$x = \left(\frac{dy}{dx}\right)((2y)\log y + y)$

Multiply both sides with y

$xy = (2y^2 \log y + y^2)\frac{dy}{dx}$

We know, $x^2 = 2y^2 \log y$. So replace $2y^2 \log y$ with x^2 in the above equation.

$xy = (x^2 + y^2)\frac{dy}{dx}$

$(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Conclusion: Therefore $x^2 = 2y^2 \log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

2. Question

Verify that y = e^x cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^x \cos bx$

On differentiating with x, we get

$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$

On differentiating again with x, we get

$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx) - 2e^x \cos bx - 2e^x(-b \sin bx) + 2e^x \cos bx$$

$= e^x \cos bx - e^x(b^2 \cos bx)$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

2. Question

Verify that y = e^x cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^x \cos bx$

On differentiating with x, we get

$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$
$$= e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx) - 2e^x \cos bx - 2e^x(-b \sin bx) + 2e^x \cos bx$$

$$= e^x \cos bx - e^x(b^2 \cos bx)$$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

3. Question

Verify that y = $e^{m \cos^{-1} x}$ is a solution of the differential equation (1 - x^2)$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$

Answer

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$$

3. Question

Verify that y = $e^{m \cos^{-1} x}$ is a solution of the differential equation (1 - x^2)$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$

Answer

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$$

4. Question

Verify that y = (a + bx) e^{2x} is the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$

Answer

Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let’s see what is the value of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x}$$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

4. Question

Verify that y = (a + bx) e^{2x} is the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$

Answer

Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let’s see what is the value of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x}$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

5. Question

Verify that y = e^x(A cos x + B sin x) is the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$$+ e^x(-A\cos x - B\sin x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

5. Question

Verify that y = e^x(A cos x + B sin x) is the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$$+ e^x(-A\cos x - B\sin x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

6. Question

Verify that y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$.

Answer

Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

6. Question

Verify that y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$.

Answer

Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

7. Question

Verify that y = $ae^{2x} + be^{-x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

Answer

Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

7. Question

Verify that y = ae^{2x} + be^{-x} is the general solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$

Answer

Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

8. Question

Show that y = e^{x}(A cos x + B sin x) is the solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^{x}(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{x}(A\cos x + B\sin x) + e^{x}(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

8. Question

Show that y = e^x(A cos x + B sin x) is the solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

9. Question

Verify that $y^2 = 4a(x + a)$ is a solution of the differential equation $y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$

Answer

Given, $y^2 = 4a(x + a)$

On differentiating with x, we get

$$2y\frac{dy}{dx} = 4a$$

Now let's see what is the value of $y(1-(\frac{dy}{dx})^2)-2x\frac{dy}{dx}$

$= y\left(1-\left(\frac{2a}{y}\right)^2\right)-4\frac{ax}{y}$

$= y-\frac{4a^2}{y}-4\frac{ax}{y}$

$= \frac{y^2-4a(a+x)}{y}$

$= \frac{4a(a+x)-4a(a+x)}{y}$

= 0

Conclusion: Therefore, $y^2 = 4a(x+a)$ is the solution of $y(1-(\frac{dy}{dx})^2) = 2x\frac{dy}{dx}$

9. Question

Verify that $y^2 = 4a(x + a)$ is a solution of the differential equation $y\left\{1-\left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$

Answer

Given, $y^2 = 4a(x+a)$

On differentiating with x, we get

$2y\frac{dy}{dx} = 4a$

Now let's see what is the value of $y(1-(\frac{dy}{dx})^2)-2x\frac{dy}{dx}$

$= y\left(1-\left(\frac{2a}{y}\right)^2\right)-4\frac{ax}{y}$

$= y-\frac{4a^2}{y}-4\frac{ax}{y}$

$= \frac{y^2-4a(a+x)}{y}$

$= \frac{4a(a+x)-4a(a+x)}{y}$

= 0

Conclusion: Therefore, $y^2 = 4a(x+a)$ is the solution of $y(1-(\frac{dy}{dx})^2) = 2x\frac{dy}{dx}$

10. Question

Verify that $y = ce^{\tan^{-1}x}$ is a solution of the differential equation $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$

Answer

Given $y = c\,e^{\tan^{-1}x}$

On differentiating with x, we get

$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1+x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1+x^2}\right)$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c(\frac{1}{1+x^2})^2 e^{\tan^{-1}x} + c\ \tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right)e^{\tan^{-1}x} + c\ (\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)e^{\tan^{-1}x}$$

$$= y(\frac{1}{1+x^2})^2 + y\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right) + y\ (\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)$$

Now let's see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1+x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1+x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) + \left(\frac{2xy}{1+x^2}\right)\tan^{-1}x - \tan^{-1}x\,(\frac{y}{1+x^2})$$

$$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$$

10. Question

Verify that y = $ce^{\tan^{-1}x}$ is a solution of the differential equation (1 + x^2)$\frac{d^2y}{dx^2}$ + (2x - 1)$\frac{dy}{dx}$ = 0

Answer

Given $y = c\,e^{\tan^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1+x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1+x^2}\right)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c(\frac{1}{1+x^2})^2 e^{\tan^{-1}x} + c\ \tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right)e^{\tan^{-1}x} + c\ (\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)e^{\tan^{-1}x}$$

$$= y(\frac{1}{1+x^2})^2 + y\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right) + y\ (\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)$$

Now let's see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1+x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1+x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) + \left(\frac{2xy}{1+x^2}\right)\tan^{-1}x - \tan^{-1}x\,(\frac{y}{1+x^2})$$

$$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$$

11. Question

Verify that y = ae^{bx} is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

Answer

Given $y = a\,e^{bx}$

On differentiating with x, we get

$\frac{dy}{dx} = ab\,e^{bx}$

On differentiating again with x, we get

$\frac{d^2y}{dx^2} = ab^2\,e^{bx}$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$= ab^2\,e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$

$= ab^2\,e^{bx} - ab^2\,e^{bx}$

= 0

Conclusion: Therefore, $y = a\,e^{bx}$ is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

11. Question

Verify that y = ae^{bx} is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

Answer

Given $y = a\,e^{bx}$

On differentiating with x, we get

$\frac{dy}{dx} = ab\,e^{bx}$

On differentiating again with x, we get

$\frac{d^2y}{dx^2} = ab^2\,e^{bx}$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$= ab^2\,e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$

$= ab^2\,e^{bx} - ab^2\,e^{bx}$

= 0

Conclusion: Therefore, $y = a\,e^{bx}$ is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

12. Question

Verify that y = $\frac{a}{x} + b$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

Answer

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let’s see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

12. Question

Verify that y = $\frac{a}{x} + b$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

Answer

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let’s see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

13. Question

Verify that y = $e^{-x} + Ax + B$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Answer

Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$= e^x(e^{-x})$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

13. Question

Verify that y = $e^{-x} + Ax + B$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Answer

Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$= e^x(e^{-x})$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

14. Question

Verify that $Ax^2 + By^2 = 1$ is a solution of the differential equation $x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$

Answer

Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

14. Question

Verify that $Ax^2 + By^2 = 1$ is a solution of the differential equation $x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$

Answer

Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

15. Question

Verify that $y = \frac{c-x}{1+cx}$ is a solution of the differential equation $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$.

Answer

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1+cx)^2}$$

Now let's see what is the value of $(1+x^2)\frac{dy}{dx} + (1+y^2)$

$$= -\frac{(1+x^2)(1+c^2)}{(1+cx)^2} + \left(1+\left(\frac{c-x}{1+cx}\right)^2\right)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1+c^2x^2+2cx+c^2+x^2-2cx)}{(1+cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1+x^2)\frac{dy}{dx} + (1+y^2) = 0$

15. Question

Verify that y = $\frac{c-x}{1+cx}$ is a solution of the differential equation $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$.

Answer

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1+cx)^2}$$

Now let's see what is the value of $(1+x^2)\frac{dy}{dx} + (1+y^2)$

$$= -\frac{(1+x^2)(1+c^2)}{(1+cx)^2} + \left(1+\left(\frac{c-x}{1+cx}\right)^2\right)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1+c^2x^2+2cx+c^2+x^2-2cx)}{(1+cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1+x^2)\frac{dy}{dx} + (1+y^2) = 0$

16. Question

Verify that y = log (x + $\sqrt{x^2+a^2}$) satisfies the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

Answer

Given $y = \log\left(x + \sqrt{x^2+a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(1 + \frac{x}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{1}{\sqrt{x^2+a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

16. Question

Verify that y = log (x + $\sqrt{x^2 + a^2}$) satisfies the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

Answer

Given $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{x}{\sqrt{x^2 + a^2}}\right)$$

$$= \frac{1}{\sqrt{x^2 + a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

17. Question

Verify that y = e^{-3x} is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Answer

Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9\,e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$= 9\,e^{-3x} - 3e^{-3x} - 6e^{-3x}$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

17. Question

Verify that y = e^{-3x} is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Answer

Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9\,e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$= 9\,e^{-3x} - 3e^{-3x} - 6e^{-3x}$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Exercise 18C

1. Question

Form the differential equation of the family of straight lines y=mx+c, where m and c are arbitrary constants.

Answer

The equation of a straight line is represented as,

Y = mx + c

Differentiating the above equation with respect to x,

$$\frac{dy}{dx} = m$$

Differentiating the above equation with respect to x,

$$\frac{d^2y}{dx^2} = 0$$

This is the differential equation of the family of straight lines y=mx+c, where m and c are arbitrary constants

2. Question

Form the differential equation of the family of concentric circles $x^2+y^2=a^2$, where a>0 and a is a parameter.

Answer

Now, in the general equation of of the family of concentric circles $x^2+y^2=a^2$, where a>0, 'a' represents the radius of the circle and is an arbitrary constant.

The given equation represents a family of concentric circles centered at the origin.

$x^2+y^2=a^2$

Differentiating the above equation with respect to x on both sides, we have,

$2x + 2y\frac{dy}{dx} = 0$ (As a>0, derivative of a with respect to x is 0.)

$$x + y\frac{dy}{dx} = 0$$

3. Question

Form the differential equation of the family of curves, y=a sin (bx+c), Where a and c are parameters.

Answer

Equation of the family of curves, y=a sin (bx+c), Where a and c are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$y = a\sin(bx + c)$ (1)

$$\frac{dy}{dx} = ab\cos(bx + c)$$

$\frac{d^2y}{dx^2} = -ab^2\sin(bx + c)$ (Substituting equation 1 in this equation)

$$\frac{d^2y}{dx^2} = -b^2y$$

$$\frac{d^2y}{dx^2} + b^2y = 0$$

This is the required differential equation.

4. Question

Form the differential equation of the family of curves x=A cos nt+ Bsin nt, where A and B are arbitrary constants.

Answer

Equation of the family of curves, x=A cos nt+ Bsin nt, where A and B are arbitrary constants.

Differentiating the above equation with respect to t on both sides, we have,

$x = A\cos(nt) + B\sin(nt)$ (1)

$$\frac{dx}{dt} = -An\sin(nt) + Bn\cos(nt)$$

$$\frac{d^2x}{dt^2} = -An^2\cos(nt) - Bn^2\sin(nt)$$

$\frac{d^2x}{dt^2} = -n^2\left(A\cos(nt) + B\sin(nt)\right)$ (Substituting equation 1 in this equation)

$$\frac{d^2x}{dt^2} = -n^2x$$

$$\frac{d^2x}{dt^2} + n^2x = 0$$

This is the required differential equation.

5. Question

Form the differential equation of the family of curves $y=ae^{bx}$, where a and b are arbitrary constants.

Answer

Equation of the family of curves, $y=ae^{bx}$, where a and b are arbitrary constants.

Differentiating the above equation with respect to x on both sides, we have,

$y = ae^{bx}$ (1)

$\frac{dy}{dx} = abe^{bx}$ (2)

$$\frac{d^2y}{dx^2} = ab^2e^{bx}$$

$y\frac{d^2y}{dx^2} = ab^2e^{bx}(ae^{bx})$ (Multiplying both sides of the equation by y)

$y\frac{d^2y}{dx^2} = \left(abe^{bx}\right)^2$ (Substituting equation 2 in this equation)

$$y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

This is the required differential equation.

6. Question

Form the differential equation of the family of curves $y^2=m(a^2-x^2)$, where a and m are parameters.

Answer

Equation of the family of curves, $y^2=m(a^2-x^2)$, where a and m are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$$2y\frac{dy}{dx} = m(-2x)$$

$$y\frac{dy}{dx} = -mx$$

$m = -\frac{y}{x}\frac{dy}{dx}$ (1)

Differentiating the above equation with respect to x on both sides,

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ (2)

From equations (1) and (2),

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

This is the required differential equation.

7. Question

Form the differential equation of the family of curves given by $(x-a)^2+2y^2=a^2$, where a is an arbitrary constant.

Answer

Equation of the family of curves, $(x-a)^2+2y^2=a^2$, where a is an arbitrary constant.

$$x^2 - 2ax + a^2 + 2y^2 = a^2$$

$x^2 - 2ax + 2y^2 = 0$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2x - 2a + 4y\frac{dy}{dx} = 0$$

$$x - a + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{a - x}{2y}$$

$$\frac{dy}{dx} = \frac{a - x}{2y}\left(\frac{2x}{2x}\right)$$

$\frac{dy}{dx} = \frac{2ax - 2x^2}{4xy}$ (Substituting 2ax from equation 1)

$$\frac{dy}{dx} = \frac{x^2 + 2y^2 - 2x^2}{4xy}$$

$$\frac{dy}{dx} = \frac{2y^2 - x^2}{4xy}$$

This is the required differential equation.

8. Question

Form the differential equation of the family of curves given by $x^2+y^2-2ay=a^2$, where a is an arbitrary constant.

Answer

Equation of the family of curves, $x^2+y^2-2ay=a^2$, where a is an arbitrary constant.

$$x^2 - 2ax + a^2 + 2y^2 = a^2$$

$x^2 - 2ax + 2y^2 = 0$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2x - 2a + 4y\frac{dy}{dx} = 0$$

$$x - a + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{a - x}{2y}$$

$$\frac{dy}{dx} = \frac{a - x}{2y}\left(\frac{2x}{2x}\right)$$

$\frac{dy}{dx} = \frac{2ax - 2x^2}{4xy}$ (Substituting 2ax from equation 1)

$$\frac{dy}{dx} = \frac{x^2 + 2y^2 - 2x^2}{4xy}$$

$$\frac{dy}{dx} = \frac{2y^2 - x^2}{4xy}$$

This is the required differential equation.

9. Question

Form the differential equation of the family of all circles touching the y-axis at the origin.

Answer

Equation of the family of all circles touching the y-axis at the origin can be represented by

$(x-a)^2+y^2=a^2$, where a is an arbitrary constants.

$\mathbf{(x-a)^2+y^2 = a^2}$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2(x-a)+2y\frac{dy}{dx}=0$$

$$x-a+y\frac{dy}{dx}=0$$

$$a=x+y\frac{dy}{dx}$$

Substituting the value of a in equation (1)

$$\left(y\frac{dy}{dx}\right)^2+y^2=\left(x+y\frac{dy}{dx}\right)^2$$

$$\left(y\frac{dy}{dx}\right)^2+y^2=x^2+xy\frac{dy}{dx}+\left(y\frac{dy}{dx}\right)^2$$

Rearranging the above equation

$$y^2-x^2-xy\frac{dy}{dx}=0$$

This is the required differential equation.

10. Question

From the differential equation of the family of circles having centers on y-axis and radius 2 units.

Answer

Equation of the family of circles having centers on y-axis and radius 2 units can be represented by

$(x)^2+(y-a)^2=4$, where a is an arbitrary constant.

$(y-a)^2+x^2 = 4$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$\mathbf{2}(x)+2(y-a)\frac{dy}{dx}=0$$

$$x-a\frac{dy}{dx}+y\frac{dy}{dx}=0$$

$$a=\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2+\left(y-\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = 4$$

$$x^2+\left(\frac{y\frac{dy}{dx}-x-y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = 4$$

$$x^2 + \left(\frac{x}{\frac{dy}{dx}}\right)^2 = 4$$

Rearranging the above equation

$$x^2\left(1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}\right) = 4$$

This is the required differential equation.

11. Question

Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes

Answer

Equation of the family of circles in the second quadrant and touching the coordinate axes

can be represented by

$(x - (-a))^2+(y - a)^2= a^2$, where a is an arbitrary constants.

$\mathbf{(x + a)^2 + (y - a)^2 = a^2}$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2(x + a) + 2(y - a)\frac{dy}{dx} = 0$$

$$x + a - a\frac{dy}{dx} + y\frac{dy}{dx} = 0$$

$$a = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}$$

Substituting the value of a in equation (1)

$$\left(x + \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 + \left(y - \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 = \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2$$

$$\left(\frac{x\frac{dy}{dx} - x + x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 + \left(\frac{y\frac{dy}{dx} - y - x - y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 = \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2$$

$$\left(x\frac{dy}{dx} - x + x + y\frac{dy}{dx}\right)^2 + \left(y\frac{dy}{dx} - y - x - y\frac{dy}{dx}\right)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2 (x + y)^2 + (-y - x)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2 (x + y)^2 + (y + x)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

Rearranging the above equation

$$(x + y)^2\{\left(\frac{dy}{dx}\right)^2 + 1\} = \left(x + y\frac{dy}{dx}\right)^2$$

This is the required differential equation.

12. Question

Form the differential equation of the family of circles having centers on the x-axis and radius unity.

Answer

Equation of the family of circles having centers on the x-axis and radius unity can be represented by

$(x - a)^2+(y)^2= 1$, where a is an arbitrary constants.

$\mathbf{(x-a)^2+y^2 = 1}$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2(x-a)+2(y)\frac{dy}{dx}=0$$

$$x-a+y\frac{dy}{dx}=0$$

$$a=x+y\frac{dy}{dx}$$

Substituting the value of a in equation (1)

$$\left(x-x-y\frac{dy}{dx}\right)^2+y^2 = 1$$

$$\left(y\frac{dy}{dx}\right)^2+y^2 = 1$$

This is the required differential equation.

13. Question

Form the differential equation of the family of circles passing through the fixed point (a,0) and (-a,0), where a is the parameter.

Answer

Now, it is not necessary that the centre of the circle will lie on origin in this case. Hence let us assume the coordinates of the centre of the circle be (0, h). Here, h is an arbitrary constant.

Also, the radius as calculated by the Pythagoras theorem will be $a^2 + h^2$.

Hence, the equation of the family of circles passing through the fixed point (a,0) and (-a,0), where a is the parameter can be represented by

$(x)^2+(y – h)^2= a^2 + h^2$, where a is an arbitrary constants.

$x^2+ (y-h)^2= a^2+h^2$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2(x)+2(y-h)\frac{dy}{dx}=0$$

$$x-h\frac{dy}{dx}+y\frac{dy}{dx}=0$$

$$h=\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2 + \left(y - \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = a^2 + \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2$$

$$x^2 + \left(\frac{y\frac{dy}{dx} - x - y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = a^2 + \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 + (x)^2 = a^2\left(\frac{dy}{dx}\right)^2 + \left(x + y\frac{dy}{dx}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 + (x)^2 = a^2\left(\frac{dy}{dx}\right)^2 + x^2 + 2xy\frac{dy}{dx} + \left(y\frac{dy}{dx}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 = a^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + \left(y\frac{dy}{dx}\right)^2$$

$$(x^2 - a^2 - y^2)\left(\frac{dy}{dx}\right) = 2xy$$

This is the required differential equation.

14. Question

Form the differential equation of the family of parabolas having a vertex at the origin and axis along positive y-axis.

Answer

Equation of the family of parabolas having a vertex at the origin and axis along positive y-axis can be represented by

$(x)^2 = 4ay$, where a is an arbitrary constants.

$x^2 = 4ay$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2(x) = 4(a)\frac{dy}{dx}$$

$$x = 2a\frac{dy}{dx}$$

$$a = \frac{x}{2\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2 = 4\frac{x}{2\frac{dy}{dx}}y$$

$$x\frac{dy}{dx} = 2y$$

This is the required differential equation.

15. Question

Form the differential equation of the family of an ellipse having foci on the y-axis and centers at the origin.

Answer

Equation of the family of an ellipse having foci on the y-axis and centers at the origin can be represented by

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$\frac{2x}{b^2}+\frac{2y}{a^2}\frac{dy}{dx}=0$$

$$\frac{x}{b^2}+\frac{y}{a^2}\frac{dy}{dx}=0$$

$$\frac{y}{a^2}\frac{dy}{dx}=-\frac{x}{b^2}$$

$$\frac{y}{x}\frac{dy}{dx}=-\frac{a^2}{b^2}$$

Again differentiating the above equation with respect to x on both sides, we have,

$$\frac{y}{x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{\frac{dy}{dx}x-y\frac{dx}{dx}}{x^2}\right)=0$$

$$xy\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{dy}{dx}x-y\frac{dx}{dx}\right)=0$$

Rearranging the above equation

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

This is the required differential equation.

16. Question

Form the differential equation of the family of hyperbolas having foci on the x-axis and centers at the origin.

Answer

Equation of the family of an ellipse having foci on the y-axis and centers at the origin can be represented by

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{x}{a^2}-\frac{y}{b^2}\frac{dy}{dx}=0$$

$$\frac{y}{b^2}\frac{dy}{dx}=\frac{x}{a^2}$$

$$\frac{y}{x}\frac{dy}{dx}=\frac{b^2}{a^2}$$

Again differentiating the above equation with respect to x on both sides, we have,

$$\frac{y}{x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{\frac{dy}{dx}x-y\frac{dx}{dx}}{x^2}\right)=0$$

$$xy\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{dy}{dx}x-y\frac{dx}{dx}\right)=0$$

Rearranging the above equation

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

This is the required differential equation.