0$$

$$\Rightarrow \boxed{\frac{x}{y} = \frac{-1}{2}}$$

$$\text{or } \boxed{\frac{z}{x} = -4}$$

$$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+K = \frac{1}{2}$$

Q.2 The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :

(1) (–6, 4) (2) (4, –2) (3) (–4, 6) (4) (6, –2)

Ans. [4]

Sol. circle $x^2 + y^2 = 4$

$\Rightarrow c_1(0, 0)$; $r_1 = 2$

and circle $x^2 + y^2 + 6x + 8y - 24 = 0$

$\Rightarrow c_2(-3, -4)$; $r_2 = 7$

$\Rightarrow d = c_1c_2 = 5$

also $d = |r_1 - r_2|$

circles touch externally

equation of common tangent $s_1 - s_2 = 0$

$\Rightarrow 6x + 8y - 20 = 0$

$\Rightarrow 3x + 4y - 10 = 0$

Point (6, –2) satisfy it

Q.3 Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :

(1) 157 (2) 262 (3) 225 (4) 190

Ans. [4]

Sol. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow n^2 - 9n - 190 = 0$

$\Rightarrow (n - 19)(n + 10) = 0$

$\Rightarrow n = 19$

number of balls = $\frac{19.20}{2} = 190$

Q.4 If $\int e^{\sec x}\left(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x)\right)dx = e^{\sec x} f(x) + C$, then a possible choice of f(x) is :

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $\sec x + x\tan x - \frac{1}{2}$

(3) $\sec x + \tan x + \frac{1}{2}$ (4) $x\sec x + \tan x + \frac{1}{2}$

Ans. [3]

Sol. $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))\, dx$

$= e^{\sec x} f(x) + C$

Diff both side w.r.t x

$\Rightarrow e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))$

$= e^{\sec x} . \sec x \tan x\, f(x) + e^{\sec x} f'(x)$

$\Rightarrow f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec + C$

Q.5 If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

(1) $\sqrt{2} + 1$ (2) $2\sqrt{2} + 1$ (3) $\sqrt{2} - 1$ (4) $2\sqrt{2} - 1$

Ans. **[1]**

Sol. Equation of tangent to the parabola $y^2 = x$

at (α, β) is T = 0

$y\beta = \frac{x+\alpha}{2}$

$\Rightarrow y\beta = \frac{x+\beta^2}{2}$ $(\therefore \beta^2 = \alpha)$

$\Rightarrow y = \frac{1}{2\beta}x + \frac{\beta}{2}$

$\left(m = \frac{2}{2\beta}, c = \frac{\beta}{2}\right)$

this is also a tangent to ellipre $x^2 + 2y^2 = 1$

$\therefore C = \pm\sqrt{a^2m^2 + b^2}$

$\Rightarrow \frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\Rightarrow \frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$\Rightarrow (\beta - 1)^2 = 2$

$\Rightarrow \beta^2 - 1 = \sqrt{2}$ $(\because \beta > 0)$

$\Rightarrow \beta^2 = \sqrt{2} + 1$

$\alpha = \beta^2 = \sqrt{2} + 1$

Q.6 The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in R, x \neq y)$ for which $A^TA = 3I_3$ is :

(1) 2 (2) 4 (3) 3 (4) 6

Ans. **[2]**

Sol. $(A^T)(A) = 3I_3$

$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow 8x^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{8}}$

$\Rightarrow 6y^2 = 3 \Rightarrow y = \pm\sqrt{\frac{1}{2}}$

4 materices are possible

Q.7 If $f: R \to R$ is a differentiable function and f(2) = 6, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is :

(1) 0 (2) $24 f'(2)$ (3) $12 f'(2)$ (4) $2 f'(2)$

Ans. **[3]**

Sol. $\lim_{x\to 2} \int_6^{f(x)} \frac{2tdt}{(x-2)} dx$ {given that f(2) = 6}

$\frac{0}{0}$ form. So we use L-Hospital Rule

$= \lim_{x\to 2} \frac{f'(x).2f(x)}{1}$

= f '(2) . 2f(2)

= 12f ' (2)

Q.8 If a unit vector $\vec{a}$ makes angles π/3 with $\hat{i}$, π/4 with $\hat{j}$ and θ ∈(0, π) with $\hat{k}$, then a value of θ is :

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

Ans. **[4]**

Sol. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\gamma = \pm\frac{1}{2}$

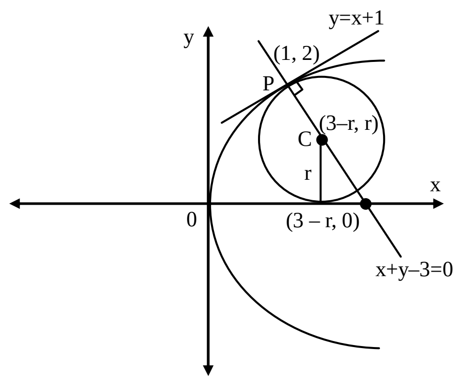
$\Rightarrow \gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

Q.9 The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :

(1) $4\pi (2 - \sqrt{2})$ (2) $8\pi (3 - 2\sqrt{2})$ (3) $4\pi (3 + \sqrt{2})$ (4) $8\pi (2 - \sqrt{2})$

Ans. **[2]**

Sol.



equation of tangent to the perabola $y^2 = 4x$ at (1, 2) is

$2y = 4\left(\frac{x+1}{2}\right)$

$\Rightarrow y = x + 1$

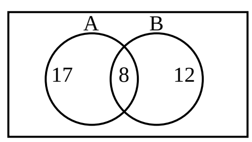
equation of normal y = –x + 3
Let center be c(3 – r, r)
Now $PC^2 = r^2$
$\Rightarrow (3 - r - 1)^2 + (r - 2)^2 = r^2$
$\Rightarrow 2(2 - r)^2 = r^2$
$\Rightarrow r^2 - 8r + 8 = 0$
$\Rightarrow r = 4 \pm 2\sqrt{2}$
for $r = 4 + 2\sqrt{2}$ (3 – r < 0) Not possible
So $r = 4 - 2\sqrt{2}$
Area $= \pi r^2 = \pi(16 + 8 - 16\sqrt{2})$
$= 8\pi(3 - 2\sqrt{2})$

Q.10 Two newspapers A and B are published in a city. It is known that 25 % of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

(1) 13.9 (2) 13.5 (3) 12.8 (4) 13

Ans. **[1]**

Sol. Let population = 100
n(A) = 25
n(B) = 20
$n(A \cap B) = 8$
$n(A \cap \overline{B}) = 17$
$n(\overline{A} \cap B) = 12$



Now % of the population who look advertiesement
$= \frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$
= 13.9

Q.11 The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + ... upto 11th term is :

(1) 946 (2) 916 (3) 915 (4) 945

Ans. **[1]**

Sol. S = 1 + 2 × 3 + 3 × 5 + 4 × 7 + ...+ upto 11th terms
nth term of the sevies is
$T_n = n(2n - 1)$
$\Rightarrow S = \sum_{n=1}^{11} Tn = \sum_{n=1}^{11} (2n^2 - n)$
$\Rightarrow S_n = \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$
put n = 1
$\Rightarrow S_{11} = \frac{2(11)(12)(23)}{6} - \frac{11(12)}{2}$
$\Rightarrow S_{11} = 946$

Q.12 If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$

is continous at x = 5, then value of a – b

(1) $\frac{2}{\pi+5}$ (2) $\frac{-2}{\pi+5}$ (3) $\frac{2}{\pi-5}$ (4) $\frac{2}{5-\pi}$

Ans. **[4]**

Sol. $f(x) = \begin{cases} a|\pi - x| + 1 & ,x < 5 \\ b|\pi - x| + 3 & ,x > 5 \end{cases}$

cortinous at x = 5

$\therefore$ L.H.L = R.H.L = f(5)

$\Rightarrow b|\pi - 5| + 3 = a|\pi - 5| + 1$

$\Rightarrow -b(\pi - 5) + 3 = -a(5 - \pi) + 1$

$\Rightarrow (a - b)(\pi - 5) = -2$

$\Rightarrow a - b = \frac{-2}{\pi-5} = \frac{2}{5-\pi}$

Q.13 Let $z \in C$ be such that |z| < 1. If $\omega = \frac{5+3z}{5(1-z)}$ then :

(1) 4 Im (ω) > 5 (2) 5 Re (ω) > 1 (3) 5 Im (ω) < 1 (4) 5 Re (ω) > 4

Ans. **[2]**

Sol. $\omega = \frac{5+32}{5(1-z)}$

$\Rightarrow 5\omega - 5\omega z = 5 + 3z$

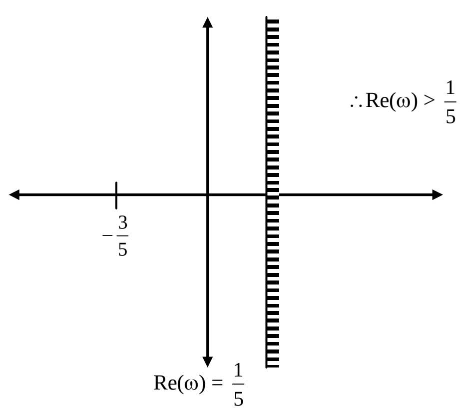
$\Rightarrow z = \frac{5\omega-5}{3+5\omega}$

given |z| < 1

$\Rightarrow \left|\frac{5\omega-5}{3+5\omega}\right| < 1$

$\Rightarrow |5\omega - 5| < |3 + 5\omega|$

$\Rightarrow |\omega - 1| < \left|\frac{3}{4} + \omega\right|$



Q.14 If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim_{x\to 4-} f(x)$ exists but $\lim_{x\to 4+} f(x)$ does not exist.

(2) Both $\lim_{x\to 4-} f(x)$ and $\lim_{x\to 4+} f(x)$ exist but are not equal.

(3) $\lim_{x\to 4+} f(x)$ exists but $\lim_{x\to 4-} f(x)$ does not exist.

(4) f is continuous at x = 4.

Ans. **[4]**

Sol. $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left\{[x] - \left[\frac{x}{4}\right]\right\} = 4 - 1 = 3$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \left\{[x] - \left[\frac{x}{4}\right]\right\} = 3 - 0 = 3$$

f(4) = 3

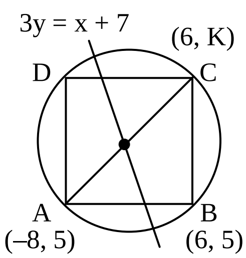
∴ continous at x = 4

Q.15 A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :

(1) 98 (2) 56 (3) 72 (4) 84

Ans. **[4]**

Sol.



Let vertex c is (6, K) then center of circle $\left(-1, \frac{5+K}{2}\right)$ it lies on diameter 3y = x + 7

$\Rightarrow 3\left(\frac{5+K}{2}\right) = -1 + 7$

$\Rightarrow K = -1$

So AB = 14 and BC = 6

Area = 14 × 6 = 84

Q.16 The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :

(1) $(1, 2) \cup (2, \infty)$

(2) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

Ans. **[3]**

Sol. $f(x) = \dfrac{1}{4 - x^2} + \log_{10}(x^3 - x)$

Let $f_1 = \dfrac{1}{4 - x^2}$

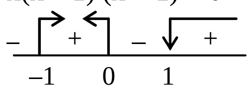
$\Rightarrow 4 - x^2 \neq 0$

$\Rightarrow x \neq \pm 2$

and $f_2 = \log_{10}(x^3 - x)$

$x^3 - x > 0$

$\Rightarrow x(x - 1)(x + 1) > 0$

$\Rightarrow$ 

$x \in (-1, 0) \cup (1, \infty) - \{2\}$

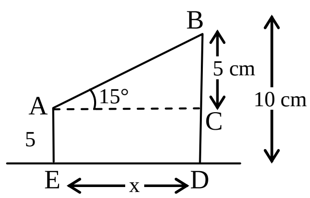
$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

Q.17 Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes and angle of 15° with the ground. Then the distance (in m) between the poles, is :

(1) $10(\sqrt{3} - 1)$ (2) $5(2 + \sqrt{3})$ (3) $5(\sqrt{3} + 1)$ (4) $\dfrac{5}{2}(2 + \sqrt{3})$

Ans. **[2]**

Sol.



in $\Delta ABC \Rightarrow \tan 15° = \dfrac{5}{x}$

$\Rightarrow 2 - \sqrt{3} = \dfrac{5}{x}$

$\Rightarrow x = 5(2 + \sqrt{3})$

Q.18 If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x - (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$ (3) $10\sqrt{5}$ (4) $8\sqrt{5}$

Ans. **[4]**

Sol. $(m^2 + 1)x^2 - 3x + (m + 1)^2 = 0$

$\Rightarrow \alpha + \beta = \dfrac{3}{m^2 + 1}$

$\alpha\beta = \dfrac{(m + 1)^2}{m^2 + 1}$

$\because \alpha + \beta$ is maximum $\therefore m^2 + 1$ is minimum

$\Rightarrow m = 0$

$\therefore \alpha + \beta = 3$ and $\alpha\beta = 1$

$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)|$

$= \left|\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}\,\{(\alpha + \beta)^2 - \alpha\beta\}\right|$

$= \left|\sqrt{9 - 4}(9 - 1)\right|$

$= 8\sqrt{5}$

Q.19 The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$ is :

(1) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2}-\log_e 2$ (3) $\frac{\pi}{4}-\log_e 2$ (4) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$

Ans. **[1]**

Sol. $I = \int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$

$I = \int_0^1 x\tan^{-1}\left(\frac{1}{1-x^2+x^4}\right)dx$

$I = \int_0^1 x\tan^{-1}\left\{\frac{x^2-(x^2-1)}{1+x^2(x^2-1)}\right\}dx$

$I = \int_0^1 x\,\{\tan^{-1}x^2-\tan^{-1}(x^2-1)\}dx$

let $x^2 = t \Rightarrow 2x\,dx = dt$

$I = \frac{1}{2}\int_0^1 \left\{\tan^{-1}t-\tan^{-1}(t-1)\right\}dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt-\frac{1}{2}\int_0^1\tan^{-1}(t-1)\,dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt-\frac{1}{2}\int_0^1\tan^{-1}(-t)\,dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt+\frac{1}{2}\int_0^1\tan^{-1}(t)\,dt$

$= \int_0^1 \tan^{-1}(t)\,dt$

$= (\tan.\tan^{-1})_0^1-\int_0^1\frac{t}{1+t^2}\,dt$

$= \left(\frac{\pi}{4}\right)-\frac{1}{2}\left[\log(1+t^2\right]_0^1$

$= \frac{\pi}{4}-\frac{1}{2}\log_e 2$

Q.20 Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :

(1) $205\sqrt{5}$ (2) $11/\sqrt{5}$ (3) $63\sqrt{5}$ (4) $17/\sqrt{5}$

Ans. **[2]**

Sol. Equation of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0$

$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + \lambda) - 6 + 5\lambda = 0$

This plane is $\perp$ to xy - plane

$\therefore\ 1 + \lambda = 0 \Rightarrow \lambda = -1$

So, equation of plane

$-x - 2y - 11 = 0$

$\Rightarrow x + 2y + 11 = 0$

distance of the point (0, 0, 256) from this plane

$= \left|\frac{0+0+1}{\sqrt{1+4}}\right| = \frac{11}{\sqrt{5}}$

Q.21 If some three coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is :

(1) 232 (2) 964 (3) 625 (4) 227

Ans. **[1]**

Sol. given : $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$

$\Rightarrow 15r = 2n - 2r + 2$

$\Rightarrow 17r = 2n + 2$ ......(1)

also given $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70} \Rightarrow \frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 3n - 3r = 14r + 14$

$\Rightarrow 17r = 3n - 14$

Solving (1) & (2)

$n = 16, r = 2$

average of coefficient $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$= \frac{16+120+560}{3}$

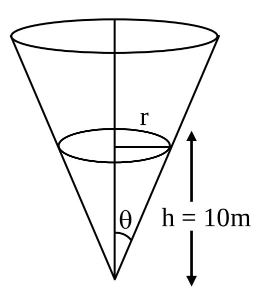
$= 232$

Q.22 A water tank has the shape of an inverted right cirular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

(1) 1/5 π (2) 1/10 π (3) 1/15 π (4) 2/π

Ans. **[1]**

Sol. given $\theta = \tan^{-1}\left(\frac{1}{2}\right)$



$\Rightarrow \tan\theta = \frac{1}{2} = \frac{r}{h}$

$\Rightarrow r = \frac{h}{2}$

$v = \frac{1}{3}\pi r^2 h$

$v = \frac{1}{3}\pi \frac{h^3}{4}$

$\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

$S = \frac{\pi}{4}(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$

Q.23 The value of sin10° sin30° sin50° sin70° is :

(1) $\frac{1}{16}$ (2) $\frac{1}{36}$ (3) $\frac{1}{18}$ (4) $\frac{1}{32}$

Ans. **[1]**

Sol. = sin10° sin30° sin50° sin70°

= sin30°{sin10° sin(60°–10°) sin(60° + 10°)}

$= \sin 30° \left\{\frac{1}{4}\sin 3(10)°\right\}$

$= \frac{1}{2}\left(\frac{1}{4}\times\frac{1}{2}\right)$

$= \frac{1}{16}$

Q.24 If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :

(1) F, F, F (2) T, F, F (3) F, T, T (4) T, T, F

Ans. **[2]**

Sol. $p \Rightarrow (q \vee r)$ is false.

($\because$ T $\Rightarrow$ F = F)

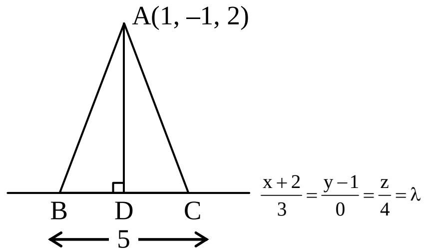
So, P = T, q = F and r = F

Q.25 The vertices B and C of ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :

(1) $5\sqrt{17}$ (2) $\sqrt{34}$ (3) $2\sqrt{34}$ (4) 6

Ans. **[2]**

Sol.



Let any point on given line is

D(3λ –2, 1, 4λ)

Now AD ⊥ BC

D.R. of BC $\Rightarrow a_1 = 3, b_1 = 0, c_1 = 4$

D.R. of AD $\Rightarrow a_2 = 3\lambda - 3, b_2 = 2, c_2 = 4\lambda - 2$

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$\Rightarrow 25\lambda = 17$

$\Rightarrow \lambda = \frac{17}{25}$

co-oridnation of point D $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

AD = $\sqrt{\frac{576}{625} + 4 + \frac{324}{25}} = \frac{2}{5}\sqrt{34}$

Area of ΔABC = $\frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{5}\sqrt{34}$

$= \sqrt{34}$

Q.26 If $\cos x \frac{dy}{dx} - y\sin x = 6x$, $(0<x<\frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{\pi^2}{4\sqrt{3}}$ (2) $-\frac{\pi^2}{2}$ (3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

Ans. **[4]**

Sol. $\cos x \frac{dy}{dx} - y\sin x = 6x$

$\Rightarrow \frac{dy}{dx} - y\tan x = 6x \sec x$

I.F = $e^{-\int \tan x dx} = e^{-\log_e \sec x} = \frac{1}{\sec x}$

$\therefore$ solution of equation

$\Rightarrow y.\frac{1}{\sec x} = \int 6x \sec x.\frac{1}{\sec x} dx$

$\Rightarrow \frac{y}{\sec x} = 3x^2 + c$ ....(1)

given $y\left(\frac{\pi}{3}\right) = 0$

So, $0 = \frac{3\pi^2}{9} + C$

$\Rightarrow C = -\frac{\pi^2}{3}$

Now from (1)

$\Rightarrow \frac{y}{\sec x} = 3x^2 - \frac{\pi^2}{3}$

at $x = \frac{\pi}{6}$

$\Rightarrow \frac{\sqrt{3}y}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$

$\Rightarrow y = -\frac{\pi^2}{2\sqrt{3}}$

Q.27 If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is :

(1) –25 (2) –35 (3) –36 (4) 25

Ans. **[1]**

Sol. Let the three numbers in A.P. are

$a - d, a, a + d$

given that : $a - d + a + a + d = 33$

$\Rightarrow a = 11$

and $(a - d)(a)(a + d) = 1155$

$\Rightarrow a(a^2 - d^2) = 1155$

$\Rightarrow 11(121 - d^2) = 1155$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

If $d = 4$ then first term $a - d = 7$

If $d = -4$ then first term $a - d = 15$

$T_{11} = 7 + 10(4) = 47$

or $T_{11} = 15 + 10(-4) = -25$

Q.28 The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34 x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

(1) 7/2 (2) 9/4 (3) 8/3 (4) 7/3

Ans. **[4]**

Sol. mean = 42

$$\Rightarrow \frac{10 + 22 + 26 + 29 + 34 + x + 42 + 67 + 70 + y}{10} = 45$$

$\Rightarrow 420 = 300 + x + y$

$\Rightarrow x + y = 120$ ......(1)

and medium = 35

$$\Rightarrow \frac{34 + x}{2} = 35$$

$\Rightarrow x = 36$

From (1) $y = 84$

$$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$$

Q.29 If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1$ ($a \in R - \{0, 1\}$) are perpendicular, then the distance of their point of intersection from the origin is :

(1) $\frac{2}{5}$ (2) $\sqrt{\frac{2}{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{2}}{5}$

Ans. **[2]**

Sol. Two lines are perpendicular

$\therefore m_1m_2 = -1$

$$\Rightarrow \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a + 1)(a^2 - 2a + 2) = 0$

$\therefore$ a = –1

so lines are L_1 : x – 2y + 1 = 0

L_2 : 2x + y – 1 = 0

Solving there equation we get point of intersetion P $\left(\frac{1}{5}, \frac{3}{5}\right)$

Now distance of P from origin

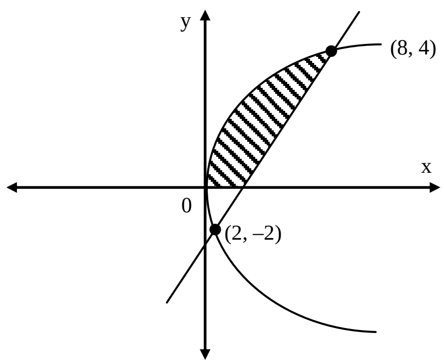
OP = $\sqrt{\frac{1}{25}+\frac{9}{25}} = \sqrt{\frac{2}{5}}$

Q.30 The area (in sq. units) of the region A = $\{(x, y) : \frac{y^2}{2} \le x \le y + 4\}$ is :

(1) 18 (2) 30 (3) $\frac{53}{3}$ (4) 16

Ans. **[1]**

Sol.



$y^2 = 2x$ ....(1)

and x – y – 4 = 0 ....(2)

solving (1) & (2)

$(x - 4)^2 = 2x$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x = 8_1 + 2$ and $y = 4_1 - 2$

$$A = \int_{-2}^{4}\left(y + 4 - \frac{y^2}{2}\right)dy$$

$$A = \left(\frac{y^2}{4} + 4y - \frac{y^3}{3}\right)_{-2}^{4}$$

$$A = \left(4 + 16 - \frac{64}{6}\right) - \left(1 - 8 + \frac{8}{6}\right) = 18$$