CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - I

PHYSICS

Q.1 An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. The produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are –

(1) 0.67 kΩ, 300 (2) 0.67 kΩ, 200 (3) 0.33 kΩ, 1.5 (4) 0.33 kΩ, 300

Ans. **[1]**

Sol.

Current input = 15×10^{-6}

Current output = 3×10^{-3}

$R_0 = 1000$

$V_{in} = 10 \times 10^{-3}$

$V_{in} = r_{in} \times I_{in}$

$r_{in} = \frac{2000}{3} = .67\ k\Omega$

$$\text{Voltage gain} = \frac{V_0}{V_i} = \frac{1000 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

Q.2 For a given at 1 atm pressure, rms speed of the molecules is 200 m/s at 127ºC. At 2 atm pressure and at 227ºC, the rms speed of the molecules will be –

(1) 100m/s (2) $80\sqrt{5}$ m/s (3) $100\sqrt{5}$ m/s (4) 80 m/s

Ans. **[3]**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_w}}$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{V_2}{200} = \sqrt{\frac{500}{400}}$$

$V_2 = 100\sqrt{5}$

Q.3 A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which is has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is -

(1) $\frac{k}{I}\theta$ (2) $\frac{k}{2I}\theta$ (3) $\frac{k}{4I}\theta$ (4) $\frac{2k}{I}\theta$

Ans. **[4]**

Sol. Given that energy ⇒ $\frac{1}{2}I\omega^2 = k\theta^2$

$$\frac{1}{2}I\omega^2 = k\theta^2$$

By differentiate

$$\frac{1}{2}\ I \times 2\omega\ \frac{d\omega}{dt} = 2k\theta\ \frac{d\theta}{dt}$$

$$I\ \omega\ \frac{d\omega}{d\theta} = 2k\theta$$

$$\alpha = \frac{2k\theta}{I}$$

Q.4 The pressure wave, P = 0.01 sin [1000t – 3x] Nm^{-2}, corresponds to the sound produced by vibrating blade on a day when atmospheric temperature is 0ºC. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}/ Approximate value of T is -

(1) 4ºC (2) 12ºC (3) 11ºC (4) 15ºC

Ans. **[1]**

Sol. Speed of sound $= \frac{w}{k}$

$$= \frac{1000}{3}$$

$$v \propto \sqrt{T}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{336}{1000/3} = \sqrt{\frac{T}{273}}$$

T = 277 K (appr.)

T = 4ºC

Q.5 An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be -

(1) $\frac{m\bar{v}^2}{7k_B}$ (2) $\frac{m\bar{v}^2}{6k_B}$ (3) $\frac{m\bar{v}^2}{5k_B}$ (4) $\frac{m\bar{v}^2}{3k_B}$

Ans. **[Bonus]**

Sol. Energy of molecules = $\frac{f}{2}$ kT

For translational $\frac{1}{2}\ mV^2 = \frac{3}{2}kT$

$$T = \frac{mV^2}{3k}$$

According to Translational equation

Ans should be (4) $\frac{mV^2}{3K}$

NTA has given $\frac{mV^2}{6K}$ (2)

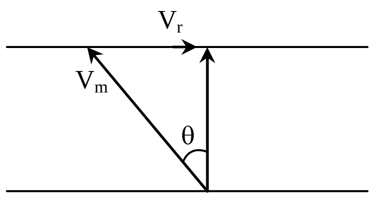
So NTA should bonus this question and give full mark's to students.

Q.6 The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

(1) 60º (2) 90º (3) 150º (4) 120º

Ans. **[4]**

Sol.



$V_m \sin\theta = V_r$

$4 \sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$

Angle from river flow = 30º + 90º = 120º

Q.7 The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is -

(1) 14 : 15 : 20 (2) 10 : 15 : 7 (3) 2 : 3 : 4 (4) 4 : 3 : 2

Ans. **[Bonus]**

Sol. By energy conservation

$$\frac{1}{2}mv^2\left(1+\frac{k^2}{r^2}\right) = mgh$$

$$h \propto \left(1+\frac{k^2}{r^2}\right)$$

$$h_1 : h_2 : h_3 \rightarrow (1+1) : \left(1+\frac{1}{2}\right) : \left(1+\frac{2}{5}\right)$$

$$\Rightarrow \quad 2 : \frac{3}{2} : \frac{7}{5}$$

$$\Rightarrow \quad 20 : 15 : 14$$

Ans. Should be 20 : 15 : 14

NTA has give 10 : 15 : 7

So (Q) should bonus by NTA

Q.8 The magnetic field of a plane electromagnetic wave is given by :

$\vec{B} = B_0\,\hat{i}\,[\cos(kz - \omega t)] + B_1\,\hat{j}\,\cos(kz + \omega t)$

Where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$T. The rms value of the force experienced by a stationary charge Q = 10^{-4} C at z = 0 is closest to -

(1) 0.6 N (2) 3×10^{-2} N (3) 0.9 N (4) 0. 1N

Ans. **[1]**

Sol. Maximum Electric Field = BC

$F_{net} = q\,\vec{E}_{net}$

$= qC(-3 \times 10^{-5}\,\hat{j} - 2 \times 10^{-6}\,\hat{i})$

$= 10^{-4} \times 3 \times 10^{8}\,\sqrt{(3\times10^{-5})^2 + (2\times10^{-6})^2}$

$= .9$ N

$F_{rms} = \dfrac{F_0}{\sqrt{2}} = .6$ N

Q.9 A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be -

(1) $\dfrac{GM}{9a^2}$ (2) $\dfrac{2GM}{9a^2}$ (3) $\dfrac{2GM}{3a^2}$ (4) $\dfrac{GM}{3a^2}$

Ans. **[4]**

Sol. According to gauss theorem

$g4\pi\,(3a)^2 = 3M\,4\pi G$

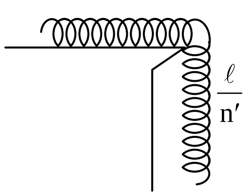
$g = \dfrac{GM}{3a^2}$

Q.10 A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\dfrac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be -

(1) nMgL (2) $\dfrac{MgL}{2n^2}$ (3) $\dfrac{2MgL}{n^2}$ (4) $\dfrac{MgL}{n^2}$

Ans. **[2]**

Sol.



Work done against gravity

= mgh

$= \dfrac{m}{n}\,g\,\dfrac{\ell}{2n}$

$= \dfrac{mg\ell}{2n^2}$

Q.11 A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45º about Z-axis, then the torque on the coil is -

(1) 0.55 Nm (2) 0.38Nm (3) 0.42 Nm (4) 0.27 Nm

Ans. **[4]**

Sol. $\iota = M \times B$

$= NIA\ B \sin\theta$

$= 100 \times 3 \times 5 \times 2.5 \times 10^{-4} \times 1 \times \sin 45^{\circ}$

$= .27$ Nm

Q.12 The electric field of light wave is given as $\vec{E} = 10^{-3} \cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\ \frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is : Given, E (in eV) = $\frac{12375}{\lambda(\text{in Å})}$.

(1) 0.72 V (2) 2.0 V (3) 0.48 V (4) 2.48 V

Ans. **[3]**

Sol. $\omega = 2\pi n = 2\pi \times 6 \times 10^{14}$

$n = 6 \times 10^{14}$

$V = n\lambda$

$\lambda = \frac{3\times10^{8}}{6\times10^{14}} = 5000$ Å

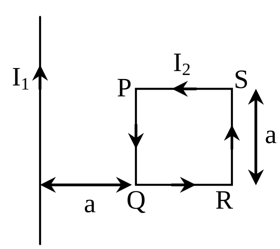
$E_n = \frac{12375}{5000} = 2.475$

$KE_{rmx} = E - \phi$

$eV_s = (2.475 - 2)$ eV

$V_s = .475\ V = .48\ V$

Q.13 A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be –



(1) zero (2) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(3) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$ (4) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

Ans. **[2]**

Sol. Force of PQ

$F_1 = I_2 B_1 A$

$= I_2 \frac{\mu_0 I_1}{2\pi a} a$

Force of RS

$F_2 = I_2B_2a$

$= I_2 \dfrac{\mu_0 I_1}{2\pi(2a)} a$

Net force $F_1 - F_2$

$= \dfrac{\mu_0 I_1 I_2}{4\pi}$ (Repulsive)

Q.14 In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is -

(1) 0.31 kg/m^3 (2) 0.10 kg/m^3 (3) 0.01 kg/m^3 (4) 0.07 kg/m^3

Ans. **[Bonus]**

Sol. $M = 10 \pm .10$

$\ell = .10 \pm .01$

$\rho = \dfrac{M}{\ell^3} = \dfrac{10}{(.1)^3} = 10^4$

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta M}{M} + \dfrac{3\Delta\ell}{\ell}$

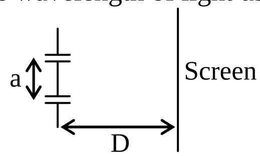
$= \dfrac{.10}{10} + \dfrac{3\times.01}{.10}$

$= \dfrac{1}{100} + \dfrac{3}{10}$

$\dfrac{\Delta\rho}{\rho} = \dfrac{31}{100}$

$\dfrac{\Delta\rho}{\rho} = .31$ This Ans. is not for error in density it is relative error. So (Q) should be Bonus.

Q.15 The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be :



(1) $\dfrac{2nD\lambda}{a(\mu-1)}$ (2) $\dfrac{nD\lambda}{a(\mu-1)}$ (3) $\dfrac{D\lambda}{a(\mu-1)}$ (4) $\dfrac{2D\lambda}{a(\mu-1)}$

Ans. **[Bonus]**

Sol. Path difference $\Delta = (\mu - 1)t$

$(\mu - 1)t\dfrac{D}{d} = \dfrac{n\lambda D}{d}$

$f = \dfrac{n\lambda}{\mu-1}$

correct ans. Should be $\dfrac{n\lambda}{\mu-1}$

NTA has given $\dfrac{nD\lambda}{a(\mu-1)}$, So it should be bonus

Q.16 A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4mA current. A voltmeter is made using this galvanometer and a 5kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to :

(1) 40 V (2) 10 V (3) 15 V (4) 20 V

Ans. **[4]**

Sol. $V = I_g (G + S)$

$V = 4 \times 10^{-3} (50 + 5000)$

$= 4 \times 10^{-3} \times 5050$

$= 20$ V (approximate)

Q.17 Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm (2) 388.9 nm (3) 488.9 nm (4) 642.7 nm

Ans. **[3]**

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

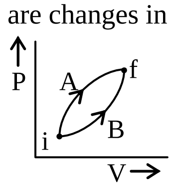
$\frac{1}{660} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$

$\frac{\lambda}{660} = \frac{5/36}{3/16}$

$\lambda = 488.88$

Q.18 Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then :

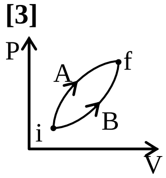


(1) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$ (2) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$

(3) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$ (4) $\Delta Q_A = \Delta Q_B ; \Delta U_A = \Delta U_B$

Ans. **[3]**

Sol.



Initial & final point are same

So, $dU_A = dU_B$

$dW_A > dW_B$ (Area)

So, $dQ_A > dQ_B$ $(dQ = dU + dW)$

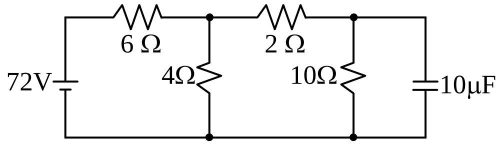
Q.19 If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) M (2) 4 M (3) $\frac{M}{2}$ (4) 2 M

Ans. **[1]**

Sol. Height will change with change in radius mass remain same

Q.20 Determine the charge on the capacitor in the following circuit :

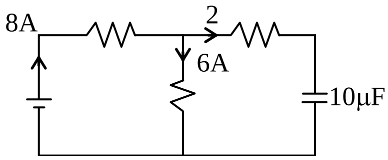


(1) 200 μC (2) 10 μC (3) 60 μC (4) 2 μC

Ans. **[1]**

Sol. Total Resistance = 9

Total current 8 cm



Voltage at 10Ω = 20V

q = CV

= 10 μF × 20 = 200 μC

Q.21 The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to :

(1) L (2) L^2 (3) $1/L^2$ (4) 1/L

Ans. **[4]**

Sol. $\phi = NBA = LI$

$N\mu_0 nI\pi r^2 = LI$

$N\mu_0 \frac{N}{\ell} \pi r^2 I = LI$

$L \propto \frac{1}{\ell}$

Q.22 A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is :

(1) 1.60 m (2) 0.16 m (3) 0.32 m (4) 0.24 m

Ans. **[3]**

Sol. $m = \frac{f}{f-u} = \frac{-40}{-40-u}$

u = – 32 cm

Q.23 A capacitor with capacitance 5 μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done?

(1) 3.75×10^{-6} J (2) 6.25×10^{-6} J (3) 2.55×10^{-6} J (4) 2.16×10^{-6} J

Ans. **[1]**

Sol. Work = $U_f - U_i$

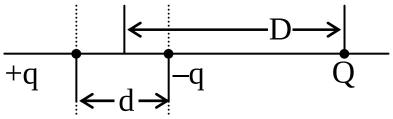
$= \frac{q^2}{2C_2} - \frac{q^2}{2C_1}$

$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$

$= \frac{15}{4} \times 10^{-6}$

$= 3.75 \times 10^{-6}$

Q.24 A system of three charges are placed as shown in the figure :



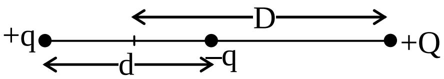
If D >> d, the potential energy of the system is best given by :

,

(1) $\frac{1}{4\pi \in_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$ (2) $\frac{1}{4\pi \in_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$ (3) $\frac{1}{4\pi \in_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$ (4) $\frac{1}{4\pi \in_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$

Ans. **[2]**

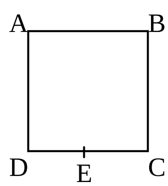
Sol.



$$u = -\frac{kq^2}{d} - \frac{kQ(qd)}{D^2}$$

$$= -k\left[\frac{q^2}{d} + \frac{qQd}{D^2}\right)$$

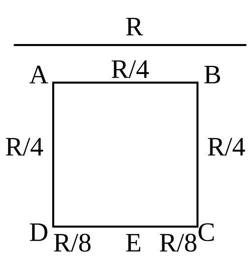
Q.25 A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is : (E is mid-point of arm CD)



(1) $\frac{3}{4}R$ (2) $\frac{1}{16}R$ (3) $\frac{7}{64}R$ (4) R

Ans. **[3]**

Sol.



$$R_{EDABC} = \frac{R}{4} + \frac{R}{4} + \frac{R}{4} + \frac{R}{8} = \frac{7R}{8}$$

$$R_{EC} = \frac{R}{8}$$

Effective resistance = $\frac{\frac{R}{8}\times\frac{7R}{8}}{\frac{R}{8}+\frac{7R}{8}} = \frac{7R}{64}$

Q.26 A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\frac{1}{14}}$ (2) $2T\sqrt{\frac{1}{10}}$ (3) $4T\sqrt{\frac{1}{15}}$ (4) $4T\sqrt{\frac{1}{14}}$

Ans. **[3]**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

When immersed in liquid

Tension $\Rightarrow T' = mg - m'g$

$T' = mg\left(1 - \frac{m'}{m}\right)$

$T' = T\left(1 - \frac{Vd\ell}{Vd_0}\right)$

$T' = T\left(1 - \frac{1}{16}\right)$

$T' = \frac{15T}{16}$

$g' = \frac{15}{16}g$

$\frac{T'}{T} = \sqrt{\frac{16}{15}}$

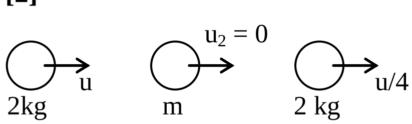
$T' = \frac{4T}{\sqrt{15}}$

Q.27 A body of mass 2kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.5 kg (2) 1.2 kg (3) 1.0 kg (4) 1.8 kg

Ans. **[2]**

Sol.



$V_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2}$

$\frac{u}{4} = \frac{O + u(2 - m)}{2 + m}$

$2 + m = 8 - 4m$

$5m = 6$

$m = 1.2$ kg

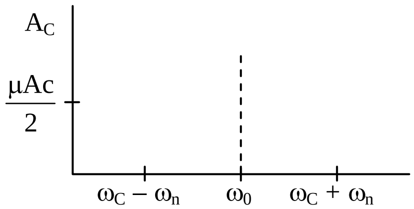
Q.28 A signal $A\cos\omega t$ is transmitted using $\nu_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is :

1. $\nu_0 \sin[\omega_0(1 + 0.01\, A\sin\omega t)t]$
2. $\nu_0 \sin\omega_0 t + A\cos\omega t$
3. $\nu_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$
4. $(\nu_0 + A)\cos\omega t \sin\omega_0 t$

Ans. **[3]**

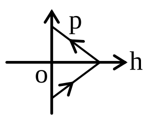
Sol. According to standard result is should be

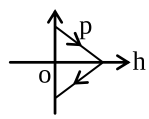
$= A_C \sin\omega_C t + \frac{\mu A_C}{2}\sin(\omega_0 - \omega)t + \frac{\mu A}{2}\sin(\omega_0 + \omega)t$

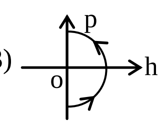


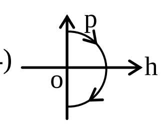
So, Ans. [3]

Q.29 A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is :

(1) 

(2) 

(3) 

(4) 

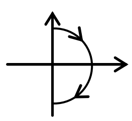
Ans. **[4]**

Sol. When thrown upward, (h = 0) V → max

$V^2 = u^2 + 2as$

$V^2 = u^2 - 2gh$ (Parabola)

When downward (h → max, V → 0)



Q.30 A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is : (All quantities are in SI units.)

(1) 40 m (2) 80 m (3) 20 m (4) 60 m

Ans. **[2]**

Sol. $y = a\sin(.157x)\cos(200\pi t)$

$\frac{2\pi}{\lambda} = .157$

$\lambda = 40$

4^{th} harmonic real

$\ell = 4\frac{\lambda}{2}$

$\ell = 2\lambda$

$= 80$ m

JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - I

CHEMISTRY

Q.1 Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statements is :

(x_M = Mole fraction of 'M' in solution ;

x_N = Mole fraction of 'N' in solution;

y_M = Mole fraction of 'M' in vapour phase;

y_N = Mole fraction of 'N' in vapour phase)

(1) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (2) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (3) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$ (4) $(x_M - y_M) < (x_N - y_N)$

Ans. **[3]**

Sol. $P_M^\circ = 450$

$P_N^\circ = 700$

$$y_M = \frac{P_M^\circ X_M}{P_S}$$

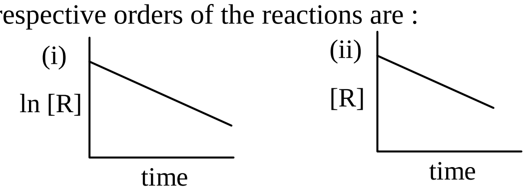
$$y_N = \frac{P_N^\circ X_N}{P_S}$$

$$\frac{y_M}{y_N} = \frac{P_M^\circ}{P_N^\circ}\frac{X_M}{X_N}$$

$$\frac{y_M}{y_N} = \frac{450}{700}\frac{X_M}{X_N}$$

$$\frac{X_M}{X_N} > \frac{y_M}{y_N}$$

Q.2 The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are :



(1) 1, 1 (2) 0, 1 (3) 1, 0 (4) 0, 2

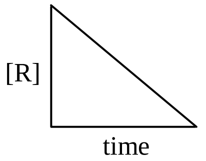
Ans. **[3]**

Sol. $R \longrightarrow P$

For zero order reaction

$(a - x) = -K_0t + a$

$[A]_t = -K_0t + [A]_0$



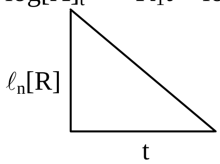
for I order reaction

$K_1 = \frac{2.3}{t} \log \frac{a}{a-x}$

$K_1 = \frac{1}{t} \ln \frac{a}{a-x}$

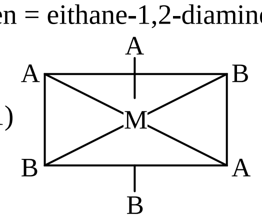
$K_1t = \log a - \log a - x$

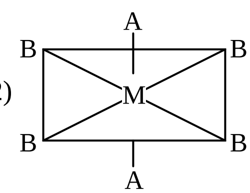
$\log[A]_t = -K_1t + \log[A_0]$

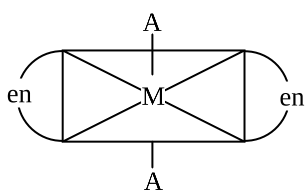


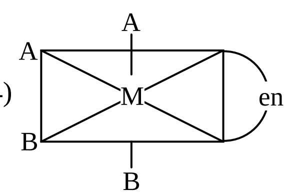
Q.3 The one that will show optical activity is :

(en = eithane-1,2-diamine)

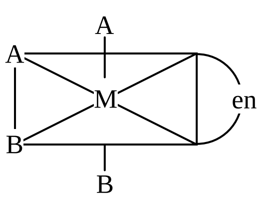
(1) 

(2) 

(3) 

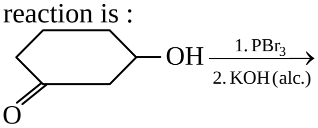
(4) 

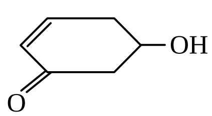
Ans. **[4]**

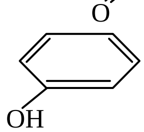
Sol. 

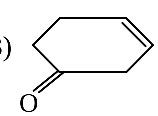
No plane of symmetry $\therefore$ optically active

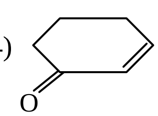
Q.4 The major product of the following reaction is :



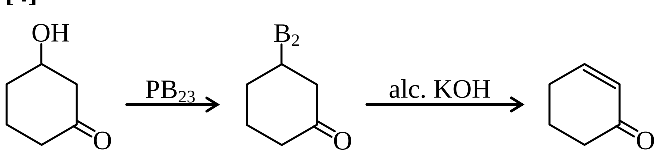
(1) 

(2) 

(3) 

(4) 

Ans. **[4]**

Sol. 

Q.5 The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is :

(1) $NO_2 < NO < N_2O_3 < N_2O$ (2) $N_2O < NO < N_2O_3 < NO_2$

(3) $NO_2 < N_2O_3 < NO < N_2O$ (4) $N_2O < N_2O_3 < NO < NO_2$

Ans. **[2]**

Sol. order of oxidation state –

$$\overset{+1}{N}_2O < \overset{+2}{N}O < \overset{+3}{N}_2O_3 < \overset{+4}{N}O_2$$

Q.6 Magnesium powder burns in air to give :

(1) MgO only   (2) MgO and $Mg(NO_3)_2$

(3) MgO and Mg_3N_2 (4) $Mg(NO_3)_2$ and Mg_3N_2

Ans. **[3]**

Sol. Air contain both N_2 and O_2

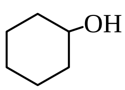
$Mg + O_2 \xrightarrow{\Delta} MgO$

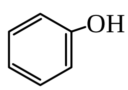
$Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$

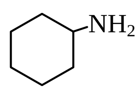
$\therefore$ Both MgO and Mg_3N_2 are formed.

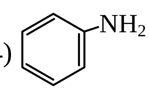
Q.7 The organic compound that gives following qualitative analysis is :

	Test	Inference
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2/water	Decolourization

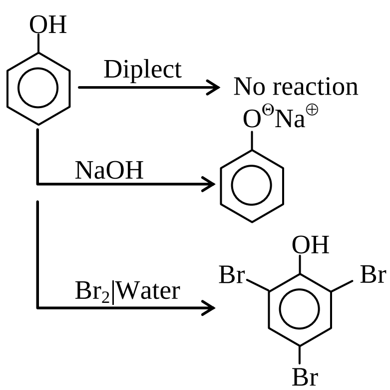
(1) 

(2) 

(3) 

(4) 

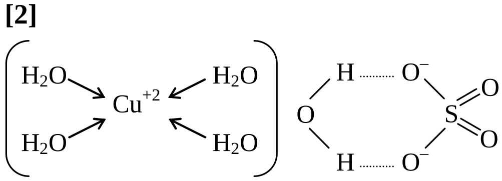
Ans. **[2]**

Sol. 

Q.8 The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is :

(1) 2   (2) 1   (3) 3   (4) 4

Ans. **[2]**

Sol. 

$\therefore$ 4, H_2O molecules are coordinated directly.

Q.9 Match the catalysts (Column I) with products (Column II).

Column I Catalyst	Column II Product
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) ethanal
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron Oxide	(iv) NH_3

(1) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii) (2) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)
(3) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i) (4) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

Ans. **[4]**

Sol. (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

Q.10 Excessive release of CO_2 into the atmosphere results in :
(1) depletion of ozone (2) polar vortex (3) global warming (4) formation of smog

Ans. **[3]**

Sol. CO_2 trap solar radiation $\therefore$ results in global warming.

Q.11 C_{60}, an allotrope of carbon contains :
(1) 16 hexagons and 16 pentagons. (2) 12 hexagons and 20 pentagons.
(3) 18 hexagons and 14 pentagons. (4) 20 hexagons and 12 pentagons.

Ans. **[4]**

Sol. C_{60} contains
20 hexagons and 12 pentagons

Q.12 Among the following, the molecule expected to be stabilized by anion formation is :
C_2, O_2, NO, F_2
(1) NO (2) O_2 (3) C_2 (4) F_2

Ans. **[3]**

Sol. The molecule in which B.O. ↑ with formation of anion, will get stabilize.

	F_2 ——	F_2^-		C_2 $\rightarrow$	C_2^{-1}	
BO	1	0.5	BO	2	2.5	
	NO ——	NO^-		O_2 $\rightarrow$	O_2^{-1}	Increase in BO results
BO	2.5	2.0	BO	2	1.5	in increase in stability

$\therefore$ Ans. C_2 (3)

Q.13 The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is :
(1) 4×10^{-4} (2) 4×10^{-2} (3) 16×10^{-4} (4) 6×10^{-2}

Ans. **[4]**

Sol. $\pi_{xy} = 4\pi_{BaCl_2}$
iCRT = 4(iCRT)
2C = 4 × 3(0.01)
C = 0.06 = 6×10^{-2} M

Q.14 The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is :

$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$,

$E^o = 2V$ at 298 K

(Faraday's constant, $F = 96000 \text{ C mol}^{-1}$)

(1) – 192 (2) 192 (3) – 384 (4) 384

Ans. **[3]**

Sol. $\Delta G = -nF E_{cell}$

$= -2(96000) \times 2$

$= -384000$ J/mole

$= -384$ KJ/mole

Q.15 The element having greatest difference between its first and second ionization energies, is :

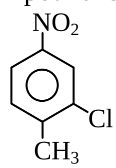
(1) Ca (2) Sc (3) K (4) Ba

Ans. **[3]**

Sol. K has $4s^1$ configuratioon $\therefore$ after removal of outermost e^- it acquire inert gas conf.

$\therefore$ Greatest jump in I.E is observes

Q.16 The correct IUPAC name of the following compound is :

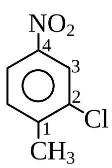


(1) 5-chloro-4-methyl-1-nitrobenzene (2) 2-chloro-1-methyl-4-nitrobenzene

(3) 2-methyl-5-nitro-1-chlorobenzene (4) 3-chloro-4-methyl-1- nitrobenzene

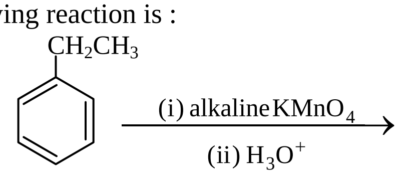
Ans. **[2]**

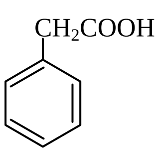
Sol.

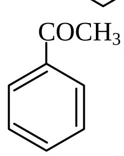


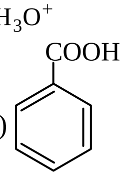
2-Chloro-1-methyl-4-nitrobenzene

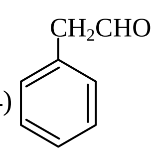
Q.17 The major product of the following reaction is :



(1) 

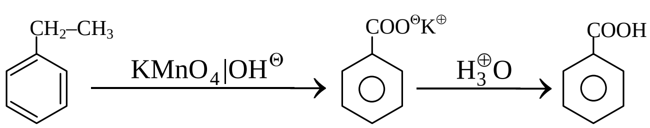
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



Q.18 The aerosol is a kind of colloid in which :

(1) gas is dispersed in liquid (2) solid is dispersed in gas

(3) liquid is dispersed in water (4) gas is dispersed in solid

Ans. **[2]**

Sol. Aerosol = solid is dispersed in gas

Q.19 The ore that contains the metal in the form of fluoride is :

(1) cryolite (2) magnetite (3) malachite (4) sphalerite

Ans. **[1]**

Sol. Cryolite = Na_3AlF_6 As magnetite = Fe_3O_4

Contain fluoride Malachite = $CuCO_3 . Cu(OH)_2$ Sphalerite = Zns

Q.20 The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are :

(1) d_{z^2} and d_{xz} (2) d_{yz} and d_{z^2} (3) d_{xz} and d_{yz} (4) $d_{x^2-y^2}$ and d_{xy}

Ans. **[3]**

Sol. $[Cr(H_2O)_6]^{+3}$ has sp^3d^2 hybridization orbitals involved are S, P_x, P_y, P_z, $d_{x^2-y^2}$ and d_{z^2}.

None of the options are correct.

Q.21 Consider the van der Waals constants, a and b, for the following gases.

Gas	Ar	Ne	Kr	Xe
$a/(atm\ dm^6\ mol^{-2})$	1.3	0.2	5.1	4.1
$b/(10^{-2}\ dm^3\ mol^{-1})$	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(1) Xe (2) Ne (3) Kr (4) Ar

Ans. **[3]**

Sol. $T_C = \dfrac{8a}{27Rb}$

$T_C \propto \dfrac{a}{b}$

Kr has highest ratio or $\dfrac{a}{b}$

Q.22 For a reaction,

$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$; identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures.

(1) 14 g of N_2 + 4g of H_2 (2) 28 g of N_2 + 6g of H_2

(3) 56 g of N_2 + 10g of H_2 (4) 35 g of N_2 + 8g of H_2

Ans. **[3]**

Sol. $N_2 + 3H_2 \longrightarrow 2NH_3$

1. 14gm 4gm

$\frac{1}{2}$ mole 2 mole

L.R $\underset{(N_2)}{\left[\dfrac{\frac{1}{2}}{1}\right]}$ $\left(\dfrac{2}{3}\right)$

2. 28 gm 6 gm

1 mole 3 mole

$\left(\frac{1}{1}\right)$ $\left(\frac{3}{3}\right)$

3. 56 gm 10 gm

2 mole 5 mole

$\left(\frac{2}{1}\right)$ $\left(\frac{5}{3}\right) = (1.66)\ H_2$

4. 35 gm 8 gm

$\frac{35}{28}$ mole 4 mole

$\frac{35}{28}$ $\frac{4}{3}$

$= 1.25$ 1.33

(N_2)

Q.23 For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman} / \Delta\bar{v}_{Balmer}$ is :

(1) 4 : 1 (2) 9 : 4 (3) 27 : 5 (4) 5 : 4

Ans. **[2]**

Sol. $\frac{\Delta\bar{v}_{Lyman}}{\Delta\bar{v}_{Balmar}}$ $\bar{v} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

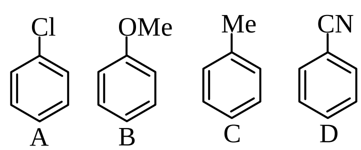
$= \frac{\Delta\bar{v}_{max} - \Delta\bar{v}_{min}}{\Delta\bar{v}_{max} - \Delta\bar{v}_{min}}$

$= \frac{\left[\frac{1}{1} - \frac{1}{\infty}\right] - \left[\frac{1}{1} - \frac{1}{4}\right]}{\left[\frac{1}{4} - \frac{1}{\infty}\right] - \left[\frac{1}{4} - \frac{1}{9}\right]}$

$= \frac{\frac{1}{4}}{\frac{1}{9}} = \frac{9}{4}$

$= 9 : 4$

Q.24 The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is :

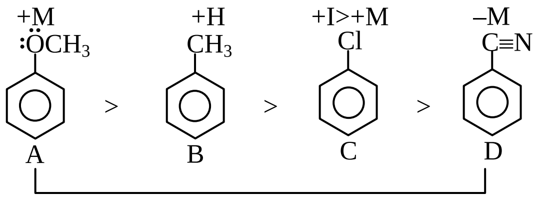


(1) D < A < C < B (2) B < C < A < D

(3) A < B < C < D (4) D < B < A < C

Ans. **[1]**

Sol.



Order of rate of ESR
Order of rate of activation

Q.25 Among the following, the set of parameters that represents path functions, is :

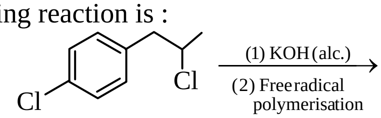
(A) q + w (B) q (C) w (D) H – TS

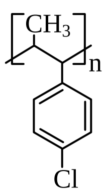
(A) (A), (B) and (C) (B) (A) and (B) (C) (A) and (D) (D) (B), (C) and (D)

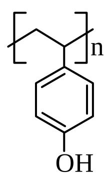
Ans. **[2]**

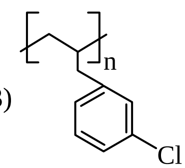
Sol.
(A) $q + w = \Delta E$ state function
(B) q = Path function
(C) w = Path function
(D) H – TS = G = State function

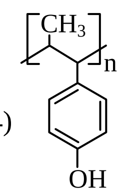
Q.26 The major product of the following reaction is :



(1) 

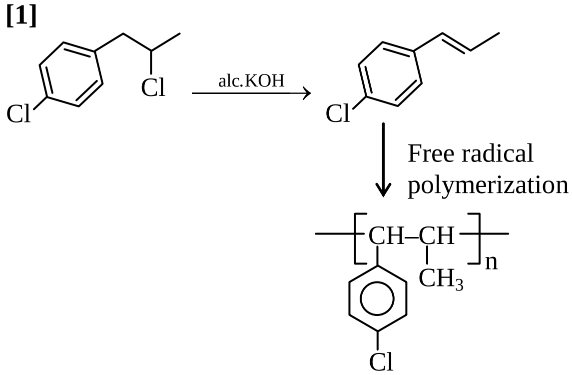
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.27 The major product of the following reaction is :

$$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl(1equiv.)}}$$

(1) $CH_3CD(Cl)CHD(I)$ (2) $CH_3CD_2CH(Cl)(I)$ (3) $CH_3CD(I)CHD(Cl)$ (4) $CH_3C(I)(Cl)CHD_2$

Ans. **[4]**

Sol. $CH_3–C \equiv C–H \xrightarrow{DCl} CH_3–C(Cl) = CH–D$

$\xrightarrow{DI}$ $CH_3–C(I)(Cl)–CHD_2$

Q.28 The major product of the following reaction is :

$$CH_3CH = CHCO_2CH_3 \xrightarrow{LiAlH_4}$$

(1) $CH_3CH_2CH_2CO_2CH_3$ (2) $CH_3CH_2CH_2CH_2OH$

(3) $CH_3CH = CHCH_2OH$ (4) $CH_3CH_2CH_2CHO$

Ans. **[3]**

Sol. $CH_3–CH = CH – \underset{\underset{O}{||}}{C} – O – CH_3$

$\downarrow$ LiA|H$_4$ (Reduction)

$CH_3–CH = CH – CH_2 – OH$

(1º-Alcohol)

Q.29 Which of the following statements is not true about sucrose?

(1) On hydrolysis, it produces glucose and fructose

(2) It is also named as invert sugar. The glycosidic linkage is present

(3) between C_1 of α-glucose and C_1 of β-fructose

(4) It is a non reducing sugar

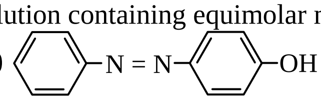
Ans. **[3]**

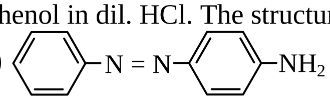
Sol. Sucrose $\xrightarrow{+H_2O}$ α-D-Glucose + β-D-Fructose

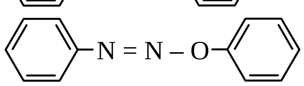
In β-D-Fructose $\rightarrow$ C-2 in involved

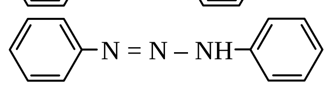
In β-D-Glucose $\rightarrow$ C-1 in involved

Q.30 Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0ºC. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is

(1) 

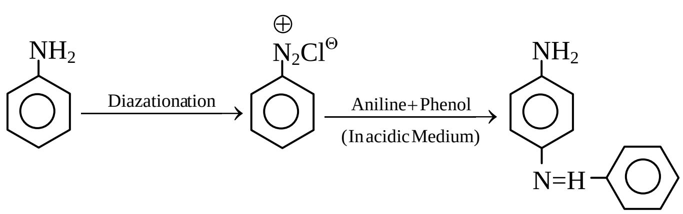
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



JEE Main Online Exam 2019

Questions & Solutions

9th April 2019 | Shift - I

MATHEMATICS

Q.1 Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to :

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ (3) $3\hat{i} - 9\hat{j} - 5\hat{k}$ (4) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

Ans. **[2]**

Sol. $\vec{\beta_1}$ is parallel to $\vec{\alpha}$

$\therefore \vec{\beta_1} = \lambda \vec{\alpha}$

$\Rightarrow \vec{\beta_1} = \lambda(3\hat{i} + \hat{j})$

Given that $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$

$\Rightarrow \vec{\beta_2} = \vec{\beta_1} - \vec{\beta}$

$\Rightarrow \vec{\beta_2} = \lambda(3\hat{i} + \hat{j}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$\Rightarrow \vec{\beta_2} = \hat{i}(3\lambda - 2) + \hat{j}(\lambda + 1) - 3\hat{k}$

Also given that $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$

$\therefore \vec{\beta_2} \cdot \vec{\alpha}$

$\Rightarrow 3(3\lambda - 2) + (\lambda + 1) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

So, $\vec{\beta_1} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$ and $\vec{\beta_2} = \frac{-1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$$\vec{\beta_1} \times \vec{\beta_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3/2 & 1/2 & 0 \\ 1/2 & 3/2 & -3 \end{vmatrix} = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$$

Q.2 Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :

(1) $\left\{\frac{1}{3}, 1\right\}$ (2) $\left\{\frac{1}{3}, -1\right\}$ (3) $\left\{-\frac{1}{3}, -1\right\}$ (4) $\left\{-\frac{1}{3}, 1\right\}$

Ans. [4]

Sol. $y = f(x) = x^3 - x^2 - 2x$

slope of tangent $\frac{dy}{dx} = f'(x) = 3x^2 - 2x - 2$

This tangent is parallel to line segment joining points (1, f(1)) and (–1, f(–1))

$\therefore\ m_1 = m_2$

$\Rightarrow\ 3x^2 - 2x - 2 = \frac{f(-1) - f(1)}{-1-1}$

$\Rightarrow\ 3x^2 - 2x - 2 = \frac{(-1-1+2)-(1-1-2)}{-2}$

$\Rightarrow 3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (3x + 1)(x - 1) = 0$

$\Rightarrow\ x = -\frac{1}{3}, 1$

Q.3 Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(1) π (2) $\frac{13\pi}{6}$ (3) 2π (4) $\frac{5\pi}{3}$

Ans. [3]

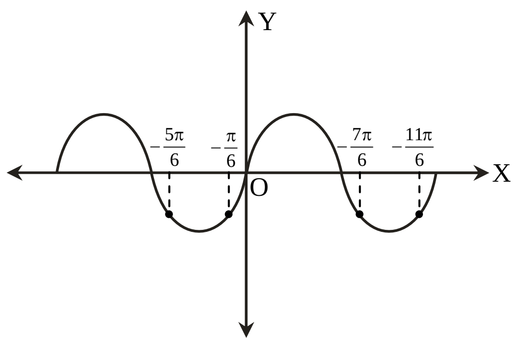
Sol. $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow\ \sin\theta = -\frac{1}{2}$



in $\theta \in [-2\pi, 2\pi]$

$\Rightarrow\ \theta = -\frac{5\pi}{6}, \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum of all roots $= \frac{-5\pi - \pi + 7\pi + 11\pi}{6} = 2\pi$

Q.4 For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is :

(1) $\sim p \vee \sim q$ (2) $p \leftrightarrow q$ (3) $p \wedge q$ (4) $\sim p \wedge \sim q$

Ans. [4]

Sol. $\sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q$

Q.5 The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with $y(1) = 1$, is :

(1) $y = \frac{x^3}{5} + \frac{1}{5x^2}$ (2) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (3) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$ (4) $y = \frac{x^2}{4} + \frac{3}{4x^2}$

Ans. **[4]**

Sol. $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$

$\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$

I.F. = $e^{\int \frac{2}{x}dx} = e^{\log_e x^2} = x^2$

$\therefore$ Solution is

$\Rightarrow yx^2 = \int x^2 \cdot x\,dx$

$\Rightarrow yx^2 = \frac{x^4}{4} + C$

at $y(1) = 1 \Rightarrow 1 = \frac{1}{4} + C$

$\Rightarrow C = \frac{3}{4}$

$\Rightarrow yx^2 = \frac{x^4}{4} + \frac{3}{4}$

$\Rightarrow y = \frac{x^2}{4} + \frac{3}{4x^2}$

Q.6 If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

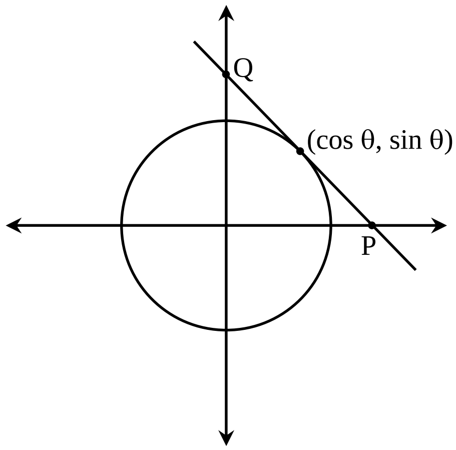
(1) $x^2 + y^2 - 2x^2y^2 = 0$ (2) $x^2 + y^2 - 4x^2y^2 = 0$ (3) $x^2 + y^2 - 16x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

Ans. **[2]**

Sol. Let the equation of tangent is $x \cos\theta + y \sin\theta = 1$

co-ordinates of P and Q are

$P\left(\frac{1}{\cos\theta}, 0\right)$ and $Q\left(0, \frac{1}{\sin\theta}\right)$



Let mid point of P and Q is (h, k)

so, $h = \dfrac{\frac{1}{\cos\theta} + 0}{2}$ and $k = \dfrac{0 + \frac{1}{\sin\theta}}{2}$

$\Rightarrow \cos\theta = \dfrac{1}{2h}$ and $\sin\theta = \dfrac{1}{2k}$

squaring and adding we get

$$\frac{1}{4h^2} + \frac{1}{4k^2} = 1$$

$\therefore$ locus $\dfrac{1}{4x^2} + \dfrac{1}{4y^2} = 1$

$\Rightarrow x^2 + y^2 - 4x^2 y^2 = 0$

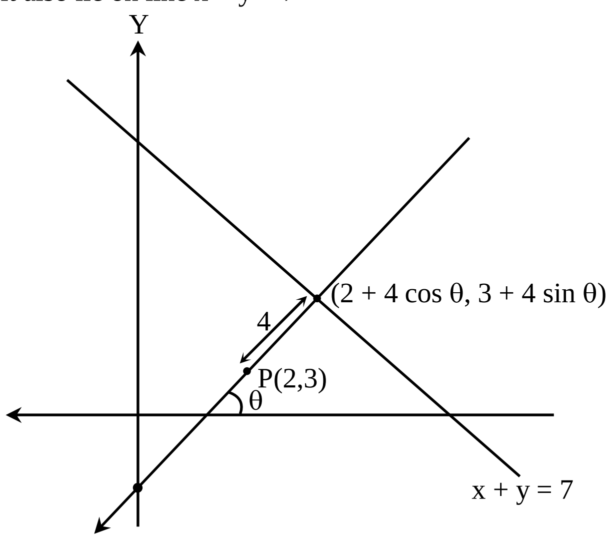
Q.7 Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is :

(1) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\dfrac{\sqrt{7}-1}{\sqrt{7}+1}$ (3) $\dfrac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\dfrac{1-\sqrt{5}}{1+\sqrt{5}}$

Ans. **[3]**

Sol. Let any point on the line is P(2 ± 4 cos θ, 3 ± 4 sin θ)

it also lie on line x + y = 7



$\therefore$ (2 ± 4 cos θ) + (3 ± 4 sin θ) = 7

$\Rightarrow (\sin\theta + \cos\theta) = \pm\dfrac{1}{2}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = \dfrac{1}{4}$

$\Rightarrow 1 + \sin 2\theta = \dfrac{1}{4}$

$\Rightarrow \sin 2\theta = -\dfrac{3}{4}$

$\Rightarrow \dfrac{2\tan\theta}{1+\tan^2\theta} = -\dfrac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \dfrac{-8 \pm 2\sqrt{7}}{6} = \dfrac{8-2\sqrt{7}}{6} = \dfrac{(1-\sqrt{7})^2}{1-7} = \dfrac{1-\sqrt{7}}{1+\sqrt{7}}$

Q.8 If f(x) is a non-zero polynomial of degree four, having local extreme points at x = – 1, 0, 1; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly :

(1) four rational numbers. (2) two irrational and two rational numbers.

(3) two irrational and one rational number. (4) four irrational numbers.

Ans. **[3]**

Sol. Four degree polynomial function f(x) have local extreme points at x = –1, 0, 1

$\therefore f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + K$

Now, f(x) = f(0)

$\Rightarrow \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + K = K$

$\Rightarrow \frac{x^4}{4} - \frac{x^2}{2} = 0$

$\Rightarrow x = 0, \pm\sqrt{2}$

Two irrational and one rational number.

Q.9 A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m + n = 68 (2) m = n = 78 (3) m = n = 68 (4) n = m – 8

Ans. **[2]**

Sol. Given : (8 males, 5 females)

Committee to be selected = 11 members

m = no. of ways the committee is formed with at least 6 males.

$\Rightarrow$ (6M, 5F) or (7M, 4F) or (8M, 3F)

$= {}^8C_6 \times {}^5C_5 + {}^8C_7 \times {}^5C_4 + {}^8C_8 \times {}^5C_3 = 78$

n = no. of ways the committee is formed with atleast 3 female

$\Rightarrow$ (8M, 3F) or (7M, 4F) or (6M, 5F)

$= {}^8C_8 \times {}^5C_3 + {}^8C_7 \times {}^5C_4 + {}^8C_6 \times {}^5C_5$

= 10 + 40 + 28 = 78

$\Rightarrow$ m = n = 78

Q.10 The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\,dx$ is equal to : (Here C is a constant of integration)

(1) $-3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$ (3) $3\tan^{-1/3}x + C$ (4) $-3\cot^{-1/3}x + C$

Ans. **[1]**

Sol. $I = \int \sec^{2/3} x \operatorname{cosec}^{4/3} x\,dx$

$I = \int \frac{dx}{(\sin x)^{4/3}(\cos x)^{2/3}}dx$

$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} \cdot \cos^2 x}$

$$I = \int \frac{\sec^2 x \ dx}{(\tan x)^{4/3}}$$

put tan x = t $\Rightarrow \sec^2 x \ dx = dt$

$$I = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + C$$

$$I = \frac{-3}{(\tan x)^{1/3}} + C$$

Q.11 Let p, q $\in$ R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $p^2 - 4q + 12 = 0$ (2) $q^2 - 4p - 16 = 0$ (3) $q^2 + 4p + 14 = 0$ (4) $p^2 - 4q - 12 = 0$

Ans. **[4]**

Sol. If one root of equation $x^2 + px + q = 0$ is $2 - \sqrt{3}$

then other root will be $2 + \sqrt{3}$

$\therefore$ equation $x^2 - 4x + 1 = 0$

$\Rightarrow p = -4$ and $q = 1$

$\Rightarrow p^2 - 4q - 12 = 0$

Q.12 Let f(x) = 15 – |x – 10|; x $\in$ R. Then the set of all values of x, at which the function, g(x) = f(f(x)) is not differentiable, is :

(1) {10, 15} (2) {5, 10, 15} (3) {10} (4) {5, 10, 15, 20}

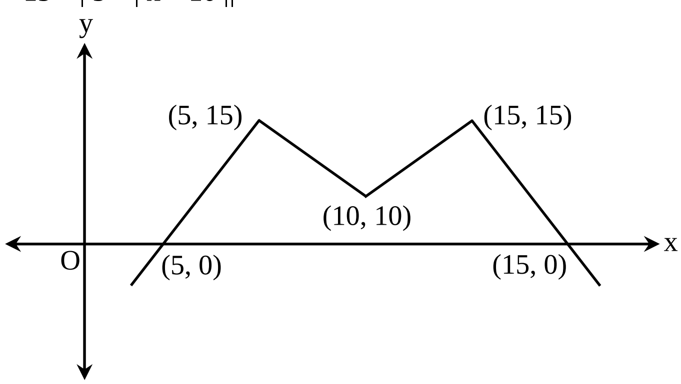
Ans. **[2]**

Sol. f(x) = 15 – |x – 10|

g(x) = f [f(x)] = 15 – | f(x) – 10 |

= 15 – | 15 – | x – 10 | – 10 |

= 15 – | 5 – | x – 10 ||



$\therefore$ g(x) is not differentiable at x = 5, 10, 15

Q.13 Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies f(x + y) = f(x)f(y) for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is :

(1) 2 (2) 3 (3) 16 (4) 4

Ans. **[2]**

Sol. Given f(1) = 2 and f(x + y) = f(x) · f(y)

at x = 1, y = 1 $\Rightarrow$ f(2) = f(1) · f(1) = 2^2

x = 2, y = 1 $\Rightarrow$ f(3) = f(2) · f(1) = 2^3

…………………

…………………

$f(n) = 2^n$

Now $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$

$\Rightarrow f(a + 1) + f(a + 2) + \ldots + f(a + 10) = 16(2^{10} - 1)$

$\Rightarrow 2^{a+1} + 2^{a+2} + \ldots + 2^{a+10} = 16(2^{10} - 1)$

$\Rightarrow 2^a [2^1 + 2^2 + \ldots + 2^{10}] = 16(2^{10} - 1)$

$\Rightarrow 2^a \left[\frac{2(2^{10}-1)}{2-1}\right] = 16(2^{10}-1)$

$\Rightarrow 2^{a+1} = 16$

$\Rightarrow a = 3$

Q.14 If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to :

(1) $\sqrt{6}$ (2) $2\sqrt{\frac{10}{3}}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $2\sqrt{6}$

Ans. **[4]**

Sol. $\text{S.D.} = \sqrt{\frac{1}{n}\sum x_i^2 - (\overline{x})^2}$

Now mean $\overline{x} = \frac{-1+0+1+k}{4} = \frac{k}{4}$

Given that S.D. = $\sqrt{5}$

$\Rightarrow \sqrt{5} = \sqrt{\frac{1}{4}(1+0+1+k^2) - \frac{k^2}{16}}$

$\Rightarrow 5 = \frac{2+k^2}{4} - \frac{k^2}{16}$

$\Rightarrow 80 = 8 + 4k^2 - k^2$

$\Rightarrow 3k^2 = 72 \quad \Rightarrow k^2 = 24$

$\Rightarrow k = \pm 2\sqrt{6}$

$k = 2\sqrt{6}$ ($\because$ k > 0)

Q.15 If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(1) $\frac{3}{\sqrt{5}}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{\sqrt{15}}{2}$

Ans. **[2]**

Sol. Equation of normal of hyperbola in slope form is $y = mx \pm \frac{m(a^2+b^2)}{\sqrt{a^2 - b^2m^2}}$

$\therefore \quad 7\sqrt{3} = \frac{42m}{\sqrt{24-18m^2}}$

$\Rightarrow 72 - 54\ m^2 = 36\ m^2$

$\Rightarrow 72 = 90\ m^2$

$\Rightarrow\ m^2 = \frac{72}{90} = \frac{4}{5}$

$\Rightarrow\ m = \pm\frac{2}{\sqrt{5}}$

$m = \frac{2}{\sqrt{5}}$

Q.16 If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is :

(1) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\13 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

Ans. **[4]**

Sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+....+n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow 1 + 2 + 3 + \ldots\ldots + (n - 1) = 78$

$\Rightarrow \frac{n(n-2)}{2} = 78$

$\Rightarrow n = 13$

Now inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ i.e. $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$ is $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ Ans.

Q.17 If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by $f(x) = \begin{cases}\frac{\sqrt{2}\cos x - 1}{\cot x - 1} & x \neq \frac{\pi}{4}\\ k, & x = \frac{\pi}{4}\end{cases}$ is continuous, then k is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) 1 (3) $\frac{1}{2}$ (4) 2

Ans. **[3]**

Sol. $\lim_{x\to\frac{\pi}{4}}\left(\frac{\sqrt{2}\cos x - 1}{\cot x - 1}\right) = f\left(\frac{\pi}{4}\right)$

$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$

using L-Hospital Rule

$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}(-\sin x)}{-\operatorname{cosec}^2 x} = k$

$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = k$

$\Rightarrow k = \frac{1}{2}$ Ans.

Q.18 Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(1) $\frac{25}{192}$ (2) $\frac{25}{32}$ (3) $\frac{7}{32}$ (4) $\frac{1}{192}$

Ans. **[2]**

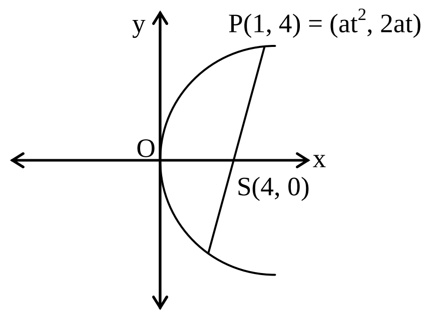
Sol. Let four persons are A, B, C, D.
Probability of Hitting target = 1 – (None of four person Hit the target)
$= 1 - P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C}) \cdot P(\overline{D})$
$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{7}{8}$
$= \frac{25}{32}$ Ans.

Q.19 If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

(1) 24 (2) 20 (3) 25 (4) 22

Ans. **[3]**

Sol. Parabola $y^2 = 16x$
$\{4a = 16 \Rightarrow a = 4\}$



One end $(at^2, 2at) = (1, 4)$
$\Rightarrow 2at = 4$
$\Rightarrow 2(4)\, t = 4$
$\Rightarrow t = 1/2$

Length of focal chord $= a\left(t + \frac{1}{t}\right)^2 = 4\left(\frac{1}{2} + 2\right)^2 = 25$ Ans.

Q.20 Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to :

(1) $y(y^2 - 3)$ (2) y^3 (3) $y(y^2 - 1)$ (4) $y^3 - 1$

Ans. **[2]**

Sol. Roots of eq^n. $x^2 + x + 1 = 0$ are α and β
$\alpha, \beta = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$
$\Rightarrow \alpha = \omega, \beta = \omega^2$ (complex cube root of unity)

$$\Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = \begin{vmatrix} y & y & y \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix} \quad (\because\ 1 + \omega + \omega^2 = 0)$$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$\Delta = y(y^2)$

$\Delta = y^3$ Ans.

Q.21 All the points in the set S = $\left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\}$ ($i = \sqrt{-1}$) lie on a :

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is –1.

(4) circle whose radius is $\sqrt{2}$.

Ans. **[1]**

Sol. Let $\frac{\alpha+i}{\alpha-i} = z$

$\Rightarrow \left|\frac{\alpha+i}{\alpha-i}\right| = |z|$

$\Rightarrow |z| = 1$

$\Rightarrow$ Circle of radius = 1

Q.22 The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is :

(1) $\frac{\pi-2}{8}$ (2) $\frac{\pi-1}{2}$ (3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-2}{4}$

Ans. **[3]**

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ …(1)

$$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos x + \sin x} dx$ …(2)

Adding (1) & (2) we get

$$\Rightarrow 2I = \int_0^{\pi/2} \left(\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\right) dx$$

$\Rightarrow 2I = \int_0^{\pi/2} \frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)}{(\sin x + \cos x)} dx$

$\Rightarrow 2I = \int_0^{\pi/2} (1 - \sin x \cos x) dx$

$\Rightarrow 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin 2x\right) dx$

$\Rightarrow 2I = \left(x + \frac{\cos 2x}{4}\right)_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \left(\frac{\pi}{2} - \frac{1}{4}\right) - \left(\frac{1}{4}\right) = \frac{\pi}{2} - \frac{1}{2}$

$\Rightarrow I = \left(\frac{\pi - 1}{4}\right)$

Q.23 The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is :

(1) $\frac{3}{2}(1 + \cos 20^\circ)$ (2) 3/2 (3) 3/4 (4) $\frac{3}{4} + \cos 20^\circ$

Ans. **[3]**

Sol. $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$

$= \frac{1}{2}[2\cos^2 10^\circ - 2\cos 10^\circ \cos 50^\circ + 2\cos^2 50^\circ]$

$= \frac{1}{2}[(1 + \cos 20^\circ) - (\cos 60^\circ + \cos 40^\circ) + (1 + \cos 100^\circ)]$

$= \frac{1}{2}[2 - \cos 60^\circ + \cos 20^\circ + (\cos 100^\circ - \cos 40^\circ)]$

$= \frac{1}{2}[2 - \frac{1}{2} + \cos 20^\circ + 2\sin 70^\circ \sin(-30^\circ)]$

$= \frac{1}{2}[\frac{3}{2} + \cos 20^\circ - \sin 70^\circ]$

$= \frac{1}{2}[\frac{3}{2} + \cos 20^\circ - \sin(90^\circ - 20^\circ)]$

$= \frac{3}{4}$ Ans.

Q.24 Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :

(1) (50, 50 + 46A) (2) (50, 50 + 45A) (3) (A, 50 + 45A) (4) (A, 50 + 46A)

Ans. **[4]**

Sol. $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$T_n = 50n + \left(\frac{n(n-7)}{2}\right)A - 50(n-1) - \left(\frac{(n-1)(n-8)}{2}\right)A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

= 50 + A(n – 4)
Now, d = $T_n - T_{n-1}$
= 50 + A(n – 4) – 50 – A(n – 5)
= A
and T_{50} = 50 + 46A
(d, A_{50}) = (A, 50 + 46A) Ans.

Q.25 If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is :

(1) $\sqrt{5}/2$ (2) 7/2 (3) $2\sqrt{5}$ (4) 9/2

Ans. **[4]**

Sol. Line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ = k (say)
any point on this line P(2k + 1, 3k – 1, 4k + 2)
This point P lies on plane x + 2y + 3z = 15
$\therefore$ (2k + 1) + 2(3k – 1) + 3(4k + 2) = 15
$\Rightarrow$ 20k + 5 = 15
$\Rightarrow$ 20k = 10
$\Rightarrow$ k = 1/2 $\therefore$ $P\left(2, \frac{1}{2}, 4\right)$
Distance of P from origin is
$= \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$ Ans.

Q.26 A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point :

(1) ($\sqrt{2}$, –1, 4) (2) (–$\sqrt{2}$, 1, –4) (3) (–$\sqrt{2}$, –1, –4) (4) ($\sqrt{2}$, 1, 4)

Ans. **[4]**

Sol. Let ax + by + cz = 1 be the eq^n. of plane
it passed through (0, –1, 0) and (0, 0, 1)
$\Rightarrow$ b = – 1 and c = 1
other plane is y – z + 5 = 0
Given that angle b/w them is $\frac{\pi}{4}$
$\cos\theta = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right|$
$\Rightarrow \frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{a^2+1+1}\sqrt{0+1+1}}$
$\Rightarrow a^2 + 2 = 4$
$\Rightarrow a = \pm\sqrt{2}$
$\therefore$ eq^n. of plane $\pm\sqrt{2}$ x – y + z = 1
Now for –ve sign
$-\sqrt{2}(\sqrt{2}) - 1 + 4 = 1$
$\therefore$ ($\sqrt{2}$, 1, 4) satisfy the eq^n. of plane.

Q.27 If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, – 5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve?

(1) (2, – 2) (2) (– 2, 2) (3) (– 2, 1) (4) (2, – 1)

Ans. **[1]**

Sol. $y = x^3 + ax - b$

(1, – 5) lies on curve

$\therefore -5 = 1 + a - b$

$\Rightarrow a - b = -6$ …(1)

$\frac{dy}{dx} = 3x^2 + a$

Slope of tangent at (1, – 5)

$\Rightarrow \frac{dy}{dx} = 3 + a$

This tangent is perpendicular to $-x + y + 4 = 0$

$\therefore (3 + a)(1) = -1$

$\Rightarrow a = -4$ …(2)

By (1) & (2) a = – 4, b = 2

So, eq^n. of curve $y = x^3 - 4x - 2$

(2, – 2) lies on this curve

Q.28 If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8^2 (2) 8^3 (3) 8 (4) 8^{-2}

Ans. **[1]**

Sol. $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ (x > 0)

$\Rightarrow T_4 = 20 \times 8^7$

$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3 \left(x^{\log_8 x}\right)^3 = 20 \times 8^7$

$\Rightarrow \frac{160}{x^3} x^{3\log_8 x} = 20 \times 8^7$

$\Rightarrow x^{3\log_8 x - 3} = 8^6$

$\Rightarrow x^{\log_2 x - 3} = 8^6 = 2^{18}$

$\Rightarrow \log_2\left(x^{\log_2 x - 3}\right) = \log_2 2^{18}$

$\Rightarrow (\log_2 x - 3)(\log_2 x) = 18$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow t = 6, -3$

$\Rightarrow \log_2 x = 6 \quad \Rightarrow x = 2^6 = 8^2$

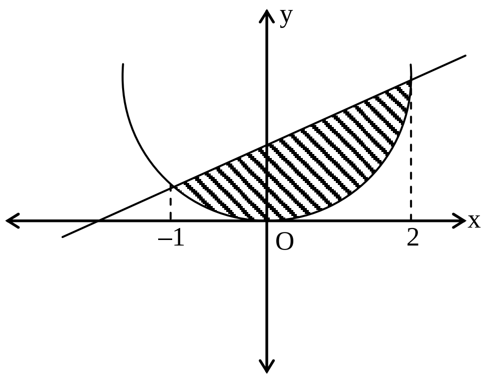
$\Rightarrow \log_2 x = -3 \quad \Rightarrow x = 2^{-3} = 1/8$

Q.29 The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is :

(1) $\frac{9}{2}$ (2) $\frac{31}{6}$ (3) $\frac{10}{3}$ (4) $\frac{13}{6}$

Ans. **[1]**

Sol.



$x^2 \le y \le x + 2$

$x^2 = y;\ y = x + 2$

$\Rightarrow x^2 = x + 2$

$\Rightarrow x = 2, -1$

So, area = $\int_{-1}^{2} \{(x+2) - x^2\}dx = \frac{9}{2}$

Q.30 If the function f : R – {1, – 1} → A defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to :

(1) R – {– 1} (2) R – [– 1, 0) (3) R – (– 1, 0) (4) [0, ∞)

Ans. **[2]**

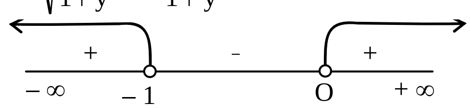
Sol. $y = \frac{x^2}{1-x^2}$

$\Rightarrow y - x^2y = x^2$

$\Rightarrow x^2 = \frac{y}{1+y}$

$\Rightarrow x = \sqrt{\frac{y}{1+y}} \Rightarrow \frac{y}{1+y} \ge 0$

$\Rightarrow$



Range of y is R – [–1, 0)

For surjective function codomain = Range

∴ A is R – [–1, 0)