# CAREER POINT

# JEE Main Online Exam 2019

## Questions & Solution

9th January 2019 | Shift - II

## PHYSICS

**Q.1** In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)

(1) $3.75 \times 10^6$ (2) $4.87 \times 10^5$ (3) $6.25 \times 10^5$ (4) $3.86 \times 10^6$

**Ans.** **[3]**

No. Of channels = $\frac{f}{B.W}$

$$n = \frac{\frac{3\times10^8}{800\times10^{-9}}\times\frac{1}{100}}{6\times10^6}$$

$$n = \frac{3\times10^{17}}{8\times6\times10^6\times10^4} = \frac{10}{2\times8} = \frac{100}{16}\times10^5$$

$= 6.25 \times 10^5$.

**Q.2** The magnetic field associated with a light wave is given, at the origin, by

$$B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct].$$

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons?

($c = 3 \times 10^8$ ms$^{-1}$, $h = 6.6 \times 10^{-34}$ J-s)

(1) 6.82 eV (2) 8.52 eV (3) 12.5 eV (4) 7.72 eV

**Ans.** **[4]**

**Sol.** $B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$

$\phi = 4.7$eV

$$K_{max} = \frac{hc}{\lambda} - \phi$$

$= hf - \phi$

$$= \frac{6.6\times10^{-34}\times10^7 c}{1.6\times10^{-19}} - \phi$$

$$K_{max} = \frac{6.6\times10^{-27}\times3\times10^8}{1.6\times10^{-19}} - 4.7 \text{ eV}$$

$$= \frac{19.6}{1.6} - 4.7$$

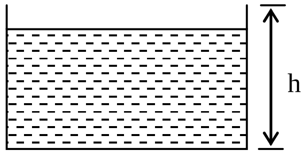
$= 12.25 - 4.70 = 7.55$ eV

**Q.3** The top of a water tank is open to air and its water level maintained. It is giving out 0.74 $m^3$ water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :

(1) 6.0 m (2) 9.6 m (3) 2.9 m (4) 4.8 m

**Ans.** **[4]**

**Sol.**




r = 2 cm

$a = \pi(2 \times 10^{-2})^2 = \pi \times 4 \times 10^{-4}\ m^2$

$\because$ A >> a

$\Rightarrow V = \sqrt{2gh}$

Volume flow rate = Av = 0.74 $m^3$/min = $\frac{0.74}{60}$ $m^3$/sec

$4\pi \times 10^{-4}\ v = \frac{0.74}{60}$

$v = \frac{0.74}{4\pi \times 10^{-4} \times 60} = \frac{0.74 \times 10^4}{4 \times 3.14 \times 60}$

$v = \frac{7400}{12.56 \times 60} = \sqrt{2gh}$

$\frac{740}{75.36} = \sqrt{2gh}$

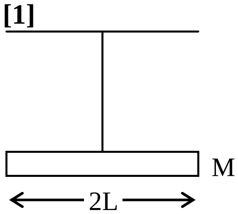
$h \approx 4.8$ m

**Q.4** A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

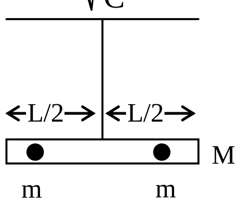
(1) 0.37 (2) 0.57 (3) 0.77 (4) 0.17

**Ans.** **[1]**

**Sol.**




$T = 2\pi\sqrt{\frac{I}{C}}$




$T^1 = 2\pi\sqrt{\frac{I^1}{C}}$

$$\frac{T}{T^1} = \sqrt{\frac{I}{I^1}} = \frac{f^1}{f} = \frac{0.8f}{f} = 0.8$$

$$\frac{I}{I^1} = 0.64$$

$$\frac{M(2L)^2}{12} = 0.64\left[\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2\right]$$

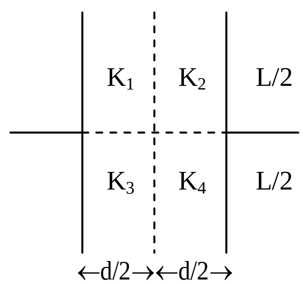
$$\frac{ML^2}{3\times 0.64} = \frac{ML^2}{3} + \frac{ML^2}{2}$$

$$\frac{M}{1.92} - \frac{M}{3} = \frac{m}{2}$$

$$\frac{1.08}{3\times 1.92}M = \frac{m}{2}$$

$$\Rightarrow \frac{m}{M} = \frac{1.08\times 2}{3\times 1.92} = \frac{2.16}{5.76} \approx 0.37$$

**Q.5** A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants $K_1$, $K_2$, $K_3$, $K_4$ arranged as shown in the figure. The effective dielectric constant K will be :




(1) $K = \dfrac{(K_1+K_2)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

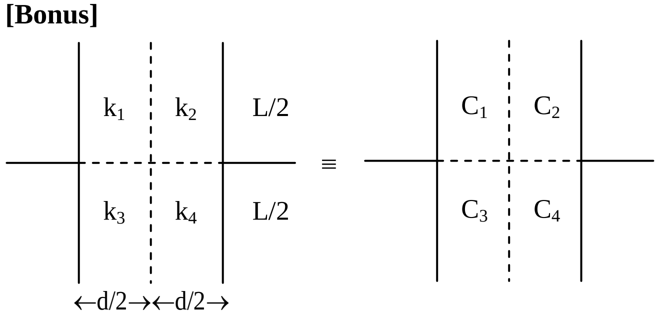
(2) $K = \dfrac{(K_1+K_2)(K_2+K_4)}{2(K_1+K_2+K_3+K_4)}$

(3) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(4) $K = \dfrac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

**Ans.** **[Bonus]**

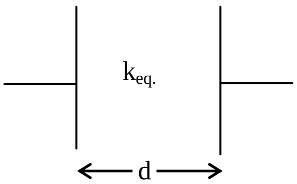
**Sol.**




$$C_{eq.} = \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4}$$

$C_1 = \dfrac{k_1 \in_0 A/2}{d/2} = \dfrac{k_1 \in_0 A}{d}$

Similarly $C_2 = \dfrac{k_2 \in_0 A}{d}$, $C_3 = \dfrac{k_3 \in_0 A}{d}$, $C_4 = \dfrac{k_4 \in_0 A}{d}$



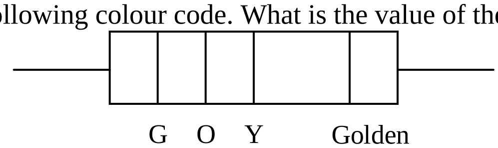

$C_{eq.} = \dfrac{k_{eq.}A}{d}$

$$C_{eq.} = \frac{\frac{k_1 \in_0 A}{d} \cdot \frac{k_2 \in_0 A}{d}}{\frac{k_1 \in_0 A}{d} + \frac{k_2 \in_0 A}{d}} + \frac{\frac{k_3 \in_0 A}{d} \cdot \frac{k_4 \in_0 A}{d}}{\frac{k_3 \in_0 A}{d} + \frac{k_4 \in_0 A}{d}}$$

$$C_{eq.} = \frac{k_1 k_2}{k_1 + k_2} \frac{\in_0 A}{d} + \frac{k_3 k_4}{k_3 + k_4} \frac{\in_0 A}{d}$$

Now, $k_{eq.} = \dfrac{k_1 k_2}{k_1 + k_2} + \dfrac{k_3 k_4}{k_3 + k_4}$

**Q.6** A carbon resistance has a following colour code. What is the value of the resistance?




(1) 5.3 MΩ ± 5% (2) 530 kΩ ± 5% (3) 64 kΩ ± 10% (4) 6.4 MΩ ± 5%

**Ans.** **[2]**

**Sol.** From colour coding table :-

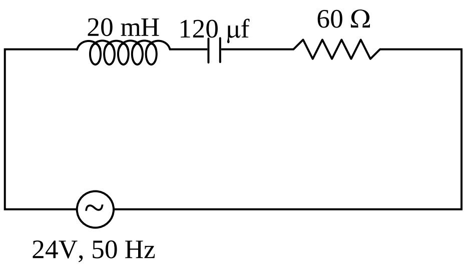
$R = 53 \times 10^4\ \Omega \pm 5\%$

**Q.7** A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is :

(1) $5.65 \times 10^2$ J (2) $2.26 \times 10^3$ J (3) $5.17 \times 10^2$ J (4) $3.39 \times 10^3$ J

**Ans.** **[3]**

**Sol.**




$X_L = 100\pi \times 20 \times 10^{-3}$

$X_L = 2\pi = 6.28\ \Omega$

$X_C = \dfrac{1}{100\pi \times 120 \times 10^{-6}} = \dfrac{250}{3\pi}$

$X_C = \dfrac{10^6}{12000\pi} = \dfrac{10^3}{12\pi}$

$|X_L - X_C| = 20.25$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Energy = $I_{rms}^2\ R \times t$

$= 5.17 \times 10^2$ J

**Q.8** A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(1) 950 J (2) 900 J (3) 875 J (4) 850 J

**Ans.** **[2]**

**Sol.** $x = 3t^2 + 5$

$V = \dfrac{dx}{dt} = 6t$

at $t = 0$, $u = 0$

at $t = 5$, $v = 30$ m/s

$W = \Delta K = \dfrac{1}{2} \times 2\,(30)^2 = 900$ J

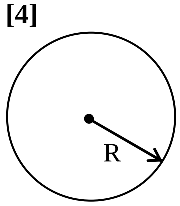
Option (2) is correct.

**Q.9** One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop ($B_L$), i.e. $\dfrac{B_L}{B_C}$ will be :

(1) $\dfrac{1}{N}$ (2) N (3) $N^2$ (4) $\dfrac{1}{N^2}$

**Ans.** **[4]**

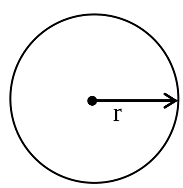
**Sol.**




$2\pi R = L$

$R = \dfrac{L}{2\pi}$

$B_L = \dfrac{\mu_0 I \cdot 2\pi}{2 \cdot L}$




$N2\pi\, r = L$

$r = \dfrac{L}{N \cdot 2\pi}$

$B_C = \dfrac{\mu_0 \cdot NIN2\pi}{2 \cdot L}$

$$\frac{B_L}{B_C} = \frac{\dfrac{\mu_0 I 2\pi}{2L}}{\dfrac{\mu_0 N^2 I \cdot 2\pi}{2L}} = \frac{1}{N^2}$$

Option (4) is correct.

**Q.10** In a young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda$ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \le \theta \le 30^\circ$ is :

(1) 640 (2) 641 (3) 320 (4) 321

**Ans.** **[2]**

**Sol.** $d \sin\theta = n\lambda$

$0.320 \times 10^{-3} \cdot \sin 30^\circ = n \cdot 500 \times 10^{-9}$

$n = \frac{320\times10^{-6}}{500\times10^{-9}} \times \frac{1}{2} = \frac{16}{50}\times1000 = 320$

So, total no. of bright fringes = 2·(320) + 1

= 641

**Q.11** Two Carnot engines A and B are operated in series. The first one, A, receives heat at $T_1$(= 600 K) and rejects to a reservoir at temperature $T_2$. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at $T_3$(= 400 K). Calculate the temperature $T_2$ if the work outputs of the engines are equal :

(1) 500 K (2) 300 K (3) 600 K (4) 400 K

**Ans.** **[1]**

**Sol.** $W = Q_1 - Q_2$

$W = Q_2 - Q_3$

$Q_1 - Q_2 = Q_2 - Q_3$

$T_1 - T' = T' - T^3$

$2T' = T_1 - T_3$

$T' = \frac{T_1 + T_3}{2} = 500$ K

**Q.12** At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as $e^{-3t}$. If the half-life of A is *l*n2, the half-life of B is -

(1) $\frac{\ln 2}{4}$ (2) 4*l*n2 (3) 2*l*n2 (4) $\frac{\ln 2}{2}$

**Ans.** **[1]**

**Sol.** $\because$ A = $\lambda$N

$\frac{R_B}{R_A} = \frac{A_0 e^{-\lambda_B t}}{A_0 e^{-\lambda_A t}} = e^{-3t}$

$e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$

$\Rightarrow \lambda_B - \lambda_A = 3$

$\because \lambda_A = 1$

$\Rightarrow \lambda_B = 4$

$(T_{1/2})_A = \ell n2 = \frac{\ell n2}{\lambda_A}$

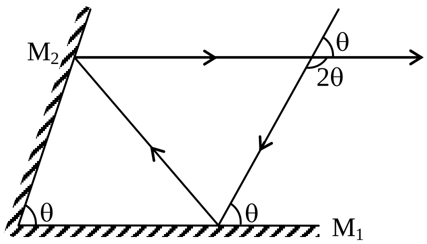
$\Rightarrow (T_{1/2})_B = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

**Q.13** Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror ($M_1$) and parallel to the second mirror ($M_2$) is finally reflected from the second mirror ($M_2$) parallel to the first mirror ($M_1$). The angle between the two mirrors will be :

(1) 60º (2) 75º (3) 45º (4) 90º

**Ans.** **[1]**

**Sol.**

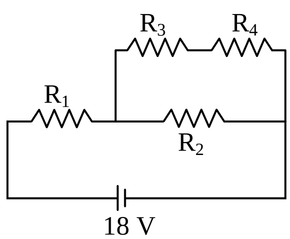



$2\theta + \theta = 180$

$\theta = 60^o$

Option (1)

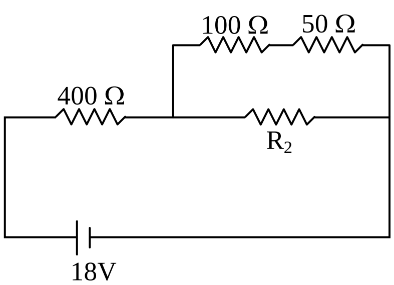
**Q.14** In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across $R_4$ is 5 V, then the value of $R_2$ will be :




(1) 550 Ω (2) 450 Ω (3) 230 Ω (4) 300 Ω

**Ans.** **[4]**

**Sol.**




$V_{R_4} = 5$ Volt

$IR_4 = 5$

or $I = \frac{5}{500} = \frac{1}{100}$ Amp.

$\Rightarrow V_{R_2} = \frac{1}{100}(100 + 500) = 6$ volt.

So, V across 400 Ω will be 18 – 6 = 12 volt

$\Rightarrow$ Total current = $\frac{12}{400}$ Amp.

$\Rightarrow$ I across $R_2$ = $\frac{12}{400} - \frac{1}{100} = \frac{8}{400}$ Amp.

Now, $\frac{8}{400} R_2 = 6$

So, $R_2 = 300\ \Omega$.

**Q.15** Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2} e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

(1) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (2) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (3) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (4) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

**Ans.** **[2]**

**Sol.** $\int \rho(r) 4\pi r^2 dr = Q$

$$\Rightarrow \int_0^R \frac{A}{r^2} e^{-2r/a} 4\pi r^2 dr = Q$$

$$\frac{4\pi A\left[e^{-2r/a}\right]_0^R}{\left[\frac{-2}{a}\right]} = Q$$

$2a\pi A[1 - e^{-2R/a}] = Q$

$$R = \frac{a}{2}\ln\left[\frac{2\pi aA}{2\pi aA - Q}\right] = \frac{a}{2}\ln\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

Option (2)

**Q.16** Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

(1) $\sqrt{\frac{Gh}{c^5}}$ (2) $\sqrt{\frac{hc^5}{G}}$ (3) $\sqrt{\frac{Gh}{c^3}}$ (4) $\sqrt{\frac{c^3}{Gh}}$

**Ans.** **[1]**

**Sol.** $T = kG^a h^b c^c$

$[T] = [M^{-1}L^3T^{-2}]^a\,[ML^2T^{-1}]^b\,[LT^{-1}]^c$

$= M^{-a+b}\,L^{3a+2b+c}T^{-2a-b-c}$

$-a + b = 0$

$\Rightarrow a = b$ …(1)

$3a + 2b + c = 0$ …(2)

$-2a - b - c = 1$ …(3)

$a + b = 1$

$a = b = \frac{1}{2}$

$C = -2 \times \frac{1}{2} - \frac{1}{2} - 1 = -\frac{5}{2}$

$T \propto \frac{G^{\frac{1}{2}} h^{\frac{1}{2}}}{C^{\frac{5}{2}}} = \sqrt{\frac{Gh}{C^5}}$

Option (1)

**Q.17** In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration $a_1$ and $a_2$ respectively. Then 'v' is equal to :

(1) $\sqrt{a_1 a_2}\ t$ (2) $\frac{2a_1a_2}{a_1 + a_2}\ t$ (3) $\frac{a_1 + a_2}{2}\ t$ (4) $\sqrt{2a_1a_2}\ t$

**Ans.** **[1]**

**Sol.** $S = \frac{1}{2} a_1 t_0^2 = \frac{1}{2} a_2(t_0 + t)^2$

$V = \left(\sqrt{a_1} - \sqrt{a_2}\right) \times \sqrt{2S}$

$\left(\frac{1}{\sqrt{a_2}} - \frac{1}{\sqrt{a_1}}\right) \times \sqrt{2S} = t$

$\sqrt{2S} = \frac{\sqrt{a_1}\sqrt{a_2}\,t}{\sqrt{a_1} - \sqrt{a_2}}$

$\frac{V}{\sqrt{a_1} - \sqrt{a_2}} = \frac{\sqrt{a_1}\sqrt{a_2}\,t}{\left(\sqrt{a_1} - \sqrt{a_2}\right)}$

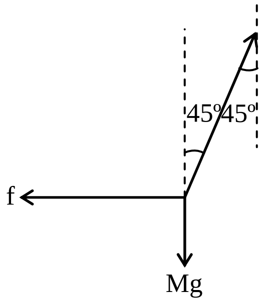
$V = \sqrt{a_1 a_2}\,t$

**Q.18** A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45º at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$)

(1) 100 N (2) 70 N (3) 140 N (4) 200 N

**Ans.** **[1]**

**Sol.**




$$\frac{f}{\sin(180^\circ - 45^\circ)} = \frac{Mg}{\sin(90^\circ + 45^\circ)}$$

$$\frac{f}{\sin 45^\circ} = \frac{Mg}{\cos 45^\circ}$$

$\Rightarrow$ f = Mg = 10g = 100 N

**Q.19** The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :

(1) 5.725 mm (2) 5.950 mm (3) 5.755 mm (4) 5.740 mm

**Ans.** **[3]**

**Sol.** Least count = $\frac{0.5}{100}$ mm

Actual value = 5.5 mm + (48 + 3) $10^{-3}$ mm

= 5.755 mm

**Q.20** A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is devlivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

(1) 45 A (2) 35 A (3) 25 A (4) 50 A

**Ans.** **[1]**

**Sol.** $V_{in}$ = 2300 volt, $I_{in}$ = 5A

$N_1$ = 4000

$V_0$ = 230 V

$$(2300 \times 5) \times \frac{90}{100} = (230) \times I$$

$$I = \frac{50 \times 90}{100} = 45 \text{ A}$$

**Q.21** A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27ºC. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about : [Take R = 8.3 J/K mole]

(1) 10 kJ (2) 0.9 kJ (3) 14 kJ (4) 6 kJ

**Ans.** **[1]**

**Sol.** To double the rms speed, temp is increased to 4 times

$T_1 = 300$ K

$T_2 = 300 \times 4 = 1200$ K

$\Delta T = 1200 - 300 = 900$ K

$Q = nC_v\Delta T = \frac{15}{28} \times \frac{5}{2} \times R \times 900$

Q = 10 KJ

**Q.22** The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is :

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$ (3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

**Ans.** **[3]**

**Sol.** $V_x = a\omega\sin\omega t$

$V_y = a\omega\cos\omega t$

$V_z = a\omega$

$|V| = \sqrt{V_x^2 + V_y^2 + V_2^2}$

$= \sqrt{a^2\omega^2\sin^2\omega t + a^2\omega^2\cos^2\omega t + a^2\omega^2}$

$|V| = \sqrt{2}a\omega$

**Q.23** A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2\sqrt{2}}$ (2) $\frac{A}{\sqrt{2}}$ (3) $\frac{A}{2}$ (4) A

**Ans.** **[2]**

**Sol.** KE = PE

$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

$A^2 = 2x^2$

$x = \frac{A}{\sqrt{2}}$

**Q.24** The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = $6.4 \times 10^3$ km) is $E_1$ and kinetic energy required for the satellite to be in a circular orbit at this height is $E_2$. The value of h for which $E_1$ and $E_2$ are equal, is :

(1) $3.2 \times 10^3$ km (2) $1.6 \times 10^3$ km (3) $1.28 \times 10^4$ km (4) $6.4 \times 10^3$ km

**Ans.** **[1]**

**Sol.** $E_2 = \dfrac{GMm}{2(R+h)}$

$E_1 = \dfrac{GMm}{R} - \dfrac{GMm}{(R+h)}$

$\dfrac{GMm}{2(R+h)} = GMm\left[\dfrac{1}{R} - \dfrac{1}{R+h}\right]$

$\dfrac{1}{2(R+h)} = \dfrac{R+h-R}{R(R+h)}$

$h = \dfrac{R}{2} = \dfrac{6.4\times10^3}{2} = 3.2 \times 10^3$ km

**Q.25** A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :

(1) 753 Hz (2) 666 Hz (3) 500 Hz (4) 333 Hz

**Ans.** **[2]**

**Sol.** Frequency of sound source = $\dfrac{2V}{2L} = \dfrac{330}{50\times10^{-2}} = 660$ Hz

Speed of observer = $\dfrac{10\times5}{18}$ m/s

Apparent frequency = $660\left(\dfrac{330+\frac{50}{18}}{330}\right)$

$= 2\left[330+\dfrac{50}{18}\right) = 666$ Hz

**Q.26** A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = $1.6 \times 10^{-19}$C)

(1) $1.6 \times 10^{-19}$ kg (2) $1.6 \times 10^{-27}$ kg (3) $9.1 \times 10^{-31}$ kg (4) $2.0 \times 10^{-24}$ kg

**Ans.** **[4]**

**Sol.** $R = \dfrac{mV}{qB}$

$(qVB) = qE$

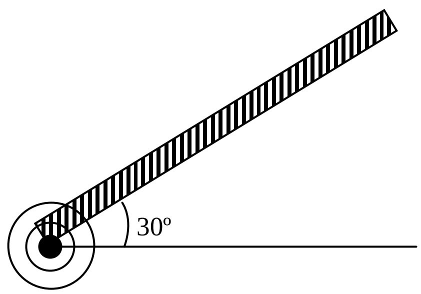
$V = \dfrac{E}{B}$

$R = \dfrac{mE}{qB^2}$

$m = \dfrac{qRB^2}{E} = \dfrac{1.6\times10^{-19}\times0.5\times10^{-2}\times0.25}{100}$

$= 2 \times 10^{-24}$ kg

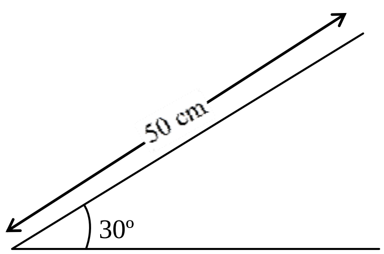
**Q.27** A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30º from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad $s^{-1}$) will be ($g = 10\ ms^{-2}$)




(1) $\sqrt{\dfrac{30}{2}}$ (2) $\dfrac{\sqrt{20}}{3}$ (3) $\dfrac{\sqrt{30}}{2}$ (4) $\sqrt{30}$

**Ans.** **[4]**

**Sol.**




$mg\dfrac{L}{2}\sin30º = \dfrac{1}{2}I\omega^2$

$\dfrac{mgL}{4} = \dfrac{1}{2}\times\dfrac{1}{3}mL^2\,\omega^2$

$\omega^2 = \dfrac{3g}{2L}$

$\omega = \sqrt{30}$ rad/s

**Q.28** The energy associated with electric field is ($U_E$) and with magnetic field is ($U_B$) for an electromagnetic wave in free space. Then :

(1) $U_E < U_B$ (2) $U_E = \dfrac{U_B}{2}$ (3) $U_E = U_B$ (4) $U_E > U_B$

**Ans.** [3]

**Sol.** Energy density in electric field $U_E = \frac{1}{2}\varepsilon_0 E^2$

Energy density in magnetic field $U_B = \frac{1}{2}\frac{B^2}{\mu_0}$

$$\frac{U_E}{U_B} = \frac{\mu_0\varepsilon_0 E^2}{B^2} = \frac{C^2}{C^2} = 1$$

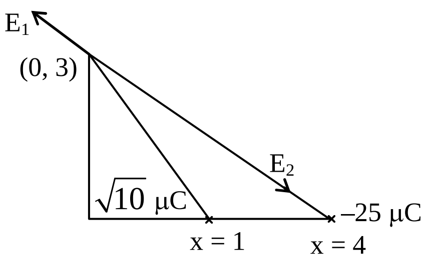
$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \text{ and } \frac{E}{B} = C$$

**Q.29** Two point charges $q_1(\sqrt{10}\ \mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is, [take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$]

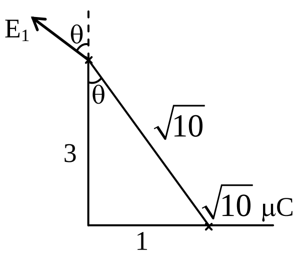
(1) $(-81\hat{i} + 81\hat{j}) \times 10^2$ (2) $(81\hat{i} - 81\hat{j}) \times 10^2$ (3) $(63\hat{i} - 27\hat{j}) \times 10^2$ (4) $(-63\hat{i} + 27\hat{j}) \times 10^2$

**Ans.** [3]

**Sol.**




Electric field due to $\sqrt{10}\ \mu C$ charge

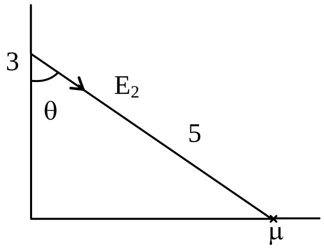



$$|E_1| = \frac{k \times \sqrt{10} \times 10^{-6}}{10} = \frac{9 \times 10^3}{\sqrt{10}}$$

$$\vec{E}_1 = E_1\sin\theta\ i + E_1\cos\theta j$$

$$\vec{E}_1 = \frac{9 \times 10^3}{\sqrt{10}}\left[\frac{-3}{\sqrt{10}}i + \frac{1}{\sqrt{10}}j\right] = 9 \times 10^2\ (-3i + j)$$

Electric field due to –25μC charge




$|E_2| = \dfrac{K \times 25 \times 10^{-6}}{25} = 9 \times 10^3$

$E_2 = E_2\sin\theta i - E_2\cos\theta j$

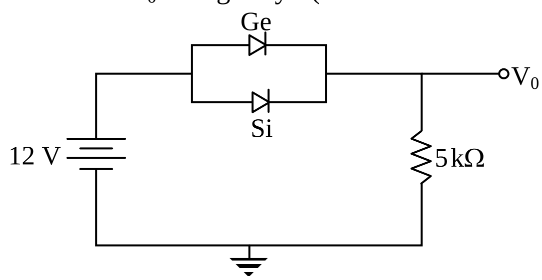
$\vec{E}_2 = 9 \times 10^3 \left[\dfrac{4}{5}i - \dfrac{3}{5}j\right]$

$= 18 \times 10^2\ [4i - 3j]$

$\vec{E}_2 = \vec{E}_1 + \vec{E}_2 = 10^2[63i - 27j]$

Option (3)

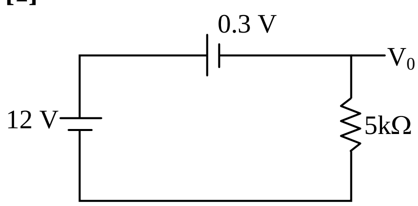
**Q.30** Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of $V_0$ changes by : (assume that the Ge diode has large breakdown voltage)




(1) 0.4 V (2) 0.6 V (3) 0.8 V (4) 0.2 V

**Ans.** **[1]**

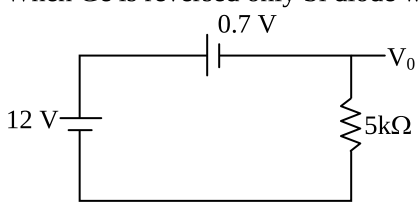
**Sol.**




Initial output $V_0 = 12 - 0.3$

$= 11.7$ Volt

When Ge is reversed only Si diode will work




Output voltage = 12 – 0.7 = 11.3 V

Change in output voltage = 11.7 – 11.3

= 0.4 volt

# JEE Main Online Exam 2019

## Questions & Solutions

9th January 2019 | Shift - II

## CHEMISTRY

**Q.1** Which of the following combination of statements is true regarding the interpretation of the atomic orbitals ?

(a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.

(b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.

(c) According to wave mechanics, the ground state angular momentum is equal to $\frac{h}{2\pi}$.

(d) The plot of $\psi$ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value.

(1) (b), (c) (2) (a), (d) (3) (a), (b) (4) (a), (c)

**Ans.** **[2]**

**Sol.** In ground state (1s orbital)

$\therefore$ Orbital angular momentum = $\sqrt{\ell(\ell+1)}\frac{h}{2\pi}$.

**Q.2** A solution containing 62 g ethylene glycol in 250 g water is cooled to –10ºC. If $K_f$ for water is 1.86 K kg mol$^{-1}$, the amount of water (in g) separated as ice is :

(1) 32 (2) 16 (3) 64 (4) 48

**Ans.** **[3]**

**Sol.** $\Delta T_f = K_f \times m$

$$10 = 1.86 \times \frac{62/62}{W_{(gm)}} \times 1000$$

W = 186 gm

$\therefore$ Amount of water separated as ice = (250 – 186) = 64 gm

**Q.3** For coagulation of arsenious sulphide sol which one of the following salt solution will be most effective ?

(1) $AlCl_3$ (2) NaCl (3) $Na_3PO_4$ (4) $BaCl_2$

**Ans.** **[1]**

**Sol.** Arsenic sulphide is a negatively changed sol so cation having maximum charge will be most effective for coagulation of $As_2S_3$ sol.

**Q.4** In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic?

(1) $NO \rightarrow NO^+$ (2) $O_2 \rightarrow O_2^+$

(3) $N_2 \rightarrow N_2^+$ (4) $O_2 \rightarrow O_2^{2-}$

**Ans.** **[1]**

**Sol.**

| Process | Change in Magnetic nature | Bond order change |
|---|---|---|
| $N_2 \rightarrow N_2^+$ | Dia → Para | 3 → 2.5 |
| $NO \rightarrow NO^+$ | Para → dia | 2.5 → 3 |
| $O_2 \rightarrow O_2^{-2}$ | Para → dia | 2 → 1 |
| $O_2 \rightarrow O_2^+$ | Para → Para | 2 → 2.5 |

**Q.5** The complex that has highest crystal field splitting energy (Δ), is :

(1) $K_3[Co(CN)_6]$ (2) $K_2[CoCl_4]$

(3) $[Co(NH_3)_5Cl]\ Cl_2$ (4) $[Co(NH_3)_5(H_2O)Cl_3]$

**Ans.** **[1]**

**Sol.** Complex $K_3\ [Co(CN)_6]$ have $CN^-$ ligand which is a strong field ligand amongst the given ligands in all complexes, therefore CFSE is highest.

**Q.6** For the following reaction, the mass of water produced from 4.45 g of $C_{57}H_{110}O_6$ is :

$2C_{57}H_{110}O_6(s) + 163O_2(g) \rightarrow 114CO_2(g) + 110\ H_2O(\ell)$

(1) 445g (2) 490 g (3) 495g (4) 890 g

**Ans.** **[3]**

**Sol.** Moles of $C_{57}H_{10}O_6 = \frac{445}{890} = 0.5$

$$\frac{n_{C_{57}H_{110}O_6}}{2} = \frac{n_{H_2O}}{110}$$

$$n_{H_2O} = 55 \times 0.5 \times 18 = 495 \text{ gm}$$

**Q.7** If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction

$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$

At 300 K is approximately

($R = 8JK^{-1}\ mol^{-1}$, $F = 96000\ C\ mol^{-1}$)

(1) $e^{160}$ (2) $e^{-160}$ (3) $e^{-80}$ (4) $e^{320}$

**Ans.** **[1]**

**Sol.** $\Delta G^o = -RT\ \ell nk$

$$-nF\ E^o_{cell} = -\ RT\ \ell nK$$

$$-\ 2 \times 96000 \times 2 = -8 \times 300 \times \ell nk$$

$$k = e^{160}$$

**Q.8** The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is : (Specific heat of water liquid and water vapour are 4.2 kJ $K^{-1}$ $kg^{-1}$ and 2.0 kJ $K^{-1}$ $kg^{-1}$ heat of liquid fusion and vapourisation of water are 334 kJ $kg^{-1}$ and 2491 kJ $kg^{-1}$, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)

(1) 2.64 kJ $kg^{-1}$ $K^{-1}$ (2) 9.26 kJ $kg^{-1}$ $K^{-1}$ (3) 7.90 kJ $kg^{-1}$ $K^{-1}$ (4) 8.49 kJ $kg^{-1}$ $K^{-1}$

**Ans.** **[2]**

**Sol.**

$$\underset{273K}{H_2O_{(s)}} \rightarrow \underset{273K}{H_2O_{(\ell)}} \quad \Delta S_1 = \frac{\Delta H_{fusion}}{T} = \frac{334}{273} = 1.22$$

$$\underset{273K}{H_2O_{(\ell)}} \rightarrow \underset{373K}{H_2O_{(\ell)}} \quad \Delta S_2 = 4.2 \times \ell n\left(\frac{373}{273}\right) = 1.31$$

$$\underset{373K}{H_2O_{(\ell)}} \rightarrow \underset{373K}{H_2O_{(g)}} \quad \Delta S_3 = \frac{\Delta H_{vap}}{T} = \frac{2491}{373} = 6.67$$

$$\underset{373K}{H_2O_{(g)}} \rightarrow \underset{383K}{H_2O_{(g)}} \quad \Delta S_4 = 2\ell n\left(\frac{383}{373}\right) = .0529$$

$\Delta S_{total}$ = 9.26 kJ $kg^{-1}$ $K^{-1}$

**Q.9** For the reaction, 2A + B $\rightarrow$ products, when the concentrations of A and B both were doubled, the rate of the reaction increased from 0.3 mol $L^{-1}$ $s^{-1}$ to 2.4 mol $L^{-1}$ $s^{-1}$. When the concentration of A alone is doubled, the rate increased from 0.3 mol $L^{-1}$ $s^{-1}$ to 0.6 mol $L^{-1}$ $s^{-1}$. Which one of the following statements is correct?

(1) Order of the reaction with respect to A is 2.
(2) Total order of the reaction is 4.
(3) Order of the reaction with respect to B is 1.
(4) Order of the reaction with respect to B is 2.

**Ans.** **[4]**

**Sol.** $r = k [A]^x [B]^y$

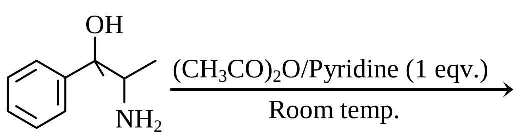
$8 \propto (2)^x (2)^y$ ….(1)

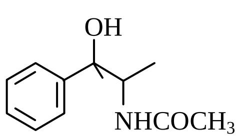
$2 \propto (2)^x$ ….(2)

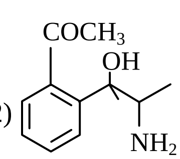
$\therefore$ x = 1 y = 2

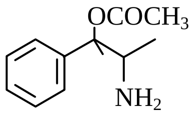
$\therefore$ order w.r.t. B is two

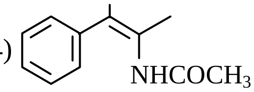
**Q.10** The major product obtained in the following reaction is :




(1) 

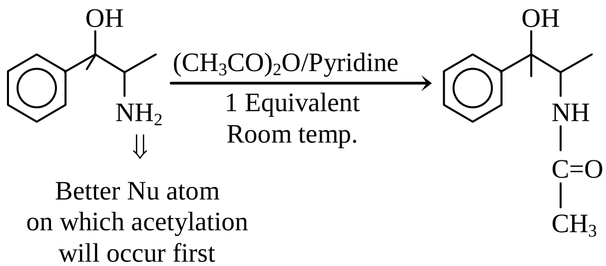

(2) 

(3) 


(4) 


**Ans.** **[1]**

**Sol.**




**Q.11** The increasing basicity order of the following compounds is :

(A) $CH_3CH_2NH_2$ (B) $CH_3CH_2\underset{|}{N}H$ with $CH_2CH_3$ on N (C) $H_3C-\underset{|}{N}-CH_3$ with $CH_3$ on N (D) $Ph-\underset{|}{N}-H$ with $CH_3$ on N

(1) (A) < (B) < (C) < (D) (2) (D) < (C) < (B) < (A)

(3) (D) < (C) < (A) < (B) (4) (A) < (B) < (D) < (C)

**Ans.** **[3]**

**Sol.** Basic strength

$$\underbrace{CH_3-CH_2-CH_2-\overset{2^\circ}{NH}-CH_2CH_3 > CH_3-CH_2-\overset{1^\circ}{NH_2}}_{\ell.p.\text{ do not participate in reason}} > CH_3-\overset{CH_3}{\overset{|}{N}}-CH_3 > Ph-\overset{CH_3}{\overset{|}{N}}-H$$

$\ell$.p. participate in resonance

**Q.12** Homoleptic octahedral complexes of a metal ion '$M^{3+}$' with three monodentate ligands $L_1$, $L_2$ and $L_3$ absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is -

(1) $L_3 < L_2 < L_1$ (2) $L_3 < L_1 < L_2$ (3) $L_1 < L_2 < L_3$ (4) $L_2 < L_1 < L_3$

**Ans.** **[2]**

**Sol.** Order of $\lambda_{absorbed}$ = Red > Green > Blue (corresponding to $L_3 > L_1 > L_2$)

$$\left[\Delta_0 \propto \frac{1}{\lambda_{abs}}\right]$$

$\therefore$ Order of $\Delta_0$ will be $\Rightarrow$ $L_2 > L_1 > L_3$

**Q.13** When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is –141 kJ/mol, Its second electron gain enthalpy is :

(1) a positive value (2) a more negative value than the first

(3) almost the same as that of the first (4) negative, but less negative than the first

**Ans.** **[1]**

**Sol.** Second electron gain enthalpy is always positive for every element due to repulsion between some charge.

$O(g) + 1e^- \rightarrow e^-(g)\ \Delta H_{eg}^-\ –141$ kJ/mol

$O^-(g) + 1e^- \rightarrow O^{-2}(g)\ \Delta H = \oplus ve$

**Q.14** Consider the following reversible chemical reactions :

$$A_2(g) + B_2(g) \underset{}{\overset{K_1}{\rightleftharpoons}} 2AB(g) \quad \ldots(1)$$

$$6AB(g) \overset{K_2}{\rightleftharpoons} 3A_2(g) + 3B_2(g) \quad \ldots(2)$$

The relation between $K_1$ and $K_2$ is :

(1) $K_2 = K_1^{-3}$ (2) $K_1K_2 = \frac{1}{3}$ (3) $K_1K_2 = 3$ (4) $K_2 = K_1^{3}$

**Ans.** **[1]**

**Sol.** $K_1 = \frac{[AB]^2}{[A_2][B_2]}$ ....(1)

$K_2 = \frac{[A_2]^3[B_2]^3}{[AB]^6}$ ....(2)

From Eq. (1) & (2)

$K_2 = \left(\frac{1}{k_1}\right)^3$

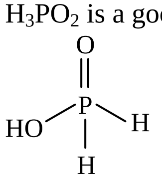
$\boxed{K_2 = K_1^{-3}}$

**Q.15** Good reducing nature of $H_3PO_2$ is attributed to the presence of :

(1) One P–H bond (2) Two P–OH bonds (3) One P–OH bond (4) Two P–H bonds

**Ans.** **[4]**

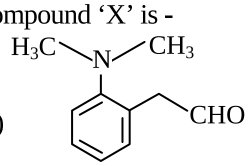
**Sol.** $H_3PO_2$ is a good reducing agent due to presence of two P–H bonds

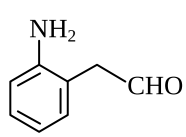


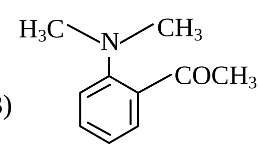

**Q.16** The tests performed on compound X and their inferences are :

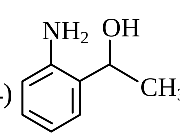
| | Test | Inference |
|---|---|---|
| (a) | 2, 4-DNP test | Coloured Precipitate |
| (b) | Iodoform test | Yellow Precipitate |
| (c) | Azo-dye test | No dye formation |

Compound 'X' is -

(1) 


(2) 


(3) 


(4) 

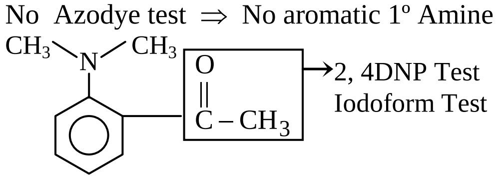

**Ans.** **[3]**

**Sol.**

2, 4 DNP Test $\Rightarrow$ Carbonyl confirm

Iodoform test $\Rightarrow$ $\underset{\underset{O}{\|}}{-C}-CH_3$ confirm

No Azodye test $\Rightarrow$ No aromatic 1° Amine



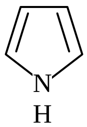
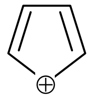
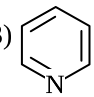
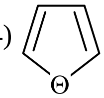
**Q.17** The pH rain water, is approximately :

(1) 5.6 (2) 6.5 (3) 7.5 (4) 7.0

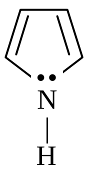
**Ans.** **[1]**

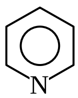
**Sol.** pH of rain water is approximately 5.6.

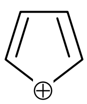
**Q.18** Which of the following compounds is not aromatic?

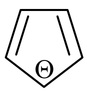
(1) 
(2) 
(3) 
(4) 

**Ans.** **[2]**

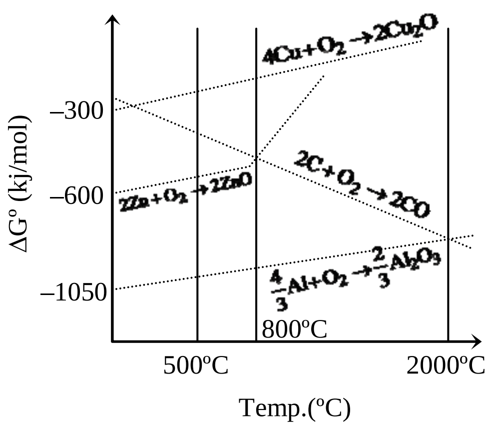
**Sol.** (1)  6π $e^-$ aromatic

(2)  6 π $e^-$ aromatic

(3)  4 π$e^-$ antiaromatic

(4)  6 π$e^-$aromatic

**Q.19** The correct statement regarding the given Ellingham diagram is :




(1) Coke cannot be used for the extraction of Cu from $Cu_2O$

(2) At 500ºC, coke can be used for the extraction of Zn from ZnO

(3) At 1400ºC, Al can be used for the extraction of Zn from ZnO

(4) At 800ºC Cu can be used for the extraction of Zn from ZnO

**Ans.** **[3]**

**Sol.** The element whichpresent below in diagram can replace element from oxide which is present above in the diagram. So according to diagram Al can reduce ZnO.

$3ZnO + 2Al \rightarrow 3Zn + Al_2O_3$

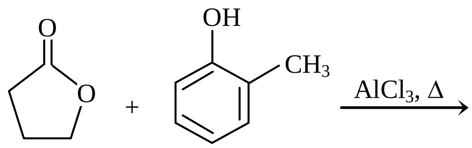
**Q.20** The temporary hardness of water is due to :

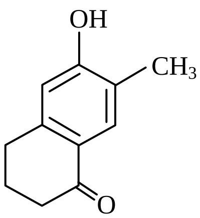
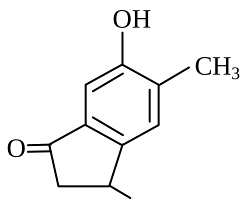
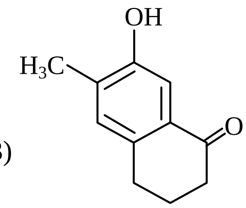
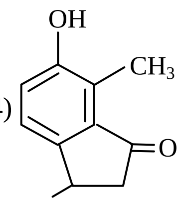
(1) $Na_2SO_4$ (2) $CaCl_2$ (3) NaCl (4) $Ca(HCO_3)_2$

**Ans.** **[4]**

**Sol.** Temporary hardness of water is due to presence of $Ca(HCO_3)_2$

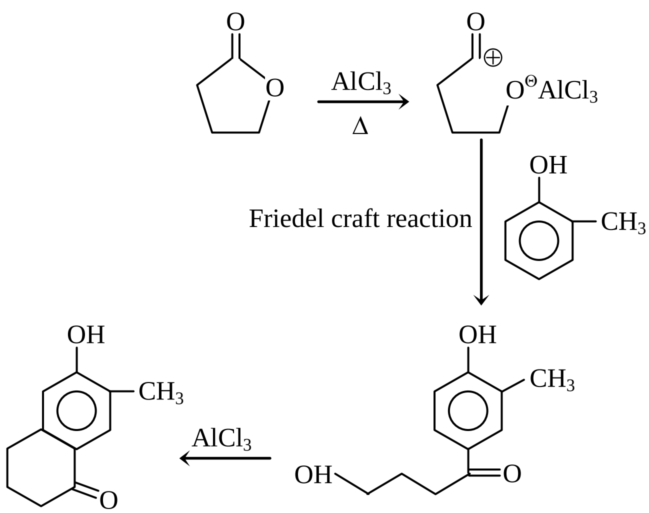
**Q.21** The major product of the following reaction is :



(1)  (2)  (3)  (4) 

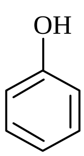
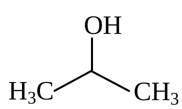
**Ans.** **[1]**

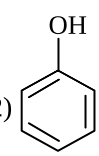
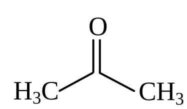
**Sol.**

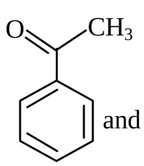


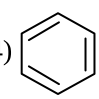
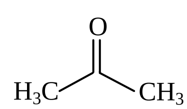

**As shown answer is 1 but NTA has given ans. 3 which is incorrect.**

**Q.22** The products formed in the reaction of cumene with $O_2$ followed by treatment with dil. HCl are :

(1)  and 

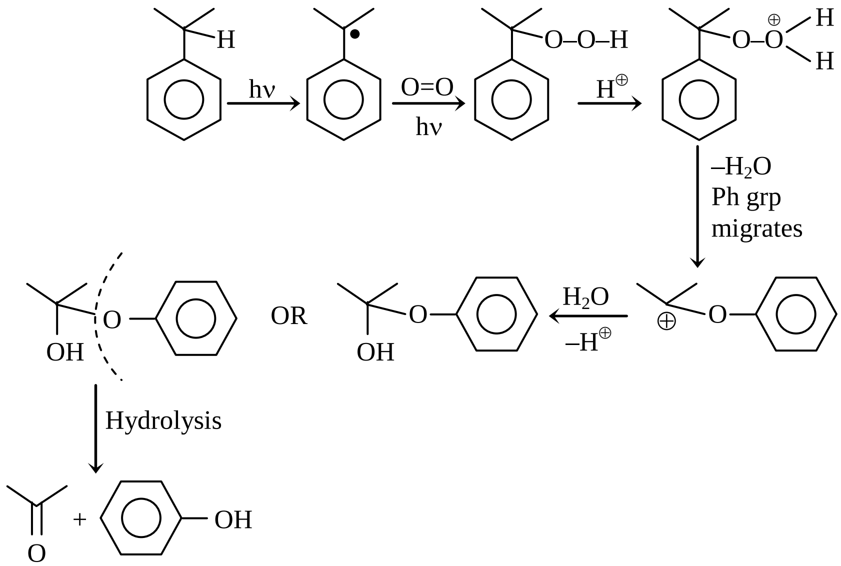
(2)  and 

(3)  and $CH_3 - OH$

(4)  and 

**Ans.** **[2]**

**Sol.**




**Q.23** At 100ºC, copper (Cu) has FCC unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g $cm^{-3}$) at this temperature? [Atomic Mass of Cu = 63.55u]

(1) $\frac{422}{x^3}$ (2) $\frac{205}{x^3}$ (3) $\frac{211}{x^3}$ (4) $\frac{105}{x^3}$

**Ans.** **[1]**

**Sol.** $d = \frac{Z \times M}{N_A \times a^3}$

For fcc unit cell Z = 4

$d = \frac{4 \times 63.55}{6.023 \times 10^{23} \times (x \times 10^{-8})^3}$

$\therefore \quad d \cong \frac{422}{x^3}$

**Q.24** The transition element that has lowest enthalpy of atomization, is :

(1) Zn (2) Fe (3) Cu (4) V

**Ans.** **[3]**

**Sol.** Zn is not a transition element

So transition element having lowest atomization energy out of Cu, V, Fe is Cu and therefore answer should be (3)

**but NTA has given Ans. 1 (Zn)**

**but Zn is not transition element.**

**Q.25** Which of the following conditions in drinking water causes methemoglobinemia?

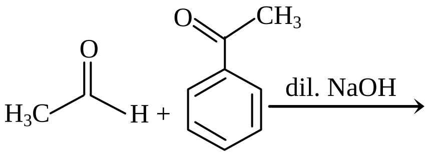
(1) > 50 ppm of nitrate (2) > 50 ppm of chloride

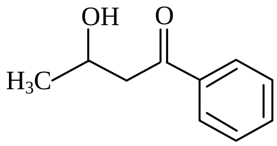
(3) > 50 ppm of lead (4) > 100 ppm of sulphate

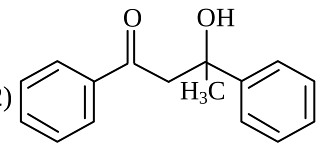
**Ans.** **[1]**

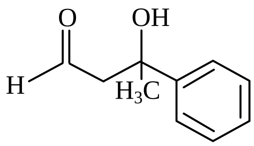
**Sol.** Concentration of nitrate > 50 ppm in drinking water cause methemoglobinemia.

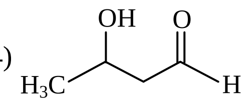
**Q.26** The major product formed in the following reaction is :




(1) 

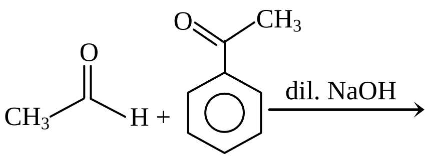

(2) 


(3) 

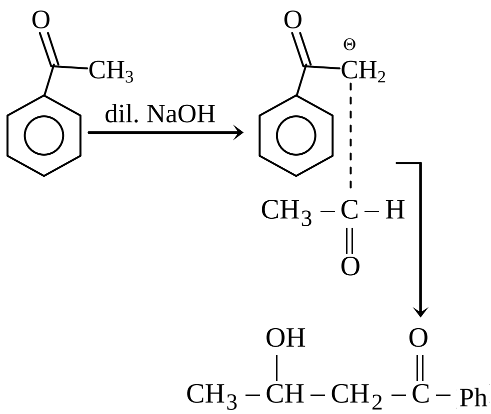

(4) 


**Ans.** **[1]**

**Sol.**




When one aldhyde and one ketone is present then in major product ketone attack on aldehyde




(β-Hydroxy carbonyl)

**Q.27** The correct match between Item-I and Item-II is :

| Item-I | | Item-II | |
|---|---|---|---|
| (A) | Benzaldehyde | (P) | Mobile phase |
| (B) | Alumina | (Q) | Adorbent |
| (C) | Acetonitrile | (R) | Adsorbate |

(1) (A) → (Q); (B) → (P) ; (C) → (R)
(2) (A) → (Q); (B) → (R) ; (C) → (P)
(3) (A) → (P); (B) → (R) ; (C) → (Q)
(4) (A) → (R); (B) → (Q) ; (C) → (P)

**Ans.** **[4]**

**Sol.**

| | |
|---|---|
| Benzaldehyde | Adsorbate |
| Alumina | Adsorbent |
| Acetonitrile | Mobile phase |

**Q.28** The metal that forms nitride by reacting directly with $N_2$ of air, is :

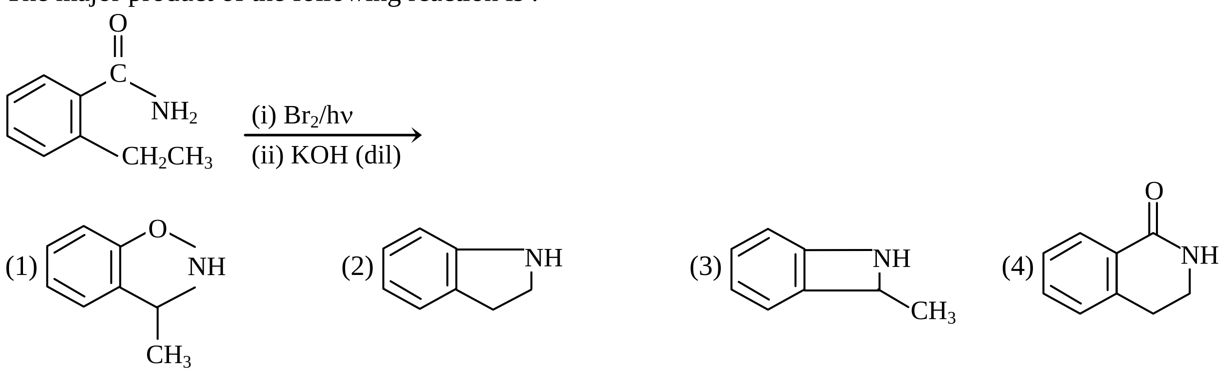
(1) Li (2) Cs (3) K (4) Rb

**Ans.** **[1]**

**Sol.** Only Li directly react with $N_2$

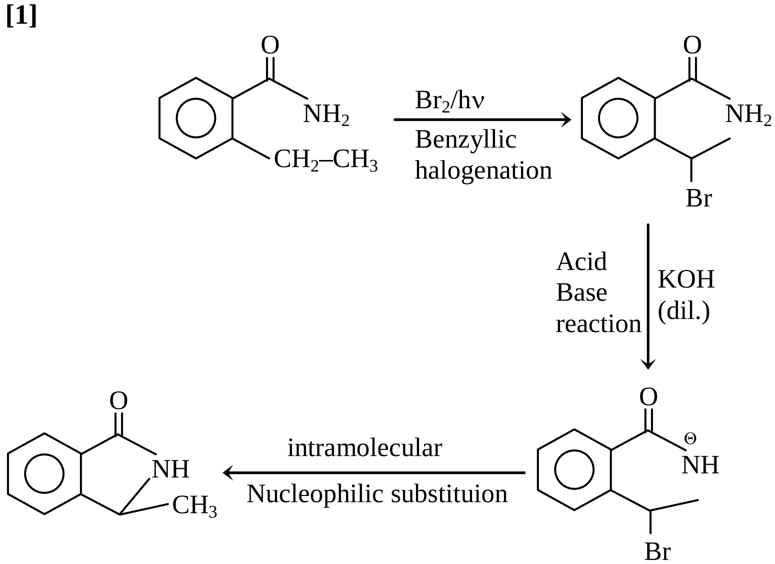
$6Li + N_2 \rightarrow 2Li_3N$

**Q.29** The major product of the following reaction is :

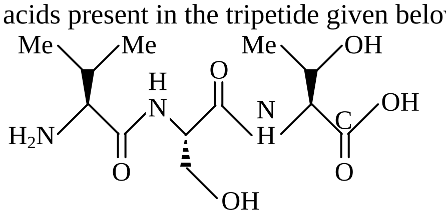


**Ans.** **[1]**

**Sol.**



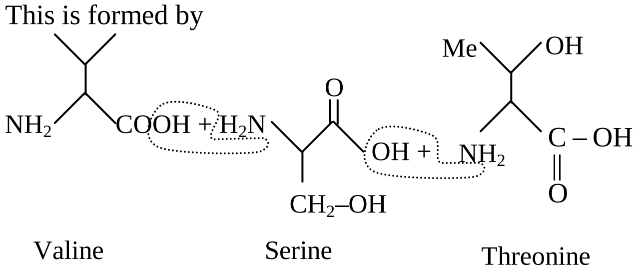
**Q.30** The correct sequence of amino acids present in the tripetide given below is :



(1) Val - Ser - Thr (2) Thr - Ser - Val (3) Thr - Ser - Leu (4) Leu - Ser - Thr

**Ans.** **[1]**

**Sol.** This is formed by



# CAREER POINT

# JEE Main Online Exam 2019

## Questions & Solutions

9th January 2019 | Shift - II

## MATHEMATICS

**Q.1** Let $z_0$ be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i\,z_0^{81} - 3i\,z_0^{93}$, then arg z is equal to :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{4}$

**Ans.** **[4]**

**Sol.** $1 + x + x^2 = 0$

$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

$z_0 = w, w^2$

Now $z = 3 + 6i\,z_0^{81} - 3i\,z_0^{93}$

$z = 3 + 6iw^{81} - 3iw^{93}$ $(w^{93} = w^{81} = 1)$

$\Rightarrow$ $z = 3 + 3i$

then $\arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

**Q.2** Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to :

(1) 2 (2) 3 (3) 5 (4) 4

**Ans.** **[2]**

**Sol.** If $f(xy) = f(x)\,f(y)\ \forall\ x, y \in R$ and $f(0) \neq 0$

put $x = y = 0$

$\Rightarrow f(0) = [f(0)]^2$

$\Rightarrow f(0) = 1$

put $y = 0 \Rightarrow f(0) = f(x)\,f(0)$

$\Rightarrow f(x) = 1$

given that $\frac{dy}{dx} = f(x)$

$\therefore \frac{dy}{dx} = 1 \Rightarrow y = x + k$

given that y(0) = 1

$\therefore k = 1$

hence y = x + 1

$y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \left(\frac{1}{4}+1\right) + \left(\frac{3}{4}+1\right) = 3$

**Q.3** The coefficient of $t^4$ in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is :

(1) 15 (2) 10 (3) 14 (4) 12

**Ans.** **[1]**

**Sol.** $\left(\frac{1-t^6}{1-t}\right)^3$

$= (1 - t^6)^3 (1 - t)^{-3}$

$= (1 - {}^3C_1t^6 + {}^3C_2t^{12} - {}^3C_3t^{18}) \times (1 - t)^{-3}$

coefficient of $t^4$ is 1 × coefficient of $t^4$ in $(1 - t)^{-3}$

$= 1 \times {}^{3+4-1}C_4$ (By multinomial theorem)

$= {}^6C_4 = 15$

**Q.4** Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to :

(1) 4 (2) $\sqrt{22}$ (3) 6 (4) $\sqrt{32}$

**Ans.** **[3]**

**Sol.** Projection of $\vec{b}$ on $\vec{a}$ is $\vec{a}$

$\therefore \frac{\vec{b}.\vec{a}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow \frac{b_1 + b_2 + 2}{2} = 2$

$\Rightarrow b_1 + b_2 = 2$ … (1)

and $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$

$\Rightarrow (\vec{a} + \vec{b}).\vec{c} = 0$

$\Rightarrow 5(b_1 + 1) + (b_2 + 1) + \sqrt{2}(2\sqrt{2}) = 0$

$\Rightarrow 5b_1 + b_2 + 10 = 0$ … (2)

solving (1) & (2)

$b_1 = -3$ and $b_2 = 5$

$\Rightarrow |\vec{b}| = \sqrt{9+25+2} = 6$

**Q.5** If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, $(x \ge 0)$, and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

**Ans.** **[3]**

**Sol.** $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$

$f(x) = \int \frac{5x^8 + 7x^6}{x^{14}(x^{-5} + x^{-7} + 2)^2} dx$

$f(x) = \int \frac{5x^{-6} + 7x^{-8}}{(x^{-5} + x^{-7} + 2)^2} dx$

$x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8})dx = dt$

$(5x^{-6} + 7x^{-8})dx = -dt$

$f(x) = \int \frac{-dt}{t^2} = \frac{1}{t} + c$

$f(x) = \frac{1}{x^{-5} + x^{-7} + 2} + c$

$f(x) = \frac{x^7}{x^2 + 1 + 2x^7} + c$

$f(0) = 0 \quad \therefore \quad c = 0$

$f(x) = \frac{x^7}{(x^2 + 1 + 2x^7)}$

$f(1) = \frac{1}{1 + 1 + 2} = \frac{1}{4}$

**Q.6** If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is :

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{6}$ (3) $\frac{1}{6\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

**Ans.** **[3]**

**Sol.** $x = 3 \tan t$ and $y = 3 \sec t$

So that $\frac{dx}{dt} = 3\sec^2 t$ and $\frac{dy}{dt} = 3 \sec t \tan t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \sin t$

$\frac{d^2y}{dx^2} = (\cos t).\ \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = (\cos t)\cdot\frac{1}{3\sec^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3}(\cos^3 t)$$

$$\left(\frac{d^2y}{dx^2}\right)_{t=\pi/4} = \frac{1}{3}\times\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{6\sqrt{2}}$$

**Q.7** The sum of the following series

$1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+.....+5^2)}{11} + .....$ up to 15 terms, is :

(1) 7830 (2) 7520 (3) 7820 (4) 7510

**Ans.** **[3]**

**Sol.** $1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+.....+5^2)}{11} + .....$ 15 terms

$= \frac{3(1^2)}{3} + \frac{6(1^2+2^2)}{5} + \frac{9(1^2+2^2+3^2)}{7} + \frac{12}{9}(1^2+2^2+3^2+4^2) + .....$

$T_r = \frac{3r}{2r+1}(1^2+2^2+...+r^2)$

$T_r = \frac{3r}{2r+1}\,\frac{r(r+1)(2r+1)}{6} = \frac{1}{2}r^2(r+1)$

Sum of n terms $= \sum_{r=1}^{n} T_r = \frac{1}{2}\sum_{r=1}^{n}(r^3+r^2)$

$= \frac{1}{2}\left[\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}\right]$

Sum upto 15 terms $\Rightarrow$ then put n = 15

$= \frac{1}{2}\left(\frac{(15\times16)^2}{4} + \frac{15\times16\times31}{6}\right) = 7820$

**Q.8** Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to :

(1) $\frac{7}{13}$ (2) 2 (3) 4 (4) $\frac{1}{2}$

**Ans.** **[3]**

**Sol.** $T_7 = A + 6d = a$; $T_{11} = A + 10d = b$; $T_{13} = A + 12d = c$

Now a, b, c are in G.P.

$\therefore b^2 = ac$

$\Rightarrow (A + 10d)^2 = (A + 6d)(A + 12d)$

$\Rightarrow A^2 + 100d^2 + 20Ad = A^2 + 18Ad + 72d^2$

$\Rightarrow A + 14d = 0,\ A = -14d$

$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{-8d}{-2d} = 4$

**Q.9** A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{3}{2}$ (2) $\frac{2}{\sqrt{3}}$ (3) 2 (4) $\sqrt{3}$

**Ans.** **[2]**

**Sol.** Let the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given 2a = 4

$\Rightarrow a = 2$

It passes through (4, 2)

$$\therefore \frac{16}{4} - \frac{4}{b^2} = 1$$

$$\Rightarrow b^2 = \frac{4}{3}$$

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4/3}{4}}$$

$$= \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$$

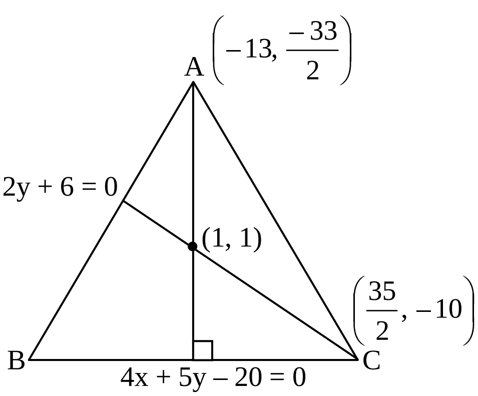
**Q.10** Let the equations of two sides of a triangle be 3x – 2y + 6 = 0 and 4x + 5y – 20 = 0. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is :

(1) 26x – 122y – 1675 = 0 (2) 26x + 61y + 1675 = 0

(3) 122y – 26x – 1675 = 0 (4) 122y + 26x + 1675 = 0

**Ans.** **[1]**

**Sol.**




4x + 5y – 20 = 0 … (1)

3x – 2y + 6 = 0 … (2)

orthocentre is (1, 1)

line perpendicular to 4x + 5y – 20 = 0 and passes through (1, 1) is $(y - 1) = \frac{5}{4}(x - 1)$

$\Rightarrow 5x - 4y = 1$ …(3)

and line ⊥ to 3x – 2y + 6 = 0 and passes through (1, 1)

$y - 1 = -\frac{2}{3}(x - 1)$

$\Rightarrow 2x + 3y = 5$ … (4)

Solving (1) and (4) we get $C\left(\frac{35}{2}, -10\right)$

Solving (2) and (3) we get $A\left(-13, \frac{-33}{2}\right)$

Side BC is $y + 10 = \frac{\frac{-33}{2} + 10}{-13 - \frac{35}{2}}\left(x - \frac{35}{2}\right)$

$\Rightarrow y + 10 = \frac{13}{61}\left(x - \frac{35}{2}\right)$

$\Rightarrow 26x - 122y - 1675 = 0$

**Q.11** A data consists of n observations :

$x_1, x_2, \ldots.., x_n$ . If $\sum_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5n$, then the standard deviation of this data is :

(1) 2 (2) $\sqrt{5}$ (3) 5 (4) $\sqrt{7}$

**Ans.** **[2]**

**Sol.** $\sum_{i=1}^{n}(x_i + 1)^2 = 9n \Rightarrow \sum_{i=1}^{n}x_i^2 + 2\sum_{i=1}^{n}x_i + n = 9n$ …(1)

$\sum_{i=1}^{n}(x_i - 1)^2 = 5n \Rightarrow \sum_{i=1}^{n}x_i^2 - 2\sum_{i=1}^{n}x_i + n = 5n$ … (2)

(1) + (2)

$\Rightarrow 2\sum_{i=1}^{n}x_i^2 + 2n = 14n$

(1) – (2)

$\Rightarrow 4\sum_{i=1}^{n}x_i = 4n$

$\Rightarrow \sum_{i=1}^{n}x_i = n$

S.D. $\sigma = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$

$\sigma = \sqrt{\frac{6n}{n} - (1)}$

$\sigma = \sqrt{5}$

**Q.12** If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x – sin 2x + sin 3x = 0, is :

(1) 2 (2) 4 (3) 3 (4) 1

**Ans.** **[1]**

**Sol.** $\sin x - \sin 2x + \sin 3x = 0 \qquad x \in \left[0, \frac{\pi}{2}\right)$

$\Rightarrow (\sin 3x + \sin x) - \sin 2x = 0$

$\Rightarrow 2\sin 2x.\cos 2x - \sin 2x = 0$

$\Rightarrow \sin 2x\,(2\cos x - 1) = 0$

$\sin 2x = 0$ and $\cos x = \frac{1}{2}$

$x = 0$ and $x = \frac{\pi}{3}$

two solutions

**Q.13** If the lines x = ay + b, z = cy + d and x = a′z + b′, y = c′z + d′ are perpendicular then :

(1) cc′ + a + a′ = 0 (2) aa′ + c + c′ = 0 (3) bb′ + cc′ + 1 = 0 (4) ab′ + bc′ + 1 = 0

**Ans.** **[2]**

**Sol.** Equation of 1st line is

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

Equation of 2nd line is

$$\frac{x-b'}{a'} = \frac{y-b'}{c'} = \frac{z}{1}$$

Lines are perpendicular

so that aa′ + c + c′ = 0

**Q.14** The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is :

(1) 3x + 2y – 3z = 0 (2) x + 2y – 2z = 0 (3) x – 2y + z = 0 (4) 5x + 2y – 4z = 0

**Ans.** **[3]**

**Sol.** Vector ⊥ to given plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}$

$= \hat{i}(12-4) - \hat{j}(9-8) + \hat{k}(6-16)$

$= 8\hat{i} - \hat{j} - 10\hat{k}$ … (1)

Vector parallel to given line $= 2\hat{i} + 3\hat{j} + 4\hat{k}$ … (2)

Vector ⊥ to both (1) & (2) vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(-4 + 30) - \hat{j}(32 + 20) + \hat{k}(24 + 2)$

$= 26\hat{i} - 52\hat{j} + 26\hat{k}$

Dr's of normal of required plane is (26, –52, 26) ⇒ (1, –2, 1)

Equation of plane whose Dr's of Normal is (1, –2, 1) and passes through origin

$1.(x - 0) - 2(y - 0) + 1.(z - 0) = 0$

$x - 2y + z = 0$

**Q.15** Let f be a differentiable function from R to R such that $|f(x) - f(y)| \le 2|x - y|^{3/2}$, for all x, y ∈ R. If f(0) = 1 then $\int_0^1 f^2(x)\,dx$ is equal to :

(1) 2 (2) $\frac{1}{2}$ (3) 0 (4) 1

**Ans.** **[4]**

**Sol.** $|f(x) - f(y)| \le 2[x - y]^{3/2}$

$$\left|\frac{f(x) - f(y)}{x - y}\right| \le 2|x - y|^{1/2}$$

$$\lim_{y \to x}\left|\frac{f(x) - f(y)}{x - y}\right| \le \lim_{y \to x} 2|x - y|^{1/2}$$

$\Rightarrow |f'(x)| \le 0 \Rightarrow f'(x) = 0$

⇒ f(x) = constant

as f(0) = 1 ⇒ f(x) = 1

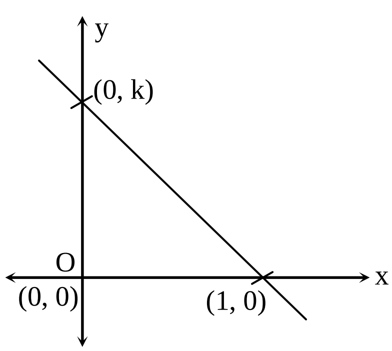
$$\int_0^1 f^2(x)\,dx = 1$$

**Q.16** Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is :

(1) 32 (2) 36 (3) 9 (4) 18

**Ans.** **[2]**

**Sol.**



Area = $\frac{1}{2}$ h. k = 50

h. k = 100

h. k = $2^2 . 5^2$

Total divisors = (2 + 1) (2 + 1) = a

if h > 0, k > 0

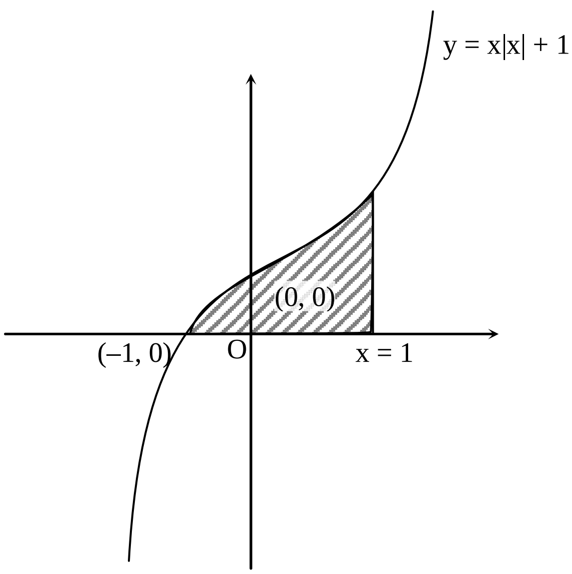
But $\left.\begin{matrix} h > 0, & k < 0 \\ h < 0, & k > 0 \\ h < 0, & k < 0 \end{matrix}\right\}$ all are possible so that total no. of positive case 9 + 9 + 9 + 9 = 36

**Q.17** The area of the region A = {(x, y) : $0 \le y \le x|x| + 1$ and $-1 \le x \le 1$} in sq. units, is :

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) 2

**Ans.** **[4]**

**Sol.**




Required area

$$= \int_{-1}^{1} (x\,|x| + 1)\,dx$$

$$= 0 + (x)_{-1}^{1} = 2$$

**Q.18** If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then

(1) r > 11 (2) r = 11 (3) 1 < r < 11 (4) 0 < r < 1

**Ans.** **[3]**

**Sol.** Circles are $x^2 + y^2 - 16x - 20y + 164 = r^2 \Rightarrow c_1$ (8, 10)

and $(x - 4)^2 + (y - 7)^2 = 36$

they intersect at two distinct points

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$ $\{c_1c_2 = \sqrt{16+9} = 5\}$

Now $|r - 6| < 5 < r + 6$

| | |
|---|---|
| $|r - 6| < 5$ | $5 < r + 6$ |
| $\Rightarrow -5 < r - 6 < 5$ | $-1 < r$ … (ii) |

$\Rightarrow 1 < r < 11$ … (i)

from (i) and (ii)

$r \in (1, 11)$

**Q.19** An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{27}{49}$ (2) $\frac{26}{49}$ (3) $\frac{32}{49}$ (4) $\frac{21}{49}$

**Ans.** **[3]**

**Sol.** 5 Red and 2 green balls

P(one red ball) $= \frac{5}{7}$

P(one green ball) $= \frac{2}{7}$

**Case I :**

If drawn ball is green than a red ball is added

$\begin{pmatrix} 6\,\text{Red} \\ 1\,\text{Green} \end{pmatrix}$ P (red ball) $= \frac{6}{7}$

**Case II :**

If drawn ball is red then a green ball is added

$\begin{pmatrix} 4\,\text{Red} \\ 3\,\text{Green} \end{pmatrix}$ P(red ball) $= \frac{4}{7}$

P($2^{nd}$ red ball) $= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$

**Q.20** If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ then A is :

(1) invertible only if $t = \pi$

(2) invertible for all $t \in R$

(3) invertible only if $t = \frac{\pi}{2}$

(4) not invertible for any $t \in R$

**Ans.** **[2]**

**Sol.** $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$

$|A| = e^t.\, e^{-t}.\, e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$

Apply operations $R_2 < R_2 - R_1$, $R_3 < R_3 - R_1$, $R_1 < R_1$

$|A| = e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t - 2\cos t & -2\sin t + \cos t \\ 0 & 2\sin t - \cos t & -2\cos t - \sin t \end{vmatrix}$

Open the determinant by $R_1$

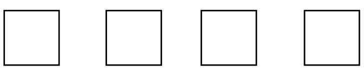
$|A| = 5e^{-t}$

Invertible for all $t \in R$

**Q.21** The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to:

(1) 372 (2) 375 (3) 250 (4) 374

**Ans.** **[4]**

**Sol.** 

There are 3 ways to fill I position (0, 1, 3)

for remaining digits there are 5 ways

hence ans = 3. 5. 5. 5 – 1 (0000 not included)

= 375 – 1 = 374

**Q.22** If $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\, d\theta = 1 - \frac{1}{\sqrt{2}}$, (k > 0), then the value of k is :

(1) 2 (2) 1 (3) 4 (4) $\frac{1}{2}$

**Ans.** **[1]**

**Sol.** $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\, d\theta = 1 - \frac{1}{\sqrt{2}}$ (k > 0)

$\Rightarrow \int_0^{\pi/3} \frac{\sin\theta}{\sqrt{2k}\sqrt{\cos\theta}} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2k}} \left(-2\sqrt{\cos\theta}\right)_0^{\pi/3} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2k}}(-\sqrt{2}+2) = 1-\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{2}(\sqrt{2}-1)}{\sqrt{2k}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\Rightarrow k = 2$

**Q.23** The number of all possible positive integral values of $\alpha$ for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :

(1) 4 (2) 2 (3) 5 (4) 3

**Ans.** **[4]**

**Sol.** For rational D must be perfect square

$D = 121 - 24\alpha$

for $121 - 24\alpha$ to be perfect square a must be 3, 4, 5 So, ans $\alpha = 3$

**Q.24** The logical statement $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to :

(1) $(\sim p \wedge \sim q) \wedge r$ (2) $(p \wedge \sim q) \vee r$ (3) $(p \wedge r) \wedge \sim q$ (4) $\sim p \wedge r$

**Ans.** **[3]**

**Sol.**

| p | q | r | $\sim p$ | $\sim p \vee q$ | $\sim(\sim p \vee q)$ | $(p \wedge r)$ | $\sim q$ | $\sim q \wedge r$ | $[\sim(\sim p \vee q) \vee (p \wedge r)]$n $(\sim q \wedge r)$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | F | T | F | F | F |
| T | T | F | F | T | F | F | F | F | F |
| T | F | T | F | F | T | T | T | T | T |
| T | F | F | F | F | T | F | T | F | F |
| F | T | T | T | T | F | F | F | F | F |
| F | T | F | T | T | F | F | F | F | F |
| F | F | T | T | T | F | F | T | T | F |
| F | F | F | T | T | F | F | T | F | F |

By option

| $(p \wedge r) \wedge \sim q$ |
|---|
| F |
| F |
| T |
| F |
| F |
| F |
| F |
| F |

**Q.25** If the system of linear equations

$$x - 4y + 7z = g$$
$$3y - 5z = h$$
$$-2x + 5y - 9z = k$$

is consistent, then :

(1) $g + 2h + k = 0$ (2) $g + h + 2k = 0$ (3) $g + h + k = 0$ (4) $2g + h + k = 0$

**Ans.** **[4]**

**Sol.**

$$x - 4y + 7z = g$$
$$3y - 5z = h$$
$$-2x + 5y - 9z = k$$

$$D = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix}$$

$D = 1(-27 + 25) - 2(20 - 21)$

$D = -2 + 2 = 0$

If system is consistent then $D_1 = D_2 = D_3 = 0$

$$\begin{vmatrix} 1 & -4 & g \\ 0 & 3 & h \\ -2 & 5 & k \end{vmatrix} = 0$$

$1(3k - 5h) - 2(-4h - 3g) = 0$

$3k - 5h + 8h + 6g = 0$

$6g + 3h + 3k = 0$

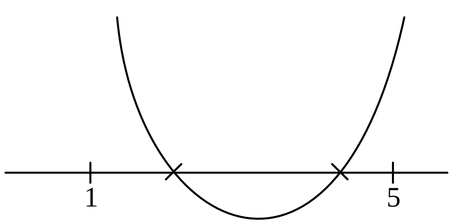
$2g + h + k = 0$

**Q.26** If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval :

(1) (4, 5) (2) (–5, –4) (3) (3, 4) (4) (5, 6)

**Ans.** **[1]**

**Sol.** $x^2 - mx + 4 = 0$




**Case-I :**

$D > 0$

$m^2 - 16 > 0$

$\Rightarrow m \in (-\infty, -4), (4, \infty)$

**Case-II :**

$\Rightarrow 1 < \frac{-b}{2a} < 5$

$\Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2, 10)$

**Case-III :**

$f(1) > 0$ and $f(5) > 0$

$1 - m + 4 > 0$ and $25 - 5m + 4 > 0$

$m < 5$ and $m < \frac{29}{5}$

**Case-IV :**

Let one root is $x = 1$

$1 - m + 4 = 0$

$m = 5$

Now equation $x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1$ i.e. $m = 5$ is also included

hence $m \in (4, 5]$

So given option is (4, 5)

**Q.27** If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to :

(1) 0 (2) $7\pi$ (3) $\pi$ (4) 10

**Ans.** **[3]**

**Sol.** $x = \sin^{-1} \sin 10 = 3\pi - 10$

$y = \cos^{-1} \cos 10 = 4\pi - 10$

$y - x = (4\pi - 10) - (3\pi - 10) = \pi$

**Q.28** Let $A = \{x \in R : x$ is not a positive integer$\}$. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is :

(1) not injective (2) surjective but not injective

(3) injective but not surjective (4) neither injective nor surjective

**Ans.** **[3]**

**Sol.** $f(x) = \frac{2x}{x-1}$

$f(x) = 2 + \frac{2}{x-1}$

$f'(x) = -\frac{2}{(x-1)^2} < 0 \ \forall \ x \in R$

Hence f(x) is strictly decreasing

So, f(x) is one-one

**Range :** let $y = \dfrac{2x}{x-1}$

$xy - y = 2x$

$\Rightarrow x(y - 2) = y$

$\Rightarrow x = \dfrac{y}{y-2}$

given that $x \in R$ : x is not a +ve integer

$\therefore \dfrac{y}{y-2} \neq N$ (N → Natural number)

$\Rightarrow y \neq Ny - 2N$

$\Rightarrow y \neq \dfrac{2N}{N-1}$

So range $\not\subseteq$ R (in to function)

**Q.29** For each $x \in R$, let [x] be the greatest integer less than or equal to x. Then $\lim\limits_{x \to 0^-} \dfrac{x([x]+|x|)\sin[x]}{|x|}$ is equal to :

(1) 1 (2) 0 (3) sin 1 (4) – sin 1

**Ans.** **[4]**

**Sol.** $\lim\limits_{x \to 0^-} \dfrac{x([x]+|x|)\sin[x]}{|x|}$

$= \lim\limits_{h \to 0} \dfrac{(0-h)([0-h]+|0-h|)\sin[0-h]}{|0-h|}$

$= \lim\limits_{h \to 0} \dfrac{-h(-1+h)\sin(-1)}{h}$

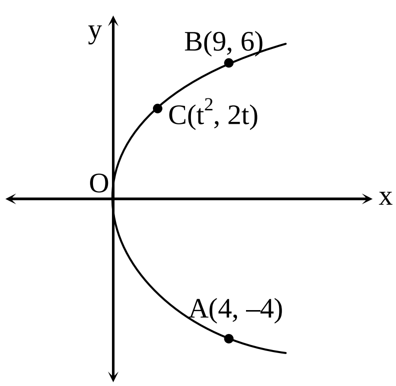
$= \lim\limits_{h \to 0} (-1 + h) \sin(1) = -\sin 1$

**Q.30** Let A(4, –4) and B(9, 6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is :

(1) 32 (2) $31\frac{3}{4}$ (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

**Ans.** **[4]**

**Sol.**



$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -4 & 1 \\ 9 & 6 & 1 \\ t^2 & 2t & 1 \end{vmatrix}$$

$D = 60 + 10t - 10t^2$

$\frac{d\Delta}{dt} = 0 \Rightarrow t = \frac{1}{2}$

$\frac{d^2\Delta}{dt^2} = -20 < 0$

$\therefore$ max at $t = \frac{1}{2}$

max area $\Delta = 65 - \frac{5}{2}$

$= \frac{125}{2} = 31\frac{1}{4}$