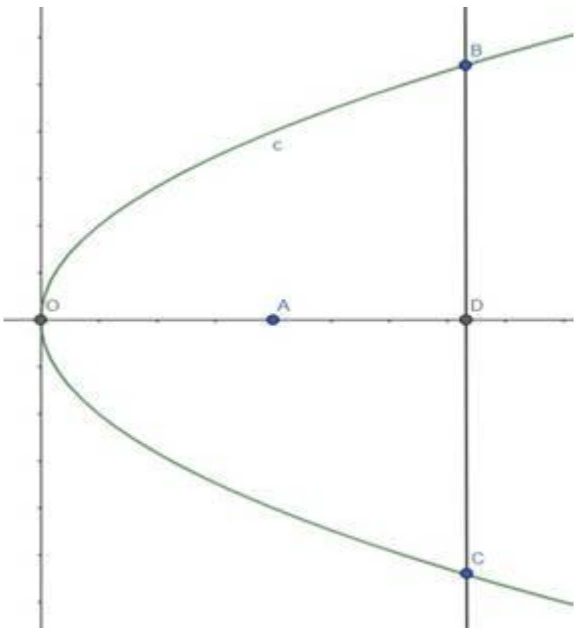
Applications of Conic Sections

Exercise 25

Q. 1. The focus of a parabolic mirror is at a distance of 6 cm from its vertex. If the mirror is 20 cm deep, find its diameter.

Answer : Given: The focus of a parabolic mirror is at a distance of 6 cm from its vertex. And the mirror is 20 cm deep.

Need to find: Diameter of the mirror.



Here O is the vertex and A is the Focus. So, OA = a = 6 cm.

OD is the deep of the mirror = 20 cm

BC is the diameter of the mirror.

Equation of the parabola is, $y^2 = 4ax$

$\Rightarrow y^2 = 24x$

The mirror is 20 cm deep. That means the x-coordinate of the points B, C and D is 20

Both the points, B and C are on the parabola. Hence, the points satisfies the equation of the parabola.

Therefore, $y^2 = 24 \times 20 = 480$

$\Rightarrow y = \pm 21.9$

So, the coordinate of B is (20, 21.9) and the coordinate of C is (20, -21.9).

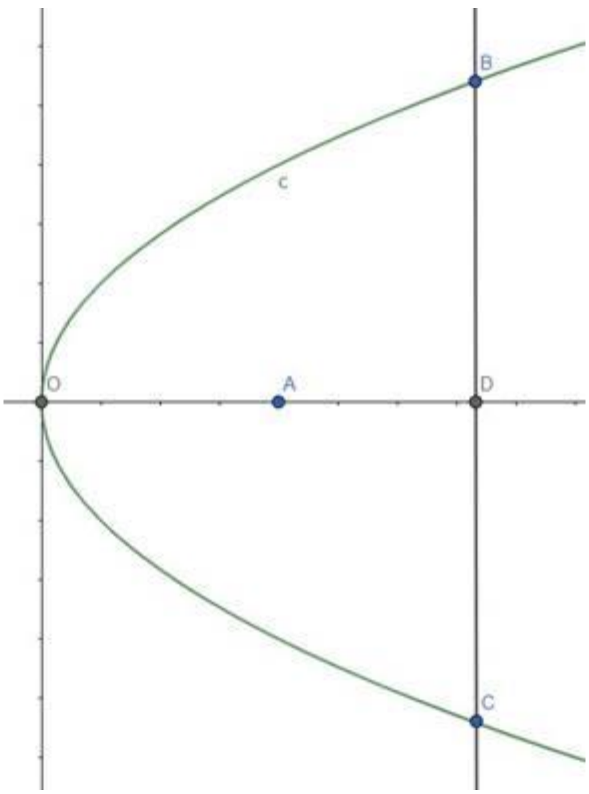
Therefore, the diameter of the mirror is = (21.9 + 21.9) cm

= 43.8 cm

Q. 2. A parabolic reflector is 5 cm deep and its diameter is 20 cm. How far is its focus from the vertex?

Answer : Given: Parabolic reflector is 5 cm deep & its diameter is 20 cm

Need to find: Distance of its focus from the vertex.



Reflector is 5 cm deep, i.e., OD = 5 cm

Diameter of the mirror is 20 com, i.e., BC = 20 cm

Let, the equation of the parabola is $y^2 = 4ax$, where a is the distance of the focus from the vertex.

The x-coordinate of the points B and C is 5.

D is the middle point of BC which is upon the x-axis.

So, we can say that BD = CD = 10 cm.

So, the coordinate of the point B is (5, 10)

Putting the values of the equation,

$y^2 = 4ax$

$\Rightarrow 100 = 4a \times 5$

$\Rightarrow 20a = 100$

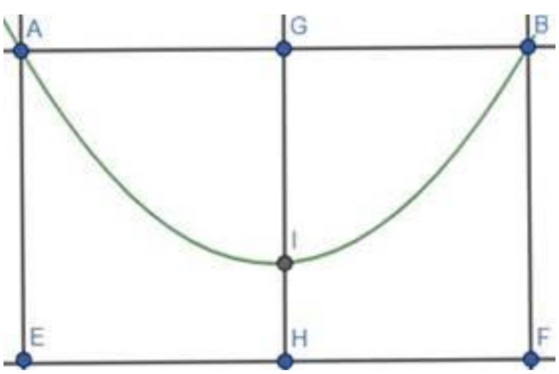
$\Rightarrow a = 5$

That means, the focus is 5 cm far from the vertex.

Q. 3. The towers of bridge, hung in the form of a parabola, have their tops 30 m above the roadway, and are 200 m apart. If the cable is 5 m above the roadway at the center of the bridge, find the length of the vertical supporting cable, 30 m from the center.

Answer : Given: Top of the towers are 30 m above the roadway and are 200 m apart. Cable is 5 m above the roadway at center.

Need to find: Length of the vertical supporting cable, 30 m from the center.



A and B are the top of the towers. AE and BF are the height of the towers. H is the center of the bridge. HI is the 5 m above from the roadway.

Let, the equation of the parabola be: $x^2 = 4a(y - b)$

Here b = 5. So, $x^2 = 4a(y - 5)$

Here, AB = 200 m and BF = 30 m.

So, the coordinate of the point B is (100, 30)

The point is on the parabola.

Hence, $x^2 = 4a(y - 5)$

$\Rightarrow 10000 = 4a (30 - 5)$

$\Rightarrow 10000 = 4a \times 25$

$\Rightarrow a = 100$

Now we need to find, the length of the vertical supporting cable, 30 m from the center.

The x-coordinate of the point, 30 m from the center, is 30.

So, $30 \times 30 = 4a (y - 5)$

$\Rightarrow 900 = 400 (y - 5)$

$$\Rightarrow y - 5 = \frac{9}{4}$$

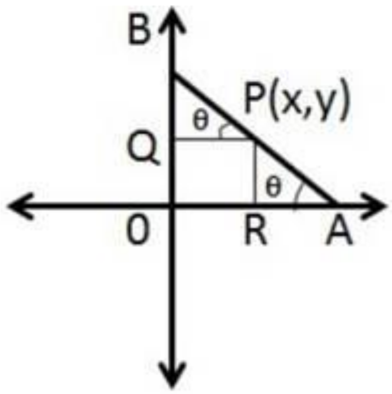
$$\Rightarrow y = \frac{9}{4} + 5 = \frac{29}{4}$$

So, the length of the vertical supporting cable is $\frac{29}{4}$ m = 7.25 m

Q. 4. A rod of length 15 cm moves with its ends always touching the coordinate axes. Find the equation of the locus of a point P on the rod, which is at a distance of 3 cm from the end in contact with the x-axis.

Answer : Given: A rod of length 15 cm moves with its ends always touching the coordinate axes. A point P on the rod, which is at a distance of 3 cm from the end in contact with the x-axis

Need to find: Find the equation of the locus of a point P



Here AB is the rod making an angle θ with the x-axis.

Here AP = 3.

PB = AB – AP = 12 – 3 = 9 cm

Here, PQ is the perpendicular drawn from the x-axis and RP is the perpendicular drawn from y-axis.

Let, the coordinates of the point P is (x, y).

Now, in the triangle ΔBPQ,

$$\cos\theta = \frac{x}{PB} = \frac{x}{9}$$

And in the triangle ΔPAR,

$$\sin\theta = \frac{y}{AP} = \frac{y}{3}$$

We know, $\sin^2\theta + \cos^2\theta = 1$

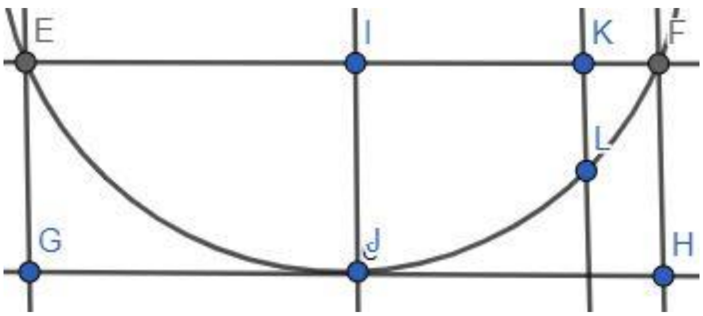
$$\Rightarrow \frac{x^2}{81}+\frac{y^2}{9}=1$$

This is the locus of the point P.

Q. 5. A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its center, there is a deflection of 3 cm at the center, and the deflected beam is in the shape of a parabola. How far from the center is the deflection 1 cm?

Answer : Given: A beam is supported at its ends by supports which are 12 m apart. There is a deflection of 3 cm at the center, and the deflected beam is in the shape of a parabola.

Need to find: How far from the center is the deflection 1 cm



Here EF are the ends of the beam and they are 12 m apart.

IJ is the deflection of 3 cm at the center.

We know, that the distance IF= $\frac{12}{2}$ = 6 m = 600 cm and the deflection IJ = FH = 3 cm.

So, the coordinate of the point F is (600, 3)

Let, the equation of the parabola is: $x^2 = 4ay$

F point is on the parabola. So, putting the coordinates of F in the equation we get,

$x^2 = 4ay$

$\Rightarrow 3600 = 4a \times 3$

$\Rightarrow a = 300$

Here KL denotes the deflection of 1 cm.

So, at the point L the value of y-coordinate is (3 – 1) = 2

So, by the equation,

$\Rightarrow x^2 = 4ay = 4 \times 300 \times 2 = 2400$

$\Rightarrow x = 49$ cm.

So, the distance of the point of 1 cm deflection from the center is 49 cm.