CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

11th January 2019 | Shift - II

PHYSICS

Q.1 The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :

(1) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$

(2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$

(3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

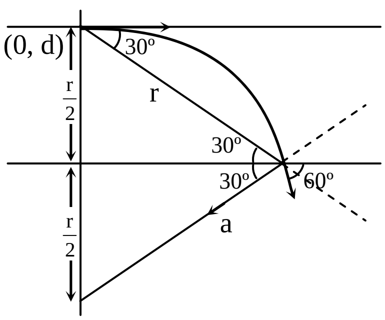
(4) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Ans. **[Bonus]**

Sol. In question entry point of particle is not given

Assuming particle enters from (0, d)

$r = \frac{mv}{qB}$; $d = \frac{r}{2}$



$\vec{a} = a\cos30(-\hat{i}) + a\sin30(-\hat{j})$

$\Rightarrow a\frac{\sqrt{3}}{2}(-\hat{i}) - \frac{a}{2}\hat{j}$

$\Rightarrow \frac{qvB}{m}\left(\frac{-\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

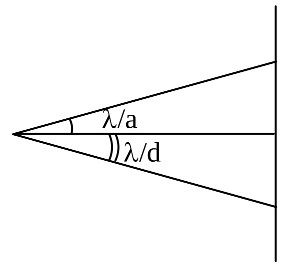
this option also not given in above choices.

Q.2 In a double-slit experiment, green light (5303Å) falls on a double slit having a separation of 19.44 μm and a widht of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :

(1) 04 (2) 05 (3) 10 (4) 09

Ans. **[1]**

Sol.



Angle subtended by first and second diffraction minima on the screen = $\frac{\lambda}{a}$

angular fringe width = $\frac{\lambda}{d}$

no. of bright fringes = $\frac{(\lambda/a)}{(\lambda/d)} = \frac{d}{a} = \frac{19.44}{4.05} = 4.81$

Q.3 A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :

(1) 1 rad/s (2) 10^{-3} rad/s (3) 10^{-1} rad/s (4) 10^{-5} rad/s

Ans. **[2]**

Sol. $\omega = \sqrt{\frac{g}{\ell}}$

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g}{g}$$

Δg is due to oscillation of support

$\Delta g = 2\omega_1^2 A_1$

where ω_1 is the angular frequency of oscillation of support, A_1 amplitude of oscillation

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{2\omega_1^2 A_1}{g}$$

$$\Rightarrow \frac{1}{2} \times \frac{2\times1\times10^{-2}}{10} = 10^{-3}$$

Q.4 An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole?

(1) -7×10^{-27} J (2) -9×10^{-20} J (3) -10×10^{-29} J (4) -20×10^{-18} J

Ans. **[1]**

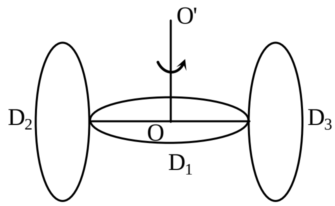
Sol. Electric potential energy of dipole is

$U = -PE\cos\theta$

$U = -10^{-29} \times 1000 \cos 45^\circ$

$\Rightarrow -\frac{1}{\sqrt{2}} \times 10^{-26} = -5\sqrt{2} \times 10^{-27} = -7 \times 10^{-27}$ J

Q.5 A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1 as shown in the figure, will be :



(1) $3MR^2$ (2) MR^2 (3) $\frac{2}{3}MR^2$ (4) $\frac{4}{5}MR^2$

Ans. **[1]**

Sol. I = (M.I. of D_2 and D_3) + m.I of D_1

$\Rightarrow \left[\frac{mR^2}{4} + MR^2\right] \times 2 + \frac{mR^2}{2}$

$\Rightarrow \frac{5}{4}mR^2 \times 2 + \frac{mR^2}{2}$

$\Rightarrow \frac{6mR^2}{2} = 3mR^2$

Q.6 Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is

(1) 200°C (2) 270°C (3) 230°C (4) 250°C

Ans. **[3]**

Sol. Changing in length in both rods are same

$\Delta\ell = \alpha\ell\Delta\theta$

$\therefore \alpha_1\ell_1\Delta\theta_1 = \alpha_2\ell_2\Delta\theta_2$

$4 \times (180 - 30) = (T - 30) \times 3$

$600 = (T - 30) \times 3$

$T = 230^\circ C$

Q.7 When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :

(1) 60°C (2) 70°C (3) 85°C (4) 80°C

Ans. **[4]**

Sol. **Case-I**

$100 \times S_A \times (100 - 90) = 50 \times S_B(90 - 75)$ … (1)

Case-II

$100 \times S_A(100 - \theta) = 50 \times S_B(\theta - 50)$ … (2)

By $\frac{(2)}{(1)}$

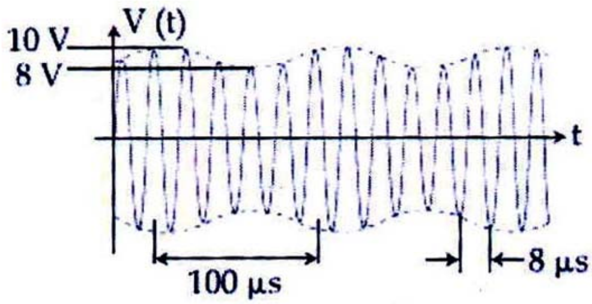
$\frac{(100-\theta)}{10} = \frac{\theta - 50}{15}$

$1500 - 15\theta = 10\theta - 500$

$2000 = 25\theta$

$\theta = 80°C$

Q.8 An amplitude modulated signal is plotted below :



Which one of the following best describes the above signal ?

(1) $(1 + 9\sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V

(2) $(9 + \sin(4\pi \times 10^4 t))\sin(5\pi \times 10^5 t)$ V

(3) $(9 + \sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V

(4) $(9 + \sin(2.5\pi \times 10^4 t))\sin(2\pi \times 10^4 t)$ V

Ans. **[3]**

Sol. $\omega_s = \frac{2\pi}{100 \times 10^{-6}} = 2\pi \times 10^4\ s^{-1}$

$\omega_C = \frac{2\pi}{8 \times 10^{-6}} = 2.5\ \pi \times 10^5\ s^{-1}$

$V_{max} = V_C + V_s = 10$

$V_{min} = V_C - V_s = 8$

$V_c = 9$ mV

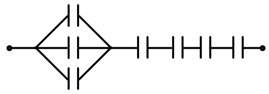
$V_s = 1$ mV

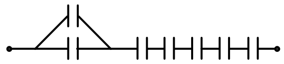
Equation of AM wave

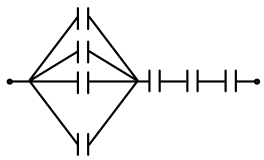
$V_{AM} = (V_C + V_s \sin \omega_s t) \sin \omega_C t$

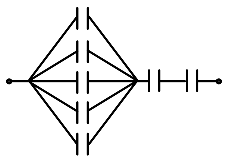
$= \{9 + \sin(2\pi \times 10^4)t\} \sin(2.5\ \pi \times 10^5\ t)$ (in mV)

Q.9 Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value

(1) 

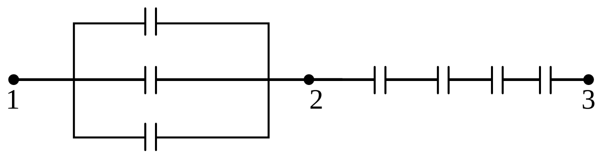
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.

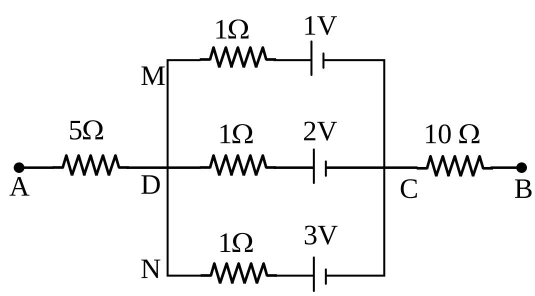


$C_{12} = 6$ μF

$C_{23} = \frac{2}{4} = \frac{1}{2}$ μF

$C_{13} = \frac{6 \times \frac{1}{2}}{6 + \frac{1}{2}} = \frac{3}{\frac{13}{2}} = \frac{6}{13}$ μF

Q.10 In the circuit shown, the potential difference between A and B is :



(1) 6 V (2) 3 V (3) 2 V (4) 1 V

Ans. **[3]**

Sol. $V_{AB} = \frac{\frac{1}{1} + \frac{2}{1} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}} = \frac{6}{3} = 2$ volt

Q.11 A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

(1) 60 (2) 35 (3) 25 (4) 40

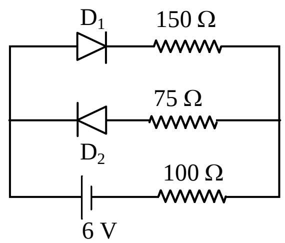
Ans. **[3]**

Sol. $$\frac{\frac{x_0}{2}-\frac{x_0}{3}}{x_0-\frac{x_0}{3}} = \frac{C-0}{100-0}$$

$$\frac{1}{4} = \frac{C}{10}$$

C = 25ºC

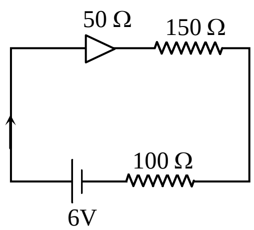
Q.12 The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in Amperes) is :



(1) 0.027 (2) 0.030 (3) 0.036 (4) 0.020

Ans. **[4]**

Sol. Since the second diode is reverse biased the simplified circuit is as shown in the figure



$$i = \frac{6}{300} = 0.02A$$

Q.13 In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to: ($\frac{hc}{e}$ = 1240 nm–V)

(1) 0.5 V (2) 1.0 V (3) 2.0 V (4) 1.5 V

Ans. **[2]**

Sol. $$V_{s1} = \frac{1240}{300} - \phi$$

$$V_{s2} = \frac{1240}{400} - \phi$$

$$V_{s1} - V_{s2} = \frac{1240}{300} - \frac{1240}{400}$$

= 4.13 – 3.1

= 1.03

= 1

Q.14 A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :

(1) $K_2 = \frac{K_1}{2}$ (2) $K_2 = 2K_1$ (3) $K_2 = K_1$ (4) $K_2 = \frac{K_1}{4}$

Ans. **[2]**

Sol. Maximum kinetic energy = $\frac{1}{2}m\omega^2A^2$

$\omega = \sqrt{\frac{g}{\ell}}$

$A = L\theta$

$KE = \frac{1}{2}m\left(\frac{g}{\ell}\right) \times L^2\theta^2 = \frac{1}{2}mgL\theta^2$

$K_1 = \frac{1}{2}mgL\theta_1^2$

if length is doubled

$K_2 = \frac{1}{2}mg\,(2L)\theta_2{}^2$ (here we assuming angular amplitude is same)

$\Rightarrow mg\ell\theta_2{}^2$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}mgL\theta_2^2}{\frac{1}{2}mg(2L)\theta_2^2} = \frac{1}{2} \qquad (\because \theta_1 = \theta_2)$$

$K_2 = 2K_1$

Q.15 The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be :

(1) $\frac{\sqrt{3}}{2}s$ (2) $\frac{3}{2}s$ (3) $\frac{2}{\sqrt{3}}s$ (4) $2\sqrt{3}\,s$

Ans. **[4]**

Sol. $g = \frac{Gm}{R^2}$

$$\frac{g_p}{g_e} = \frac{m_p}{m_e} \times \left(\frac{R_e}{R_p}\right)^2 = 3 \times \frac{1}{9} = \frac{1}{3}$$

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$T_p = \sqrt{3}\ T_e$

Time period at earth for seconds pendulum = 2 sec

$T_p = 2\sqrt{3}$ sec

Q.16 A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/ T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is

(1) 3.3×10^{-4} (2) 2.3×10^{-2} (3) 4.3×10^{-2} (4) 3.3×10^{-2}

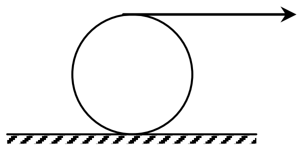
Ans. **[1]**

Sol. $I = \chi H$

$$\frac{20 \times 10^{-6}}{10^{-6}} = \chi(60 \times 10^3)$$

$$\chi = \frac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$$

Q.17 A string is wound around a hollow cylinder of mass 5 kg and radius 0.5m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :



(1) 16 rad/s^2 (2) 20 rad/s^2 (3) 10 rad/s^2 (4) 12 rad/s^2

Ans. **[1]**

Sol. Taking torque about the point of contact

$40 \times 1 = (mr^2 + mr^2)\ \alpha$

$$40 = 2m \times \frac{1}{4}\ \alpha$$

$$40 = 2 \times 5 \times \frac{1}{4} \alpha$$

$\alpha = 16$ rad/s^2

Q.18 A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Specific Heat Capacities of water and metal are, respectively, 4200 Jkg^{-1} and 400 Jkg^{-1} K^{-1}]

(1) 20% (2) 25% (3) 15% (4) 30%

Ans. **[1]**

Sol. $\Delta Q_{ball} = (\Delta Q)_{water} + (\Delta Q)_{vessel}$

$0.1 \times 400(500 - T) = 0.5 \times 4200 \times (T - 30) + 800(T - 30)$

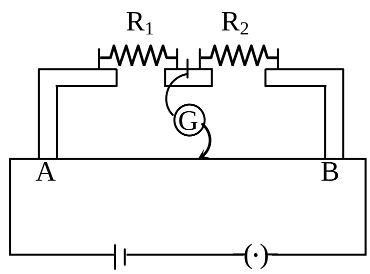
After solving

T = 36.4 C

% rise in temp $\Rightarrow \frac{6.4}{30} \times 100\% = 21\%$

Approx. 20%

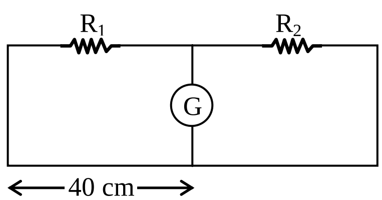
Q.19 In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)$ Ω such that the null point shifts back to its initial position is :



(1) 40 Ω (2) 30 Ω (3) 20 Ω (4) 60 Ω

Ans. **[4]**

Sol.



$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$

$\frac{R_1 + 10}{R_2} = \frac{50}{50} = 1$

$R_1 + 10 = R_2$

$R_1 + 10 = \frac{3}{2} R_1$

$R_1 = 20$

$R_2 = 30$

$\frac{\left(\frac{(R_1+10)R}{R_1+10+R}\right)}{R_2} = \frac{2}{3}$

$\frac{\left(\frac{30R}{30+R}\right)}{30} = \frac{2}{3}$

$\frac{30R}{30+R} = 20$

30R = 600 + 20R

10R = 600

R = 60

Q.20 A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :

(1) $2\sqrt{\frac{k}{p}}$ (2) $2\sqrt{\frac{p}{k}}$ (3) $\sqrt{\frac{2p}{2}}$ (4) $\sqrt{\frac{2k}{p}}$

Ans. **[2]**

Sol. $F = \frac{dP}{dt}$

$kt = \frac{dP}{dt}$

$\int_{P}^{3P} dP = \int_{0}^{t} kt\, dt$

$2P = \frac{kt^2}{2}$

$t = \sqrt{\frac{4P}{k}} = 2\sqrt{\frac{P}{k}}$

Q.21 A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by :

[Given permittivity of space $\in_0 = 9 \times 10^{-12}$ SI units, Speed of light $c = 3 \times 10^8$ m/s]

(1) 2 kV/m (2) 1 kV/m (3) 1.4 kV/m (4) 0.7 kV/m

Ans. **[3]**

Sol. $I = \frac{P}{A} = \frac{1}{2} \in_0 E_0^2 C$

$E_0 = \sqrt{\frac{2P}{\in_0 CA}}$

$\Rightarrow \sqrt{\frac{2 \times 27 \times 10^{-3} \times 36\pi \times 10^9}{3 \times 10^8 \times 10 \times 10^{-6}}}$

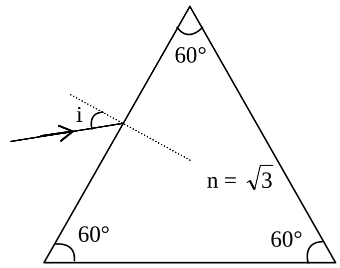
$\Rightarrow$ 1.4 kV/m

Q.22 A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is:

(1) 60° (2) 45° (3) 90° (4) 30°

Ans. **[1]**

Sol.



$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sqrt{3} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{60+\delta_m}{2}\right)$$

$$60 = \left(\frac{60+\delta_m}{2}\right)$$

$\delta_m = 60°$

$$i = \frac{60+\delta_m}{2} = 60°$$

Q.23 In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be:

(1) $\frac{25}{16}\lambda$ (2) $\frac{27}{20}\lambda$ (3) $\frac{16}{25}\lambda$ (4) $\frac{20}{27}\lambda$

Ans. **[4]**

Sol. from M orbit to L orbit

$$\frac{hc}{\lambda_1} = (13.6 \text{ eV})\, z^2 \left(\frac{1}{4}-\frac{1}{9}\right) \quad ....(i)$$

From N orbit to L orbit

$$\frac{hc}{\lambda_2} = (13.6 \text{ eV})\, z^2 \left(\frac{1}{4}-\frac{1}{16}\right) \quad ....(ii)$$

diving (i) by (ii)

$$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \times \frac{64}{12} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27}\lambda_1$$

Q.24 A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is:

(1) 120 Ω (2) 125 Ω (3) 80 Ω (4) 100 Ω

Ans. **[3]**

Sol. Full deflection current

$i_g = nk$

$= 30 \times 0.005$

$15 = 0.005 \times 30 \times 20 + 30 \times 0.005 \times R$

$R = 80\ \Omega$

Q.25 A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i} + 3\hat{j};\ \vec{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from he origin to the point P(x = 1; y = 1) along a straight path. The magnitude of the total work done is :

(1) (2.5) q (2) (0.35) q (3) (0.15) q (4) 5q

Ans. **[4]**

Sol. Work done by magnetic force = 0

work done by electric force = [(2×1) + (3×1)]q

= 5q

Q.26 A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$ m, at t = 0, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})$ ms^{-2}. What is the distance of the particle from the origin at time 2 s?

(1) 15 m (2) $20\sqrt{2}$ m (3) $10\sqrt{2}$ m (4) 5 m

Ans. **[2]**

Sol. $r_2 - r_1 = ut + \frac{1}{2}at^2$

$r_2 = \vec{r}_1 + ut + \frac{1}{2}at^2$

$= (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2 + \frac{1}{2}(4\hat{i} + 4\hat{j})\, 2^2$

$\Rightarrow 2\hat{i} + 4\hat{j} + 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\Rightarrow 20\hat{i} + 20\hat{j}$

$|\vec{r}_2| = 20\sqrt{2}$

Q.27 If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young,s modulus will be:

(1) $V^{-2}A^2F^2$ (2) $V^{-4}A^{-2}F$ (3) $V^{-4}A^2F$ (4) $V^{-2}A^2F^{-2}$

Ans. **[3]**

Sol. $Y = K\, v^x\, F^y\, A^z$; K is dimension less constant

$ML^{-1}T^{-2} = [LT^{-1}]^x\, [MLT^{-2}]^y\, [LT^{-2}]^z$

$ML^{-1}T^{-2} = L^{x+y+z}\, T^{-x-2y-2z}\, M^y$

y =1; x + y + z = –1; x + z = –2

and –x –2y –2z = –2

–x –2z = 0; x + 2z = 0;

so x = – 4, y = 1, z = 2

$Y = V^{-4}A^2F$

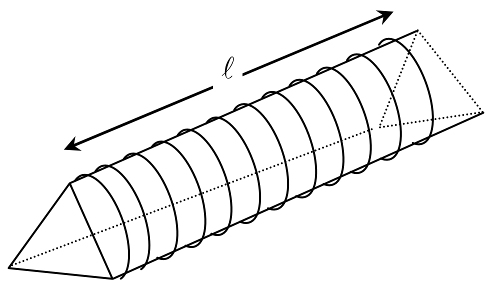
Q.28 A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil:

(1) decreases by a factor of $9\sqrt{3}$ (2) increases by a factor of 27

(3) decreases by a factor of 9 (4) increases by a factor of 3

Ans. **[2]**

Sol.



$L = \frac{\rho N^2 A}{\ell}$

$\frac{N}{\ell}$ = constant

$N \propto \ell$

$L \propto \ell A$

$L \propto \ell a^2$ $A = \frac{\sqrt{3}}{4}a^2$

All dimesions becomes 3 times

Hence L become 27 times

Q.29 The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Ans. **[2]**

Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = rF \sin\theta$

$2.5 = 5 \times 1 \times \sin\theta$

$\theta = 30°$

Q.30 In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant) :

(1) $\frac{1}{2}KR\Delta T$ (2) $\frac{1}{2}R\Delta T$ (3) $\frac{3}{2}R\Delta T$ (4) $\frac{2K}{3}\Delta T$

Ans. **[2]**

Sol. VT = K

From ideal gas equation PV = nRT

PV^2 = constant

For polytropic $W = \frac{P_1V_1 - P_2V_2}{x-1}$

$W = \frac{nR(T_1 - T_2)}{x-1}$

$W = \frac{-nR(\Delta T)}{2-1} = -nR\Delta T$

$\Delta Q = \Delta U + \Delta W$

$= \frac{3}{2}R\Delta T - nR\Delta T$

$\Delta Q = \frac{1}{2}R\Delta T$

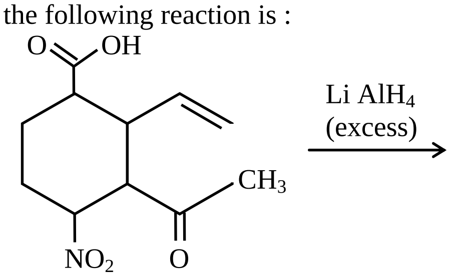
JEE Main Online Exam 2019

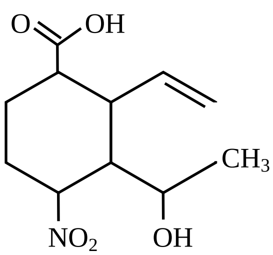
Questions & Solutions

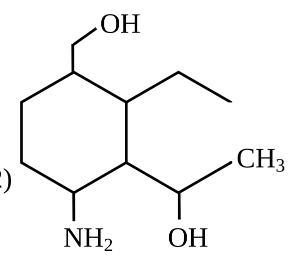
11th January 2019 | Shift - II

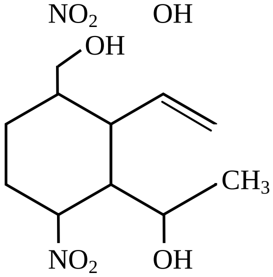
Part B – CHEMISTRY

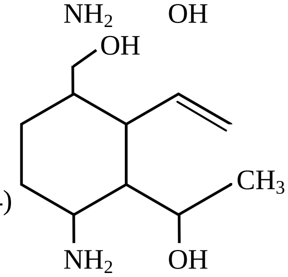
Q.1 The major product otained in the following reaction is :



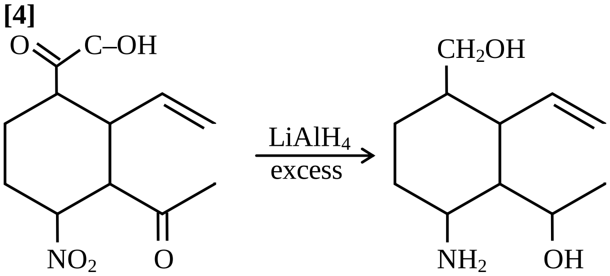
(1) 

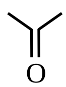
(2) 

(3) 

(4) 

Ans. **[4]**

Sol. 

$LiAHl_4$ reduces – COOH, $–NO_2$,  but do not reduces C = C

Q.2 The correct option with respect to the Pauling electronegativity values of the element is :

(1) Ga < Ge (2) Si < Al (3) Te > Se (4) P > S

Ans. **[1]**

Sol. Ga < Ge

Theoritical question

Along the period, electronegativity increases.

Q.3 The correct match between Item I and Item II is :

Item I	**Item II**
(A) Ester test	(P) Tyr
(B) Carbylamine test	(Q) Asp
(C) Phthalein dye test	(R) Ser
	(S) Lys

(1) (A)→(Q); (B)→(S); (C)→(P)
(2) (A)→(R); (B)→(S); (C)→(Q)
(3) (A)→(R); (B)→(Q); (C)→(P)
(4) (A)→(Q); (B)→(S); (C)→(R)

Ans. **[1]**

Sol.

(A) Ester Test	Asp
(B) Carbylamine Test	Lys
(C) Ptnalein Dye Test	Tyr

Q.4 For the equlibrium,

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of $\Delta G°$ at 298 K is approximately :

(1) –80 kJ mol^{-1} (2) 100 kJ mol^{-1} (3) –100 kJ mol^{-1} (4) 80 kJ mol^{-1}

Ans. **[4]**

Sol. $2H_2O \rightleftharpoons H_3O^+ + OH^-$ $K = 10^{-14}$

$\Delta G° = -RTlnK$

$$= -\frac{8.314}{1000} \times 298 \ln 10^{-14}$$

= 80 KJ/mole.

Q.5 The radius of the largest sphere which fits properly at the centre of the edge of a body centred cubic unit cell is : (Edge length is represented by 'a')

(1) 0.134 a (2) 0.067 a (3) 0.047 a (4) 0.027 a

Ans. **[2]**

Sol. a = 2(R + r)

$\frac{a}{2} = (R + r)$ ....(i)

$\sqrt{3}a = 4R$ ....(ii)

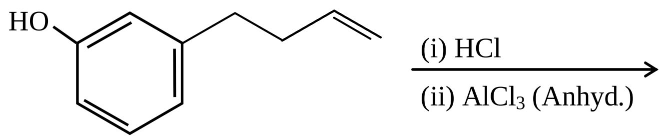
Using (i) & (ii)

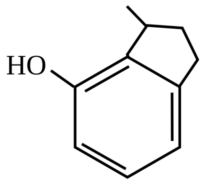
$$\frac{a}{2} = \frac{a\sqrt{3}}{4} = r$$

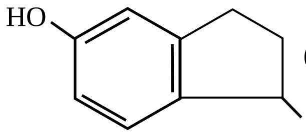
$$\Rightarrow a\left(\frac{2-\sqrt{3}}{4}\right) = r$$

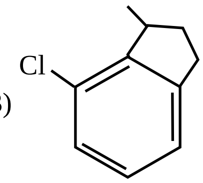
$\Rightarrow$ r = 0.067 a.

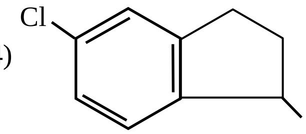
Q.6 The major product of the following reaction is :



(1) 

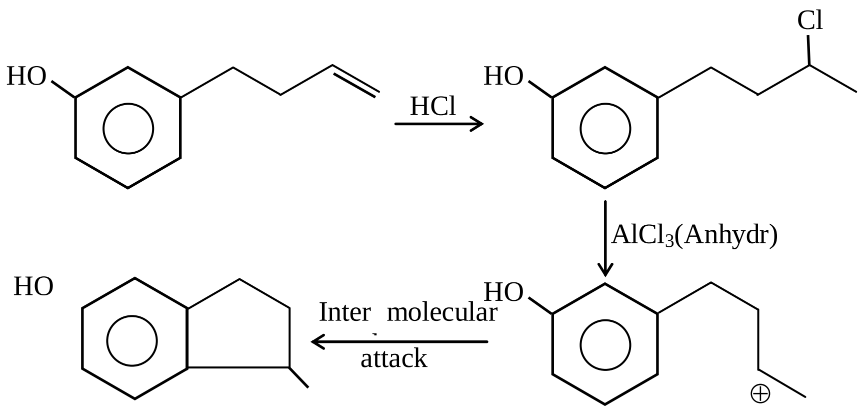
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Q.7 The higher concentration of which gas in air can cause stiffness of flower buds ?

(1) CO_2 (2) SO_2 (3) NO_2 (4) CO

Ans. **[2]**

Sol. SO_2

Due to acid rain in plants, high conc. of SO_2 makes the flower buds stift and makes then fall.

Q.8 K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is:

(1) 1.6 (2) 2.2 (3) 2.0 (4) 1.8

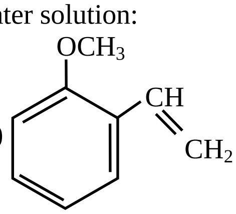
Ans. **[4]**

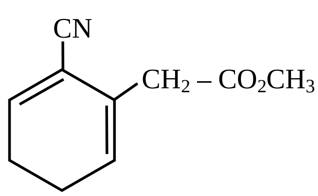
Sol. For $K_2[HgI_4]$

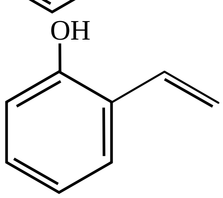
$i = 1 + 0.4(3 - 1)$

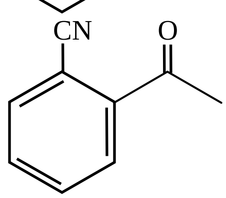
$\Rightarrow i = 1.8$

Q.9 Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution:

(1) 

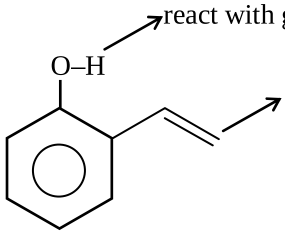
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



Q.10 Match the following items in column I with the corresponding items in column II.

Column I	**Column II**
(i) $Na_2CO_3.10H_2O$	(A) Portland cement ingredient
(ii) $Mg(HCO_3)_2$	(B) Castner-Kellner process
(iii) NaOH	(C) Solvay process
(iv) $Ca_3Al_2O_6$	(D) Temporary hardness

(1) (i)→(D); (ii)→(A); (iii)→(B); (iv)→(C)
(2) (i)→(B); (ii)→(C); (iii)→(A); (iv)→(D)
(3) (i)→(C); (ii)→(B); (iii)→(D); (iv)→(A)
(4) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

Ans. **[4]**

Sol. $Na_2CO_3.\,10H_2O \rightarrow$ Solvay process
$Mg(HCO_3)_2 \rightarrow$ Temporary Hardness
$NaOH \rightarrow$ Castner-Kellner process.
$Ca_3Al_2O_6 \rightarrow$ Portland cement ingredient.

Q.11 The reaction, $MgO(s) + C(s) \rightarrow Mg(s) + CO(g)$, for which $\Delta_rH° + 491.1$ kJ mol^{-1} and $\Delta_rS° = 198.0$ JK^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaciton will be feasible is :

(1) 2480.3 K (2) 2040.5 K (3) 2380.5 K (4) 1890.0 K

Ans. **[1]**

Sol. $T_{eq} = \dfrac{\Delta H}{\Delta S}$

$= \dfrac{491.1 \times 1000}{198}$

$= 2480.3$ K

Q.12 The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (v) of the incident radiation as, [v_0 is threshold frequency] :

(1) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{3}{2}}}$ (2) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{1}{4}}}$ (3) $\lambda \propto \dfrac{1}{(v - v_0)}$ (4) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{1}{2}}}$

Ans. **[4]**

Sol. For electron ;

$\lambda = \dfrac{h}{\sqrt{2mK.E}}$

By photo electric effect ;

$hv = hv_0 + KE$

$\Rightarrow K.E = hV\,.\,hV_0$

$\Rightarrow \lambda = \dfrac{h}{\sqrt{2m(hv - hv_0)}}$

$\lambda \propto \dfrac{1}{(v - v_0)^{1/2}}$

Q.13 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solutions ?

(1) 50 mL (2) 12.5 mL (3) 25 mL (4) 75 mL

Ans. **[3]**

Sol. $Na_2CO_3 + 2HCl \rightarrow 2NaCl + H_2CO_3$

eq. of HCl = eq. of Na_2CO_3

$$\frac{25}{1000} \times M \times 1 = \frac{30}{1000} \times 0.1 \times 2$$

$$\Rightarrow M = \frac{6}{25} M$$

eq. of HCl = eq. of NaOH

$$\frac{6}{25} \times 1 \times \frac{V}{1000} = \frac{30}{1000} \times 0.2 \times 1$$

$\Rightarrow v = 25$ mL

Q.14 $$\underline{A} \xrightarrow{4KOH, O_2} \underset{(Green)}{2\underline{B}} + 2H_2O$$

$$3\underline{B} \xrightarrow{4HCl} \underset{(Purple)}{2\underline{C}} + MnO_2 + 2H_2O$$

$$2\underline{C} \xrightarrow{H_2O.KJ} 2\underline{A} + 2KOH + \underline{D}$$

In the above sequence of reactions, $\underline{A}$ and $\underline{D}$, respectively, are :

(1) Kl and $KMnO_4$ (2) Kl and K_2MnO_4 (3) KlO_3 and MnO_2 (4) MnO_2 and KlO_3

Ans. **[4]**

Sol. $$MnO_2 \xrightarrow{4KOH, O_2} \underset{(Green)}{2K_2MnO_4} + 2H_2O$$

$$3K_2MnO_4 \xrightarrow{4HCl} \underset{(Purple)}{2KMnO_4} + 2H_2O$$

$$2KMnO_4 \xrightarrow{H_2O, Kl} 2MnO_4 + 2KOH + KIO_3$$

$A \rightarrow MnO_2$

$D \rightarrow KIO_3$

Q.15 The reaction that does NOT define calcination is :

(1) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$ (2) $CaCO_3 . MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(3) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$ (4) $Fe_2O_3 . XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

Ans. **[1]**

Sol. Calciration is carried out for carbonates and oxide ores in absence of oxygen. Roasting is carried out mainly for sulphide ores in preserce of excess of oxygen.

Q.16 Taj Mahal is being slowly disfigured and discoloured. This is primarily due to :

(1) acid rain (2) soil pollution (3) global warming (4) water pollution

Ans. **[1]**

Sol. Taj-Mahol is slowly disfigured and discolured due to acid rain.

Q.17 The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$\Delta_r G° = A - BT$

Where A and B are non-zero constants. Which of the following is TRUE about this reaction ?

(1) Exothermic if $B < 0$
(2) Endothermic if $A > 0$
(3) Exothermic if $A > 0$ and $B < 0$
(4) Endothermic if $A < 0$ and $B > 0$

Ans. **[2]**

Sol. Thcoritical.

Q.18 Among the colloids cheese (C), milk (M), and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is :

(1) C : liquid in solid ; M : liquid in liquid ; S : solid in gas
(2) C : liquid in solid ; M : liquid in solid ; S : solid in gas
(3) C : solid in liquid ; M : liquid in liquid ; S : gas in solid
(4) C : solid in liqid ; M : solid in liquid ; S : solid in gas

Ans. **[1]**

Sol.

Dispered phase		Dispersion medium
Cheese	Liquid	Solid
Milk	Liquid	Liquid
Smoke	Solid	Gas

Q.19 A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is :

(1) $CH_3CH_2CH_2CONH_2$
(2) $CH_3CON(CH_3)_2$
(3) $CH_3CH_2COCH_2NH_2$
(4) $CH_3COCH_2NHCH_3$

Ans. **[1]**

Sol. $X \xrightarrow[\text{Hoffmann Bromamide}]{Br_2/NaOH} C_3H_9N$ [1° amine)
Positive carbylamine test

$CH_3-CH_2-CH_2-C(=O)-NH_2$

Q.20 The correct match between Item I and Item II is :

Item I	**Item II**
(A) Allosteric effect	(P) Molecule binding to the active site of enzyme
(B) Competitive inhibitor	(Q) Molecule crucial for communication in the body
(C) Receptor	(R) Molecule binding to a site other than the active site of enzyme
(D) Poison	(S) Molecule binding to the enzyme covalently

(1) (A)→(P); (B)→(R); (C)→(Q); (D)→(S)
(2) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)
(3) (A)→(R); (B)→(P); (C)→(S); (D)→(Q)
(4) (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

Ans. **[4]**

Sol. Theoritical

Q.21 The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be:

(1) 18.0 h (2) 9.0 h (3) 7.2 h (4) 12.0 h

Ans. **[1]**

Sol. For zero order reaction.

$[A]_t = [A_0] - Kt$

$\Rightarrow [A_0] - [A_t] = Kt$

$\Rightarrow 0.2 - 0.1 = K \times 6$

$\Rightarrow K = \frac{1}{60}$ M/hr

$0.5 - 0.2 = \frac{1}{60} \times t$

$\Rightarrow t = 18$ hrs.

Q.22 Given the equilibrium constant:

K_C of the reaction :

$Cu(s) + 2Ag+ (aq) \rightarrow Cu^{2+} (aq) + 2Ag(s)$ is 10×10^{15}, calculate the E^0_{cell} of this reaciton at 298 K

$\left[2.303\frac{RT}{F} \text{at} 298\,K = 0.059V\right]$

(1) 0.4736 mV (2) 0.04736 V (3) 0.4736 V (4) 0.04736 mV

Ans. **[3]**

Sol. $E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n}\log Q$

At equilibrium ;

$E^{\circ}_{cell} = \frac{0.059}{2}\log 10^{16}$

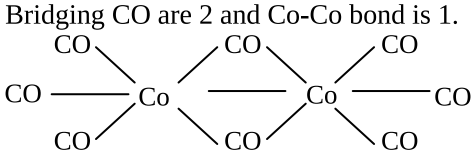
$\Rightarrow E^{\circ}_{cell} = 0.472$

Q.23 The number of bridging CO ligand(s) and Co-Co bond(s) in $Co_2(CO)_8$, respectively are :

(1) 2 and 0 (2) 0 and 2 (3) 4 and 0 (4) 2 and 1

Ans. **[4]**

Sol. Bridging CO are 2 and Co-Co bond is 1.



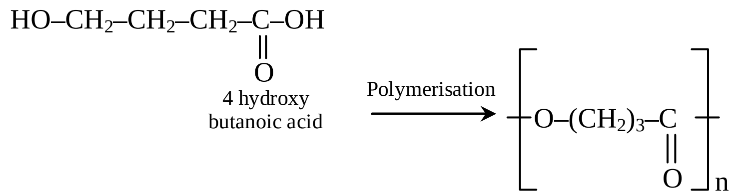
Q.24 The homopolymer formed from 4-hydroxy-butanoic acid is :

(1) $\left[\overset{O}{\overset{||}{C}} - (CH_2)_2\overset{O}{\overset{||}{C}} - O \right]_n$ (2) $\left[O\overset{O}{\overset{||}{C}}(CH_2)_3 - O \right]_n$

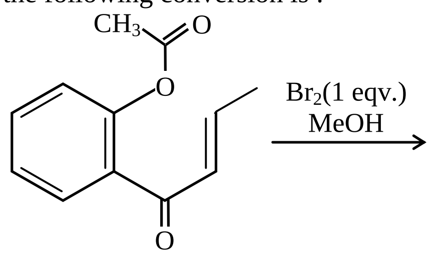
(3) $\left[\overset{O}{\overset{||}{C}}(CH_2)_3 - O \right]_n$ (4) $\left[\overset{O}{\overset{||}{C}} - (CH_2)_2\overset{O}{\overset{||}{C}} \right]_n$

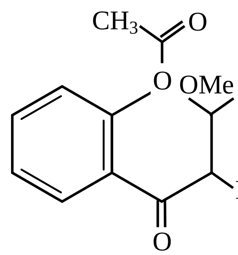
Ans. **[3]**

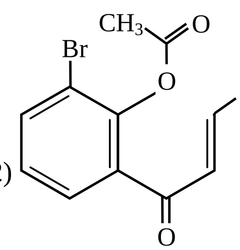
Sol.

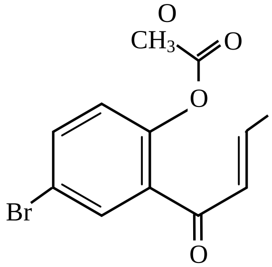


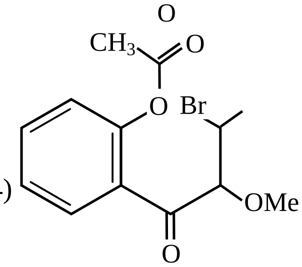
Q.25 The major product obtained in the following conversion is :



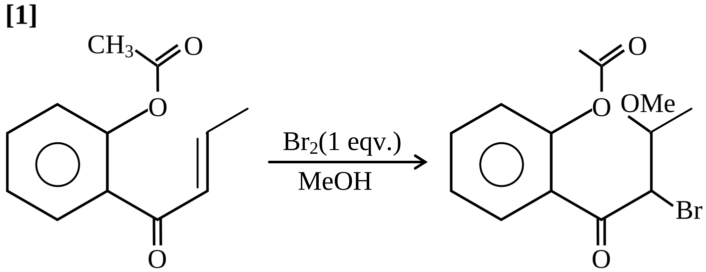
(1) 

(2) 

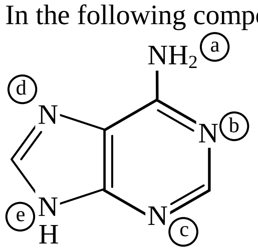
(3) 

(4) 

Ans. **[1]**

Sol. 

Q.26 In the following compound,



the favourable site/s for protonation is/are :

(1) (b), (c) and (d) (2) (a) and (e) (3) (a) and (d) (4) (a)

Ans. **[1]**

Sol. In case of b, c, d lone pair do not participate in resonance (localise ℓp) so it is favourable site for protonation

Q.27 The hydride that is NOT electron deficient

(1) B_2H_6 (2) AlH_3 (3) GaH_3 (4) SiH_4

Ans. **[4]**

Sol. $B_2H_6 \rightarrow$ Electron deficient.

$AlH_3 \rightarrow$ Electron deficient.

$SiH_4 \rightarrow$ Electron precise.

$GaH_3 \rightarrow$ Electron deficient.

Q.28 The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is :

$\left(C_2O_4^{2-} = \text{Oxalato}\right)$

(1) 14 (2) 10 (3) 8 (4) 6

Ans. **[2]**

Sol. $C_2O_4^{-2} \rightarrow$ bidentate

$H_2O \rightarrow$ Monodentate

Q.29 The relative stability of +1 oxidation state of group 13 elements follows the order :

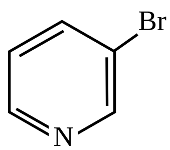
(1) Tl < In < Ga < Al (2) Al < Ga < TI < In (3) AI < Ga < In < TI (4) Ga < AI < In < TI

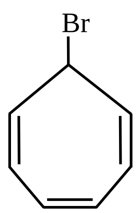
Ans. **[3]**

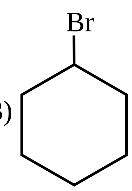
Sol. Al < Ga < In < TL

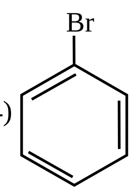
Due to Inert pair effect, as we move down the group; in 13th group, lower oxidation state becomes more stable.

Q.30 Which of the following compounds will produce a precipitate with $AgNO_3$?

(1) 

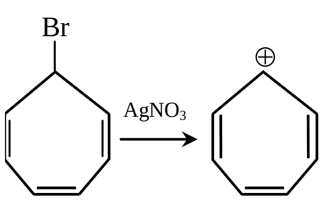
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. Those AlKyl halide which forms stable carbocation gives precipitate with $AgNO_3$



Stable carbocation
$6\pi e^-$ aromatic

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

11th January 2019 | Shift - II

MATHEMATICS

Q.1 The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when y(1) = 1, is :

(1) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$

(2) $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

(3) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

Ans. **[4]**

Sol. $\frac{dy}{dx} = (x-y)^2$ ...(1)

Put x – y = v

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dv}{dx}$

Now from (1)

$\Rightarrow 1 - \frac{dv}{dx} = v^2$

$\frac{dv}{dx} = 1 - v^2$

$\Rightarrow \frac{dv}{1-v^2} = dx$

$\int \frac{dv}{1-v^2} = \int dx$

$\frac{1}{2\times 1}\log\left|\frac{1+v}{1-v}\right| = x + c$

$\because x - y = v$

$\frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x + c$ ....(i)

y(1) = 1

at x = 1, y = 1

$\frac{1}{2}\log\left|\frac{1}{1}\right| = 1 + C$

C = –1

From (i)

$\frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x - 1$

$\log\left|\frac{1+x-y}{1-x+y}\right| = 2(x-1)$

$\Rightarrow -\log\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

Q.2 Let K be the set of all real values of x where the function f(x) = sin |x| – |x| + 2(x – π) cos |x| is not differentiable. Then the set K is equal to :

(1) {0, π} (2) ϕ (an empty set) (3) {π} (4) {0}

Ans. **[2]**

Sol. f(x) = sin|x| – |x| + 2(x – π) cos x

∵ sin|x| – |x| is differentiable function at x = 0

∴ k = ϕ

Q.3 The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :

(1) $6^5 \times (15)!$ (2) $5^6 \times 15$ (3) (15)! × 6 ! (4) 5! × 6!

Ans. **[3]**

Sol. k = {4, 8, 12, 16, 20}

f(k) can take value from set {3, 6, 9, 12, 15, 18} this can be done is ${}^6C_5 \times \lfloor 5 = \lfloor 6$ ways for remaining elements $\lfloor 15$

So total number of onto functions = $\lfloor 6 \times \lfloor 15$

Q.4 If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :

(1) 2 : 1 (2) 4 : 1 (3) 1 : 3 (4) 3 : 1

Ans. **[4]**

Sol. If $a_{19} = 0$

a + 18d = 0

Then $\frac{a_{49}}{a_{29}} = \frac{a+48d}{a+28d}$

$= \frac{a+18d+30d}{a+18d+10d}$

$= \frac{0+30d}{0+10d} = 3 : 1$

Q.5 Let z be a complex number such that |z| + z = 3 + i (where i = $\sqrt{-1}$). Then |z| is equal to

(1) $\frac{\sqrt{34}}{3}$ (2) $\frac{5}{3}$ (3) $\frac{5}{4}$ (4) $\frac{\sqrt{41}}{4}$

Ans. **[2]**

Sol. $|z| + z = 3 + i$

Let z = x + iy

$\Rightarrow \sqrt{x^2+y^2} + x + iy = 3 + i$

$\Rightarrow (x + \sqrt{x^2+y^2}) + iy = 3 + i$

y = 1 $x + \sqrt{x^2+y^2} = 3$

$x + \sqrt{1+x^2} = 3$

$1 + x^2 = 9 + x^2 - 6x$

6x = 8

$x = \frac{4}{3}$

$|z| = \sqrt{x^2+y^2} = \sqrt{\frac{16}{9}+1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

Q.6 Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ (0 < θ < 45º), and α < β. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :

(1) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$ (2) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Ans. **[2]**

Sol. $D = (1 + \sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta = (1 - \sin\theta\cos\theta)^2$

roots are β = coseθ and α = cosθ

$\Rightarrow \sum_{n=0}^{\infty}\left\{\alpha^n + \left(-\frac{1}{\beta}\right)^n\right\} = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{\infty}(-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

Q.7 If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$, x ≠ 0 and a + b + c ≠ 0, then x is equal to :

(1) –2(a + b + c) (2) 2(a + b + c) (3) abc (4) –(a + b + c)

Ans. **[1]**

Sol. $R \to R_1 + R_2 + R_3$

$$D = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-b-a \end{vmatrix}$$

$$D = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-b-a \end{vmatrix}$$

$C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_1$

$$D = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -a-b-c & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$D = (a + b + c)(a + b + c)^2 = (a + b + c)^3$

Now, $(a + b + c)(x + a + b + c)^2 = (a + b + c)^3$

$(x + a + b + c)^2 = (a + b + c)^2$

When $(x + a + b + c) = -(a + b + c)$

$\Rightarrow x = -2(a + b + c)$

Q.8 $\lim\limits_{x\to 0} \dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to :

(1) 0 (2) 4 (3) 1 (4) 2

Ans. **[3]**

Sol. $\lim\limits_{x\to 0} \dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$

$$= \lim_{x\to 0} \frac{x\tan^2 2x}{\tan 4x.\sin^2 x}$$

$$= \lim_{x\to 0} \frac{x\left(\dfrac{\tan^2 2x}{4x^2}\right).4x^2}{\left(\dfrac{\tan 4x}{4x}\right)4x\left(\dfrac{\sin^2 x}{x^2}\right)x^2} = 1$$

Q.9 Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for all $x \in R$; then $\dfrac{a_2}{a_0}$ is equal to

(1) 12.25 (2) 12.75 (3) 12.00 (4) 12.50

Ans. [1]

Sol. $(10 + x)^{50} + (10 - x)^{50}$

$\Rightarrow a_2 = 2 \,.\, {}^{50}C_2\, 10^{48}$

and $a_0 = 2 \,.\, 10^{50}$

$$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10} = 12.25$$

Q.10 The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :

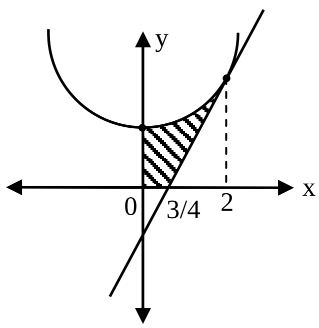
(1) $\frac{8}{3}$ (2) $\frac{14}{3}$ (3) $\frac{187}{24}$ (4) $\frac{37}{24}$

Ans. [4]

Sol. tangent

$y - 5 = 4 (x - 2)$

$y = 4x - 3$



$$\text{Area} = \int_0^2 (x^2 + 1)\,dx - \frac{1}{2} \times 5 \times \left(2 - \frac{3}{4}\right)$$

$$= \left(\frac{x^3}{3} + x\right)_0^2 - \frac{25}{8}$$

$$= \frac{8}{3} + 2 - \frac{25}{8}$$

$$= \frac{14}{3} - \frac{25}{8} = \frac{37}{24}$$

Q.11 Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :

(1) (2, 4, 7) (2) (2, –4, –7) (3) (2, –4, 7) (4) (–2, 4, 7)

Ans. [2]

Sol. Point on $L_1(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\lambda + 3 = 7\mu - 5$ ...(1)

$3\lambda - 1 = -6\mu + 2$ ....(2)

By Point (1) and (2)

$\lambda = -1, \mu = 1$

R(2, –4, 7)

Reflection is (2, –4, –7)

Q.12 Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :

(1) $\frac{1}{4}$ (2) 16 (3) $\frac{1}{16}$ (4) 1

Ans. [3]

Sol. $|A||B||A^T| = 8$

$|A|^2 |B| = 8$ ....(i)

$|A||B^{-1}| = 8$

$|A| = 8|B|$ ....(ii)

From (i) & (ii)

$64|B|^2 |B| = 8$

$|B|^3 = \frac{1}{8}$

$|B| = \frac{1}{2}$

$|A| = 4$

$\det(BA^{-1}B^T)$

$= |B|\frac{1}{|A|}|B| = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Q.13 If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to :

(1) $\frac{2}{3}(x-4)$ (2) $\frac{1}{3}(x+4)$ (3) $\frac{1}{3}(x+1)$ (4) $\frac{2}{3}(x+2)$

Ans. [2]

Sol. $\int \frac{x+1}{\sqrt{2x-1}}dx$

Let $\sqrt{2x-1} = t$

$2x - 1 = t^2$

$2dx = 2t\,dt$

$dx = t\,dt$

$\int \left(\frac{t^2+1}{2}+1\right) \times \frac{1}{t}.t.dt$

$$\int\left(\frac{t^2+1+2}{2}\right)dt$$

$$\frac{1}{2}\int(t^2+3)\,dt$$

$$\frac{1}{2}\left[\frac{t^3}{3}+3t\right]+c$$

$$\frac{t^3}{6}+\frac{3t}{2}+c$$

$$\frac{1}{6}(2x-1)^{3/2}+\frac{3}{2}(2x-1)^{1/2}+c$$

$$\sqrt{2x-1}\left[\frac{1}{6}(2x-1)+\frac{3}{2}\right]+c$$

$$\sqrt{2x-1}\left[\frac{2x-1}{6}+\frac{3}{2}\right]+c$$

$$\sqrt{2x-1}\left[\frac{2x-1+9}{6}\right]+c$$

$$\sqrt{2x-1}\left[\frac{2x+8}{6}\right]+c$$

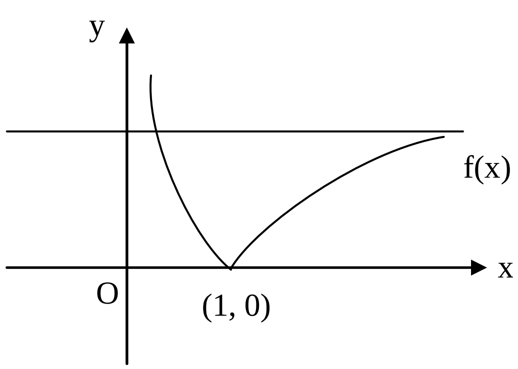
$$f(x)=\frac{x+4}{3}$$

Q.14 Let a function $f:(0,\infty)\to(0,\infty)$ be defined by $f(x)=\left|1-\frac{1}{x}\right|$. Then f is :

(1) not injective but it is surjective
(2) neiter injective nor surjective
(3) injective only
(4) both injective as well as surjective

Ans. **[Bonus]**

Sol. $f(x)=\left|1-\frac{1}{x}\right|=\frac{|x-1|}{x}$



$$f(x)=\frac{|x-1|}{x}=\begin{cases}\frac{1-x}{x}; & 0<x\le 1\\ \frac{x-1}{x}; & x\ge 1\end{cases}$$

f(x) is not injective but range of function is $[0,\infty)$

Remark : If co domain is $[0,\infty)$, then f(x) will be surjective.

Q.15 Contrapositive of the statement " If two numbers are not equal, then their squares are not equal." is :

(1) If the squares of two numbers are equal, then the numbers are not equal

(2) If the squares of two numbers are equal, then the numbers are equal

(3) If the squares of two numbers are not equal, then the numbers are equal

(4) If the squares of two numbers are not equal, then the numbers are not equal

Ans. **[2]**

Sol. Let p : two numbers are not equal

q : there squares are not equal

Contrapositive of $p \to q$ is

$$\sim q \to \sim p$$

If the squares of two numbers are equal, then the number are equal.

Q.16 Let $f(x) = \dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$, where a, b and d are non-zero real constants. Then :

(1) f is an increasing function of x

(2) f is neither increasing nor decreasing function of x

(3) f' is not a continuous function of x

(4) f is a decreasing function of x

Ans. **[1]**

Sol. $$f'(x) = \frac{\sqrt{x^2+a^2} - x\dfrac{1}{2\sqrt{x^2+a^2}}2x}{(x^2+a^2)} - \frac{(-1)\sqrt{b^2+(d-x)^2} - (d-x)\dfrac{2(d-x)(-1)}{2\sqrt{b^2+(d-x)^2}}}{b^2+(d-x)^2}$$

$$f'(x) = \frac{x^2+a^2-x^2}{(x^2+a^2)^{3/2}} - \frac{-(b^2+(d-x)^2)+(d-x)^2}{(b^2+(d-x)^2)^{3/2}}$$

$$f'(x) = \frac{a^2}{(x^2+a^2)^{3/2}} + \frac{b^2}{(b^2+(d-x)^2)^{3/2}} > 0 \ \forall\ x \in R$$

f (x) is an increasing function.

Q.17 The integral $\displaystyle\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals :

(1) $\dfrac{\pi}{40}$

(2) $\dfrac{1}{20}\tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)$

(3) $\dfrac{1}{10}\left(\dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)\right)$

(4) $\dfrac{1}{5}\left(\dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)\right)$

Ans. **[3]**

Sol. $$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x(\tan^5 x + \cot^5 x)}\,dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{(1+\tan^2 x)}{2\tan x(\tan^5 x + \cot^5 x)}\,dx$$

Let tan x = t

$\tan\frac{\pi}{6} = t,\qquad t = \tan\frac{\pi}{4}$

$\frac{1}{\sqrt{3}} = t,\qquad t = 1$

$$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{dt}{2t\left(t^5 + \frac{1}{t^5}\right)}$$

$$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{t^5\,dt}{2t(t^{10}+1)} = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{t^4 dt}{2(1+t^{10})}$$

$t^5 = u$

$5.t^4\,dt = du$

$$\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{du}{5.2(1+u^2)} = \frac{1}{10}\left[\tan^{-1}u\right]_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}$$

$$= \frac{1}{10}\left[\frac{\pi}{4} - \tan^{-1}\frac{1}{(\sqrt{3})^5}\right]$$

Q.18 Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $(4\sqrt{2}, 2\sqrt{3})$ (2) $(4\sqrt{3}, 2\sqrt{3})$ (3) $(4\sqrt{3}, 2\sqrt{2})$ (4) $(4\sqrt{2}, 2\sqrt{2})$

Ans. **[3]**

Sol. $\frac{2b^2}{a} = 8$

$\Rightarrow b^2 = 4a$

$2ae = 2b$

$b = ae$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2 - a^2e^2$

$b^2 = a^2 - b^2$

$2b^2 = a^2$

$2 \times 4a = a^2$

$a = 8$

$a^2 = 64$

$b^2 = 32$

$$\frac{x^2}{64} + \frac{y^2}{32} = 1$$

Q.19 If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :

(1) $\frac{13}{6}$ (2) 2 (3) $\frac{13}{12}$ (4) $\frac{13}{8}$

Ans. **[3]**

Sol. 2ae = 13

2b = 5

$\Rightarrow b = \frac{5}{2}$

$\because b^2 = a^2(e^2 - 1)$

$b^2 = a^2e^2 - a^2$

$\frac{25}{4} = \frac{169}{4} - a^2$

$a^2 = \frac{144}{4} = 36$

a = 6

$\frac{x^2}{6^2} - \frac{y^2}{\frac{25}{4}} = 1$

2ae = 13 $\because$ a = 6

$2 \times 6 \times e = 13$

$e = \frac{13}{12}$

Q.20 Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randonly chosen subset of S is "nice" is :

(1) $\frac{5}{2^{20}}$ (2) $\frac{7}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{6}{2^{20}}$

Ans. **[1]**

Sol. 7

1, 6

2, 5

3, 4

1, 2, 4

$\because \; p = \frac{5}{2^{20}}$

Q.21 Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

Ans. **[2]**

Sol. $$\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$$

$$\frac{x^m\ \ y^n}{x^m\left(\frac{1}{x^m}+x^m\right)\ y^n\left(\frac{1}{y^n}+y^n\right)}$$

For max $\frac{1}{(\min)\ (\min)}$

$$\frac{1}{(2)\ (2)} = \frac{1}{4}$$

Q.22 If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to

(1) 12 (2) 7 (3) 17 (4) 5

Ans. **[2]**

Sol. Normal vector of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 0 \\ -4 & -4 & 4 \end{vmatrix} = -4(5\hat{i}+2\hat{j}-3\hat{k})$$

equation of plane is

$5(x - 7) + 2y - 3(z - 6) = 0$

$5x + 2y - 3z = 17$

then $(2\alpha - 3\beta) = 17 - 10 = 7$

Q.23 Let $S_n = 1 + q + q^2 + ... + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 . S_1 + ... + {}^{101}C_{101} . S_{100} = \alpha T_{100}$ then α is equal to

(1) 202 (2) 200 (3) 2^{100} (4) 2^{99}

Ans. **[3]**

Sol. $${}^{101}C_1 + {}^{101}C_2\,(1 + q) + {}^{101}C_3(1 + q + q^2) + ... + {}^{101}C_{101}(1 + q + ... + q^{100}) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow {}^{101}C_1\,(1 - q) + {}^{101}C_2\,(1 - q^2) + ... + {}^{101}C_{101}(1 - q^{101}) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1-\left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

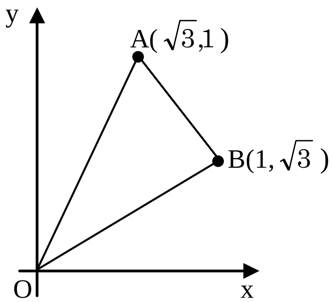
$2\alpha = 2^{101} \Rightarrow \alpha = 2^{100}$

Q.24 Let $\sqrt{3}\,\hat{i}+\hat{j},\ \hat{i}+\sqrt{3}\,\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is :

(1) 4 (2) 1 (3) 2 (4) 3

Ans. **[2]**

Sol.



Now, $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$\Rightarrow 2\beta - 1 = \pm 3$

$\beta = 2, -1$

Q.25 All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval :

(1) $(\cot 2, \infty)$ (2) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

(3) $(\cot 5, \cot 4)$ (4) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

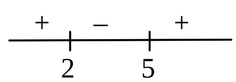
Ans. **[1]**

Sol. $(\cot^{-1}x)^2 - 7\cot^{-1}x + 10 > 0$

$(\cot^{-1}x)^2 - 5\cot^{-1}x - 2\cot^{-1}x + 10 > 0$

$(\cot^{-1}x)(\cot^{-1}x - 5) - 2(\cot^{-1}x - 5) > 0$

$(\cot^{-1}x - 5)(\cot^{-1}x - 2) > 0$



$\cot^{-1}x > 5$ or $\cot^{-1}x < 2$

$\Rightarrow \cot^{-1}x < 2$ (as $\cot^{-1} \in (0, \pi)$)

$\Rightarrow x \in (\cot 2, \infty)$

Q.26 A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of } X}{\text{standard deviation of } X}\right)$ is equal to :

(1) 4 (2) $3\sqrt{2}$ (3) $\frac{4\sqrt{3}}{3}$ (4) $4\sqrt{3}$

Ans. **[2]**

Sol. P(probability of getting white ball) = $\frac{30}{40}$

$q = \frac{1}{4}$, n = 16

mean = np = $16 \times \frac{3}{4} = 12$

(S.D) = $\sqrt{npq} = \sqrt{3}$

$$\frac{\text{Mean of } x}{(\text{standerd deviation of } x)} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

Q.27 Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value

(1) (19, 7, 25) (2) (7, 19, 25) (3) (5, 12, 13) (4) (3, 4, 5)

Ans. **[1]**

Sol. b + c = 11k

a + c = 12k

a + b = 13k

2(a + b + c) = 36k

a + b + c = 18k

a = 7k, c = 5k, b = 6k

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{36k^2+25k^2-49k^2}{2\times 6k\times 5k} = \frac{12k^2}{2\times 6k\times 5k} = \frac{1}{5}$$

$$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{25k^2+49k^2-36k^2}{2\times 5k\times 7k} = \frac{38k^2}{2\times 5k\times 7k} = \frac{19}{35}$$

$$\cos C = \frac{b^2+a^2-c^2}{2ab} = \frac{36k^2+49k^2-25k^2}{2\times 7k\times 6k} = \frac{60k^2}{2\times 7k\times 6k} = \frac{60}{12\times 7} = \frac{5}{7}$$

$$\frac{1}{5\alpha} = \frac{19}{35\beta} = \frac{5}{7\gamma} = \lambda$$

$$\alpha = \frac{1}{5\lambda},\ \beta = \frac{19}{35\lambda},\ \gamma = \frac{5}{7\lambda}$$

$\alpha : \beta : \gamma$

$$\frac{1}{5\lambda} : \frac{19}{35\lambda} : \frac{5}{7\lambda}$$

7 : 19 : 25

Q.28 A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :

(1) an ellipse (2) a parabola (3) a hyperbola (4) a straight line

Ans. **[2]**

Sol. Let the equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through (0, 2b)

$0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$4b^2 + 4f + c = 0$ ...(1)

$2\sqrt{g^2 - c} = 4a$ ...(2)

$g^2 - c = 4a^2 \quad \therefore\ c = (g^2 - 4a^2)$

Putting in eq. (1)

$4b^2 + 4f + g^2 - 4a^2 = 0$

$x^2 + 4y + 4(b^2 - a^2) = 0$

It represent a parabola.

Q.29 If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is :

(1) $5x + 3y - 11 = 0$ (2) $5x - 3y + 1 = 0$ (3) $3x - 5y + 7 = 0$ (4) $3x + 5y - 13 = 0$

Ans. **[2]**

Sol. Coordinates of point D are (4, 7)

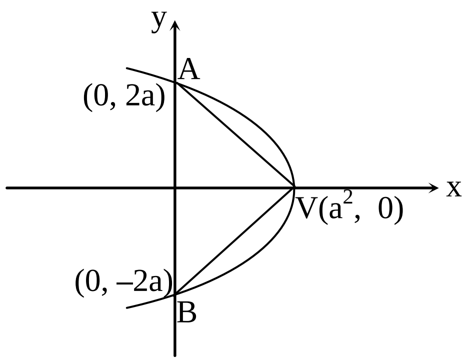
Equation of line AD is $5x - 3y + 1 = 0$

Q.30 If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

Ans. **[4]**

Sol. $y^2 = -4(x - a^2)$



$\because$ ar $\Delta ABV = 250$

$\frac{1}{2} \times 4a \times a^2 = 250$

$a^3 = 125$

$a = 5$