Therefore,

f(x) is not derivable at x = 2

7. Question

Show that function

f(x) = $\begin{cases} (1-x), & \text{when } x < 1; \\ (x^2-1), & \text{when } x \geq 1. \end{cases}$ is continuous but not differentiable at x=1

Answer

Given function f(x) = $\begin{cases} (1-x), \text{when} x < 1; \\ (x^2-1), \text{when} x \geq 1. \end{cases}$

Left hand limit at x = 1:

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1}(1-x) = 1 - 1 = 0$

Right hand limit at x = 1:

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1}(x^2-1) = 1^2 - 1 = 0$

Also, f(1) = 1^2 – 1 = 0

As,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

Therefore,

f(x) is continuous at x = 1

Now, let's see the differentiability of f(x):

LHD at x = 2:

$\lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2} \frac{(1-x)-(1-2)}{x-2} = \lim_{x \to 2} \frac{1-x-1+2}{x-2} = \lim_{x \to 2} \frac{-(x-2)}{x-2}$

$= \lim_{x \to 2} -1 = -1$

RHD at x = 2:

$\lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2} \frac{(x^2-1)-(2^2-1)}{x-2} = \lim_{x \to 2} \frac{x^2-1-3}{x-2} = \lim_{x \to 2} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2} \frac{x^2-2^2}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \to 2}(x+2)$ = 2 + 2 = 4

As, LHD ≠ RHD

Therefore,

f(x) is not differentiable at x = 2

8. Question

Let f(x) = $\begin{cases} (2+x), & \text{if } x \geq 0; \\ (2-x), & \text{if } x < 0. \end{cases}$ Show that f(x) is not derivable at x=0.

Answer

Given function f(x) = $\begin{cases} (2+x), \text{if} x \geq 0; \\ (2-x), \text{if} x < 0. \end{cases}$

LHD at x = 0:

$$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{(2-x)-(2)}{x-0} = \lim_{x\to 0}\frac{-x}{x}$$

$$= \lim_{x\to 0} -1 = -1$$

RHD at x = 0:

$$\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{(2+x)-(2)}{x-0} = \lim_{x\to 0}\frac{x}{x} = \lim_{x\to 0} 1 = 1$$

As, LHD ≠ RHD

Therefore,

f(x) is not differentiable at x = 0

9. Question

If ƒ(x) = |x| show that ƒ’(2)=1

Answer

Given function is f(x) = |x|

LHD at x = 2:

$$\lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2} = \lim_{h\to 0}\frac{f(2-h)-f(2)}{2-h-2} = \lim_{h\to 0}\frac{|2-h|-|2|}{-h} = \lim_{h\to 0}\frac{-h}{-h}$$

$$\lim_{h\to 0} 1 = 1$$

RHD at x = 2:

$$\lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{2+h-2} = \lim_{h\to 0}\frac{|2+h|-|2|}{h} = \lim_{h\to 0}\frac{h}{h}$$

$$\lim_{h\to 0} 1 = 1$$

As, LHD = RHD

Therefore, f(x) = |x| is differentiable at x = 2

Now f’(2) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|2+h|-|2|}{h} = \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0} 1 = 1$

Therefore,

f’(2) = 1

10. Question

Find the values of a and b so that the function

ƒ(x) = $\begin{cases} (x^2+3x+a), & \text{when } x \le 1; \\ (bx+2), & \text{when } x > 1 \end{cases}$ is differentiable at each x ϵ R

Answer

It is given that f(x) is differentiable at each x ϵ R

For x ≤ 1,

f(x) = x^2 + 3x + a i.e. a polynomial

for x > 1,

f(x) = bx + 2, which is also a polynomial

Since, a polynomial function is everywhere differentiable. Therefore, f(x) is differentiable for all x > 1 and for all x < 1.

f(x) is continuous at x = 1

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$$

$$\lim_{x\to1}(x^2 + 3x + a) = \lim_{x\to1}(bx + 2) = 1 + 3 + a$$

$$1^2 + 3(1) + a = b(1) + 2 = 4 + a$$

4 + a = b + 2

a – b + 2 = 0 ...(1)

As function is differentiable, therefore, LHD = RHD

LHD at x = 1:

$$\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1} = \lim_{x\to1}\frac{x^2+3x+a-(4+a)}{x-1} = \lim_{x\to1}\frac{x^2+3x-4}{x-1} = \lim_{x\to1}\frac{(x+4)(x-1)}{x-1}$$

$$= \lim_{x\to1}(x+4) = 1 + 4 = 5$$

RHD at x = 1:

$$\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1} = \lim_{x\to1}\frac{(bx+2)-(4+a)}{x-1} = \lim_{x\to1}\frac{bx-2-a}{x-1} = \lim_{x\to1}\frac{bx-b}{x-1} = \lim_{x\to1}\frac{b(x-1)}{x-1}$$

$$= \lim_{x\to1} b = b$$

As, LHD = RHD

Therefore,

5 = b

Putting b in (1), we get,

a – b + 2 = 0

a - 5 + 2 =0

a = 3

Hence,

a = 3 and b = 5