32. Binomial Distribution

Exercise 32

1. Question

A coin is tossed 6 times. Find the probability of getting at least 3 heads.

Answer

As the coin is tossed 6 times the total number of outcomes will be 2^6

And we know that the favourable outcomes of getting at least 3 heads will be $^6c_3 + {}^6c_4 + {}^6c_5 + {}^6c_6$

Thus, the probability of getting at least 3 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$\Rightarrow = \frac{\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}}{2^6}$$

$$\Rightarrow = \frac{21}{32}$$

2. Question

A coin is tossed 5 times. What is the probability that a head appears an even number of times?

Answer

As the coin is tossed 5 times the total number of outcomes will be $2^5 = 32$.

And we know that the favourable outcomes of a head appearing even number of times will be,

That either the head appears 0, 2 or 4 times so,

The respective probabilities will be:- $^5C_0 + {}^5C_2 + {}^5C_4 = 16$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$\Rightarrow = \frac{16}{32} = \frac{1}{2}$$

Hence, the probability is $1/2$.

3. Question

7 coins are tossed simultaneously. What is the probability that a tail appears an odd number of times?

Answer

As 7 coins are tossed simultaneously the total number of outcomes are 2^7=128.

The favourable number of outcomes that a tail appears an odd number of times will be, $^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7 = 64$.

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{64}{128}$$

$= \frac{1}{2}$

Hence, the probability is $1/2$.

4. Question

A coin is tossed 6 times. Find the probability of getting

(i) exactly 4 heads

(ii) at least 1 heads

(iii) at most 4 heads

Answer

(i) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting exactly 4 heads will be $^6c_4$ = 15

Thus, the probability of getting exactly 4 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 15/64

(ii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at least 1 heads will be $^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 63$

Thus, the probability of getting at least 1 head will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 63/64

(iii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at most 4 heads will be $^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 = 57$

Thus, the probability of getting at most 4 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 57/64

5. Question

10 coins are tossed simultaneously. Find the probability of getting

(i) exactly 3 heads

(ii) not more than 4 heads

(iii) at least 4 heads

Answer

(i) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting exactly 3 heads will be

$^{10}C_3 = 120$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{120}{1024}$$

$$= \frac{15}{128}$$

Hence, the probability is $\frac{15}{128}$.

(ii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting not more than 4 heads will be

${}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 386$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{386}{1024}$$

$$\Rightarrow \frac{193}{512}$$

Hence, the probability is $\frac{193}{512}$.

(iii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting at least 4 heads will be

${}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 848$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{848}{1024}$$

$$\Rightarrow \frac{53}{64}$$

Hence, the probability is $\frac{53}{64}$.

6. Question

A die is thrown 6 times. If ‘getting an even number’ is a success, find the probability of getting

(i) exactly 5 successes

(ii) at least 5 successes

(iii) at most 5 successes

Answer

(i) Using Bernoulli’s Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting exactly 5 successes will be, either getting 2, 4 or 6 i.e., 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{1}{2}$,q = $\frac{1}{2}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting exactly 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow {}^6C_5 \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow {}^6C_5.\frac{1}{64}$

$\Rightarrow \frac{3}{32}$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at least 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{3}{6}$, q = $\frac{3}{6}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at least 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^6C_5 + {}^6C_6)\ \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow ({}^6C_5 + {}^6C_6).\frac{1}{64}$

$\Rightarrow \frac{7}{64}$

(iii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at most 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability of success .

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at most 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5).\ \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow ({}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5).\frac{1}{64}$

$\Rightarrow \frac{63}{64}$

7. Question

A die is thrown 4 times. 'Getting a 1 or a 6' is considered a success, Find the probability of getting

(i) exactly 3 successes

(ii) at least 2 successes

(iii) at most 2 successes

Answer

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting exactly 3 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting exactly 3 successes will be

$= \frac{\textit{The favourable outcomes}}{\textit{The total number of outcomes}}$

$\Rightarrow (^4C_3)\ \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6}$

$\Rightarrow (^4C_3).\frac{2}{81}$

$\Rightarrow \frac{8}{81}$

(ii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at least 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at least 2 successes will be

$= \frac{\textit{The favourable outcomes}}{\textit{The total number of outcomes}}$

$\Rightarrow (^4C_2)\ )\ \frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6} + (^4C_3)\ )\ \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6} + (^4C_4)\ \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{2}{6}$

$\Rightarrow \frac{33}{81}$

$\Rightarrow \frac{11}{27}$

(iii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at most 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at most 2 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow (^4C_0)\frac{4}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + (^4C_1)\frac{2}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + (^4C_2)\frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6}$

$\Rightarrow \frac{72}{81}$

$\Rightarrow \frac{8}{9}$

8. Question

Find the probability of a 4 turning up at least once in two tosses of a fair die.

Answer

The total outcomes = 36,

The favourable outcomes are (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3) ,(4,5), (4,6)

Thus, the probability = favourable outcomes/total outcomes

$\Rightarrow \frac{11}{36}$

9. Question

A pair of dice is thrown 4 times. If ‘getting a doublet’ is considered a success, find the probability of getting 2 successes.

Answer

As the pair of die is thrown 4 times,

The total number of outcomes = 36

Using Bernoulli’s Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of 2 successes = $^4C_2.(\frac{1}{6})^2(\frac{5}{6})^2$

$\Rightarrow \frac{25}{216}$

10. Question

A pair of dice is thrown 7 times. If ‘getting a total of 7’ is considered a success, find the probability of getting

(i) no success

(ii) exactly 6 successes

(iii) at least 6 successes

(iv) at most 6 successes

Answer

(i) Using Bernoulli’s Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of no success = $^7C_0.(\frac{1}{6})^0(\frac{5}{6})^7$

⇒ $(\frac{5}{6})^7$

(ii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of exactly 6 successes = $^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1$

⇒ $35.(\frac{1}{6})^7$

(iii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of at least 6 successes =

$^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1 + {}^7C_7.(\frac{1}{6})^7(\frac{5}{6})^0$

⇒ $36.(\frac{1}{6})^7$

⇒ $(\frac{1}{6})^5$

(iv) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

$q = \frac{5}{6}$

probability of at least 6 successes =

${}^7C_0.(\frac{1}{6})^0(\frac{5}{6})^7 + {}^7C_1.(\frac{1}{6})^1(\frac{5}{6})^6 + {}^7C_2.(\frac{1}{6})^2(\frac{5}{6})^5 + {}^7C_3.(\frac{1}{6})^3(\frac{5}{6})^4 + {}^7C_4.(\frac{1}{6})^4(\frac{5}{6})^3 + {}^7C_5.(\frac{1}{6})^5(\frac{5}{6})^2 + {}^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1$

$\Rightarrow (1- (\frac{1}{6})^7)$

11. Question

There are 6% defective items in a large bulk of times. Find the probability that a sample of 8 items will include not more than one detective item.

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =8

The probability of success, i.e. the bulb is defective = $p = \frac{6}{100} = \frac{6}{100}$

$q = 1 - \frac{6}{100} = \frac{94}{100}$

probability of that there is not more than one defective piece=

P(0 defective items) + P(1 defective item) =

${}^8C_0.(\frac{6}{100})^0(\frac{94}{100})^8 + {}^8C_1.(\frac{6}{100})^1(\frac{94}{100})^7$

$\Rightarrow ((\frac{47}{50})^7 \times (\frac{71}{50}))$

12. Question

In a box containing 60 bulbs, 6 are defective. What is the probability that out of a sample of 5 bulbs

(i) none is defective

(ii) exactly 2 are defective

Answer

(i) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = $p = \frac{6}{60} = \frac{1}{10}$

$q = 1 - \frac{1}{10} = \frac{9}{10}$

probability of that no bulb is defective piece=

P(0 defective items) =

${}^5C_0.(\frac{1}{10})^0(\frac{9}{10})^5$

$\Rightarrow ((\frac{9}{10})^5)$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = $p = \frac{6}{60} = \frac{1}{10}$

$q = 1 - \frac{1}{10} = \frac{9}{10}$

probability of that there are exactly 2 defective pieces=

P(2 defective items) =

${}^5C_2.(\frac{1}{10})^2(\frac{9}{10})^3$

$\Rightarrow ((\frac{729}{10000}))$

13. Question

The probability that a bulb produced by a factory will fuse after 6 months of use is 0.05. find the probability that out of 5 such bulbs

(i) none will fuse after 6 months of use

(ii) at least one will fuse after 6 months of use

(iii) not more than one will fuse after 6 months of use

Answer

(i) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that none will fuse =

${}^5C_0.(0.05)^0(0.95)^5$

$\Rightarrow (0.95)^5$

(ii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that at least one will fuse = P(1) + P(2) + P(3) + P(4) + P(5)

${}^5C_1.(0.05)^1(0.95)^4 + {}^5C_2.(0.05)^2(0.95)^3 + {}^5C_3.(0.05)^3(0.95)^2 + {}^5C_4.(0.05)^4(0.95)^1 + {}^5C_5.(0.05)^5(0.95)^0$

$\Rightarrow (1-(0.95)^5)$

(iii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that not more than one will fuse = P(0) + P(1)

${}^5C_0.(0.05)^0(0.95)^5 + {}^5C_1.(0.05)^1(0.95)^4$

$\Rightarrow (1.20).(0.95)^5$

14. Question

In the items produced by a factory, there are 10% defective items. A sample of 6 items is randomly chosen. Find the probability that this sample contains.

(i) exactly 2 defective items

(ii) not more than 2 defective items

(iii) at least 3 defective items

Answer

(i) The probability that the item is defective $= \frac{1}{10} = p$

The probability that the bulb will not fuse $= 1 - \frac{1}{10} = \frac{9}{10} = q$

Using Bernoulli's we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that exactly 2 defective items are,

$\Rightarrow {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$\Rightarrow \frac{3}{20} \times \left(\frac{9}{10}\right)^4$

(ii) The probability that the item is defective $= \frac{1}{10} = p$

The probability that the bulb will not fuse $= 1 - \frac{1}{10} = \frac{9}{10} = q$

Using Bernoulli's we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that not more than 2 defective items are,

$\Rightarrow {}^6C_0.(\frac{1}{10})^0(\frac{9}{10})^6 + {}^6C_1.(\frac{1}{10})^1(\frac{9}{10})^5 + {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$\Rightarrow \left(\frac{81+54+15}{10^6}\right).(9^4) = \frac{150 \times 9^4}{10^6}$

(iii) The probability that the item is defective $= \frac{1}{10} = p$

The probability that the bulb will not fuse $= 1 - \frac{1}{10} = \frac{9}{10} = q$

Using Bernoulli's we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability of at least 3 defective items are,

P(3) + P(4) + P(5) + P(6)

$\Rightarrow {}^6C_3.(\frac{1}{10})^3(\frac{9}{10})^3 + {}^6C_4.(\frac{1}{10})^4(\frac{9}{10})^2 + {}^6C_5.(\frac{1}{10})^5(\frac{9}{10})^1 + {}^6C_6.(\frac{1}{10})^6(\frac{9}{10})^0$

$\Rightarrow \frac{15850}{10^6}$

15. Question

Assume that on an average one telephone number out of 15, called between 3 p.m. on weekdays, will be busy. What is the probability that if six randomly selected telephone numbers are called, at least 3 of them will be busy?

Answer

The probability that the called number is busy is $\frac{1}{15}$

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =6

The probability that at least three of them will be busy is:-

P(0) + P(1) + P(2) + P(3)

$\Rightarrow {}^6C_0(\frac{1}{15})^0(\frac{14}{15})^6 + {}^6C_1(\frac{1}{15})^1(\frac{14}{15})^5 + {}^6C_2(\frac{1}{15})^2(\frac{14}{15})^4 + {}^6C_3(\frac{1}{15})^3(\frac{14}{15})^3$

$\Rightarrow 1 - (\frac{14}{15})^4.(\frac{59}{45})$

16. Question

Three cars participate in a race. The probability that any one of them has an accident is 0.1. Find the probability that all the cars reach the finishing line without any accident.

Answer

The probability that any one of them has an accident is 0.1.

The probability any car reaches safely is 0.9.

The probability that all the cars reach the finishing line without any accident is = (0.9)(0.9)(0.9) = 0.729

17. Question

Past records show that 80% of the operations performed by a certain doctor were successful. If the doctor performs 4 operations in a day, what is the probability that at least 3 operations will be successful?

Answer

The probability that the operations performed are successful is = 0.8

The probability that at least three operations are successful is = P(3) + P(4)

$\Rightarrow {}^4C_3(0.8)^3(0.2)^1 + {}^4C_4(0.8)^4(0.2)^0$

$\Rightarrow \frac{512}{625}$

18. Question

The probability of a man hitting a target is (1/4). If he fires 7 times, what is the probability of his hitting the target at least twice?

Answer

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =7

p = � q = �

The probability of hitting the target at least twice is = P(2) + P(3) + P(4) + P(5) + P(6) + P(7)

⇒ 1-(P(0) + P(1))

⇒ $1 - ({}^7C_0(\frac{1}{4})^0(\frac{3}{4})^7 + {}^7C_1(\frac{1}{4})^1(\frac{3}{4})^6)$

⇒ $1 - (\frac{10}{4})(\ (\frac{3}{4})^6)$

⇒ $\frac{4547}{8192}$

19. Question

In a hurdles race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is (5/6). What is the probability that he will knock down fewer than 2 hurdles?

Answer

The probability that the hurdle will be cleared is 5/6

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =10

p = 5/6 q = 1/6

Probability that he will knock down fewer than 2 hurdles is =

P(0) + P(1)

⇒ ${}^{10}C_0(\frac{1}{6})^0(\frac{5}{6})^{10} + {}^{10}C_1(\frac{1}{6})^1(\frac{5}{6})^9$

⇒ $\frac{5^{10}}{2\times6^9}$

20. Question

A man can hit a bird, once in 3 shots. On this assumption he fires 3 shots. What is the chance that at least one bird is hit?

Answer

The probability that the bird will be shot, is 1/3

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 1/3 q =2/3

Probability that he will hit at least one bird is =

P(1) + P(2) + P(3)

⇒ ${}^3C_1(\frac{1}{3})^1(\frac{2}{3})^2 + {}^3C_2(\frac{1}{3})^2(\frac{2}{3})^1 + {}^3C_3(\frac{1}{3})^3(\frac{2}{3})^0$

⇒ $\frac{19}{27}$

21. Question

If the probability that a man aged 60 will live to be 70 is 0.65, what is the probability that out of 10 men, now 60, at least 8 will live to be 70?

Answer

The probability that a man aged 60 will live to be 70 is 0.65

Using Bernoulli's Trial we have,

$P(Success=x) = {}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =8

p = 0.65 q =0.35

Probability that out of 10 men, now 60, at least 8 will live to be 70 is: P(8) + P(9) + P(10)

${}^{10}C_{8}(0.65)^{8}(0.35)^{2} + {}^{10}C_{9}(0.65)^{9}(0.35)^{1} + {}^{10}C_{10}(0.65)^{10}(0.35)^{0}$

⇒ 0.2615

22. Question

A bag contains 5 white, 7 red 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

(i) None is white

(ii) All are white

(iii) At least one is white

Answer

(i) Balls are drawn at random,

So, the probability that none is white is,

In a trial the probability of selecting a non-white ball is $\frac{15}{20}$

So, in 4 trials it will be,

⇒ $(\frac{15}{20})(\frac{15}{20})(\frac{15}{20})(\frac{15}{20}) = \frac{81}{256}$

(ii) Balls are drawn at random,

So, the probability that all are white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials it will be,

⇒ $(\frac{5}{20})(\frac{5}{20})(\frac{5}{20})(\frac{5}{20}) = \frac{1}{256}$

(iii) Balls are drawn at random,

So, the probability that at least one is white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials the probability that at least one is white is,

Selecting a white and then choosing from the rest,

⇒ $1-\frac{81}{256}$ that no ball is white

is $\frac{175}{256}$.

23. Question

A policeman fires 6 bullets at a burglar. The probability that the burglar will be hit by a bullet is 0.6. what is the probability that burglar is still unhurt?

Answer

The probability that the burglar will be hit by a bullet is 0.6.

Using Bernoulli's Trial we have,

P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

p = 0.6 q =0.4

The probability that the burglar is unhurt is,

${}^{6}C_{0}(0.6)^{0}(0.4)^{6}$

⇒ 0.004096

24. Question

A die is tossed thrice. A success is 1 or 6 on a toss. Find the mean and variance of successes.

Answer

Using Bernoulli's Trial we have,

P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 2/6 = 1/3, q = 4/6 = 2/3

P(x = 0) = P (no success) = P (all failures) = $(\frac{2}{3})(\frac{2}{3})(\frac{2}{3}) = \frac{8}{27}$

P(x = 1) = P (1 success and 2 failures) = ${}^{3}C_{1}(\frac{1}{3})^{1}(\frac{2}{3})^{2} = \frac{12}{27}$

P(x = 2) = P (2 success and 1 failure) = ${}^{3}C_{2}(\frac{1}{3})^{2}(\frac{2}{3})^{1} = \frac{6}{27}$

P(x = 3) = P (all 3 success) = ${}^{3}C_{3}(\frac{1}{3})^{3}(\frac{2}{3})^{0} = \frac{1}{27}$

∴ The probability distribution of the random variable x is -

x : 0 1 2 3

P(x) : $\frac{8}{27}$ $\frac{12}{27}$ $\frac{6}{27}$ $\frac{1}{27}$

x_1 p_1 p_1x_1 $p_1{x_1}^2$

0 $\frac{8}{27}$ 0 0

1 $\frac{12}{27}$ $\frac{12}{27}$ $\frac{12}{27}$

2 $\frac{6}{27}$ $\frac{12}{27}$ $\frac{24}{27}$

3 $\frac{1}{27}$ $\frac{3}{27}$ $\frac{9}{27}$

1 $\frac{45}{27}$

Mean μ = Σp_1x_1 = 1

Variance = $\sigma^2 = \Sigma p_1{x_1}^2$ - μ

⇒ 5/3 - 1/1

⇒ 2/3

25. Question

A die is thrown 100 times. Getting an even number is considered a success. Find the mean and variance of success.

Answer

Probability of getting an even number is = 3/6 = 1/2

Probability of getting an odd number is = 3/6 = 1/2

Variance = npq

⇒ $100 \times \frac{1}{2} \times \frac{1}{2}$

⇒ 25

26. Question

Determine the binomial distribution whose mean is 9 and variance is 6?

Answer

Mean = np = 9

Variance = npq = 6

⇒ $q = \frac{6}{9} = \frac{2}{3}$

⇒ $p = 1 - \frac{6}{9} = \frac{1}{3}$

⇒ n = 27

Binomial distribution

$^{27}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(27-r)}$ where r = 0, 1, 2, 3, ..., 27

27. Question

Find the binomial distribution whose mean is 5 and variance is 2.5.

Answer

Mean = np = 5

Variance = npq = 2.5

⇒ $q = \frac{2.5}{5} = \frac{1}{2}$

⇒ $p = 1 - \frac{1}{2} = \frac{1}{2}$

⇒ n = 10

Probability distribution is:-

$^{10}C_r \cdot \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{(10-r)}, 0 \le r \le 10$

28. Question

The mean and variance of a binomial distribution are 4 and (4/3) respectively. Find P(X≥ 1).

Answer

Mean = np = 4

Variance = npq = 4/3

⇒ $q = \frac{1}{3}$

⇒ $p = 1 - \frac{1}{3} = \frac{2}{3}$

⇒ n = 6

The probability ($X \geq 1$) is

$^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5$

$= \frac{728}{729}$

29. Question

For a binomial distribution, the mean is 6 and the standard deviation is $\sqrt{2}$. Find the probability of getting 5 successes.

Answer

Mean = np = 6

Variance = npq = 2

⇒ $q = \frac{1}{3}$

⇒ $p = 1 - \frac{1}{3} = \frac{2}{3}$

⇒ n = 9

The probability of getting 5 successes,

$^9C_5(\frac{2}{3})^5(\frac{1}{3})^4$

30. Question

In a binomial distribution, the sum and the product of the mean and the variance are (25/3) and (50/3) respectively. Find the distribution.

Answer

Mean + Variance = np + npq = np(1 + q) = 25/3

Variance = n^2p^2q = n2 = 50/3 ...(i)

$n^2p^2(1 + q)^2 = 625/9$ ...(ii)

Dividing (i) by (ii), we get,

$$\frac{q}{(q+1)^2} = \frac{\frac{50}{3}}{\frac{625}{9}} = \frac{6}{25}$$

⇒ 6q2-13q + 6 = 0

⇒ q = 2/3 or 3/2

⇒ But as q can not be greater than 1 thus, q = 2/3.

⇒ p = 1/3

⇒ n = 15

Binomial distribution,

$$^{15}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(15-r)}$$

31. Question

Obtain the binomial distribution whose mean is 10 and standard deviation is $2\sqrt{2}$.

Answer

Mean is 10,

Standard deviation is $2\sqrt{2}$

So, variance is σ^2 i.e. 8

Thus,

Mean = np = 10

Variance = npq = 8

⇒ q = $\frac{4}{5}$

⇒ p = 1 - $\frac{4}{5}$ = $\frac{1}{5}$

⇒ n = 50

Thus, the binomial distribution is

$$\cdot\ ^{50}C_r \cdot \left(\frac{1}{5}\right)^r \cdot \left(\frac{4}{5}\right)^{(50-r)}, 0 \le r \le 50$$

32. Question

Bring out the fallacy, if any, in the following statement:

'The mean of a binomial distribution is 6 and its variance is 9'

Answer

Variance can not be greater than mean as then, q wll be greater than 1, which is not possible.

As, np = 6 and npq = 9

q = 3/2 ...(not possible)

Objective Questions

1. Question

Mark (√) against the correct answer in each of the following:

If A and B are mutually exclusive events such that P(A) = 0.4, P(B) =x and P(A ∪ B) = 0.5, then x = ?

A. 0.2

B. 0.1

C. $\frac{4}{5}$

D. None of these

Answer

If A and B are mutually exclusive events then,

P(A) = 0.4, P(B) = X

And $P(A \cup B) = P(A)+P(B) = 0.5 = 0.4 + P(B)$

$\Rightarrow P(B) = 0.1$

2. Question

Mark (√) against the correct answer in each of the following:

If A and B are independent events such that P(A) = 0.4, P(B) = x and $P(A \cup B) = 0.5$, then x = ?

A. $\frac{4}{5}$

B. 0.1

C. $\frac{1}{6}$

D. None of these

Answer

As A and B are independent events such that P(A) = 0.4, P(B) = x

So, $P(A \cap B) = P(A)P(B)$

And $P(A \cup B) = P(A)+P(B)+P(A \cap B)$

$P(A \cup B) = 0.4 + X - 0.4X = 0.5$

$\Rightarrow 0.4 + 0.6X = 0.5$

$\Rightarrow X = 1/6$

3. Question

Mark (√) against the correct answer in each of the following:

If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, then P(A/B) =?

A. 0.32

B. 0.64

C. 0.16

D. 0.25

Answer

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow$ And P(A) = 0.8,

$\Rightarrow P(A \cap B) = 0.32$

So, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A/B) = \frac{0.32}{0.5} = 0.64$

$\Rightarrow$ Hence, the answer is b.

4. Question

Mark (√) against the correct answer in each of the following:

If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$, then P(A/B)=?

A. $\frac{5}{6}$

B. $\frac{5}{7}$

C. $\frac{6}{7}$

D. $\frac{4}{5}$

Answer

P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

⇒ $\frac{7}{11} = \frac{6}{11} + \frac{5}{11} -$ P(A ∩ B)

⇒ P(A ∩ B) = $\frac{4}{11}$

P(A/B) = P(A ∩ B)/P(B)

⇒ P(A/B) = $\frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$

5. Question

Mark (√) against the correct answer in each of the following:

If A and B are events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P(A' ∪ B') = $\frac{1}{4}$, then A and B are

A. Independent

B. Mutually exclusive

C. Both 'a' and 'b.'

D. None of these

Answer

We are having two events A and B such that

P(A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P(A' ∪ B') = $\frac{1}{4}$,

P(A'∪ B') = P'(A ∩ B) = 1 – P(A ∩ B) = $\frac{1}{4}$

⇒ P(A ∩ B) = $\frac{3}{4}$

⇒ As P(A ∩ B) ≠ P(A).P(B) … thus, they are not independent,

⇒ And as P(A ∪ B) ≠ P(A) + P(B) … thus, they are not mutually exclusive.

Hence, the answer is option d.

6. Question

Mark (√) against the correct answer in each of the following:

It is given that the probability that A can solve a given problem is $\frac{3}{5}$ and the probability that B can solve the same problem is $\frac{2}{3}$. The probability that at least one of A and B can solve a problem is

A. $\frac{2}{5}$

B. $\frac{1}{15}$

C. $\frac{13}{15}$

D. $\frac{2}{15}$

Answer

P(A) = probability that A can solve the problem

= 3/5

And P(B) = probability that B can solve the problem = 2/3

P(A U B) = P(A) + P(B), As the events are independent

⇒ P(A ∩ B) = P(A).P(B)

Thus,

⇒ P(A) + P(B) = 3/5 +2/3 – 2/5 = 13/15

7. Question

Mark (√) against the correct answer in each of the following:

The probabilities of A, B and C of solving a problem are $\frac{1}{6}$, $\frac{1}{5}$ and $\frac{1}{3}$respectively. What is the probability that the problem is solved?

A. $\frac{4}{9}$

B. $\frac{5}{9}$

C. $\frac{1}{3}$

D. None of these

Answer

The probability that the problem is solved = P(A∪B∪ C) = P(A)+P(B)+P(C)-P(A∩B)-P(B∩C)-P(C∩A) + 3P(A∩B∩ C)

Considering independent events, $P(A\cap B) = P(A).P(B)$,

$P(B\cap C) = P(B).P(C)$, $P(C\cap A) = P(C).P(A)$,

$P(A\cap B\cap C) = P(A).P(B).P(C)$,

Thus, $P(A\cup B\cup C)$ is,

$\Rightarrow \frac{1}{6}+\frac{1}{5}+\frac{1}{3}-\frac{1}{30}-\frac{1}{15}-\frac{1}{18}+3\left(\frac{1}{90}\right)=\frac{5}{9}$

8. Question

Mark (√) against the correct answer in each of the following:

A can hit a target 4 times in 5 shots, B can hot 3 times in 4 shots, and C can hit 2 times in 3 shots. The probability that B and C hit and A does not hit is

A. $\frac{1}{10}$

B. $\frac{2}{5}$

C. $\frac{7}{12}$

D. None of these

Answer

$P(A) = \frac{4}{5}$ $P(B) = \frac{3}{4}$ $P(C) = \frac{2}{3}$

$P(B \cap C \cap A') = P(B \cap C) - P(B \cap C \cap A)$

As the events are independent, So, $P(B \cap C) = P(B).P(C) = \frac{3}{4}\times\frac{2}{3}$

And $P(B \cap C \cap A) = P(B).P(C).P(A) = \frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$

$P(B \cap C \cap A') = \frac{1}{10}$

9. Question

Mark (√) against the correct answer in each of the following:

A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third component are 0.2, 0.3 and 0.5, respectively. What is the probability that the machine will fail?

A. 0.70

B. 0.72

C. 0.07

D. None of these

Answer

The probability of failure of the first component = 0.2 =P(A)

The probability of failure of second component = 0.3 = P(B)

The probability of failure of third component = 0.5 = P(C)

As the events are independent,

The machine will operate only when all the components work, i.e.,

(1-0.2)(1-0.3)(1-0.5) = P(A')P(B')P(C')

In rest of the cases, it won't work,

So P(AUBUC) = 1 – P(A'∩B'∩C') = 1 – (0.8).(0.7).(0.5)

⇒ 1 – 0.28 = 0.72

10. Question

Mark (√) against the correct answer in each of the following:

A die is rolled. If the outcome is an odd number, what is the probability that it is prime?

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $\frac{5}{12}$

D. None of these

Answer

The probability that the outcome which is either, 1, 3 or 5 is prime is

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

Favourable outcomes = 3 or 5

Total outcomes = 1, 3, and 5

Thus, probability=

$\Rightarrow \frac{2}{3}$

11. Question

Mark (√) against the correct answer in each of the following:

If A and B are events such that P(A) = 0.3, P(B) = 0.2 and P(A ∩ B) = 0.1, then P($\overline{A}$ ∩ B) = ?

A. 0.2

B. 0.1

C. 0.4

D. 0.5

Answer

P(A) = 0.3, P(B) = 0.2 and P(A ∩ B) = 0.1

P($\overline{A}$ ∩ B) = P(B) – P(A ∩ B) = 0.2 – 0.1 = 0.1

12. Question

Mark (√) against the correct answer in each of the following:

If P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$ and P(A ∩ B) = $\frac{1}{5}$, then P($(\overline{B}/\overline{A})$ = ?

A. $\frac{11}{15}$

B. $\frac{11}{45}$

C. $\frac{23}{60}$

D. $\frac{37}{45}$

Answer

P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$ and P(A ∩ B) = $\frac{1}{5}$,

P$(\overline{B}/\overline{A})$ = $\frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})} = \frac{1-P(A \cup B)}{1-P(A)} = \frac{1-\left(\frac{1}{4}+\frac{1}{3}-\frac{1}{5}\right)}{1-\frac{1}{4}}$

⇒ P$(\overline{B}/\overline{A})$ = $\frac{23}{60}$

13. Question

Mark (√) against the correct answer in each of the following:

If A and B are events such that P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6, then P(A/B) =?

A. 0.2

B. 0.3

C. 0.4

D. 0.5

Answer

P(A) = 0.4, P(B) = 0.8 and

P(B/A) = 0.6,

P(B/A) = $\frac{P(A \cap B)}{P(A)} = 0.6$

P(A ∩ B) = 0.24

⇒ P(A/B) = $\frac{P(A \cap B)}{P(B)} = 0.3$

14. Question

Mark (√) against the correct answer in each of the following:

If A and B are independent events, then P$\left(\overline{A}/\overline{B}\right)$=?

A. 1 – P(A)

B. 1 – P(B)

C. 1 – P(A/$\overline{B}$)

D. – P($\overline{A}$/B)

Answer

$P\left(\overline{A}/\overline{B}\right) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{P(\bar{A})P(\bar{B})}{1-P(B)} = 1 - P(A)$

15. Question

Mark (√) against the correct answer in each of the following:

If A and B are two events such that $P(A \cup B) = \left(\frac{5}{6}\right)$, $P(A \cap B) = \left(\frac{1}{3}\right)$ and $P(\overline{B}) = \left(\frac{1}{2}\right)$, then the events A and B are

A. Independent

B. Dependent

C. Mutually exclusive

D. None of these

Answer

Given,

$P(A \cup B) = \left(\frac{5}{6}\right)$, $P(A \cap B) = \left(\frac{1}{3}\right)$ and

$P(\overline{B}) = \left(\frac{1}{2}\right)$, $P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow P(B) = \frac{1}{2}$

$\Rightarrow P(A) = \frac{2}{3}$

⇒ Hence, these are independent.

16. Question

Mark (√) against the correct answer in each of the following:

A die is thrown twice, and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared at least one?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{2}{7}$

D. $\frac{3}{5}$

Answer

The die is thrown twice,

So the favourable outcomes that the sum appears to be 7 are

(1,6), (2,5), (3,4), (4,3), (5,2) and (6,1)

Out of these 2 appears twice,

So the probability that 2 appears at least once is:

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

$\Rightarrow \frac{2}{6} = \frac{1}{3}$

17. Question

Mark (√) against the correct answer in each of the following:

Two numbers are selected random from integers 1 through 9. If the sum if even, what is the probability that both numbers are odd?

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{4}{9}$

D. $\frac{5}{8}$

Answer

The sum will be even when; both numbers are either even or odd,

i.e. for both numbers to be even, the total cases ${}^5C_1 X {}^4C_1$ (Both the numbers are odd)+${}^4C_1 X {}^3C_1$ (Both the numbers are even)= 32

The favourable number of cases will be,

Both odd, i.e. selecting numbers from 1, 3, 5, 7, or 9, i.e.

${}^5C_1 X {}^4C_1 = 20$

Thus, the probability that both numbers are odd will be =

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

$\Rightarrow \frac{20}{32} = \frac{5}{8}$

18. Question

Mark (√) against the correct answer in each of the following:

In a class, 60% of the students read mathematics, 25% biology and 15% both mathematics and biology. One student is selected at random. What is the probability that he reads mathematics if it is known that he reads biology?

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{3}{8}$

D. $\frac{5}{8}$

Answer

Given:

60% of the students read mathematics, 25% biology and 15% both mathematics and biology

That means,

Let the event A implies students reading mathematics,

Let the event B implies students reading biology,

Then, P(A) = 0.6

P(B) = 0.25

P(A∩B) = 0.15

We, need to find P(A/B) = P(A∩B)/ P(B)

$\Rightarrow \frac{0.15}{0.25} = \frac{3}{5}$

19. Question

Mark (√) against the correct answer in each of the following:

A couple has 2 children. What is the probability that both are boys. If it is known that one of them is a boy?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$

Answer

The couple has two children and one is known to be boy,

The probability that the other is boy will be =

$$\frac{Favorable\ outcomes}{Total\ outcomes}$$

Total outcomes are 3,

The first child is a boy, the second girl

The first child is a girl, the second boy

The first child is a boy, second boy

The favourable outcome is one,

Thus, the probability that the other is boy will be

⇒ 1/3

20. Question

Mark (√) against the correct answer in each of the following:

An unbiased die is tossed twice. What is the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{5}{6}$

Answer

A die is tossed twice,

The probability of getting a 4, 5 or 6 in the first trial is 3/6 = P(A)

The probability of getting a 1, 2, 3 or 4 in the second trial is 4/6 =P(B)

As the events are independent, the probability of these two events together will be, P(A).P(B) = 1/3.

21. Question

Mark (√) against the correct answer in each of the following:

A fair coin is tossed 6 times. What is the probability of getting at least 3 heads?

A. $\frac{11}{16}$

B. $\frac{21}{32}$

C. $\frac{1}{18}$

D. $\frac{3}{64}$

Answer

Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is thrown 6 times the total number of outcomes will be 2^6.

And we know that the favourable outcomes of getting at least 3 successes will be, getting a head

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

$^{6}C_{3}(\frac{1}{2})^{3}(\frac{1}{2})^{3} + {}^{6}C_{4}(\frac{1}{2})^{4}(\frac{1}{2})^{2} + {}^{6}C_{5}(\frac{1}{2})^{5}(\frac{1}{2})^{1} + {}^{6}C_{6}(\frac{1}{2})^{6}(\frac{1}{2})^{0}$

$\Rightarrow \frac{21}{32}$

22. Question

Mark (√) against the correct answer in each of the following:

A coin is tossed 5 times. What is the probability that tail appears an odd number of times?

A. $\frac{3}{5}$

B. $\frac{2}{15}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 5 times the total number of outcomes will be 2^5.

And we know that the favourable outcomes of getting the odd tail number of times ,successes will be, getting a tail

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

${}^5C_1(\frac{1}{2})^1(\frac{1}{2})^4 + {}^5C_3(\frac{1}{2})^3(\frac{1}{2})^2 + {}^5C_5\ (\frac{1}{2})^5(\frac{1}{2})^0$

$\Rightarrow \frac{16}{32} = \frac{1}{2}$

23. Question

Mark (√) against the correct answer in each of the following:

A coin is tossed 5 times. What is the probability that the head appears an even number of times?

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{4}{15}$

D. $\frac{1}{2}$

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 5 times the total number of outcomes will be 2^5.

And we know that the favourable outcomes of getting the head even number of times ,successes will be, getting a head,

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

the probability that head appears an even number of times =

P(0)+P(2)+P(4)

$= {}^5C_2(\frac{1}{2})^2(\frac{1}{2})^3 + {}^5C_3(\frac{1}{2})^3(\frac{1}{2})^2 + {}^5C_5\ (\frac{1}{2})^5(\frac{1}{2})^0$

$\Rightarrow \frac{16}{32} = \frac{1}{2}$

24. Question

Mark (√) against the correct answer in each of the following:

8 coins are tossed simultaneously. The probability of getting at least 6 heads is

A. $\frac{7}{64}$

B. $\frac{57}{64}$

C. $\frac{37}{256}$

D. $\frac{249}{256}$

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 8 times the total number of outcomes will be 2^8.

And we know that the favourable outcomes of getting at least 6 heads are,successes will be, getting a head,

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

the probability of getting at least 6 heads is =

P(6) +P(7) +P(8)

$= {}^8C_6(\frac{1}{2})^6(\frac{1}{2})^2 + {}^8C_7(\frac{1}{2})^7(\frac{1}{2})^1 + {}^8C_8(\frac{1}{2})^8(\frac{1}{2})^0$

$\Rightarrow \frac{28+8+1}{256} = \frac{37}{256}$

25. Question

Mark (√) against the correct answer in each of the following:

A die is thrown 5 times. If getting an odd number is a success, then what is the probability of getting at least 4 successes?

A. $\frac{4}{5}$

B. $\frac{7}{16}$

C. $\frac{3}{16}$

D. $\frac{3}{20}$

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 5 times the total number of outcomes will be 6^5.

And we know that the favourable outcomes of getting at least 4 successes will be, either getting 1, 3 or 5 i.e., 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{1}{2}$,q = $\frac{1}{2}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

the probability of getting at least 4 successes =

P(4)+P(5)

$\Rightarrow {}^5C_4(\frac{1}{2})^4(\frac{1}{2})^1 + {}^5C_5(\frac{1}{2})^5(\frac{1}{2})^0$

$\Rightarrow \frac{3}{16}$

26. Question

Mark (√) against the correct answer in each of the following:

In 4 throws of a pair of dice, what is the probability of throwing doublets at least twice?

A. $\frac{7}{36}$

B. $\frac{17}{144}$

C. $\frac{19}{144}$

D. None of these

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As we know that the favourable outcomes of getting at least doublets twice are, successes will be, getting a doublet, i.e.,

, p = $\frac{1}{6}$,q = $\frac{5}{6}$

The probability of success is $\frac{1}{6}$ and of failure is also $\frac{5}{6}$

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

the probability of getting at least 2 successes =

P(2)+P(3)+P(4)

$\Rightarrow {}^4C_2(\frac{1}{6})^2(\frac{5}{6})^2 + {}^4C_3(\frac{1}{6})^3(\frac{5}{6})^1 + {}^4C_4(\frac{1}{6})^4(\frac{5}{6})^0$

$\Rightarrow \frac{19}{144}$

27. Question

Mark (√) against the correct answer in each of the following:

A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of getting at most 6 successes?

A. $\left(\frac{5}{7}\right)^7$

B. $\left(\frac{1}{6}\right)^7$

C. $\left(1-\frac{1}{6^7}\right)$

D. None of these

Answer

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), here n = 7

As we know that the favourable outcomes of getting at most 6 success are, successes will be, getting a total of 7 is success, i.e.,

We can get 7 by, (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

, p = $\frac{6}{36}$,q = $\frac{30}{36}$

The probability of success is $\frac{1}{6}$ and of failure is also $\frac{5}{6}$

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

the probability of getting at most 6 successes =

P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6) = 1-P(7)

$\Rightarrow 1 - {}^7C_7(\frac{1}{6})^7(\frac{5}{6})^0$

$\Rightarrow 1 - (\frac{1}{6})^7$

28. Question

Mark (√) against the correct answer in each of the following:

The probability that a man can hit a target is $\frac{3}{4}$. He tries five times. What is the probability that he will hit the target at least 3 times?

A. $\frac{459}{512}$

B. $\frac{291}{364}$

C. $\frac{371}{464}$

D. None of these

Answer

The probability that the man hits the target is 3/4

Using Bernoulli’s Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

p = �, q = �

Probability that he will hit at least 3 times is =

P(3)+P(4)+P(5)

$\Rightarrow {}^5C_3(\frac{3}{4})^3(\frac{1}{4})^2+{}^5C_4(\frac{3}{4})^4(\frac{1}{4})^1+{}^5C_5(\frac{3}{4})^5(\frac{1}{4})^0$

$\Rightarrow \frac{459}{512}$

29. Question

Mark (√) against the correct answer in each of the following:

The probability of the safe arrival of one ship out of 5 is $\frac{1}{5}$. What is the probability of the safe arrival of at least 3 ships?

A. $\frac{1}{31}$

B. $\frac{3}{52}$

C. $\frac{181}{3125}$

D. $\frac{184}{3125}$

Answer

The probability of safe arrival of the ship is 1/5

Using Bernoulli’s Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

p = 1/5 , q = 4/5

Probability of safe arrival of at least 3 ships is =

P(3)+P(4)+P(5)

$\Rightarrow {}^5C_3(\frac{1}{5})^3(\frac{4}{5})^2+{}^5C_4(\frac{1}{5})^4(\frac{4}{5})^1+{}^5C_5(\frac{1}{5})^5(\frac{4}{5})^0$

$\Rightarrow \frac{181}{3125}$

30. Question

Mark (√) against the correct answer in each of the following:

The probability that an event E occurs in one trial is 0.4, Three independent trials of the experiment are performed. What is the probability that E occurs at least once?

A. 0.784

B. 0.936

C. 0.964

D. None of these

Answer

The probability of occurrence of an event E in one trial is 0.4

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 0.4 , q = 0.6

The probability that E occurs at least once is,

P(1)+P(2)+P(3)

$\Rightarrow {}^3C_1(\frac{2}{5})^1(\frac{3}{5})^2+{}^3C_2(\frac{2}{5})^2(\frac{3}{5})^1+{}^3C_3(\frac{2}{5})^3(\frac{3}{5})^0$

$\Rightarrow \frac{98}{125} = 0.784$