

Conditional Identities Involving the Angles Of a Triangle

Exercise 16

Q. 1. If $A + B + C = \pi$, prove that

$$\sin 2A + \sin 2B - \sin 2C = 4\cos A \cos B \sin C$$

Answer : $= \sin 2A + \sin 2B - \sin 2C$

$$= 2 \sin (B + C) \cos A + 2 \sin (A + C) \cos B - 2 \sin (A + B) \cos C$$

Using formula, $\sin (A + B) = \sin A \cos B + \cos A \sin B$

$$= \sin 2A + \sin 2B - \sin 2C$$

Using formula

$$\sin 2A = 2\sin A \cos A$$

$$= 2\sin A \cos A + 2\sin B \cos B - 2\sin C \cos C$$

Since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And $\sin(\pi - A) = \sin A$

$$= 2\sin(B + C)\cos A + 2\sin(A + C)\cos B - 2\sin(A + B)\cos C$$

$$= 2(\sin B \cos C + \cos B \sin C) \cos A + 2(\sin A \cos C + \cos A \sin C)\cos B - 2(\sin A \cos B + \cos A \sin B) \cos C$$

$$= 2\cos A \sin B \cos C + 2\cos A \cos B \sin C + 2\sin A \cos B \cos C + 2\cos A \cos B \sin C - 2\sin A \cos B \cos C - 2\cos A \sin B \cos C$$

$$= 2\cos A \cos B \sin C + 2\cos A \cos B \sin C$$

$$= 4\cos A \cos B \sin C$$

$$= \text{R.H.S}$$

Q. 2. If $A + B + C = \pi$, prove that

$$\cos 2A - \cos 2B - \cos 2C = -1 + 4 \cos A \sin B \sin C$$

$$\text{Answer} : = \cos 2A - (\cos 2B + \cos 2C)$$

Using formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \cos 2A - \left\{2\cos\left(\frac{2B+2C}{2}\right)\cos\left(\frac{2B-2C}{2}\right)\right\}$$

$$= \cos 2A - \{2\cos(B+C)\cos(B-C)\}$$

$$\text{Since } A + B + C = \pi$$

$$\rightarrow B + C = 180 - A$$

$$= \cos 2A - \{2\cos(\pi - A)\cos(B-C)\}$$

$$\text{And } \cos(\pi - A) = -\cos A$$

$$= \cos 2A - \{-2\cos A \cos(B-C)\}$$

$$= \cos 2A + 2\cos A \cos(B-C)$$

$$\text{Using } \cos 2A = 2\cos^2 A - 1$$

$$= 2\cos^2 A - 1 + 2\cos A \cos(B-C)$$

$$= 2\cos A \{\cos A + \cos(B-C)\} - 1$$

$$\text{Using, } \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= 2\cos A \left\{2\cos\left(\frac{A+B-C}{2}\right)\cos\left(\frac{A+C-B}{2}\right)\right\} - 1$$

$$= 2\cos A \left\{2\cos\left(\frac{\pi-C-C}{2}\right)\cos\left(\frac{\pi-B-B}{2}\right)\right\} - 1$$

$$\text{As, } \cos\left(\frac{\pi}{2} - A\right) = \sin A$$

$$= 2\cos A \left\{2\cos\left(\frac{\pi}{2} - \frac{2C}{2}\right)\cos\left(\frac{\pi}{2} - \frac{2B}{2}\right)\right\} - 1$$

$$= 2\cos A \{2\sin C \sin B\} - 1$$

$$= 4\cos A \sin B \sin C - 1$$

$$= \text{R.H.S}$$

Q. 3. If $A + B + C = \pi$, prove that

$$\cos 2A - \cos 2B + \cos 2C = 1 - 4\sin A \cos B \sin C$$

Answer : $= \cos 2A - \cos 2B + \cos 2C$

Using,

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$= \cos 2A - \left\{2\sin\left(\frac{2B+2C}{2}\right)\sin\left(\frac{2B-2C}{2}\right)\right\}$$

$$= \cos 2A - \{2\sin(B+C)\sin(B-C)\}$$

since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And $\sin(\pi - A) = \sin A$

$$= \cos 2A - \{2\sin(\pi - A)\sin(B-C)\}$$

$$= \cos 2A - \{2\sin A \sin(B-C)\}$$

$$= \cos 2A - 2\sin A \sin(B-C)$$

Using, $\cos 2A = 1 - 2\sin^2 A$

$$= -2\sin^2 A + 1 - 2\sin A \sin(B-C)$$

$$= -2\sin A \{\sin A + \sin(B-C)\} + 1$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= -2\sin A \left\{2\sin\left(\frac{A+B-C}{2}\right)\cos\left(\frac{A+C-B}{2}\right)\right\} + 1$$

$$= -2\sin A \left\{2\sin\left(\frac{\pi-C-C}{2}\right)\cos\left(\frac{\pi-B-B}{2}\right)\right\} + 1$$

$$= -2\sin A \left\{ 2\sin\left(\frac{\pi}{2} - \frac{2C}{2}\right) \cos\left(\frac{\pi}{2} - \frac{2B}{2}\right) \right\} + 1$$

$$\text{As, } \sin\left(\frac{\pi}{2} - A\right) = \cos A$$

$$= -2\sin A \{2\cos C \sin B\} + 1$$

$$= -4\sin A \cos B \sin C + 1$$

$$= \text{R.H.S}$$

Q. 4. If $A + B + C = \pi$, prove that

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$\text{Answer : } = \sin A + \sin B + \sin C$$

Using,

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$= \sin A + \left\{ 2\sin\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\text{Since } A + B + C = \pi$$

$$\rightarrow B + C = 180 - A$$

And,

$$\sin\left(\frac{\pi}{2} - A\right) = \cos A$$

$$= \sin A + \left\{ 2\sin\left(\frac{\pi-A}{2}\right) \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= \sin A + \left\{ 2\cos\left(\frac{A}{2}\right) \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\text{Using, } \sin 2A = 2\sin A \cos A$$

$$= 2\sin \frac{A}{2} \cos \frac{A}{2} + \left\{ 2\cos\left(\frac{A}{2}\right) \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2\cos\frac{A}{2}\left\{\sin\frac{A}{2} + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$\rightarrow B + C = 180 - A$$

And,

$$\sin\left(\frac{\pi}{2} - A\right) = \cos A$$

$$= 2\cos\frac{A}{2}\left\{\cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cos\frac{A}{2}\left\{2\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)\right\}$$

$$= 4\cos\frac{A}{2}\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$$

= R.H.S

Q. 5. If $A + B + C = \pi$, prove that

$$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Answer : $\cos A + \cos B + \cos C$

Using,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \cos A + \left\{2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}$$

since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And,

$$\cos\left(\frac{\pi}{2} - A\right) = \sin A$$

$$= \cos A + \left\{ 2 \cos \left(\frac{\pi - A}{2} \right) \cos \left(\frac{B - C}{2} \right) \right\}$$

$$= \cos A + \left\{ 2 \sin \left(\frac{A}{2} \right) \cos \left(\frac{B - C}{2} \right) \right\}$$

Using, $\cos 2A = 1 - 2\sin^2 A$

$$= 1 - 2\sin^2 \frac{A}{2} + \left\{ 2 \sin \left(\frac{A}{2} \right) \cos \left(\frac{B - C}{2} \right) \right\}$$

$$= 2 \sin \frac{A}{2} \left\{ -\sin \frac{A}{2} + \cos \left(\frac{B - C}{2} \right) \right\} + 1$$

$$= 2 \sin \frac{A}{2} \left\{ \cos \left(\frac{-B - C}{2} \right) + \cos \left(\frac{B - C}{2} \right) \right\} + 1$$

$$= 2 \sin \frac{A}{2} \left\{ 2 \cos \left(\frac{-C}{2} \right) \cos \left(\frac{-B}{2} \right) \right\} + 1$$

$$= 4 \sin \frac{A}{2} \cos \left(\frac{B}{2} \right) \cos \left(\frac{C}{2} \right) + 1$$

= R.H.S

Q. 6. If $A + B + C = \pi$, prove that

$$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

Answer : $\sin 2A + \sin 2B + \sin 2C$

Using,

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\sin 2A = 2 \sin A \cos A$$

$$= 2 \sin A \cos A + 2 \sin(B+C) \cos(B - C)$$

since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

$$= 2\sin A \cos A + 2\sin(\pi - A)\cos(B - C)$$

$$= 2\sin A \cos A + 2\sin A \cos(B - C)$$

$$= 2\sin A \{\cos A + \cos(B - C)\}$$

$$\text{(but } \cos A = \cos \{ 180 - (B + C) \} = -\cos(B + C)$$

$$\text{And now using } \cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{-A+B}{2}\right)$$

$$= 2\sin A \{2\sin B \sin C\}$$

$$= 4\sin A \sin B \sin C$$

$$= 32\sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{B}{2}\sin\frac{C}{2}\cos\frac{C}{2}$$

Now,

$$= \sin A + \sin B + \sin C$$

Using,

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \sin A + \left\{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= \sin A + \left\{2\sin\left(\frac{\pi-A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= \sin A + \left\{2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\sin\frac{A}{2}\cos\frac{A}{2} + \left\{2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cos\frac{A}{2}\left\{\sin\frac{A}{2} + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cos\frac{A}{2}\left\{\cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cos\frac{A}{2}\left\{2\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)\right\}$$

$$= 4\cos\frac{A}{2}\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$$

Therefore,

$$= \frac{32\sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{B}{2}\sin\frac{C}{2}\cos\frac{C}{2}}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$= 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

= R.H.S

Q. 7. If $A + B + C = \pi$, prove that

$$\sin(B + C - A) + \sin(C + A - B) - \sin(A + B - C) = 4\cos A \cos B \sin C$$

$$\text{Answer : } = \sin(B + C - A) + \sin(C + A - B) - \sin(A + B - C)$$

Using,

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= 2\sin C \cos(B-A) - \sin(A+B-C)$$

Since $A + B + C = \pi$

$$\rightarrow B + A = 180 - C$$

$$= 2\sin C \cos(B-A) - \sin(\pi - C - C)$$

$$= 2\sin C \cos(B-A) - \sin 2C$$

Since , $\sin 2A = 2\sin A \cos A$,

$$= 2\sin C \cos(B-A) - 2\sin C \cos C$$

$$= 2\sin C \{\cos(B-A) - \cos C\}$$

Using ,

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$= 2\sin C \left\{ 2\sin\left(\frac{B-A+C}{2}\right)\sin\left(\frac{C-B+A}{2}\right) \right\}$$

$$= 2\sin C \left\{ 2\sin\left(\frac{\pi-A-A}{2}\right)\sin\left(\frac{\pi-B-B}{2}\right) \right\}$$

$$= 4\cos A \cos B \sin C$$

$$= \text{R.H.S}$$

Q. 8. If $A + B + C = \pi$, prove that

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2$$

Answer :

$$= \frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B}$$

Taking L.C.M

$$= \frac{\cos A \sin A + \cos B \sin B + \cos C \sin C}{\sin B \sin C \sin A}$$

Multiplying and divide the above equation by 2, we get

$$= \frac{2\cos A \sin A + 2\cos B \sin B + 2\cos C \sin C}{2\sin B \sin C \sin A}$$

Since , $\sin 2A = 2\sin A \cos A$

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2\sin B \sin C \sin A}$$

Now,

$$= \sin 2A + \sin 2B + \sin 2C$$

$$= 2\sin A \cos A + 2\sin(B+C)\cos(B - C)$$

since $A + B + C = \pi$

$$\rightarrow B + A = 180 - C$$

$$= 2\sin A \cos A + 2\sin(\pi - A)\cos(B - C)$$

$$= 2\sin A \cos A + 2\sin A \cos(B - C)$$

$$= 2\sin A \{ \cos A + \cos(B - C) \}$$

$$\text{(but } \cos A = \cos \{ 180 - (B + C) \} = -\cos(B + C) \text{)}$$

$$\text{And now using } \cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{-A+B}{2}\right)$$

$$= 2\sin A \{ 2\sin B \sin C \}$$

$$= 4\sin A \sin B \sin C$$

Putting the above value in the equation, we get

$$= \frac{4\sin A \sin B \sin C}{2\sin B \sin C \sin A}$$

$$= 2$$

$$= \text{R.H.S}$$

Q. 9. If $A + B + C = \pi$, prove that

$$\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$$

$$\text{Answer : } = \cos^2 A + \cos^2 B + \cos^2 C$$

Using formula,

$$\frac{1 + \cos 2A}{2} = \cos^2 A$$

$$= \frac{1 + \cos 2A}{2} + \frac{1 + \cos 2B}{2} + \frac{1 + \cos 2C}{2}$$

$$= \frac{1 + \cos 2A + 1 + \cos 2B + 1 + \cos 2C}{2}$$

$$= \frac{3 + \cos 2A + \cos 2B + \cos 2C}{2}$$

Using,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{3 + \cos 2A + 2\cos\left(\frac{2B+2C}{2}\right)\cos\left(\frac{2B-2C}{2}\right)}{2}$$

$$= \frac{3 + \cos 2A + 2\cos(B+C)\cos(B-C)}{2}$$

Using, since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And, $\cos(\pi - A) = -\cos A$

$$= \frac{3 + \cos 2A + 2\cos(\pi - A)\cos(B-C)}{2}$$

$$= \frac{3 + \cos 2A - 2\cos(A)\cos(B-C)}{2}$$

Using $\cos 2A = 2\cos^2 A - 1$

$$= \frac{3 + 2\cos^2 A - 1 - 2\cos(A)\cos(B-C)}{2}$$

$$= \frac{2 + 2\cos^2 A - 2\cos(A)\cos(B-C)}{2}$$

$$= 1 + \cos^2 A - \cos A \cos(B-C)$$

$$= 1 + \cos A \{\cos A - \cos(B-C)\}$$

Using,

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$= 1 + \cos A \left(2\sin\left(\frac{A+B-C}{2}\right)\sin\left(\frac{B-C-A}{2}\right) \right)$$

Since, $A + B + C = \pi$

$$= 1 + \cos A \left(2\sin\left(\frac{\pi-C-C}{2}\right)\sin\left(\frac{B-(\pi-B)}{2}\right) \right)$$

$$= 1 + \cos A \left(2\cos C \sin\left(\frac{B}{2} - \frac{\pi}{2}\right) \right)$$

$$= 1 - 2\cos A \cos C \cos B$$

$$= \text{R.H.S}$$

Q. 10. If $A + B + C = \pi$, prove that

$$\sin^2 A - \sin^2 B + \sin^2 C = 2\sin A \cos B \sin C$$

Answer : $= \sin^2 A - \sin^2 B + \sin^2 C$

Using formula,

$$\begin{aligned} \frac{1 - \cos 2A}{2} &= \sin^2 A \\ &= \frac{1 - \cos 2A}{2} - \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2} \\ &= \frac{1 - \cos 2A - 1 + \cos 2B + 1 - \cos 2C}{2} \\ &= \frac{1 - \cos 2A + \cos 2B - \cos 2C}{2} \end{aligned}$$

Using ,

$$\begin{aligned} \cos A - \cos B &= 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right) \\ &= \frac{1 - \cos 2A + \left\{2\sin\left(\frac{2B+2C}{2}\right)\sin\left(\frac{2C-2B}{2}\right)\right\}}{2} \\ &= \frac{1 - \cos 2A + 2\sin(B+C)\sin(C-B)}{2} \end{aligned}$$

since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And $\sin(\pi - A) = \sin A$

$$\begin{aligned} &= \frac{1 - \cos 2A + 2\sin(\pi - A)\sin(C - B)}{2} \\ &= \frac{1 - \cos 2A + 2\sin A \sin(C - B)}{2} \end{aligned}$$

Using , $\cos 2A = 1 - 2\sin^2 A$

$$= \frac{1 - 1 + 2\sin^2 A + 2\sin A \sin(C-B)}{2}$$

$$= \frac{2\sin A \{\sin A + \sin(C-B)\}}{2}$$

$$= \frac{2\sin A \{\sin A + \sin(C-B)\}}{2}$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\sin A \{2\sin\left(\frac{A+C-B}{2}\right) \cos\left(\frac{A-C+B}{2}\right)\}}{2}$$

$$= \frac{1 - 2\sin A \{2\sin\left(\frac{\pi-B-B}{2}\right) \cos\left(\frac{\pi-C-C}{2}\right)\}}{2}$$

$$= \frac{2\sin A \{2\sin\left(\frac{\pi}{2} - \frac{2B}{2}\right) \cos\left(\frac{\pi}{2} - \frac{2C}{2}\right)\}}{2}$$

$$\text{As, } \sin\left(\frac{\pi}{2} - A\right) = \cos A$$

$$= \frac{2\sin A \{2\cos B \sin C\}}{2}$$

$$= 2\sin A \cos B \sin C$$

$$= \text{R.H.S}$$

Q. 11. If $A + B + C = \pi$, prove that

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

Answer :

$$= \sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2}$$

Using formula ,

$$\frac{1 - \cos 2A}{2} = \sin^2 A$$

$$\begin{aligned}
&= \frac{1 - \cos A}{2} + \frac{1 - \cos B}{2} + \frac{1 - \cos C}{2} \\
&= \frac{1 - \cos A + 1 - \cos B + 1 - \cos C}{2} \\
&= \frac{3 - \cos A - \cos B - \cos C}{2}
\end{aligned}$$

Using,

$$\begin{aligned}
\cos A + \cos B &= 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\
&= \frac{3 - \cos A - \{2 \cos\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right)\}}{2} \\
&= \frac{3 - \cos A - 2 \cos\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right)}{2}
\end{aligned}$$

Using , since $A + B + C = \pi$

$$\rightarrow B + C = 180 - A$$

And, $\cos(\pi - A) = -\cos A$

$$\begin{aligned}
&= \frac{3 - \cos A - 2 \cos\left(\frac{\pi - A}{2}\right) \cos\left(\frac{B-C}{2}\right)}{2} \\
&= \frac{3 - \cos A - 2 \sin\left(\frac{A}{2}\right) \cos\left(\frac{B-C}{2}\right)}{2}
\end{aligned}$$

Using , $\cos 2A = 1 - 2 \sin^2 A$

$$\begin{aligned}
&= \frac{3 - 1 + 2 \sin^2 \frac{A}{2} - 2 \sin \frac{A}{2} \cos \frac{B-C}{2}}{2} \\
&= \frac{2 - 2 \sin \frac{A}{2} \left\{ \sin \frac{A}{2} - \cos \left(\frac{B-C}{2}\right) \right\}}{2}
\end{aligned}$$

Since $A + B + C = \pi$

And Using ,

$$\cos A - \cos B = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{B-A}{2}\right)$$

$$= \frac{2 - 2 \sin \frac{A}{2} \left\{ 2 \sin\left(\frac{B+C}{2} + \frac{B-C}{2}\right) \sin\left(\frac{B+C}{2} - \frac{B-C}{2}\right) \right\}}{2}$$

$$= \frac{2 - 2 \sin \frac{A}{2} \left\{ 2 \sin\left(\frac{2B}{2}\right) \sin\left(\frac{2C}{2}\right) \right\}}{2}$$

Using, since $A + B + C = \pi$

$$= \frac{2 - 2 \sin \frac{A}{2} \left\{ 2 \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right) \right\}}{2}$$

$$= 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

= R.H.S

Q. 12. If $A + B + C = \pi$, prove that

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$$

Answer : $\tan 2A + \tan 2B + \tan 2C$

Since $A + B + C = \pi$

$$A + B = \pi - C$$

$$2A + 2B = 2\pi - 2C$$

$$\tan (2A+2B) = \tan (2\pi - 2C)$$

Since $\tan (2\pi - C) = -\tan C$

$$\tan (2A + 2B) = -\tan 2C$$

Now using formula,

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$$

$$\tan 2A + \tan 2B = -\tan 2C + \tan 2C \tan 2B \tan 2A$$

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$$

= R.H.S