CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Ans. Volume of cube = $(Side)^3$

According to question, $(Side)^3$ = 64

$\Rightarrow (Side)^3 = 4^3$

$\Rightarrow$ Side = 4 cm

For the resulting cuboid, length (l) = 4 + 4 = 8 cm, breadth (b) = 4 cm and height (h) = 4 cm

Surface area of resulting cuboid = $2(lb+bh+hl)$

= $2(8\times4+4\times4+4\times8)$

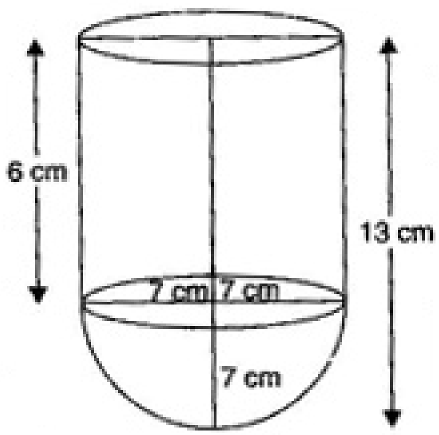
= 2 (32 + 16 + 32)

= $2\times80 = 160\ cm^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Ans. $\because$ Diameter of the hollow hemisphere = 14 cm

$\therefore$ Radius of the hollow hemisphere = $\frac{14}{2}$ = 7 cm



Total height of the vessel = 13 cm

∴ Height of the hollow cylinder = 13 – 7 = 6 cm

∴ Inner surface area of the vessel

= Inner surface area of the hollow hemisphere + Inner surface area of the hollow cylinder

= $2\pi(7)^2 + 2\pi(7)(6)$

= $98\pi + 84\pi = 182\pi$

= $182 \times \frac{22}{7}$ = $26 \times 22 = 572\ cm^2$

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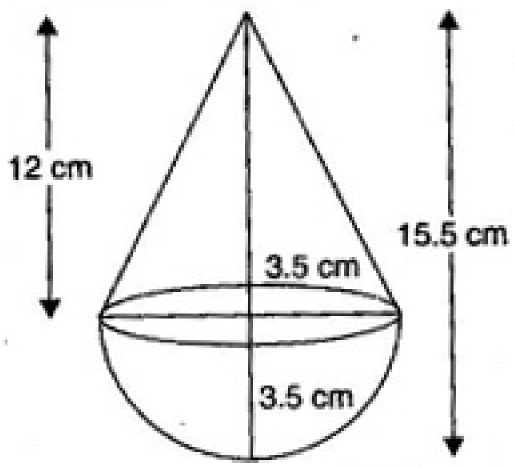
3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Ans. Radius of the cone = 3.5 cm

∴ Radius of the hemisphere = 3.5 cm



Total height of the toy = 15.5 cm

$\therefore$ Height of the cone = 15.5 – 3.5 = 12 cm

Slant height of the cone = $\sqrt{(3.5)^2 + (12)^2}$

= $\sqrt{12.25 + 144}$

= $\sqrt{156.25}$ = 12.5 cm

$\therefore$ TSA of the toy = CSA of hemisphere + CSA of cone

= $2\pi r^2 + \pi rl$

= $2\pi(3.5)^2 + \pi(3.5)(12.5)$

= $24.5\pi + 43.75\pi$ = 68.25π

= $68.25 \times \frac{22}{7}$ = $214.5\ cm^2$

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4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Ans. Greatest diameter of the hemisphere = Side of the cubical block = 7 cm

$\therefore$ TSA of the solid = External surface area of the cubical block + CSA of hemisphere

$= \left\{6(7)^2 - \pi\left(\frac{7}{2}\right)^2\right\} + 2\pi\left(\frac{7}{2}\right)^2$

$=> \left(294 - \frac{49}{4}\pi\right) + \frac{49}{2}\pi$

$= 294 + \frac{49}{4}\pi$

$= 294 + \frac{49}{4} \times \frac{22}{7}$

$= 294 + \frac{77}{2}$

= 294 + 38.5 = $332.5\ cm^2$

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5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans. $\because$ Diameter of the hemisphere = l, therefore radius of the hemisphere = $\frac{l}{2}$

Also, length of the edge of the cube = l

$\therefore$ Surface area of the remaining solid = total surface area of cubical block + curved surface area of hemispherical - area of circular base

$= 2\pi\left(\frac{l}{2}\right)^2 + 6l^2 - \pi\left(\frac{l}{2}\right)^2$

$= \pi\left(\frac{l}{2}\right)^2 + 6l^2$

$= \frac{\pi l^2}{4} + 6l^2$

$= \frac{1}{4}l^2(\pi + 24)$

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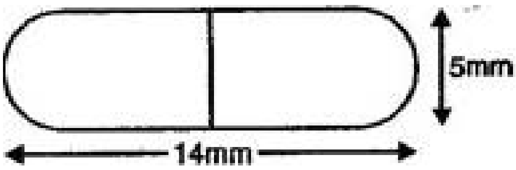
6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each

of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

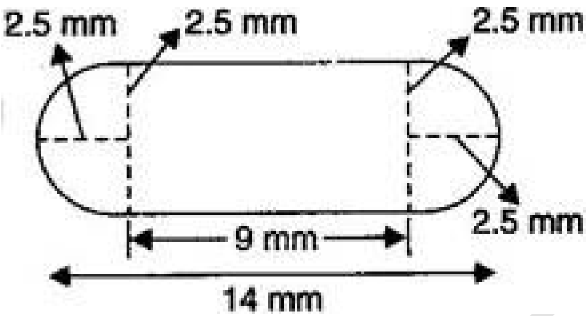


Ans. Radius of the hemisphere = $\frac{5}{2}$ mm

Let radius = r = 2.5 mm

Cylindrical height = Total height – Diameter of sphere = h = 14 – (2.5 + 2.5) = 9 mm

Surface area of the capsule = CSA of cylinder + curved Surface area of 2 hemispheres



= $2\pi rh + 2\left(2\pi r^2\right)$

= $2\pi\left(\frac{5}{2}\right)(9) + 2\left\{2\pi\left(\frac{5}{2}\right)^2\right\}$

= $45\pi + 25\pi$

= $70\pi = 70 \times \frac{22}{7}$ = $220\ mm^2$

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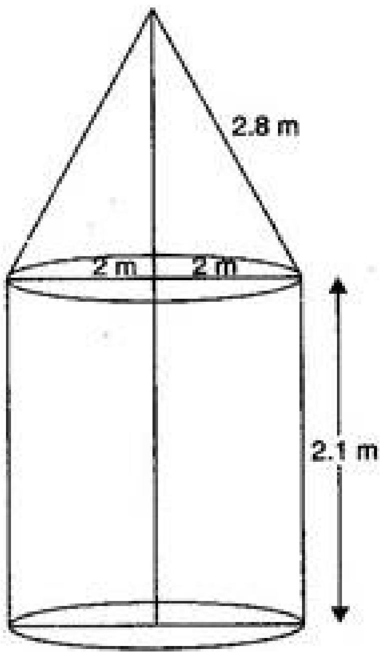
7. A tent is in the shape of a cylinder surmounted buy a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2 . (Note that the base of the tent will not be covered with canvas.)

Ans. Diameter of the cylindrical part = 4 cm

$\therefore$ Radius of the cylindrical part = 2 cm

TSA of the tent = CSA of the cylindrical part + CSA of conical top



= $2\pi(2)(2.1)+\pi(2)(2.8)$

= $8.4\pi+5.6\pi$

= 14π

= $14\times\frac{22}{7}$

= $44\ m^2$

$\therefore$ Cost of the canvas of the tent of 1 m^2 = Rs. 500

cost of canvas of the tent of 44 m^2 =

= 44×500 = Rs. 22000

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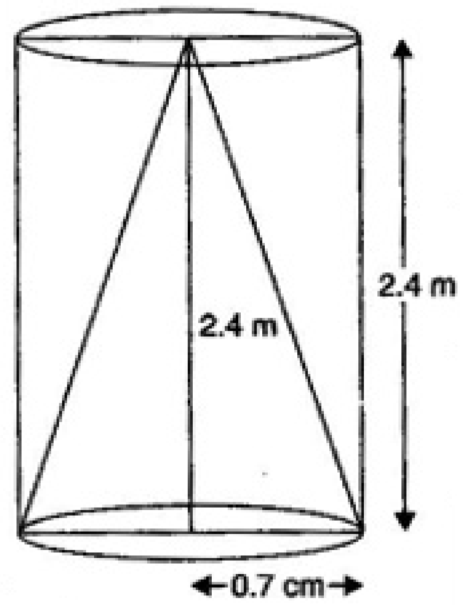
8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Ans. Diameter of the solid cylinder = 1.4 cm

$\therefore$ Radius of the solid cylinder = 0.7 cm

$\therefore$ Radius of the base of the conical cavity = 0.7 cm



Height of the solid cylinder = 2.4 cm

$\therefore$ Height of the conical cavity = 2.4 cm

$\therefore$ Slant height of the conical cavity = $\sqrt{(0.7)^2+(2.4)^2}$

= $\sqrt{0.49+5.76}$

= $\sqrt{6.25}$ = 2.5 cm

$\therefore$ TSA of remaining solid = curved surface area of cylinder + area of upper circular part + curved surface area of conical part

= $2\pi(0.7)(2.4)+\pi(0.7)^2+\pi(0.7)(2.5)$

= $3.36\pi+0.49\pi+1.75\pi$

= 5.6π

= $5.6\times\frac{22}{7}$ = 17.6 cm^2

= $18\ cm^2$ (to the nearest cm^2)

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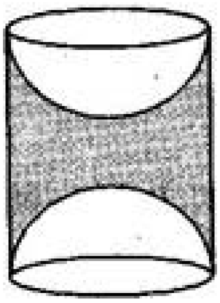
9. A wooden article was made by scooping out a hemisphere from each end of a solid

cylinder as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Ans. TSA of the article = $2\pi rH + 2\left(2\pi r^2\right)$ = curved surface area of cylinder + curved surface area of 2 hemispheres

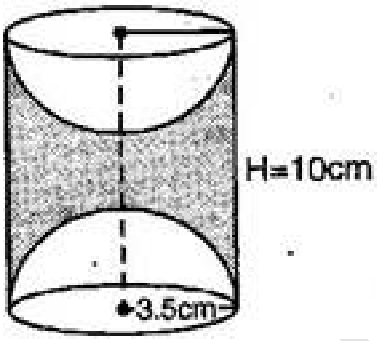
= $2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$

= $70\pi + 49\pi$

= 119π

= $119 \times \frac{22}{7}$

= $374\ cm^2$



CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

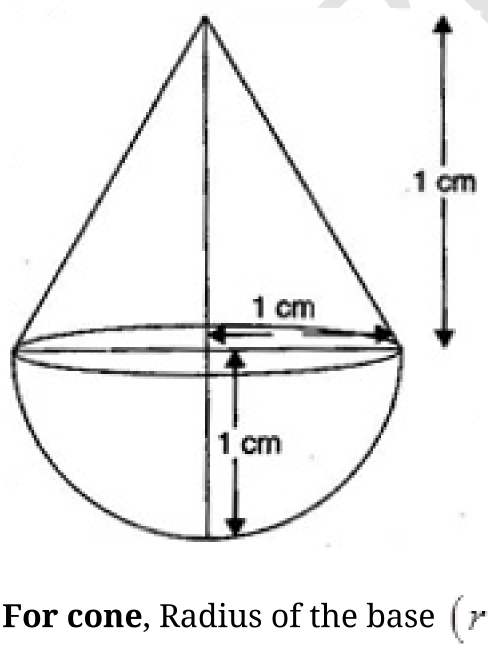
1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Ans. For hemisphere, Radius (r) = 1 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(1)^3$

= $\frac{2}{3}\pi$ cm^3



For cone, Radius of the base (r) = 1 cm

Height (h) = 1 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(1)^2 \times 1$

= $\frac{1}{3}\pi$ cm^3

$\therefore$ Volume of the solid = Volume of hemisphere + Volume of cone

= $\frac{2}{3}\pi + \frac{1}{3}\pi = \pi \ cm^3$

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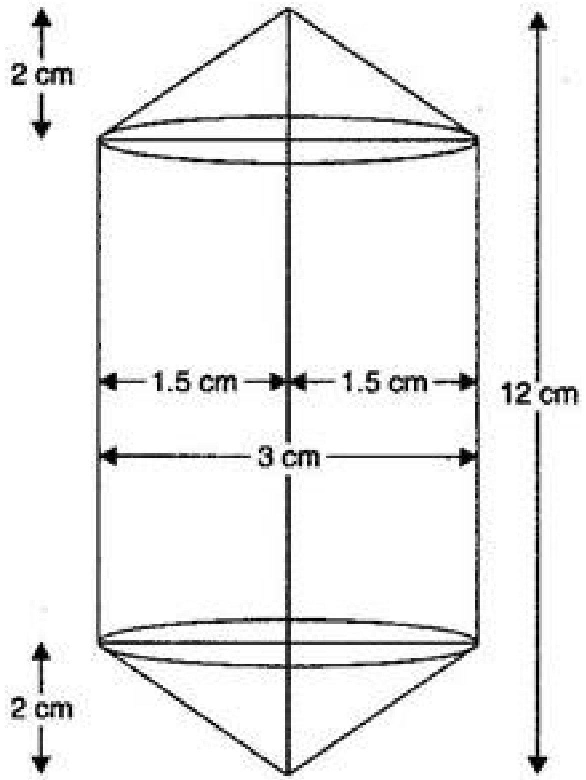
2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Ans. For upper conical portion, Radius of the base (r) = 1.5 cm

Height (h_1) = 2 cm



Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(1.5)^2 \times 2$

= $1.5\pi cm^3$

For lower conical portion, Volume = $1.5\pi cm^3$

For central cylindrical portion:

Radius of the base (r) = 1.5 cm

Height (h_2) = 12 – (2 + 2) = 8 cm

Volume = $\pi r^2 h_2$ = $\pi(1.5)^2 \times 8$ = $18\pi cm^3$

$\therefore$ Volume of the model = 1.5π + 1.5π + 18π = volume of top cone + volume of bottom cone + volume of cylindrical part

= 21π

$= 21\times\frac{22}{7} = 66\ cm^3$

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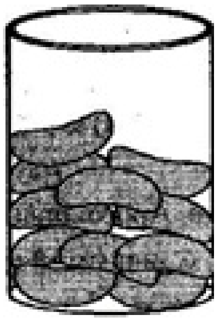
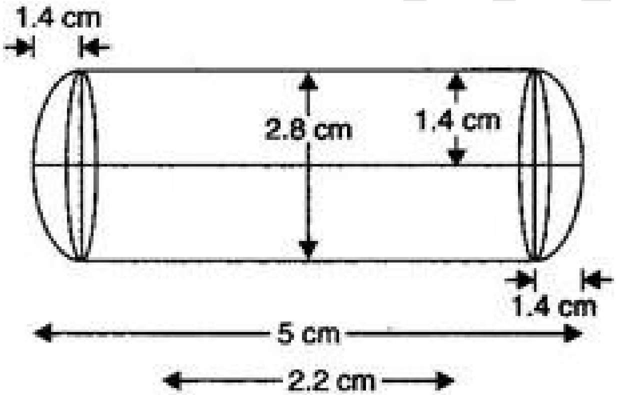
3. A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm (see figure).



Ans. Volume of a gulab jamun = $\frac{2}{3}\pi r^3+\pi r^2 h+\frac{2}{3}\pi r^3$ = volume of 2hemisphere + volume of cylinder

$= \frac{2}{3}\pi(1.4)^3+\pi(1.4)^2\times 2.2+\frac{2}{3}\pi(1.4)^3$



$= \frac{4}{3}\pi(1.4)^3+\pi(1.4)^2\times 2.2$

$= \pi(1.4)^2\left[\frac{4\times 1.4}{3}+2.2\right]$

$= \pi \times 1.96\left[\frac{5.6+6.6}{3}\right] = \frac{1.96 \times 12.2}{3}\pi cm^3$

$\therefore$ Volume of 45 gulab jamuns

$= 45 \times \frac{1.96 \times 12.2}{3}\pi$

$= 15 \times 1.96 \times 12.2 \times \frac{22}{7}$

$= 1127.28\ cm^3$

$\therefore$ Volume of syrup = $1127.28 \times \frac{30}{100}$ = 30% of volume of 45 gulab jamun

$= 338.184\ cm^3 = 338\ cm^3$ (approx.)

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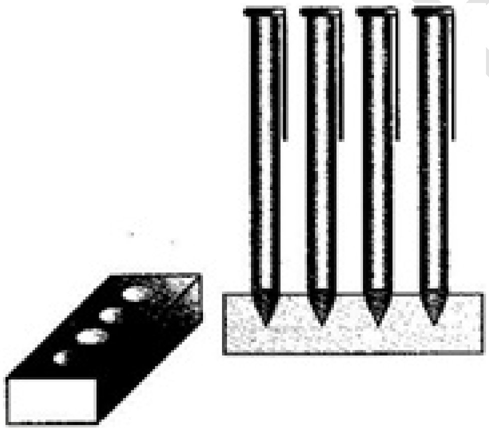
4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth id 1.4 cm. Find the volume of wood in the entire stand (see figure).



Ans: For Cuboid:

l=15 cm

b=10 cm

h=3.5 cm

Volume of the cuboid = $l \times b \times h$

= $15 \times 10 \times 3.5$

= $525\ cm^3$

For Cone: r = 0.5 cm

h = 1.4 cm

Volume of conical depression = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

= $\frac{11}{30} cm^3$

$\therefore$ Volume of four conical depressions = $4 \times \frac{11}{30}$ = $1.47\ cm^3$

$\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depression =525 – 1.47 = $523.53\ cm^3$

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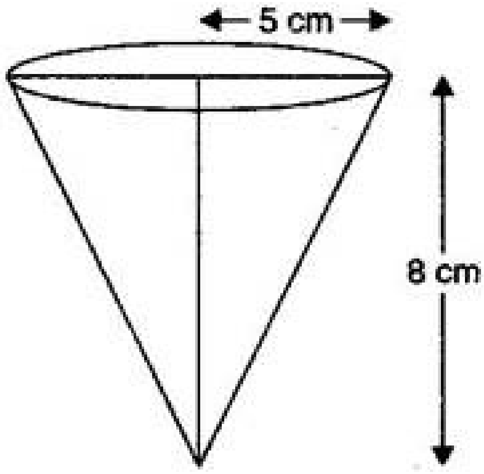
5. A vessel is in the form of inverted cone. Its height is 8 cm and the radius of the top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans. For cone, Radius of the top (r) = 5 cm and height (h) = 8 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(5)^2 \times 8$

= $\frac{200}{3}\pi$ cm^3

For spherical lead shot, Radius (R) = 0.5 cm

Volume of spherical lead shot = $\frac{4}{3}\pi R^3$

= $\frac{4}{3}\pi(0.5)^3$

= $\frac{\pi}{6} cm^3$

Volume of water that flows out = $\frac{1}{4}$ Volume of the cone

= $\frac{1}{4} \times \frac{200\pi}{3}$ = $\frac{50\pi}{3} cm^3$

Let the number of lead shots dropped in the vessel be n.

n * volume of spherical shot = volume of water flows out

$\therefore\ n \times \frac{\pi}{6} = \frac{50\pi}{3}$

$\Rightarrow\ n = \frac{50\pi}{3} \times \frac{6}{\pi}$

$\Rightarrow\ n$ = 100

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6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that $1\ cm^3$ of iron has approximately 8 g mass. $(\text{Use } \pi = 3.14)$

Ans. For lower cylinder, Base radius (r) = $\frac{24}{2}$ = 12 cm

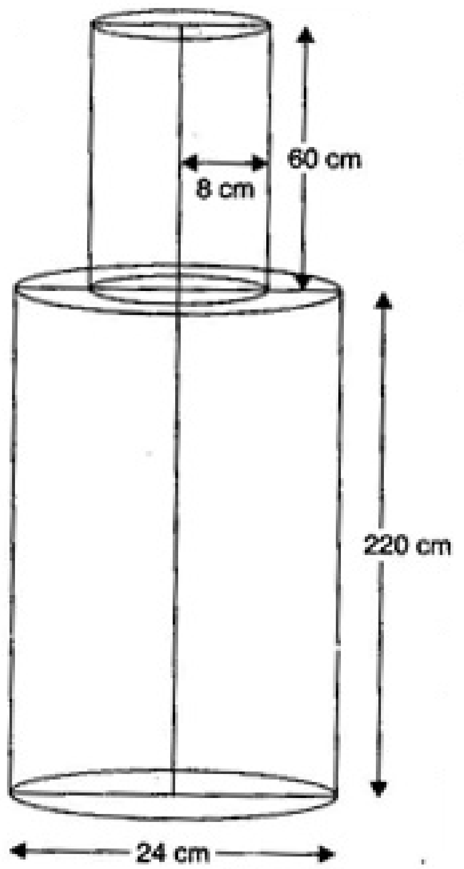
And Height (h) = 220 cm

Volume = $\pi r^2 h$

= $\pi(12)^2 \times 220$

= $31680\pi\ cm^3$

For upper cylinder, Base Radius (R) = 8 cm



And Height (H) = 60 cm

Volume = $\pi R^2 H$

= $\pi(8)^2 \times 60$

= $3840\pi \ cm^3$

$\therefore$ Volume of the solid Iron pole

= V of lower cylinder + V of upper cylinder

= $31680\pi + 3840\pi = 35520\pi$

= 35520×3.14 = 111532.8 cm^3

mass of 1 cm^3 iron = 8 gm

mass of 111532.8 cm^3 iron = 8* 111532.8 = 892262.4 gm = 892.2624 kg

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans. For right circular cone, Radius of the base (r) = 60 cm

And Height (h_1) = 120 cm

Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(60)^2 \times 120$

= $144000\pi\, cm^3$

For Hemisphere, Radius of the base (r) = 60 cm

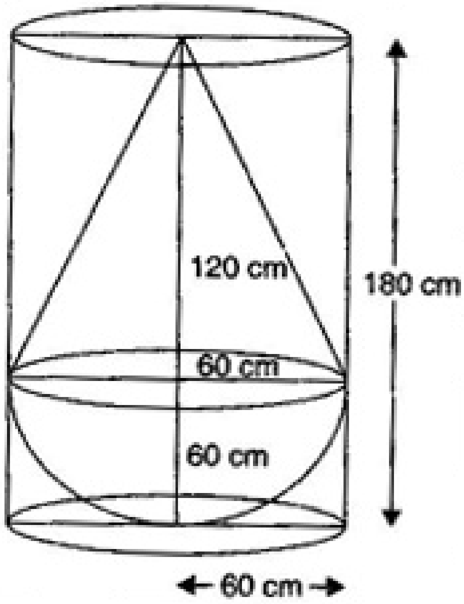
Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(60)^3$

= $144000\pi\, cm^3$

For right circular cylinder, Radius of the base (r) = 60 cm

And Height (h_2) = 180 cm



Volume = $\pi r^2 h_2$

= $\pi(60)^2 \times 180$

= $648000\pi\ cm^3$

Now, V of water left in the cylinder

= V of right circular cylinder – (V of right circular cone + V of hemisphere)

= $648000\pi - (144000\pi + 144000\pi)$

= $360000\pi\ cm^3$

= $\frac{360000}{100 \times 100 \times 100}\pi\ m^3$

= $0.36 \times \frac{22}{7}$ = $1.131\ m^3$ (approx.)

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8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be $345\ \text{cm}^3$. Check whether she is correct, taking the above as the inside measurements and π = 3.14.

Ans.

For Cylinder: diameter of cylin. = 2 cm, height of cylin. = 8 cm

For Sphere : diameter of sphere = 8.5 cm

Amount of water it holds = $\frac{4}{3}\pi r^3 + \pi r^2 h$ = volume of sphere + volume of cylinder

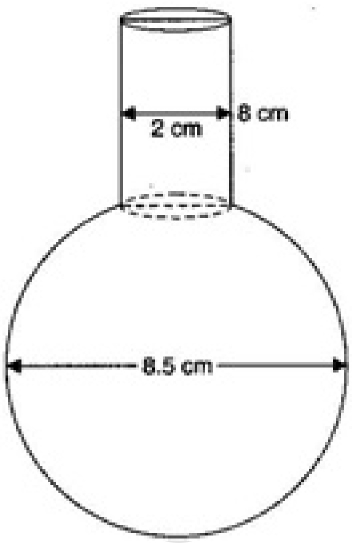
= $\frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi\left(\frac{2}{2}\right)^2 \times 8$

= $\frac{4}{3}\times 3.14\times 4.25\times 4.25\times 4.25 + 8\times 3.14$

= 321.39 + 25.12

= $346.51\ cm^3$

Hence, she is not correct. The correct volume is $346.51\ cm^3$.



CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.3

Unless stated otherwise, take $\pi = \dfrac{22}{7}$.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans. For sphere, Radius (r) = 4.2 cm

Volume = $\dfrac{4}{3}\pi r^3$ = $\dfrac{4}{3}\pi(4.2)^3\ cm^3$

For cylinder, Radius (R) = 6 cm

Let the height of the cylinder be H cm.

Then, Volume = $\pi R^2 H$ = $\pi(6)^2 H\ cm^3$

According to question, Volume of sphere = Volume of cylinder

$$\Rightarrow \frac{4}{3}\pi(4.2)^3 = \pi(6)^2 H$$

$$\Rightarrow H = \frac{4(4.2)^3}{3(6)^2}$$

$\Rightarrow$ H = 2.744 cm

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2. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans. Let the volume of resulting sphere be r cm.

According to question,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$

$\Rightarrow r^3$ = 216 + 512 + 1000

$\Rightarrow r^3$ = 1728

$\Rightarrow r$ = 12 cm

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3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. Diameter of well = 7 m

$\therefore$ Radius of well (r) = $\frac{7}{2}$ m

And Depth of earth dug (h) = 20 m

Length of platform (l) = 22 m, Breadth of platform (b) = 14 m

Let height of the platform be h' m

According to question,

Volume of earth dug = Volume of platform

$\Rightarrow \pi r^2 h = l \times b \times h'$

$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h'$

$\Rightarrow h' = \frac{22 \times 7 \times 7 \times 20}{28 \times 22 \times 14}$

$\Rightarrow h'$ = 2.5 m

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans. Diameter of well = 3 m

$\therefore$ Radius of well (r) = $\frac{3}{2}$ m and Depth of earth dug (h) = 14 m

Width of the embankment = 4 m

$\therefore$ Radius of the well with embankment r' = $\frac{3}{2}+4=\frac{11}{2}$ m

Let the height of the embankment be h' m

According to the question,

Volume of embankment = Volume of the earth dug

$\Rightarrow \pi\left[(r')^2 - r^2\right]h' = \pi r^2 h$

$\Rightarrow \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]h' = \left(\frac{3}{2}\right)^2 \times 14$

$\Rightarrow \left[\frac{121}{4} - \frac{9}{4}\right]h' = \frac{9}{4} \times 14$

$\Rightarrow \frac{112}{4} \times h' = \frac{9}{4} \times 14$

$\Rightarrow h' = \frac{9 \times 14 \times 4}{4 \times 112}$

$\Rightarrow h'$ = 1.125 m

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5. A container shaped like a right circular cylinder having diameter 12 cm and height 15

cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. For right circular cylinder, Diameter = 12 cm

$\therefore$ Radius (r) = $\frac{12}{2}$ = 6 cm and height (h) = 15 cm

For cone & Hemisphere , Diameter = 6 cm

$\therefore$ Radius (r_1) = $\frac{6}{2}$ = 3 cm and height (h_1) = 12 cm

Let n cones be filled with ice cream.

Then, According to question,

Volume of n (cones +Hemisphere) = Volume of right circular cylinder

$$\Rightarrow n \times \left(\frac{1}{3}\pi(r_1)^2(h) + \frac{2}{3}\pi(r_1)^3\right) = \pi r^2 h$$

$$\Rightarrow n\left(\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3\right) = \frac{22}{7} \times (6)^2 \times 15$$

$$\Rightarrow n = \frac{22\times36\times15\times3\times7}{(7\times22\times9\times12+7\times44\times27)} = \frac{249480}{24948}$$

$\Rightarrow n$ = 10

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6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5 \text{ cm} \times 10 \text{ cm} \times 3.5 \text{ cm}$?

Ans. For silver coin, Diameter = 1.75 cm

$\therefore$ Radius $(r) = \frac{1.75}{2}$ = $\frac{7}{8}$ cm and Thickness (h) = 2 mm = $\frac{1}{5}$ cm

For cuboid, Length (l) = 5.5 cm, Breadth (b) = 10 cm and Height (h') = 3.5 cm

Let n coins be melted.

Then, According to question,

Volume of n coins = Volume of cuboid

$\Rightarrow n\times\pi r^2 h = l\times b\times h'$

$\Rightarrow n\times\pi\left(\frac{7}{8}\right)^2\times\left(\frac{1}{5}\right)=5.5\times10\times3.5$

$\Rightarrow n\times\frac{22}{7}\times\frac{49}{64}\times\frac{1}{5}=5.5\times10\times3.5$

$\Rightarrow n=\frac{5.5\times10\times3.5\times7\times64\times5}{22\times49}$

$\Rightarrow n = 400$

---

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Ans. For cylindrical bucket, Radius of the base (r) = 18 cm and height (h) = 32 cm

$\therefore$ Volume = $\pi r^2 h = \pi(18)^2\times32$

$= 10368\pi\ cm^3$

For conical heap, Height (h') = 24 cm

Let the radius be r_1 cm.

Then, Volume = $\frac{1}{3}\pi r_1^2 h'$

$= \frac{1}{3}\times\pi\times r_1^2\times24 = 8\pi r_1^2\ cm^3$

According to question, Volume of bucket = Volume of conical heap

$\Rightarrow 10368\pi = 8\pi r_1^2$

$\Rightarrow r_1^2 = \dfrac{10368\pi}{8\pi} = 1296$

$\Rightarrow r_1 = 36$ cm

Now, Slant height $(l) = \sqrt{(r_1)^2 + (h')^2}$

$= \sqrt{(36)^2 + (24)^2} = \sqrt{1296 + 576}$

$= \sqrt{1872} = 12\sqrt{13}$ cm

---

8. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Ans. For canal, Width = 6 m and Depth = 1.5 m = $\dfrac{3}{2}$ m

Speed of flow of water = 10 km/h

$= 10 \times 1000\ m/h = 10000$ m/h

$= \dfrac{10000}{60}$ m/min $= \dfrac{500}{3}$ m/min

$\therefore$ Speed of flow of water in 30 minutes

$= \dfrac{500 \times 30}{3}$ m/min = 5000 m/min

$\therefore$ Volume of water that flows in 30 minutes

$= 6 \times \dfrac{3}{2} \times 5000 = 45000\ m^3$

$\therefore$ The area it will irrigate = $\frac{45000}{\left(\frac{8}{100}\right)} = \frac{4500000}{8}$

= $562500\ m^2$

= $\frac{562500}{10000}$ hectares = 56.25 hectares

---

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Ans. For cylindrical tank, Diameter = 10 m

$\therefore$ Radius $(r) = \frac{10}{2}$ = 5 m and Depth (h) = 2 m

$\therefore$ Volume = $\pi r^2 h$ = $\pi(5)^2 \times 2$ = $50\pi m^3$

Rate of flow of water (h') = 3 km/h = 3000 m/h = $\frac{3000}{60}$ m/min = 50 m/min

For pipe, Internal diameter = 20 cm, therefore radius (r_1) = 10 cm = 0.1 m

$\therefore$ Volume of water that flows per minute = $\pi(r_1)^2 h'$

= $\pi(0.1)^2 \times 50$ = $\frac{\pi}{2} m^3$

$\therefore$ Required time = $\frac{50\pi}{\pi/2}$ = 100 minutes

CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

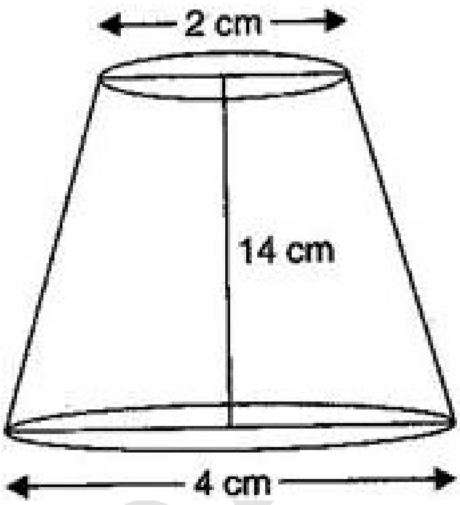
Surface Areas and Volumes -Exercise 13.4

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Ans. Here, $r_1 = \frac{4}{2}$ = 2 cm,

$r_2 = \frac{2}{2}$ = 1 cm and h = 14 cm



$\therefore$ Capacity of the glass = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

= $\frac{1}{3} \times \frac{22}{7} \times 14\left(2\times 2 + 1\times 1 + 2\times 1\right)$

= $\frac{1}{3} \times \frac{22}{7} \times 14 \times 7$

= $\frac{308}{3} = 102\frac{2}{3} cm^3$

---

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans. Let r_1 cm and r_2 cm be the radii of the ends $(r_1 > r_2)$ of the frustum of the cone.

Then, l = 4 cm

$2\pi r_1$ = 18 cm

$\Rightarrow$ πr_1 = 9 cm

$2\pi r_2$ = 6 cm

$\Rightarrow$ πr_2 = 3 cm

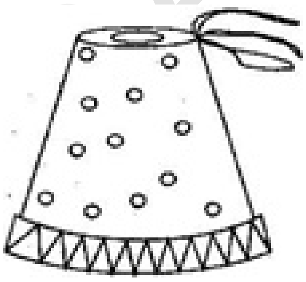
Now, CSA of the frustum = $\pi(r_1 + r_2)l$

= $(\pi r_1 + \pi r_2)l$

= $(9+3)\times 4 = 48\ cm^2$

---

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Ans. 

Here, r_1 = 10 cm,

r_2 = 4 cm and l = 15 cm

$\therefore$ Surface area = $\pi(r_1 + r_2)l + \pi r_2^2$

= $\frac{22}{7}(10+4)\times 15 + \frac{22}{7}(4)^2$

= $660 + \frac{352}{7}$ = $\frac{4972}{7} = 710\frac{2}{7}cm^2$

---

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the total cost of milk which can completely fill the container at the rate of Rs. 20 per liter. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm^2. $(\text{Take } \pi = 3.14)$

Ans. Here, r_1 = 20 cm,

r_2 = 8 cm and h = 16 cm

$\therefore$ Volume of container = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

= $\frac{1}{3} \times 3.14 \times 16 \left\{(20)^2 + (8)^2 + 20 \times 8\right\}$

= $\frac{1}{3}\times 3.14 \times 16(400+64+160)$

= $\frac{1}{3}\times 3.14\times 16\times 624$

= $10449.92\ cm^3$ = 10.44992 liters

$\therefore$ Cost of the milk = 10.44992×20

= Rs. 208.9984 = Rs. 209

Now, surface area = $\pi(r_1 + r_2)l$ + πr_2^2

= $\pi(r_1+r_2)\sqrt{h^2+(r_1-r_2)^2}$ + πr_2^2

= $3.14(20+8)\sqrt{(16)^2+(20-8)^2}$ + $3.14(8)^2$

= $3.14\times 28\sqrt{256+144}+3.14\times 64$

= 1758.4 + 200.96

= $1959.36\ cm^2$

$\therefore$ Area of the metal sheet used = $1959.36\ cm^2$

$\therefore$ Cost of metal sheet = $1959.36\times\frac{8}{100}$

= 156.7488 = Rs. 156.75

---

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans. $\tan 30^\circ = \frac{r_2}{10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{10}$

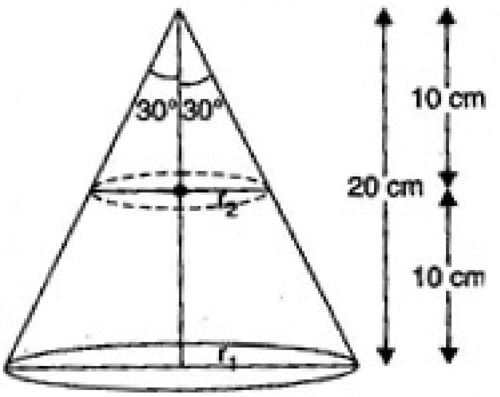
$\Rightarrow r_2 = \frac{10}{\sqrt{3}}$ cm

$\tan 30^\circ = \frac{r_1}{20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_1}{20}$

$\Rightarrow r_1 = \frac{20}{\sqrt{3}}$ cm

h = 10 cm



$\therefore$ Volume = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

= $\frac{1}{3}\times\frac{22}{7}\times 10\left\{\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)\left(\frac{10}{\sqrt{3}}\right)\right\}$

= $\frac{1}{3}\times\frac{22}{7}\times 10\times\left(\frac{400}{3}+\frac{100}{3}+\frac{200}{3}\right)$

= $\frac{1}{3}\times\frac{22}{7}\times 10\times\frac{700}{3}$ = $\frac{22000}{9} cm^3$

Diameter of the wire = $\frac{1}{16}$ cm

$\therefore$ Radius of the wire = $\frac{1}{32}$ cm

Let the length of the wire be l cm.

Then, Volume of the wire = $\pi r^2 l$ = $\frac{22}{7}\left(\frac{1}{32}\right)^2 l$ = $\frac{11l}{3584}$ cm^3

According to the question,

$$\frac{11l}{3584} = \frac{22000}{9}$$

$$\Rightarrow l = \frac{22000 \times 3584}{11 \times 9}$$

$$\Rightarrow l = \frac{2000 \times 3584}{9}$$

$\Rightarrow l =$ 796444.44 cm = 7964.4 m

CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.5

1. A copper wire, 3 mm in diameter is wound about a cylinder whose length is 12 cm and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Ans. Number of rounds to cover 12 cm, i.e. 120 mm = $\frac{120}{3}$ = 40

Here, Diameter = 10 cm, Radius $(r) = \frac{10}{2}$ cm

Length of the wire used in taking one round

= $2\pi r = 2\pi \times 5 = 10\pi$ cm

Length of the wire used in taking 40 rounds

= $10\pi \times 40 = 400\pi$ cm

Radius of the copper wire = $\frac{3}{2}$ mm

= $\frac{3}{20}$ cm

$\therefore$ Volume of wire = $\pi\left(\frac{3}{20}\right)^2(400\pi)$

= $9\pi^2 cm^3$-------------***

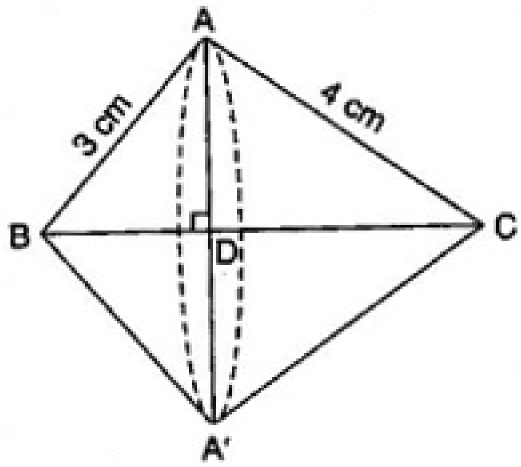
$\therefore$ Mass of the wire = $9\times(3.14)^2\times 8.88$

= 787.98 gm

2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)

Ans. Hypotenuse = $\sqrt{3^2+4^2}$ = 5 cm

In figure, $\triangle$ ADB $\sim$ $\triangle$ CAB [AA similarity]



$\therefore \frac{AD}{CA} = \frac{AB}{CB}$

$\Rightarrow \frac{AD}{4} = \frac{3}{5}$

$\Rightarrow$ AD = $\frac{12}{5}$ cm

Also, $\frac{DB}{AB} = \frac{AB}{CB}$

$\Rightarrow \frac{DB}{3} = \frac{3}{5}$

$\Rightarrow$ DB = $\frac{9}{5}$ cm

$\therefore$ CD = BC – DB = $5-\frac{9}{5}=\frac{16}{5}$ cm

Volume of the double cone

= $\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{9}{5}\right)+\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{16}{5}\right)$

= $\frac{1}{3}\times 3.14\times\frac{12}{5}\times\frac{12}{5}\times 5$ = $30.14\ cm^3$

Surface area of the double cone

= $\pi\times\frac{12}{5}\times 3+\pi\times\frac{12}{5}\times 4$

= $\pi\times\frac{12}{5}(3+4)$ = $3.14\times\frac{12}{5}\times 7$

= $52.75\ cm^2$

---

3. A cistern, internally measuring $150\ \text{cm}\times 120\ \text{cm}\times 110\ \text{cm}$ has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being $22.5\ \text{cm}\times 7.5\ \text{cm}\times 6.5\ \text{cm}$?

Ans. Volume of cistern = $150\times 120\times 110$ = $1980000\ cm^3$

Volume of water = $129600\ cm^3$

$\therefore$ Volume of cistern to be filled

= 1980000 – 129600 = $1850400\ cm^3$

Volume of a brick = $22.5\times 7.5\times 6.5$

= $1096.875\ cm^3$

Let n bricks be needed.

Then, water absorbed by n bricks = $n \times \frac{1096.875}{17} cm^3$

$\therefore \ n = \frac{1850400 \times 17}{16 \times 1096.875}$ = 1792 (approx.)

---

4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

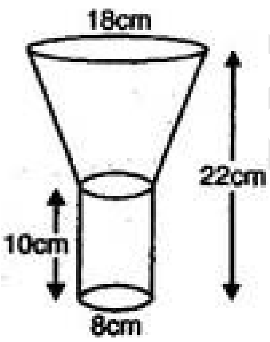
Ans. Volume of rainfall =$7280 \times \frac{10}{100 \times 1000}$ = 0.728 km^2

Volume of three rivers = $3 \times 1072 \times \frac{75}{1000} \times \frac{3}{1000}$ = 0.7236 km^2

Hence, the amount of rainfall is approximately equal to the amount of water in three rivers.

---

5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see figure).



Ans. Slant height of the frustum of the cone

$(l) = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{(22-10)^2 + \left(\frac{18}{2} - \frac{8}{2}\right)^2} = 13 \text{ cm}$$

Area of the tin sheet required

= CSA of cylinder + CSA of the frustum

$$= 2\pi(4)(10) + \pi(4+9)13$$

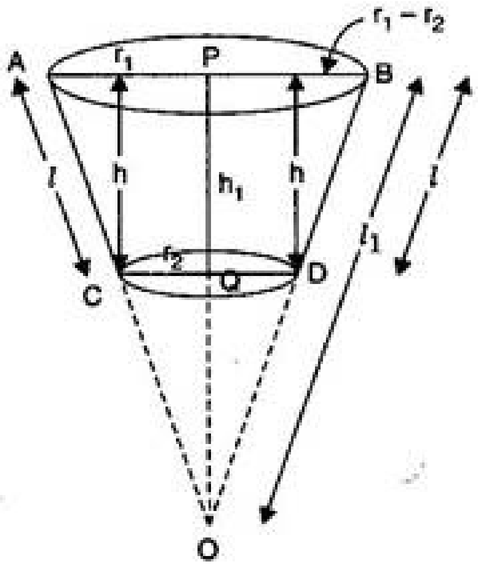
$$= 80\pi + 169\pi$$

$$= 249\pi = 249 \times \frac{22}{7} = 782\frac{4}{7} cm^2$$

---

6. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. According to the question, the frustum is the difference of the two cones OAB and OCD (in figure).



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

$\therefore$ Height of the cone OCD = $h_1 - h$

$\because$ Δ OQD ~ Δ OPB [By, AA similarity]

$\therefore$ $\frac{h_1 - h}{h_1} = \frac{r_2}{r_1}$

$\Rightarrow 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$

$\Rightarrow 1 - \frac{r_2}{r_1} = \frac{h}{h_1}$

$\Rightarrow h_1 = \frac{hr_1}{r_1 - r_2}$ ....(i)

$\therefore$ height of the cone OCD = $h_1 - h$

= $\frac{hr_1}{r_1 - r_2} - h$ = $\frac{hr_2}{r_1 - r_2}$ ....(ii)

$\therefore$ V of the frustum

= V of cone OAB – V of cone OCD

= $\frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$

= $\frac{\pi}{3}\left[r_1^2 . \frac{hr_1}{r_1 - r_2} - r_2^2 . \frac{hr_2}{r_1 - r_2}\right]$

[From eq. (i) & (ii)]

= $\frac{\pi h}{3}\left(\frac{r_1^3 - r_2^3}{r_1 - r_2}\right)$

= $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

If $A_1 \, and \, A_2$ are the surface areas of two circular bases, then

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

$\therefore$ V of the frustum

$$= \frac{h}{3}\left(\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2} \cdot \sqrt{\pi r_2^2}\right)$$

$$= \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

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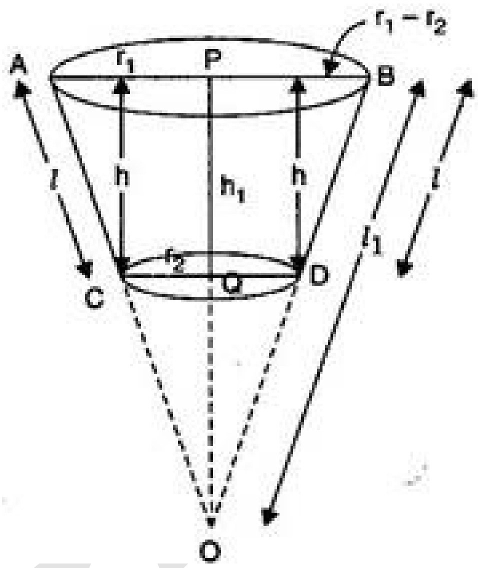
7. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans.



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

Again, from ΔDEB, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\because$ ΔOQD ~ ΔOPB [AA similarity]

$\therefore$ $\dfrac{l_1 - l}{l_1} = \dfrac{r_2}{r_1}$

$\Rightarrow l_1 = \dfrac{lr_1}{r_1 - r_2}$ ....(iii)

$\therefore$ $l_1 - l = \dfrac{lr_1}{r_1 - r_2} - l = \dfrac{lr_2}{r_1 - r_2}$ ....(iv)

Hence, CSA of the frustum of the cone = $\pi r_1 l_1 - \pi r_2 (l_1 - l)$

= $\pi r_1 . \dfrac{lr_1}{r_1 - r_2} - \pi r_2 \dfrac{lr_2}{r_1 - r_2}$ [From eq. (i) and (ii)]

= $\pi l \left(\dfrac{r_1^2 - r_2^2}{r_1 - r_2} \right)$ = $\pi l (r_1 + r_2)$,

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\therefore$ TSA of the frustum of the cone

= $\pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$