CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

9th January 2019 | Shift - I

PHYSICS

Q.1 A Mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$, is close to :

(1) 0.32 (2) 3.16 (3) 2.24 (4) 0.45

Ans. **[2]**

Sol. $\because\ V_{rms} = \sqrt{\frac{3RT}{M}}$

for same temperature

$V_{rms} \propto \sqrt{\frac{1}{M}}$

$\Rightarrow \frac{V_{rms}(\text{He})}{V_{rms}(\text{Ar})} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

Q.2 Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio :

(1) 25 : 9 (2) 4 : 1 (3) 5 : 3 (4) 16 : 9

Ans. **[1]**

Sol. $\frac{I_{max.}}{I_{min.}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = 16 = 4^2$

$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = 4$

$\sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$

$\Rightarrow 5\sqrt{I_2} = 3\sqrt{I_1}$

$\frac{I_1}{I_2} = \frac{25}{9}$

Q.3 A sample of radioactive material A, that has an activity of 10 mCi(1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively :

(1) 10 days and 40 days (2) 20 days and 10 days (3) 5 days and 10 dyas (4) 20 days and 5 days

Ans. **[4]**

Sol. $\because$ A = λN

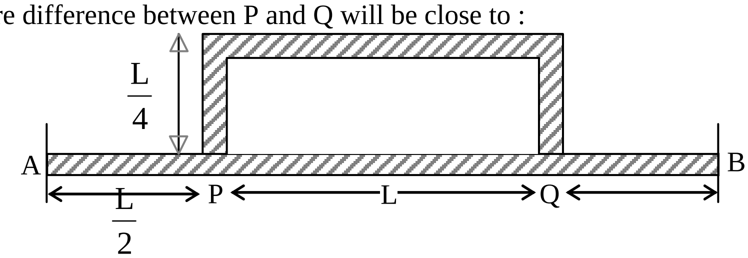
$10 = \lambda_A(2N_0)$; $20 = \lambda_B N_0$

$$\Rightarrow \frac{1}{4} = \frac{\lambda_A}{\lambda_B} = \frac{\left(\lambda_{1/2}\right)_B}{\left(\lambda_{1/2}\right)_A}$$

Ratio of half life for A and B is 4 : 1

So, option (4) is correct.

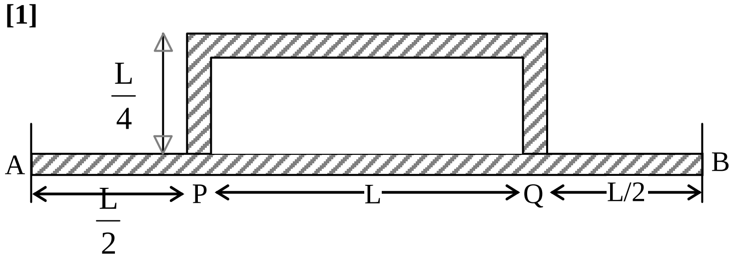
Q.4 Temperature difference of 120ºC is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to :



(1) 45 ºC (2) 75 ºC (3) 35 ºC (4) 60 ºC

Ans. **[1]**

Sol.

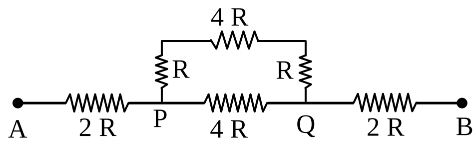


$T_A - T_B = 120$ ºC

$T_P - T_Q = ?$

$\because R_{th} = \frac{1}{K}\frac{L}{A}$

$R_{th} = \frac{1}{KA}\left(\frac{L}{4}\right)$ = R (Let us say)



$(R_{eq.})_{AB} = 6.4$ R

$i_{th} = \frac{120}{6.4R}$; $V_{PQ} = \frac{6R}{10R} \cdot \frac{120R}{6.4R} = \frac{72}{6.4R}$ = 45 ºC

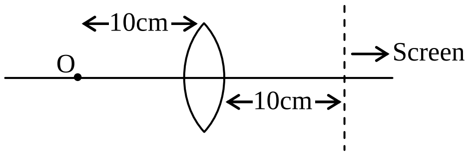
Q.5 A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

(1) 0.55 cm towards the lens (2) 0

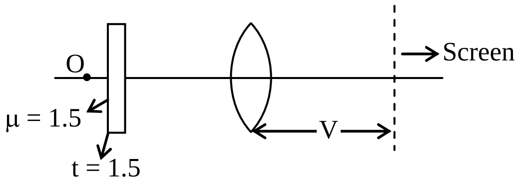
(3) 0.55 cm away from the lens (4) 1.1 cm away from the lens

Ans. **[3]**

Sol.



U = – 10
V = 10
$\Rightarrow$ f = 5 cm



Shift due to slab = $t\left(1-\frac{1}{\mu}\right)$

$= 1.5\left(1-\frac{1}{1.5}\right)$

$= 1.5 \times \frac{1}{3} = 0.5$ cm

Now for lens
u = – 9.5 cm
f = 5 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$

$\frac{1}{v} = \frac{1}{5} - \frac{1}{9.5}$

$v = \frac{5\times 9.5}{4.5}$

$v = \frac{95}{9}$ cm

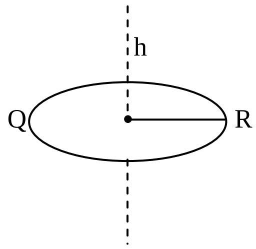
v = 10.55 cm
So, screen is shifted away from lens by 0.55 cm.

Q.6 For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :

(1) $R\sqrt{2}$ (2) R (3) $\frac{R}{\sqrt{5}}$ (4) $\frac{R}{\sqrt{2}}$

Ans. **[4]**

Sol.



$E = \dfrac{kQx}{(R^2 + x^2)^{3/2}}$

For E_{max}

$\dfrac{dE}{dx} = 0$

On solving we get $x = \dfrac{R}{\sqrt{2}}$

So, option (4) is correct.

Q.7 Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s is close to (Take charge of electron to be = 1.6×10^{-19}C)

(1) 0.02 (2) 0.2 (3) 3 (4) 2

Ans. **[1]**

Sol. Since $I = n\, e\, A\, V_d$

$1.5 = 9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}$ V

$\Rightarrow V = \dfrac{1.5}{9 \times 1.6 \times 5 \times 10^3}$ m/s

$V = \dfrac{0.3}{9 \times 1.6}$ mm/s

$V = \dfrac{1}{48}$ mm/s

$\simeq 0.02$ mm/s

Option (1) is correct.

Q.8 A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\vec{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be

(1) $18.9 \times 10^{-8}\ \hat{k}T$ (2) $2.1 \times 10^{-8}\ \hat{k}T$ (3) $18.9 \times 10^{8}\ \hat{k}T$ (4) $6.3 \times 10^{-8}\ \hat{k}T$

Ans. **[2]**

Sol. $\vec{E} = 6.3\,\vec{j}$ V/m

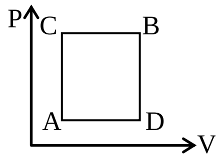
$\because\ |\vec{E}| = |\vec{B}| \cdot C$

$\Rightarrow |\vec{B}| = \dfrac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}$ T

Direction of wave is in direction of $\vec{E} \times \vec{B}$

So, option (2) is correct

Q.9 A gas can be taken from A to B via two different processes ACB and ADB.

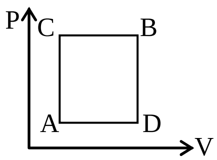


When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

(1) 40 J (2) 20 J (3) 100 J (4) 80 J

Ans. **[1]**

Sol.



For Path ACB

Q = 60 J

W = 30 J

$\because$ Q = U + W

$\Rightarrow$ U = 30 J

Now, $U_{ACB} = U_{ADB} = 30$ J

For Path ADB :-

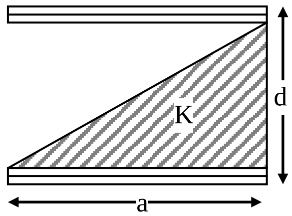
U = 30 J

W = 10 J

$\Rightarrow$ Q = U + W = 40 J

Option (1) is correct.

Q.10 A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d(d << a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is :

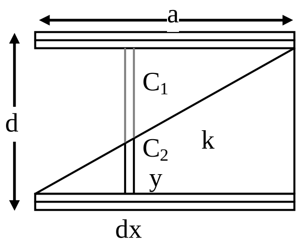


(1) $\frac{K \in_0 a^2}{2d(K+1)}$ (2) $\frac{K \in_0 a^2}{d(K-1)} \ln K$

(3) $\frac{K \in_0 a^2}{d} \ln K$ (4) $\frac{1}{2} \frac{K \in_0 a^2}{d}$

Ans. **[2]**

Sol.



$\frac{y}{x} = \frac{d}{a}$

$\Rightarrow y = \frac{d}{a}x$

$C_1 = \frac{\varepsilon_0 adx}{d-y}$; $C_2 = \frac{K\varepsilon_0 adx}{y}$

$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{K\varepsilon_0 adx}{Kd+(1-K)y}$

$\int_0^a \frac{K\varepsilon_0 adx}{Kd+(1-K)\frac{dx}{a}} = \frac{K\varepsilon_0 a^2 \ln K}{d(K-1)}$

Q.11 Surface of certain metal is first illuminated with light of wavelength λ_1 = 350 nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to : (Energy of photon = $\frac{1240}{\lambda(\text{in nm})}$ eV)

(1) 5.6 (2) 2.5 (3) 1.8 (4) 1.4

Ans. **[3]**

Sol. $\frac{V_1}{V_2} = \frac{2}{1}$

$\frac{k_1}{k_2} = \frac{4}{1}$

$k_1 = \frac{1240}{350} - W = 3.54 - W$

$k_2 = \frac{1240}{540} - W = 2.3 - W$

$\frac{k_1}{k_2} = \frac{3.5u - W}{2.3 - W} = \frac{4}{1}$

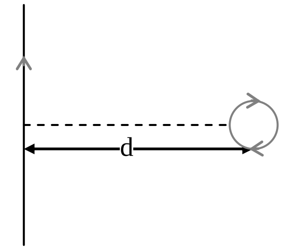
3.54 – W = 9.2 – uW

3W = 5.66

W $\simeq$ 1.8 eV

Option (3)

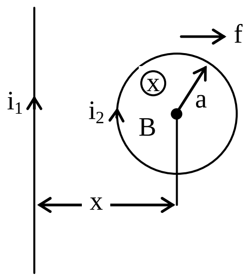
Q.12 An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d >> a). If the loop applies a force F on the wire then :



(1) $F \propto \left(\frac{a^2}{d^3}\right)$ (2) $F \propto \left(\frac{a}{d}\right)$ (3) F = 0 (4) $F \propto \left(\frac{a}{d}\right)^2$

Ans. **[4]**

Sol.



work done by magnetic force = fdx

fdx = – dU

$$f = -\frac{dU}{dx}$$

$$U = -\vec{M}\cdot\vec{B}$$

$$U = -\frac{I_2 \times \pi a^2 \times \mu_0 I_1}{2\pi x}\cos 0^\circ$$

$$\frac{dU}{dx} = \frac{\mu_0 I_1 I_2 \pi a^2}{2\pi}\left(\frac{1}{x^2}\right)$$

$$f = \frac{\mu_0 I_1 I_2 \pi a^2}{2\pi d^2} \text{ (put x = d)}$$

$$f \propto \frac{a^2}{d^2}$$

Q.13 A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to :

(1) $\frac{g}{20}$ (2) $\frac{g}{5}$ (3) $\frac{g}{10}$ (4) $\frac{g}{30}$

Ans. [2]

Sol. $V = \sqrt{\frac{Mg}{\mu}}$

$\frac{\Delta V}{V} = \frac{1}{2}\frac{\Delta g}{g}$

$\frac{0.5}{60} = \frac{1}{2}\frac{\left(\sqrt{a^2+g^2}-g\right)}{g}$

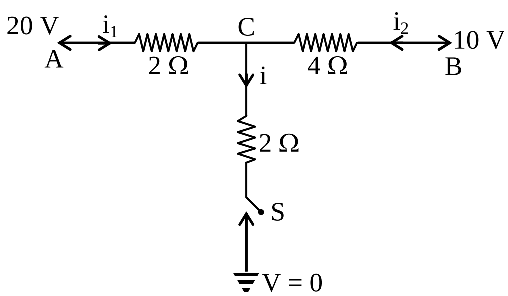
$\frac{1}{60} = 1 + \frac{1}{2}\frac{a^2}{g^2} - 1$

$\frac{1}{60} = \frac{a^2}{2g^2}$

$\Rightarrow a^2 = \frac{g^2}{30}$

$\Rightarrow a = \frac{g}{\sqrt{30}} \simeq \frac{g}{5}$

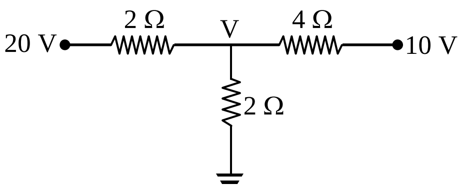
Q.14 When the switch S, in circuit shown, is closed, then the value of current i will be :



(1) 4 A (2) 5 A (3) 3 A (4) 2 A

Ans. [2]

Sol.



$\frac{V-20}{2} + \frac{V}{2} + \frac{V-10}{4} = 0$

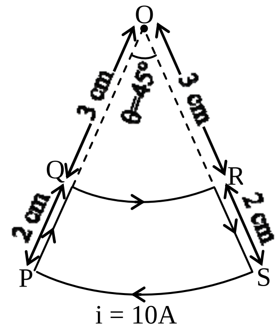
2V – 40 + 2V + V – 10 = 0

5V = 50

V = 10V

$I = \frac{V}{2} = 5A$

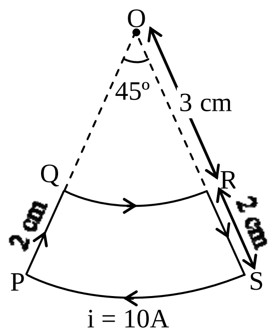
Q.15 A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to :



(1) 1.5×10^{-5} T (2) 1.0×10^{-5} T (3) 1.5×10^{-7} T (4) 1.0×10^{-7} T

Ans. **[2]**

Sol.



B due to wire QR : -

$$\mu_0 \frac{1}{8} \times \frac{10}{2\times(3\,\text{cm})}$$

B due to wire PS :-

$$\mu_0\left(\frac{1}{8}\right) \times \frac{10}{2(5\,\text{cm})}$$

$$B_{net} = \frac{\mu_0}{8}\,\frac{10}{2}\left(\frac{1}{3\times10^{-2}} - \frac{1}{5\times10^{-2}}\right)$$

$$= \frac{4\pi\times10^{-7}\times10}{16\times10^{-2}} \times \frac{2}{15}\ \text{T}$$

$$= \frac{\pi\times10^{-4}}{30}\ \text{T}$$

$$= \frac{3.14}{30} \times 10^{-4}\ \text{T}$$

$\simeq 10^{-5}$ T

Option (2) is correct.

Q.16 Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 $m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to :

(1) 0.4 Ωm (2) 2 Ωm (3) 4 Ωm (4) 0.2 Ωm

Ans. **[1]**

Sol. $\mu = \frac{V_d}{E}$

$I = n\,e\,A\,V_d$

$\Rightarrow \mu = \frac{I}{neAE}$

$\mu = \frac{I\cdot\ell}{neA(E\ell)} = \frac{I\ell}{neAV}$

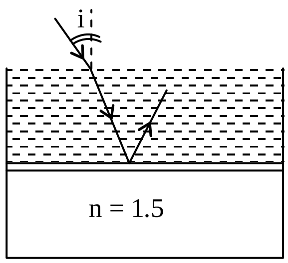
$\mu = \frac{I\ell}{neAI\cdot R}$

$\mu = \frac{\ell A}{neA\rho\ell} = \frac{1}{ne\rho}$

$\Rightarrow \rho = \frac{1}{ne\mu} = \frac{1}{10^{19}\times1.6\times10^{-19}\times1.6}$

$\rho = \frac{1}{1.6\times1.6} \simeq 0.4\ \Omega m$

Q.17 Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is :



(1) $\sqrt{\frac{5}{3}}$ (2) $\frac{4}{3}$ (3) $\frac{5}{\sqrt{3}}$ (4) $\frac{3}{\sqrt{5}}$

Ans. **[4]**

Sol. $\tan\theta = \frac{1.5}{\mu}$; $\sin\theta = \frac{1}{\mu}$

$\frac{1}{\sqrt{\mu^2-1}} = \frac{3}{2\mu}$

$4\mu^2 = 9(\mu^2 - 1)$

$5\mu^2 = 9$

$\mu = \frac{3}{\sqrt{5}}$

Q.18 If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :

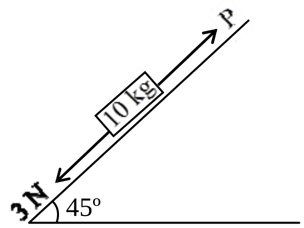
(1) $\frac{L}{m}$ (2) $\frac{L}{2m}$ (3) $\frac{4L}{m}$ (4) $\frac{2L}{m}$

Ans. **[2]**

Sol. $\frac{dA}{dt} = \frac{L}{2m}$

Option (2)

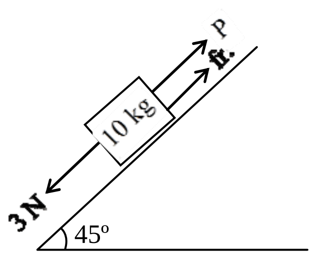
Q.19 A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward?



(1) 32 N (2) 25 N (3) 23 N (4) 18 N

Ans. **[1]**

Sol.



P + fr = 3 + mg sin 45º

$P = 3 + \frac{100}{\sqrt{2}} - 0.6\,\frac{100}{\sqrt{2}}$

$P = 3 + \frac{40}{\sqrt{2}}$

$\simeq 32$ N

Option (1) is correct.

Q.20 A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/ºC. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔTK. Young's modulus, Y, for this metal is :

(1) $\frac{F}{A\alpha\Delta T}$ (2) $\frac{F}{2A\alpha\Delta T}$ (3) $\frac{2F}{A\alpha\Delta T}$ (4) $\frac{F}{A\alpha(\Delta T - 273)}$

Ans. **[1]**

Sol. $\frac{F}{A} = Y\frac{\Delta L}{L}$

$\frac{F}{A} = Y\alpha\Delta T$

$Y = \frac{F}{A\alpha\Delta T}$

Q.21 A particle is moving with a velocity $\vec{v} = K(y\,\vec{i} + x\,\vec{j})$, where K is a constant. The general equation for its path is :

(1) $y = x^2 +$ constant (2) $y^2 = x^2 +$ constant (3) $y^2 = x +$ constant (4) $xy =$ constant

Ans. **[2]**

Sol. $\frac{dx}{dt} = y$

$\frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x}$

$xdx = ydy$

$y^2 = x^2 + C$

Q.22 A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3}m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 0 ms is close to :

(1) 6 mC (2) 14 mC (3) 7 mC (4) 21 mC

Ans. **[Bonus]**

Sol. $\Delta q = \frac{\Delta\phi}{R}$

$B = 0.4 \sin 50\,\pi t$

$\phi = B\cdot A = (0.4 \sin 50\,\pi t)\, 3.5 \times 10^{-3}$

$= 1.4 \times 10^{-3} \sin 50\,\pi t$

At $t = 0$; $\phi = 0$

At $t = 10 \times 10^{-3}$ s

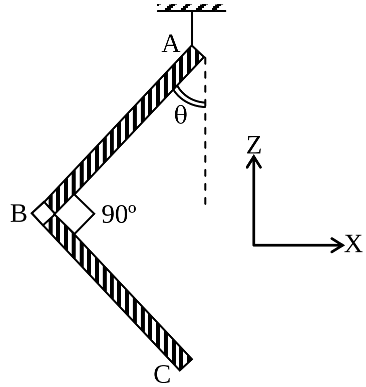
$\phi = 1.4 \times 10^{-3} \sin (50\,\pi \cdot 10^{-2})$

$= 1.4 \times 10^{-3}$

$\Rightarrow \Delta q = \frac{1.4\times10^{-3}}{10} = 0.14 \times 10^{-3}$

$= 0.14$ mC.

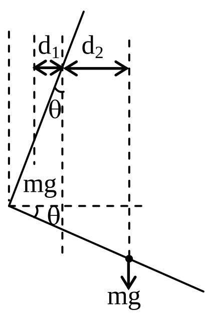
Q.23 An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then :



(1) $\tan\theta = \frac{1}{3}$ (2) $\tan\theta = \frac{1}{2}$ (3) $\tan\theta = \frac{2}{\sqrt{3}}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

Ans. **[1]**

Sol.



$$mg\frac{a}{2}d_1 = mgd_2$$

$$mg\frac{a}{2}\sin\theta = mg(\frac{a}{2}\cos\theta - a\sin\theta)$$

$$\frac{\sin\theta}{2} = \frac{\cos\theta}{2} - \sin\theta$$

$$\frac{3}{2}\sin\theta = \frac{\cos\theta}{2}$$

$$\tan\theta = \frac{1}{3}$$

Q.24 A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is :

(1) 0.5 % (2) 2.5 %

(3) 2.0 % (4) 1.0 %

Ans. **[4]**

Sol. $R = \rho \frac{\ell}{A}$

$A\ell$ = constant

$\frac{\Delta A}{A} + \frac{\Delta \ell}{\ell} = 0$

$100 \times \frac{\Delta \ell}{\ell} = 0.5\ \%$

$\frac{\Delta A}{A} \times 100 = -0.5\%$

$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$

So, $\frac{\Delta R}{R} \times 100$

$= 0.5 - (-0.5)$

$= 1\ \%$

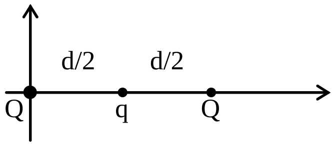
option (4)

Q.25 Three charges +Q, q, +Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by +Q, placed at x = 0, is zero, then value of q is :

(1) +Q/2 (2) –Q/4 (3) +Q/4 (4) –Q/2

Ans. **[2]**

Sol.



$\frac{kQq}{\left(\frac{d}{2}\right)^2} = \frac{kQ^2}{d^2}$

$q = \frac{Q}{4} \frac{d^2}{d^2}$

$q = -\frac{Q}{4}$

Option (2) is correct.

Q.26 A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is :

(1) 2600 A/m (2) 1200 A/m (3) 520 A/m (4) 285 A/m

Ans. **[1]**

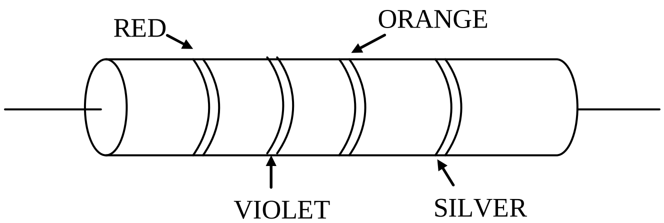
Sol. $B = \mu_0 ni$

$= \mu_0 \dfrac{100}{0.2} \times 5.2 = \mu_0 H$

H = 2600 A/m

Option (1) is correct.

Q.27 A resistance is shown in the figure. Its value and tolerance are given respectively by :



(1) 270 Ω, 10 % (2) 270 Ω, 5 % (3) 27 kΩ, 10 % (4) 27 kΩ, 20 %

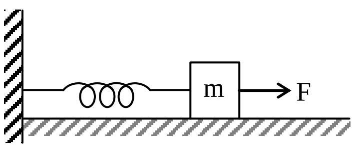
Ans. **[3]**

Sol. From colour code table

$R = 27 \times 10^3 \pm 10\%$

$R = 27\ k\Omega \pm 10\%$

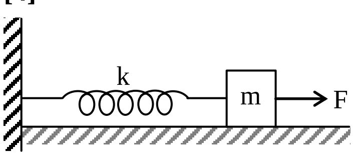
Q.28 A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is



(1) $\dfrac{2F}{\sqrt{mk}}$ (2) $\dfrac{\pi F}{\sqrt{mk}}$ (3) $\dfrac{F}{\pi\sqrt{mk}}$ (4) $\dfrac{F}{\sqrt{mk}}$

Ans. **[4]**

Sol.



For maximum speed F = kx

$x = \dfrac{f}{k}$

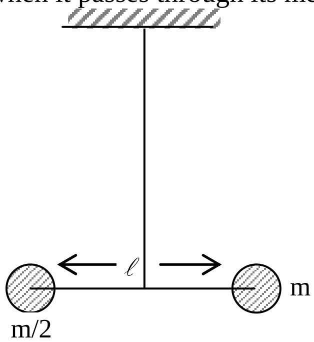
$Fx = \dfrac{1}{2}kx^2 + \dfrac{1}{2}mV^2$

$\dfrac{F^2}{k} = \dfrac{1}{2}\dfrac{kF^2}{k^2} + \dfrac{1}{2}mV^2$

$$\frac{F^2}{2k} = \frac{mV^2}{2}$$

$$V = \frac{F}{\sqrt{mk}}$$

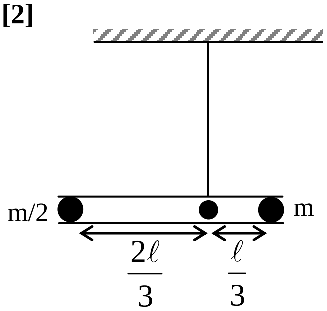
Q.29 Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length ℓ. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be :



(1) $\frac{2k\theta_0^2}{\ell}$ (2) $\frac{k\theta_0^2}{\ell}$ (3) $\frac{3k\theta_0^2}{\ell}$ (4) $\frac{k\theta_0^2}{2\ell}$

Ans. **[2]**

Sol.



$$\frac{1}{2}k\theta_0^2 = \frac{1}{2}I\omega^2$$

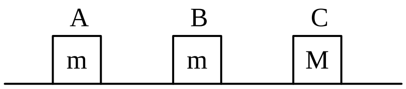
$$\frac{1}{2}k\theta_0^2 = \frac{1}{2}\left(\frac{m\ell^2}{9} + \frac{m}{2}\frac{4\ell^2}{9}\right)\omega^2$$

$$\omega^2 = m\frac{3k\theta^2}{m\ell^2}$$

$$T = m\omega^2\frac{\ell}{3} = m\frac{3k\theta^2}{m\ell^2}\cdot\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

Option (2) is correct.

Q.30 Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?



(1) 2 (2) 3 (3) 5 (4) 4

Ans. **[4]**

Sol.
$$\frac{P^2}{2m} - \frac{P^2}{2(2m+M)} = \frac{5}{6}\frac{P^2}{2m}$$
$$\frac{1}{m} - \frac{1}{(2m+M)} = \frac{5}{6m}$$
$$\frac{2m+M-m}{m(2m+M)} = \frac{5}{6m}$$
6(m + M) = 5(2m + M)
6m + 6M = 10m + 5M
4m = M
$$\frac{m}{M} = \frac{1}{4}$$
$$\Rightarrow \frac{M}{m} = 4$$
Option (4) is correct.

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

9th January 2019 | Shift - I

CHEMISTRY

Q.1 In general, the properties that decrease and increase down a group in the periodic table, respectively, are -

(1) electronegativity and atomic radius.
(2) electronegativity and electron gain enthalpy.
(3) atomic radius and electronegativity.
(4) electron gain enthalpy and electronegativity.

Ans. **[1]**

Sol. Down the group electronegativity of Atoms decreases and Atomic radius increases.

Q.2 The major product of following reaction is :

$$R{-}C{\equiv}N \xrightarrow[\text{(2) } H_2O]{\text{(1) } AlH(i{-}Bu)_2} ?$$

(1) RCHO (2)RCOOH (3) $RCONH_2$ (4) RCH_2NH_2

Ans. **[1]**

Sol. $$R{-}C{\equiv}N \xrightarrow[\text{DiBAl–H}]{\text{(1) } AlH(i{-}Bu)_2} R-\overset{\overset{\displaystyle O}{||}}{C}-H$$

DiBAL reduces cyanide, ester and acid halide upto aldehyde only.

Q.3 Which amongst the following is the strongest acid ?

(1) $CHBr_3$ (2) $CHCl_3$ (3) $CH(CN)_3$ (4) CHI_3

Ans. **[3]**

Sol. Acidic strength $\propto$ stability of Anion after loosing proton $\propto$ EWG – I, – M, –H

(1) $CHBr_3 \rightarrow \overset{\Theta}{C}Br_3$

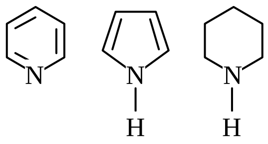
(2) $CHCl_3 \rightarrow \overset{\Theta}{C}Cl_3$

(3) $CH(CN)_3 \rightarrow N{\equiv}C-\overset{\Theta}{\underset{\underset{\displaystyle C\equiv N}{|}}{C}}-C\equiv N$ – ve charge stabilize by – M of three $C \equiv N$ group

(4) $CHI_3 \rightarrow \overset{\Theta}{C}I_3$

3 is most acidic

Q.4 Arrange the following amines in the decreasing order of basicity :



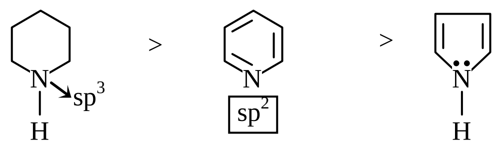
I II III

(1) I > III > II (2) III > I > II (3) I > II > III (4) III > II > I

Ans. **[2]**

Sol. Basic strength order

III > I > II



ℓ.p. of N do not participate in resonance	ℓ.p. of N do not participate in resonance but hybridization of N is sp^2 (more % s less basic)	ℓ.p. of N participate in resonance

Q.5 The ore that contains both iron and copper is -

(1) dolomite (2) malachite (3) copper pyrites (4) azurite

Ans. **[3]**

Sol.

Copper pyrities	$CuFeS_2$
Malachite	$Cu(OH)_2 . CuCO_3$
Azurite	$Cu(OH)_2 . 2\ CuCO_3$
Dolomite	$CaCO_3 . MgCO_3$

Q.6 The anodic half cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4^-$ = 303 g mol^{-1})

(1) 15.2 (2) 7.6 (3) 11.4 (4) 22.8

Ans. **[2]**

Sol. Anodic half cell : $Pb^{+2} + 2H_2O \rightarrow PbO_2 + 4H^+ + 2e^-$

$$\frac{n_{Pb^{+2}}}{1} = \frac{ne^-}{2}$$

$$\frac{n_{PbSO_4}}{1} = \frac{0.05}{2}$$

$$W_{PbSO_4} = \frac{0.05}{2} \times 303 = 7.6 \text{ g}$$

Q.7 Correct statements among a to d regarding silicones are -

(a) They are polymers with hydrophobic character.

(b) They are biocompatible.

(c) In general, they have high thermal stability and low dielectric strength.

(d) Usually, they are resistant to oxidation and used as greases.

(1) (a), (b) and (d) only (2) (a), (b) and (c) only (3) (a), (b), (c) and (d) (4) (a) and (b) only

Ans. **[1]**

Sol. These are the properties and uses of silicones.

Q.8 The following results were obtained during kinetic studies of the reaction :

$$2A + B \rightarrow \text{Products}$$

Experiment	**[A] (in mol L^{-1})**	**[B] (in mol L^{-1})**	**Initial Rate of reaction (in mol L^{-1} min^{-1})**
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is -

(1) 5 (2) 1 (3) 100 (4) 10

Ans. **[1]**

Sol. $6.93 \times 10^{-3} = k \times (0.1)^x (0.2)^y$

$6.93 \times 10^{-3} = k \times (0.1)^x (0.25)^y$

So y = 0

and $1.386 \times 10^{-2} = k (0.2)^x (0.30)^y$

$\frac{1}{2} = \left(\frac{1}{2}\right)^x \quad x = 1$

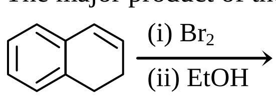
So $r = k \times (0.1) \times (0.2)^0$

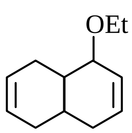
$6.93 \times 10^{-3} = k \times 0.1 \times (0.2)^0$

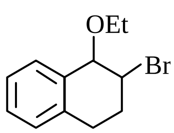
$k = 6.93 \times 10^{-2}$

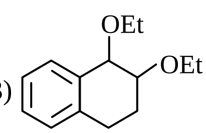
$t_{1/2} = \frac{0.693}{2k} = \frac{0.693}{0.693 \times 10^{-1} \times 2} = \frac{10}{2} = 5$

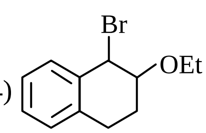
Q.9 The major product of the following reaction is :



(1) 

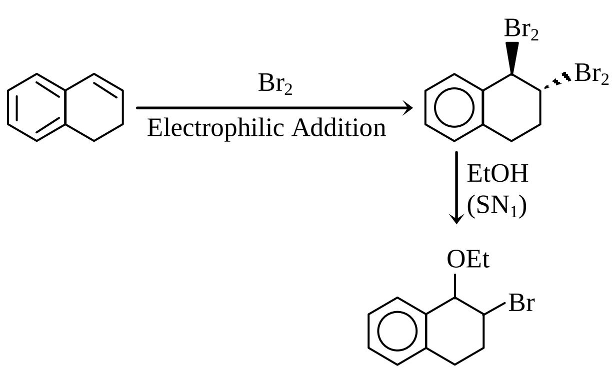
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Q.10 The one that is extensively used as a piezoelectric material is -

(1) mica (2) quartz (3) amorphous silica (4) tridymite

Ans. **[2]**

Sol. Quartz is used in Piezoelectric material (Fact)

Q.11 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in JK^{-1} mol^{-1}, x is -

(1) $\frac{2R}{4-R}$ (2) $\frac{4+R}{2R}$ (3) $\frac{4-R}{2R}$ (4) $\frac{2R}{4+R}$

Ans. **[3]**

Sol. PV = nRT

$200 \times 10 = (0.5 + x)\, R \times 1000$

$0.5 + x = \frac{2}{R}$

$x = \frac{4-R}{2R}$

Answer should be 3 but NTA has given answer 2. Which is not correct.

Q.12 For emission line of atomic hydrogen from $n_1 = 8$ to $n_f = n$, the plot of wave number ($\bar{\nu}$) against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is wave number unit)

(1) Linear with slope – R_H (2) Linear with slope R_H

(3) Non linear (4) Linear with intercept – R_H

Ans. **[2]**

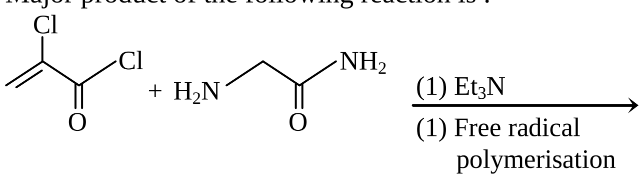
Sol. $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

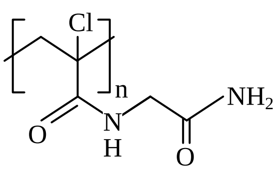
$\bar{\nu} = R_H\left(\frac{1}{n^2} - \frac{1}{8^2}\right)$

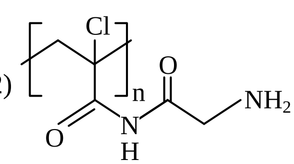
$\therefore$ Slope of the curve ($\bar{\nu}$ Vs $\frac{1}{n^2}$) is + R_H

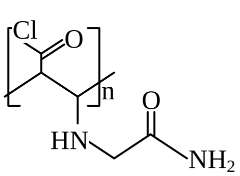
Answer should be 2 but NTA has given answer 1. Which is not correct.

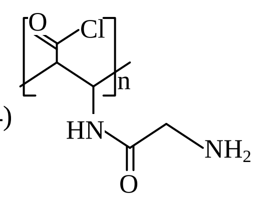
Q.13 Major product of the following reaction is :



(1) 

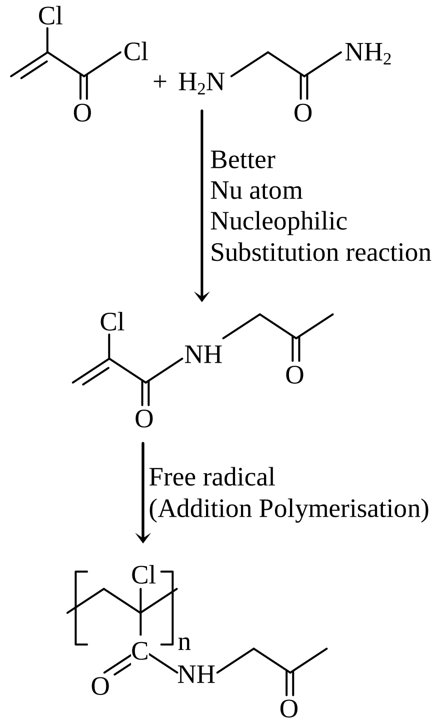
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.14 The correct decreasing order for acid strength is :

(1) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(2) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

(3) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(4) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

Ans. **[4]**

Sol. Acidic strength order $\propto$ EWG – M_1 – H_1 – I

In given example only –I of substituent is applicable

–I order – NO_2 > – C≡N >– F > –Cℓ

Acidic strength order = $NO_2–CH_2–COOH > NC–CH_2–COOH > F – CH_2 – COOH > Cl – CH_2 – COOH$

Q.15 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is : [pK_b of NH_4OH = 4.7].

(1) 5.2 (2) 9.4 (3) 9.0 (4) 5.0

Ans. **[3]**

Sol.

	H_2SO_4	+ $2NH_4OH$	$\rightarrow$ $(NH_4)_2SO_4 + 2H_2O$
t = 0	2millmole	6 millimole	
t = t	0	2 millimole	2 millimole

$\therefore \quad P^{OH} = P^{kb} + \log \dfrac{[Salt]}{[Base]}$

$P^{OH} = 4.7 + \log \dfrac{4/50}{2/50}$

$\therefore \quad P^{OH} = 4.7 + 0.3 = 5$

$\therefore \quad P^{H} = 9.$

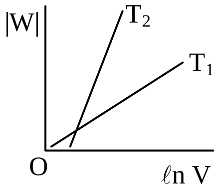
Q.16 The alkaline earth metal nitrate that does not crystallize with water molecules, is -

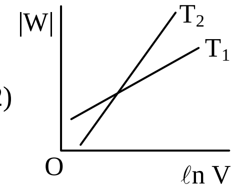
(1) $Ba(NO_3)_2$ (2) $Mg(NO_3)_2$ (3) $Ca(NO_3)_2$ (4) $Sr(NO_3)_2$

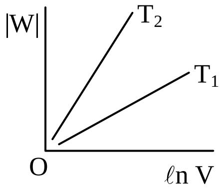
Ans. **[1]**

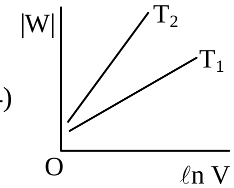
Sol. (1) Compounds of Barium does not contains water of crystallization due to large size of Barium ion.

Q.17 Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (W) on the final volume (V) is -

(1)


(2)


(3)


(4)


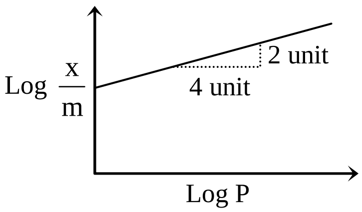
Ans. **[1]**

Sol. $|W| = nRT \ln \frac{v_2}{v_1}$

$|W| = nRT\ \ell nV_2 - nRT\ \ell nV_1$

$\therefore$ Intercept is negative and intercept of curve 2 is more negative.

Q.18 Adsorption of a gas follows Freundlich adsorption isotherm. IN the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure P. $\frac{x}{m}$ is proportional to :



(1) P (2) $P^{1/2}$ (3) P^2 (4) $P^{1/4}$

Ans. **[2]**

Sol. $\frac{x}{m} = k(P)^{1/n}$

$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$

Slope = $\frac{1}{n}$

From graph, slope = $\frac{1}{2}$

$\Rightarrow$ $n = 2 \Rightarrow \frac{x}{m} \propto (P)^{1/2}$

Q.19 The increasing order of pKa of the following amino acids in aqueous solution is :

Gly Asp Lys Arg

(1) Asp < Gly < Lys < Arg (2) Arg < Lys < Gly < Asp

(3) Gly < Asp < Arg < Lys (4) Asp < Gly < Arg < Lys

Ans. **[1]**

Sol. $\underset{\text{Ka}}{\text{Acidic strength}} \propto \dfrac{1}{\text{pKa}} \propto \text{EWG}$

pKa = – log Ka

Glycine (Gly)
NH_2-CH_2-COOH

Aspartic acid (Asp)
$HOOC-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

Lysine (Lys)
$NH_2-(CH_2)_4-\overset{\displaystyle NH_2}{\overset{|}{CH}}-COOH$

Arginine (Arg)
$\underset{\displaystyle NH_2}{\underset{|}{NH=C}}-NH(CH_2)_3-\overset{\displaystyle NH_2}{\overset{|}{CH}}-COOH$

Acidic strength in solution $\Rightarrow$ ASP > Gly > Lys > Arg

pKa order should be $\Rightarrow$ Asp < Gly < Lys < Arg

[Though data suggest this is order of Isoelectric pH value]

Q.20 According to molecular orbital theory, which of the following is true with respect to Li_2 + and Li_2^- ?

(1) Both are unstable (2) Both are stable

(3) Li_2 + is stable and Li_2^- is unstable (4) Li_2^+ is unstable and Li_2^- is stable

Ans. **[2]**

Sol. Li_2^+ $(5e^-) = \sigma 1s^2, \sigma{*}1s^2, \sigma 2s^1$

Li_2^- $(7e^-) = \sigma 1s^2, \sigma{*}1s^2, \sigma 2s^2, \sigma{*}2s^2$

Bond order of $Li_2^+ = 0.5$

Bond order of $Li_2^- = 0.5$

Both Li_2^+ and Li_2^- has non-zero bond order and hence both are stable.

Q.21 A water sample has ppm level Fe = 0.2; Mn = 5.0 ; Cu = 3.0 ; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is :

(1) Zn (2) Fe (3) Cu (4) Mn

Ans. **[4]**

Sol. Permissible values

Zn = 5 ppm

Fe = 0.2 ppm

Mn = 0.05 ppm

Cu = 3.0 ppm

Q.22 Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding, them is -

(1) Δ_0 value for (A) is less than that of (B).

(2) both absorb energies corresponding to their complementary colors.

(3) Δ_0 values of (A) and (B) are calculated. From the energies of violet and yellow light, respectively.

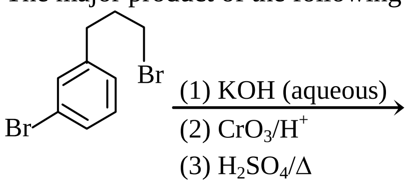
(4) both are paramagnetic with three unpaired electrons.

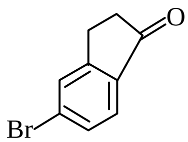
Ans. **[3]**

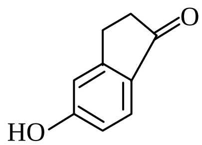
Sol. Δ_0 order will be compared by spectro-chemical series not by energies of violet and yellow light so

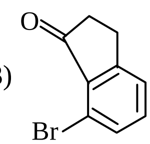
Δ_0 order is $[Cr(H_2O)_6]Cl_3 < [Cr(NH_3)_6]Cl_3$

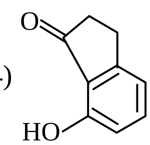
Q.23 The major product of the following reaction is :



(1) 

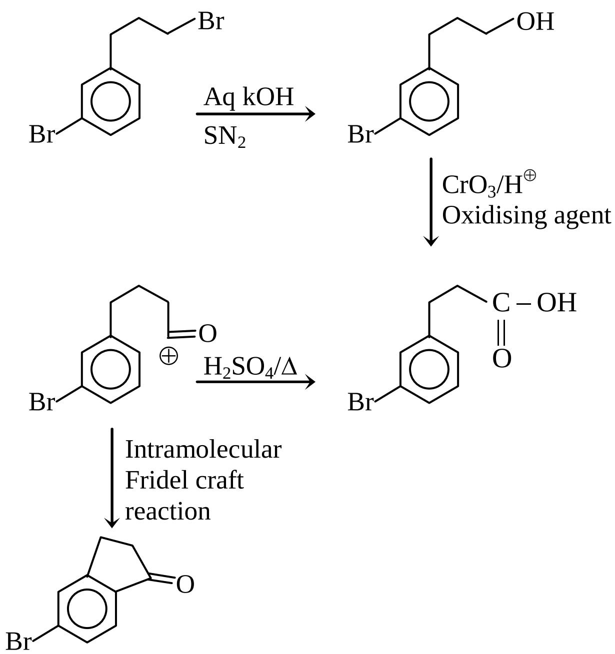
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.24 Which one of the following statements regarding Henery's law is not correct ?

(1) Different gases have different K_H (Henry's law constant) values at the same temperature.

(2) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

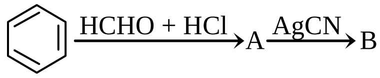
(3) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.

(4) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids.

Ans. **[4]**

Sol. Solubility of gas increases due to increase in Henery const. is wrong statement.

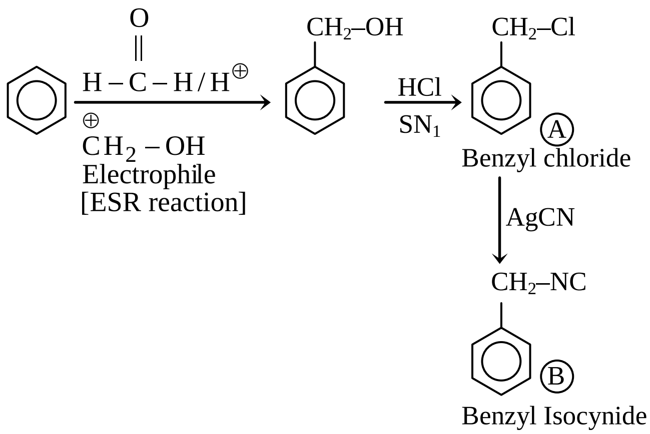
Q.25 The compound A and B in the following reaction are, respectively :



(1) A = Benzyl alcohol, B = Benzyl cyanide
(2) A = Benzyl chloride, B = Benzyl cyanide
(3) A = Benzyl alcohol, B = Benzyl isocyanide
(4) A = Benzyl chloride, B = Benzyl isocyanide

Ans. **[4]**

Sol.



Q.26 The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is -

(1) 6.93 (2) 5.92 (3) 4.90 (4) 3.87

Ans. **[2]**

Sol. Metal complex can contain maximum 5 unpaired e^- in d-subshell of metal

$\mu = \sqrt{n(n+2)}$ BM

n = no. of unpaired e^-

$\boxed{n = 5}$

$\mu = \sqrt{5(5+2)}$ BM

$\mu = \sqrt{35} = 5.92$ BM

Ex. Mn^{+2} complex

Q.27 A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is :

(1) 8 (2) 4 (3) 12 (4) 16

Ans. **[2]**

Sol. $m = \dfrac{\text{moles of } Na^+}{\text{mass of } H_2O \text{ in kg}} = \dfrac{92/23}{1} = 4$ m

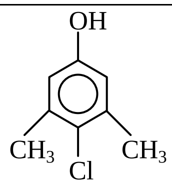
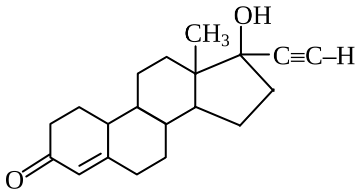
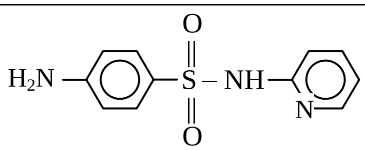
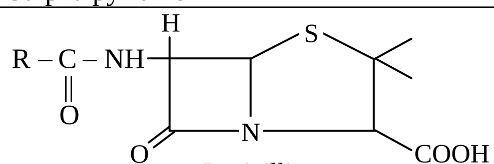
Q.28 The correct match between Item-I and Item-II is :

Item-I (drug)		Item-II (test)	
(A)	Chloroxylenol	(P)	Carbylamine test
(B)	Norethindrone	(Q)	Sodium hydrogen carbonate test
(C)	Sulphapyridine	(R)	Ferric chlorie test
(D)	Penicillin	(S)	Bayer's test

(1) A → R ; B → S ; C → P ; D → Q
(2) A → Q ; B → P ; C → S ; D → R
(3) A → Q ; B → S ; C → P ; D → R
(4) A → R ; B → P ; C → S ; D → Q

Ans. **[1]**

Sol.

(A)	 Chloroxylenol	(R)	Ferric chloride test
(B)	 Norethindrone	(S)	Bayer's Test (Unsaturation Test)
(C)	 Sulpha pyridine	(P)	Carbylamine Test
(D)	 Penicillin	(Q)	$NaHCO_3$

Q.29 The isotopes of hydrogen are :

(1) Protium and deuterium only
(2) Protium, deuterium and tritium
(3) Deuterium and tritium only
(4) Tritium and protium only

Ans. **[2]**

Sol. Protium, deuterium and Tritium are isotopes of Hydrogen.

Q.30 Aluminium is usually found in + 3 oxidation state. In contrast, thallium exists in +1 and + 3 oxidation states. This is due to **-**

(1) diagonal relationship
(2) lanthanoid contraction
(3) inert pair effect
(4) lattice effect

Ans. **[3]**

Sol. Due to inert pair effect.

JEE Main Online Exam 2019

Questions & Solutions

9th January 2019 | Shift - I

MATHEMATICS

Q.1 $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{2\sqrt{2}}$

(2) exists and equals $\frac{1}{4\sqrt{2}}$

(3) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(4) does not exist

Ans. **[2]**

Sol. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

Rationalise

$$= \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \times \frac{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0} \frac{1+\sqrt{1+y^4}-2}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)y^4}$$

$$= \lim_{y\to 0} \frac{\sqrt{1+y^4}-1}{2\sqrt{2}\,y^4}$$

$$= \frac{1}{2\sqrt{2}} \lim_{y\to 0} \frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{\left(\sqrt{1+y^4}+1\right).y^4}$$

$$= \frac{1}{2\sqrt{2}} \lim_{y\to 0} \frac{1+y^4-1}{y^4\left(\sqrt{1+y^4}+1\right)}$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{2} = \frac{1}{4\sqrt{2}}$$

Q.2 If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be :

(1) –2 (2) 2 (3) –3 (4) 4

Ans. **[2]**

Sol. Let 3 distinct numbers are a, ar, ar^2

given a + b + c = xb

$a + ar + ar^2 = xar$

$1 + r + r^2 = xr$

$r^2 + r(1 - x) + 1 = 0$

$D \geq 0$

$(1 - x)^2 - 4 \geq 0$

$((1 - x) - 2)(1 - x + 2) \geq 0$

$(x + 1)(x - 3) \geq 0$

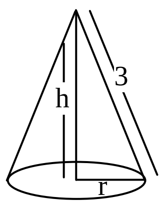
$x \leq -1$ or $x \geq 3$

Q.3 The maximum volume (in cu.m) of the right circular cone having slant height 3 m is :

(1) 6π (2) $\frac{4}{3}\pi$ (3) $2\sqrt{3}\pi$ (4) $3\sqrt{3}\pi$

Ans. **[3]**

Sol.



$V = \frac{1}{3}\pi r^2 h$

(Given $h^2 + r^2 = 9$)

$V = \frac{1}{3}\pi(9 - h^2)h$

$\frac{dV}{dh} = \frac{1}{3}\pi[9 - 3h^2]$

For maximum & minimum $\frac{dV}{dh} = 0$

$9 = 3h^2$

$h^2 = 3$

$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h) < 0$

Maximum at $h = \sqrt{3}$

$r^2 = 9 - h^2$

$r^2 = 9 - 3$

$r^2 = 6$

$V_{max} = \frac{1}{3}\pi \times 6 \times \sqrt{3} = 2\sqrt{3}\pi$

Q.4 The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4
(2) has a unique solution for $|a| = \sqrt{3}$
(3) is inconsistent when $|a| = \sqrt{3}$
(4) is inconsistent when a = 4

Ans. **[3]**

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$

$= 3(a^2 - 1) - 6 - 2(a^2 - 1) + 4 + 0$

$= a^2 - 3$

For unique solution $\Delta \neq 0$

$a \neq \pm\sqrt{3}$

$\therefore$ For inconsistent $|a| = \sqrt{3}$

Q.5 Let f : R $\to$ R be a function defined as

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Then, f is

(1) continuous if a = 0 and b = 5
(2) continuous if a = –5 and b = 10
(3) continuous if a = 5 and b = 5
(4) not continuous for any values of a and b

Ans. **[4]**

Sol. For f to be continuous at x = 1

$f(1^-) = f(1^+)$

$5 = a + b$ …(1)

For f to be continuous at x = 3

$f(3^-) = f(3^+)$

$a + 3b = b + 15$

$a + 2b = 15$ …(2)

From (1) & (2)

$5 + b = 15$

$\Rightarrow b = 10 \;\&\; a = -5$

For f to be continuous at x = 5

$f(5^-) = f(5^+)$

$b + 25 = 30$

$b = 5$

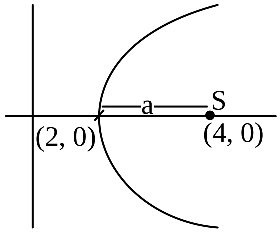
not continuous

Q.6 Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (8, 6) (2) (4, –4) (3) $(6, 4\sqrt{2})$ (4) $(5, 2\sqrt{6})$

Ans. **[1]**

Sol.



Equation of parabola is

$(y - 0)^2 = 4a(x - 2)$

$y^2 = 4.2(x - 2)$

$y^2 = 8(x - 2)$

check option (1) is correct

Q.7 Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) –256 (2) 512 (3) –512 (4) 256

Ans. **[1]**

Sol. Equation $x^2 + 2x + 2 = 0$ has roots α, β

$(x + 1)^2 + 1 = 0$

$x + 1 = \pm i$

$x = -1 \pm i$

Let $\alpha = -1 + i, \quad \beta = -1 - i$

$\alpha = \sqrt{2}\, e^{i3\pi/4}, \quad \beta = \sqrt{2}\, e^{i5\pi/4}$

$\therefore \alpha^{15} + \beta^{15} = (\sqrt{2}\, e^{i3\pi/4})^{15} + (\sqrt{2}\, e^{i5\pi/4})^{15}$

$= (\sqrt{2})^{15}\left[\cos\frac{45\pi}{4} + i\sin\frac{45\pi}{4} + \cos\frac{75\pi}{4} + i\sin\frac{75\pi}{4}\right]$

$= 2^{15/2}\left[\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right]$

$= -2^8$

$= -256$

Q.8 If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is

(1) $(\vee, \vee)$ (2) $(\wedge, \wedge)$ (3) $(\wedge, \vee)$ (4) $(\vee, \wedge)$

Ans. **[3]**

Sol. $(p \oplus q) \wedge (\sim p \odot q) = p \wedge q$

$\oplus, \odot \in \{\wedge, \vee\}$

option (3) satisfy

$(p \wedge q) \wedge (\sim p \vee q)$

p	q	$p \wedge q$	$\sim p$	$\sim p \vee q$	$(p \vee q) \wedge (\sim p \vee q)$	$p \wedge q$
T	T	T	F	T	T	T
T	F	F	F	F	F	F
F	T	F	T	T	F	F
F	F	F	T	T	F	F

same

Q.9 Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} . \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Ans. **[1]**

Sol. Given $\vec{a} = \hat{i} - \hat{j}$ …(1)

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$ …(2)

$\vec{a} . \vec{c} = 4$ …(3)

$|\vec{a}||\vec{c}|\cos\theta = 4$

$\cos\theta = \frac{2\sqrt{2}}{|\vec{c}|}$

Also $\vec{a} \times \vec{c} + \vec{b} = 0$

$\vec{a} \times \vec{c} = -\vec{b}$

$|\vec{a} \times \vec{c}| = |\vec{b}|$

$|\vec{a}||\vec{c}|\sin\theta = |\vec{b}|$

$\sqrt{2} \times |\vec{c}|\sin\theta = \sqrt{3}$

$\sqrt{2}|\vec{c}|\sqrt{1-\cos^2\theta} = \sqrt{3}$

Squaring

$2|\vec{c}|^2\left(1 - \frac{8}{|\vec{c}|^2}\right) = 3$

$2|\vec{c}|^2 - 16 = 3$

$|\vec{c}|^2 = \frac{19}{2}$

Q.10 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is :

(1) 20 (2) 18 (3) 16 (4) 22

Ans. **[1]**

Sol. $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 150$ cm

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 150 \times 5 = 750$

$\sigma = \frac{1}{5}\sum_{i=1}^{5} x_i^2 - (\bar{x})^2$ … (1)

$\Rightarrow 18 = \frac{1}{5}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2) - (150)^2$

$\Rightarrow 18 + (150)^2 = \frac{1}{5}\left(\sum_{i=1}^{5} x_i^2\right)$

$\Rightarrow 112590 = \sum_{i=1}^{5} x_i^2$ from (1)

Now new student of height 156 cm

$\sigma = \frac{1}{6}\sum_{i=1}^{6} x_i^2 - (\bar{x})^2$

$= \frac{1}{6}[x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2] - \left(\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}\right)$

$= \frac{1}{6}[x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2] - \left(\frac{750 + 156}{6}\right)^2$ … (2)

Now (2) $\frac{1}{6}[112590 + (156)^2] - \left(\frac{750 + 156}{6}\right)^2$

$= 22821 - (151)^2$

$= 22821 - 22801$

$= 20$

Q.11 For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then J(x) is equal to :

(1) $\frac{1}{x}f_3(x)$ (2) $f_2(x)$ (3) $f_3(x)$ (4) $f_1(x)$

Ans. **[3]**

Sol. Given $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$, $f_3(x) = \frac{1}{1-x}$

$f_2(J(f_1(x))) = f_3(x)$

$f_2\left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$

$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x}$

$J\left(\frac{1}{x}\right) = \frac{-x}{1-x}$

$J\left(\frac{1}{x}\right) = \frac{x}{x-1}$

Let $t = \frac{1}{x}$

$J(t) = \frac{1/t}{1/t - 1}$

$J(t) = \frac{1}{1-t}$

$\therefore J(x) = \frac{1}{1-x}$

$= f_3(x)$

Q.12 Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i \sin\theta}{1 - 2i \sin\theta} \text{ is purely imaginary}\right\}$. Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{3\pi}{4}$ (3) $\frac{2\pi}{3}$ (4) π

Ans. **[3]**

Sol. Given $\theta \in \left(-\frac{\pi}{2}, \pi\right)$

Let $f(\theta) = \frac{3 + 2i \sin\theta}{1 - 2i \sin\theta}$

$= \frac{(3 + 2i \sin\theta)(1 + 2i \sin\theta)}{(1 - 2i \sin\theta)(1 + 2i \sin\theta)}$

$f(\theta) = \frac{3 + 6i \sin\theta + 2i \sin\theta - 4\sin^2\theta}{1 + 4\sin^2\theta}$

$= \frac{(3 - 4\sin^2\theta) + 8i \sin\theta}{1 + 4\sin^2\theta}$

$\therefore$ $f(\theta)$ is purely imaginary

$\therefore 3 - 4\sin^2\theta = 0$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

Q.13 Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is :

(1) 300 (2) 500 (3) 200 (4) 350

Ans. **[1]**

Sol. 2 girls 3 boys

Boys A ✓	B ✗	$^5C_2 \times {}^5C_2 = 10 \times 10$	
A ✗	B ✓	$^5C_2 \times {}^5C_2 = 10 \times 10$	300
A ✗	B ✗	$^5C_3 \times {}^5C_2 = 10 \times 10$	

Q.14 For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(2) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(3) $13 - 4\cos^2\theta + 6\cos^4\theta$
(4) $13 - 4\cos^6\theta$

Ans. **[4]**

Sol. $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4(1 - \cos^2\theta)^3$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta\cos\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 13 + 12\sin^2\theta\cos^2\theta - 4\cos^6\theta - 12\cos^2\theta + 12\cos^4\theta$

$= 13 + 12(1 - \cos^2\theta)\cos^2\theta - 4\cos^6\theta - 12\cos^2\theta + 12\cos^4\theta$

$= 13 - 4\cos^6\theta$

Q.15 Equation of a common tangent to the circle $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :

(1) $\sqrt{3}y = x + 3$
(2) $\sqrt{3}y = 3x + 1$
(3) $2\sqrt{3}y = -x - 12$
(4) $2\sqrt{3}y = 12x + 1$

Ans. **[1]**

Sol. $y^2 = 4x$

$y = mx + \frac{1}{m}$ is tangent to $x^2 + y^2 = 6x$

$x^2 + \left(mx + \frac{1}{m}\right)^2 = 6x$

$x^2 + m^2x^2 + \frac{1}{m^2} + 2mx \cdot \frac{1}{m} = 6x$

$x^2(1 + m^2) - 4x + \frac{1}{m^2} = 0$

$D = 0$

$16 = 4(1 + m^2) \cdot \frac{1}{m^2}$

$4m^2 = 1 + m^2$

$3m^2 = 1$

$m = \pm \frac{1}{\sqrt{3}}$

if $m = +\frac{1}{\sqrt{3}}$ then equation of tangent is

$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

$\boxed{\sqrt{3}y = x + 3}$

Q.16 Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :

(1) 47 (2) 42 (3) 52 (4) 57

Ans. **[3]**

Sol. $S = a_1 + a_2 + \ldots\ldots a_{30} \Rightarrow S = \frac{30}{2}(a_1 + a_{30})$

$T = a_1 + a_3 + a_5 + \ldots .\ a_{29} \Rightarrow T = \frac{15}{2}[a_1 + a_{29}]$

$\Rightarrow 2T = 15(a_1 + a_{29})$

$S - 27 = 75$

$15(a_1 + a_{30}) - 15(a_1 + a_{29}) = 75$

$a_{30} - a_{29} = 5$

$\boxed{d = 5}$

$a_5 = 27$

$a_1 + 4d = 27$

$a_1 + 20 = 27$

$a_1 = 7$

$a_{10} = a_1 + 9d$

$= 7 + 9(5)$

$= 7 + 45$

$= 52$

Q.17 For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\,dx$ is equal to :

(where c is a constant of integration)

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$ (2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$ (4) $\frac{1}{2}\log_e|\sec(x^2-1)| + c$

Ans. **[1]**

Sol. $\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\,dx$

$= \int x\sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}}\,dx$

$= \int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}\,dx$

$$= \int x \sqrt{\frac{2\sin^2\left(\frac{x^2-1}{2}\right)}{2\cos^2\left(\frac{x^2-1}{2}\right)}}\,dx$$

$$= \int x \tan\left(\frac{x^2-1}{2}\right)dx$$

put $\frac{x^2-1}{2} = t$

$\Rightarrow x^2 - 1 = 2t$

$\Rightarrow 2x\,dx = 2dt$

$\therefore \int \tan(t)\,dt = \ell n|\sec t| + c$

$$= \ell n\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

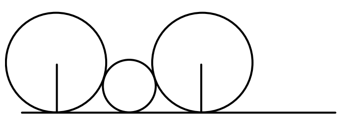
Q.18 Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$ (2) a, b, c are in A.P.

(3) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P. (4) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

Ans. **[4]**

Sol.



AB = AC + CB

$$\sqrt{(b+c)^2+(b-c)^2} = \sqrt{(b+a)^2-(b-a)^2} + \sqrt{(a+c)^2-(a-c)^2}$$

Solve $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

Q.19 If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Ans. **[3]**

Sol. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^{50} = \begin{bmatrix} \cos 50\theta & -\sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$

$(A^{-1})^{50} = \begin{bmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$

At $\theta = \frac{\pi}{12}$

$(A^{-1})^{50} = \begin{bmatrix} \cos\frac{50\pi}{12} & \sin\frac{50\pi}{12} \\ -\sin\frac{50\pi}{12} & \cos\frac{50\pi}{12} \end{bmatrix}$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Q.20 If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to

(1) $\frac{\sqrt{145}}{10}$ (2) $\frac{\sqrt{145}}{11}$ (3) $\frac{\sqrt{145}}{12}$ (4) $\frac{\sqrt{146}}{12}$

Ans. **[3]**

Sol. $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}.\frac{3}{4x} - \sqrt{1-\frac{4}{9x^2}}\sqrt{1-\frac{9}{16x^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{1}{2x^2} = \sqrt{1-\frac{4}{9x^2}}\ \sqrt{1-\frac{9}{16x^2}}$

Square

$\Rightarrow \frac{1}{4x^4} = \left(1-\frac{4}{9x^2}\right)\left(1-\frac{9}{16x^2}\right)$

$\Rightarrow \frac{1}{4x^4} = 1 - \frac{9}{16x^2} - \frac{4}{9x^2} + \frac{1}{4x^4}$

$\Rightarrow \frac{9}{16x^2} + \frac{4}{9x^2} = 1$

$\Rightarrow \frac{1}{x^2}\left(\frac{9}{16}+\frac{4}{9}\right) = 1$

$\Rightarrow \frac{81+64}{16\times 9} = x^2$

$\Rightarrow \frac{145}{16\times 9} = x^2$

$\Rightarrow x = \frac{\sqrt{145}}{12}$

Q.21 If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(1) 8 (2) 14 (3) 6 (4) 4

Ans. **[1]**

Sol. $\frac{2^{403}}{15} = 2^3 . \frac{2^{400}}{15} = \frac{2^3.(2^4)^{100}}{15}$

$= \frac{8[1+15]^{100}}{15}$

$= \frac{8[\,^{100}C_0(1) + \boxed{^{100}C_1.15 + 1 .... ^{100}C_{100}(15)^{100}}\,]}{15}$

Divisible by 15

$= \left\{\frac{8.1}{15}\right\}$

$\therefore$ k = 8

Q.22 Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ?

(1) The lines are not concurrent

(2) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(3) The lines are all parallel

(4) Each line passes through the origin

Ans. **[2]**

Sol. Given lines are px + qy + r = 0 … (1)

& 3p + 2q + 4r = 0 … (2)

$\therefore$ (2) can be written as

$\frac{3}{4}p + \frac{q}{2} + r = 0$ … (3)

from (1) & (3)

$x = \frac{3}{4}, y = \frac{1}{2}$

Q.23 Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 25/169 (2) 24/169 (3) 49/169 (4) 52/169

Ans. **[1]**

Sol. P(X = 1) + P(X = 2)

$$= \frac{4}{52} \times \frac{48}{52} \times 2 + \frac{4}{52} \times \frac{4}{52}$$

$$= 2 \times \frac{12}{(13)^2} + \left(\frac{1}{13}\right)^2$$

$$= \frac{25}{169}$$

Q.24 The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is :

(1) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(2) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(3) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Ans. **[1]**

Sol. The equation of required line passing through (–4, 3, 1) is

$$\frac{x+4}{a} = \frac{y-3}{b} = \frac{z-1}{c}$$

given line is $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$

Now the required line is parallel to plane x + 2y – z – 5 = 0

$\therefore$ given lines are coplanar

$$\because \begin{vmatrix} -3 & 0 & 1 \\ a & b & c \\ -3 & 2 & -1 \end{vmatrix} = 0$$

3(–b + 2c) + 0 + 2a + 3b = 0

–2a + 6c = 0

$\boxed{a = 3c}$ …(1)

Also a + 2b – c = 0

3c + 2b – c = 0 [use (1)]

2c + 2b = 0

b = –c

$\therefore$ Required line is

$$\frac{x+4}{(3c)} = \frac{y-3}{-c} = \frac{z-1}{c}$$

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

Q.25 The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point :

(1) (–3, 1, 1) (2) (–3, 0, –1) (3) (3, 3, –1) (4) (3, 2, 1)

Ans. **[4]**

Sol. Plane passes through

$(x + y + z - 1) + \lambda (2x + 3y - z + 4) = 0$ & parallel to y-axis is

$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) + (4\lambda - 1) = 0$

parallel to y axis $3\lambda = -1$

$$\lambda = \frac{-1}{3}$$

$$x\left(1-\frac{2}{3}\right)+z\left(1+\frac{1}{3}\right)+\left(-\frac{4}{3}-1\right)=0$$

$$x\left(\frac{1}{3}\right)+z\left(\frac{4}{3}\right)-\frac{7}{3}=0$$

$x + 4z - 7 = 0$

option check (–3, 1, 1) $= -3 + 4 - 7$ ✗

(–3, 0, –1) $= -3 - 4 - 7$ ✗

(3, 3, –1) $= 3 - 4 - 7$ ✗

(3, 2, 1) $= 3 + 4 - 7 = 0$ ✓

Q.26 If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to :

(1) $\frac{4}{9}$ (2) $\frac{7}{17}$ (3) $\frac{8}{17}$ (4) $\frac{8}{15}$

Ans. **[4]**

Sol. $y = 10 - x^2$ $y = 2 + x^2$

$10 - x^2 = 2 + x^2$

$8 = 2x^2$

$x^2 = 4$

$x = 2, -2$

points are (2, 6), (–2, 6)

$$\left(\frac{dy}{dx}\right)_{(2,6)} = 2x = 4$$

$$\left(\frac{dy}{dx}\right)_{(-2,6)} = -2x = -4$$

$$\tan\theta = \left|\frac{4-(-4)}{1+4(-4)}\right| = \left|\frac{8}{1-16}\right| = \frac{8}{15}$$

Q.27 The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is :

(1) $-\frac{4}{3}$ (2) 0 (3) $\frac{4}{3}$ (4) $\frac{2}{3}$

Ans. **[3]**

Sol. $I = \int_0^{\pi} |\cos x|^3 \, dx$

$I = 2\int_0^{\pi/2} |\cos x|^3 \, dx$

$I = 2\int_0^{\pi/2} \cos^3 x . dx$

$I = 2\int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4}\right) . dx$

$I = \frac{1}{2}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\pi/2}$

$I = \frac{1}{2}\left[\frac{1}{3}.\sin 3.\frac{\pi}{2} + 3.1\right]$

$I = \frac{1}{2}\left[\frac{-1}{3} + 3\right]$

$= \frac{1}{2}\left(\frac{8}{3}\right) = \frac{4}{3}$

Q.28 The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{56}{3}$ (2) $\frac{32}{3}$ (3) $\frac{8}{3}$ (4) $\frac{14}{3}$

Ans. **[3]**

Sol. $y = x^2 - 1$

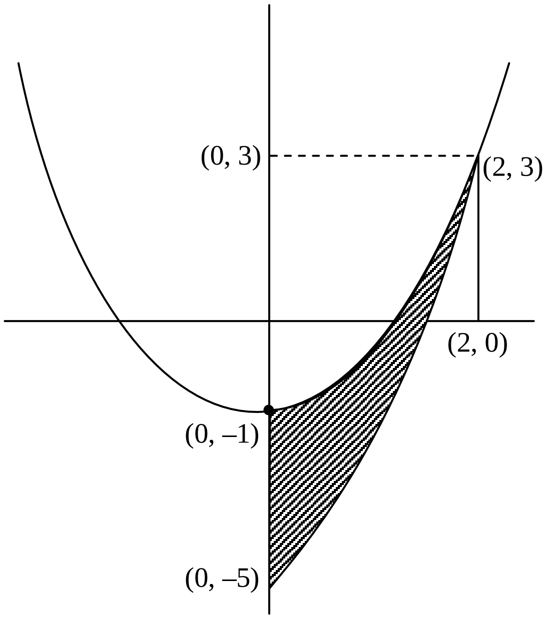
$(y + 1) = x^2$ $\quad \frac{dy}{dx} = 2x$

$\left(\frac{dy}{dx}\right)_{(2,3)} = 4$

$(y - 3) = 4(x - 2)$

$y - 3 = 4x - 8$

$y = 4x - 5$



$$\int_{-5}^{3}\left(\frac{y+5}{4}\right)dy - \int_{-1}^{3}\sqrt{y+1}\,dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2}+5y\right]_{-5}^{3} - \left[2\frac{(y+1)^{3/2}}{3}\right]_{-1}^{3}$$

$$= \frac{1}{4}\left[\left(\frac{9}{2}+15\right)-\left(\frac{25}{2}-25\right)\right] - \frac{2}{3}\,[(4)^{3/2}]$$

$$= \frac{1}{4}\left[\frac{39}{2}+\frac{25}{2}\right] - \frac{2}{3}[8]$$

$$= \frac{1}{4}\times\frac{64}{2} - \frac{16}{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3}$$

Q.29 If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{7}{64}$ (2) $\frac{49}{16}$ (3) $\frac{1}{4}$ (4) $\frac{13}{16}$

Ans. **[2]**

Sol. $x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2y}{x} = x$$

$\text{I.F.} = e^{\int \frac{2}{x}.dx} = e^{2\,\ell nx} = x^2$

$y.x^2 = \int x.x^2.dx + c$

$yx^2 = \frac{x^4}{4} + c$

$y(1) = 1$

$1 . 1 = \frac{1}{4} + c \Rightarrow c = 1 - \frac{1}{4} \Rightarrow c = \frac{3}{4}$

$yx^2 = \frac{x^4}{4} + \frac{3}{4}$

$y. \frac{1}{4} = \frac{1}{4}.\frac{1}{16} + \frac{3}{4}$

$y = \frac{1}{16} + 3$

$y = \frac{49}{16}$

Q.30 Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) $(3, \infty)$ (2) $(1, 3/2]$ (3) $(3/2, 2]$ (4) $(2, 3]$

Ans. **[1]**

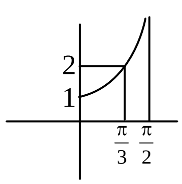
Sol. $\sin^2\theta = \cos^2\theta\,(e^2 - 1)$

$\tan^2\theta = e^2 - 1$

$1 + \tan^2\theta = e^2$

$\sec^2\theta = e^2$

$\theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow$



$e = \sqrt{\sec^2\theta}$

$e = \sec\theta$

$\sec\theta > 2 \quad \Rightarrow \quad \frac{1}{\cos\theta} > 2$

$\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ $\qquad 0 < \cos\theta < \frac{1}{2}$

length of LR = $\frac{2b^2}{a} = \frac{2.\sin^2\theta}{\cos\theta}$

$= \frac{2(1-\cos^2\theta)}{\cos\theta}$

$\left(\frac{\pi}{3}, \frac{\pi}{2}\right) \uparrow$ function

min exist at $\theta = \frac{\pi}{3}$

max exist at $\theta = \frac{\pi}{2}$ $\qquad \Rightarrow (3, \infty)$