16. Parallelograms

Exercise 16A

1. Question

ABCD is a parallelogram in which ∠A =110°. Find the measure of each of the angles ∠B, ∠C and ∠D.

Answer

Since the sum of any two adjacent angles of a parallelogram is 180°,

∠A + ∠B = 180°

110° + ∠B = 180°

∠B = (180° - 110°) = 70°

Also, ∠B + ∠C = 180° [Since, ∠B and ∠C are adjacent angles]

70° + ∠C = 180°

∠C = (180° - 70°) = 110°.

Further, ∠C + ∠D = 180° [Since, ∠C and ∠D are adjacent angles]

110° + ∠D = 180°

∠D = (180° - 110°) = 70°.

Therefore, ∠B = 70°, ∠C = 110° and ∠D = 70°.

2. Question

Two adjacent angles of a parallelogram are equal. What is the measure of each of these angles?

Answer

Let ∠A, ∠B are the two adjacent angles.

According to question,

∠A = ∠B

As we know that, sum of any two adjacent angles of a parallelogram is 180°

∠A + ∠B = 180°

∠A + ∠A = 180°

2∠A = 180°

∠A = 90°

3. Question

Two adjacent angles of a parallelogram are in the ratio 4 : 5. Find the measure of each of its angles.

Answer

Let x be the common multiple.

Since the sum of any two adjacent angles of a parallelogram is 180°,

∠A + ∠B = 180°

4x + 5x = 180°

9x = 180°

x = 20°

∠A = 80°

∠B = 100°

Also, ∠B + ∠C = 180° [Since, ∠B and ∠C are adjacent angles]

100° + ∠C = 180°

∠C = (180° - 100°) = 80°

Further, ∠C + ∠D = 180° [Since, ∠C and ∠D are adjacent angles]

80° + ∠D = 180°

∠D = (180° - 80°) = 100°

Therefore, ∠A = 80°, ∠B = 100°, ∠C = 80° and ∠D = 100°.

4. Question

Two adjacent angles of a parallelogram are (3x - 4)° and (3x + 16)°. Find the value of x and hence find the measure of each of its angles.

Answer

Let ∠A = (3x - 4)°, ∠B = (3x + 16)°

Since the sum of any two adjacent angles of a parallelogram is 180°,

∠A + ∠B = 180°

(3x - 4)° + (3x + 16)° = 180°

6x + 12° = 180°

6x = 180° - 12°

6x= 168°

X= 168/6 = 28°

∠A = (3x - 4)° = 80°

∠B = (3x + 16)° = 100°

Also, ∠B + ∠C = 180° [Since, ∠B and ∠C are adjacent angles]

100° + ∠C = 180°

∠C = (180° - 100°) = 80°

Further, ∠C + ∠D = 180° [Since, ∠C and ∠D are adjacent angles]

80° + ∠D = 180°

∠D = (180° - 80°) = 100°

Therefore, x= 28°, ∠A = 80°, ∠B = 100°, ∠C = 80° and ∠D = 100°.

5. Question

The sum of two opposite angles of a parallelogram is 130°. Find the measure of each of its angles.

Answer

Let ABCD is a parallelogram.

∠ B + ∠ D = 130°

∠B + ∠B = 130° (Opposite angles of parallelogram are equal)

∠B = 65°

∠D = 65°

Now, ∠B + ∠C = 180° (sum of any two adjacent angles of a parallelogram is 180°)

∠C = 180° - ∠B = 115°

Now, ∠A = ∠C = 115° (Opposite angles of parallelogram are equal)

Therefore, ∠A = 115°, ∠B = 65°, ∠C = 115° and ∠D = 65°.

6. Question

Two sides of a parallelogram are in the ratio 5 : 3. If its perimeter is 64 cm, find the lengths of its sides.

Answer

Let x be the common multiple.

According to question, sides will be 5x and 3x.

Perimeter = $2(l+w)$

64 = 2 (5x + 3x)

64 = 16x

x = 4

5x = 20 cm

3x = 12 cm

So, sides will be 20 cm and 12 cm.

7. Question

The perimeter of a parallelogram is 140 cm. If one of the sides is longer than the other by 10 cm, find the length of each of its sides.

Answer

Let x be the common multiple.

According to question, sides will be x and (x + 10).

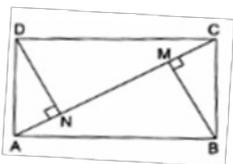Perimeter = $2(l+w)$

140 = 2 (x + (x + 10))

140 = 2 (2x + 10)

140 = 4x + 20

4x = 120

X = 30 cm

X + 10 = 40 cm

So, sides will be 30 cm and 40 cm.

8. Question

In the adjacent figure, ABCD is a rectangle. If BM and DN are perpendiculars from B and D on AC, prove that $\triangle BMC \cong \triangle DNA$. Is it true that BM = DN?
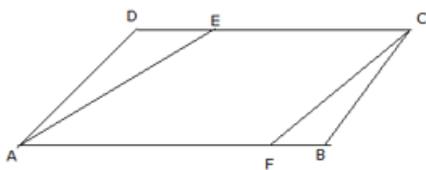

Answer

In a triangles BMC and DNABC = DA (Opposite sides) ∠BCM = ∠DAN (alternate angles)∠DNA = ∠BMC = 90° [DN and BM are perpendicular to AC]So by AAS Congruency criterionΔBMC ≅ ΔDNA BM = DN (By CPCT)

9. Question

In the adjacent figure, ABCD is a parallelogram and line segments AE and CF bisect the angles A and C respectively. Show that $AE \parallel CF$.

Answer

According to question,

$\angle A = \angle C$ (Opposite angles)

Line segments AE and CF bisect the $\angle A$ and $\angle C$ means,

$$\sqrt{100} = \frac{1}{2} \angle C$$

∠DAE = ∠BCF ----------(i)

Now, In triangles ADE and CBF,

AD = BC (Opposite sides)

∠B = ∠D (Opposite angles)

∠DAE = ∠BCF (from (i))

Therefore, Δ ADE ≅ ΔCBF (By ASA congruency)

By CPCT, DE=BF

But, CD=AB

CD - DE = AB - BF.

So, CE = AF.

Therefore, AECF is a quadrilateral having pairs of side parallel and equal,

So, AECF is a parallelogram. Hence, AE || CF.

10. Question

The lengths of the diagonals of a rhombus are 16 cm and 12 cm respectively. Find the length of each of its sides.

Answer

Let ABCD be a rhombus with diagonals AC and BD.

AC and BD bisect at O.

So, AO = $\frac{16}{2}$ = 8 cm

And BO = $\frac{12}{2}$ = 6 cm

In right angled triangle AOB,

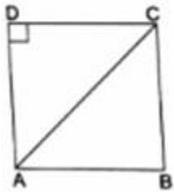$AB^2 = AO^2 + OB^2$ (According to Pythagoras theorem)

$AB^2 = 8^2 + 6^2$

$AB^2 = 64 + 36$

AB = $\sqrt{100}$

= 10 cm.

So, Length of Rhombus of each side is 10 cm.

11. Question

In the given figure ABCD is a square. Find the measure of $\angle CAD$.

Answer

Let $\angle CAD$ be x.

ABCD is a square.

So, DA = DC (every side of square is equal)

Therefore,

∠ACD = ∠CAD = x°

∠ACD + ∠CAD + ∠ADC= 180° (ACD is right angled triangle)

x° + x° + 90° = 180°

2 x° = 90°

x° = 45°

12. Question

The sides of a rectangle are in the ratio 5 : 4 and its perimeter is 90 cm. Find its length and breadth.

Answer

Let x be the common multiple.

Length = 5x

Breadth = 4x

Perimeter = $2(l+w)$

90 = 2 (5x + 4x)

18x = 90

X = 5

Length = 5x = 25 cm

Breadth = 4x = 20 cm

13. Question

Name each of the following parallelograms.

(i) The diagonals are equal and the adjacent sides are unequal.

(ii) The diagonals are equal and the adjacent sides are equal.

(iii) The diagonals are unequal and the adjacent sides are equal.

(iv) All the sides are equal and one angle is 60°.

(v) All the sides are equal and one angle is 90°.

(vi) All the angles are equal and the adjacent sides are unequal.

Answer

(i) rectangle

(ii) square

(iii) rhombus

(iv) rhombus

(v) square

(vi) rectangle

14. Question

Which of the following statements are true and which are false?

(i) The diagonals of a parallelogram are equal.

(ii) The diagonals of a rectangle are perpendicular to each other.

(iii) The diagonals of a rhombus are equal.

(iv) Every rhombus is a kite.

(v) Every rectangle is a square.

(vi) Every square is a parallelogram.

(vii) Every square is a rhombus.

(viii) Every rectangle is a parallelogram.

(ix) Every parallelogram is a rectangle.

(x) Every rhombus is a parallelogram.

Answer

(i) False

Diagonals of parallelogram bisects each other.

(ii) False

Diagonals of rectangle do not intersect at right angle. So, they are not perpendicular to each other.

(iii) False

Diagonals of rhombus bisect each other. If it's all sides are equal then only diagonals will be equal and it will be a square.

(iv) False

All kites are rhombus but every rhombus is not a kite.

(v) False

In square all sides are equal. Length of sides may vary in case of rectangle.

(vi) True

In Parallelogram, opposite sides and opposite angles are equal. In square all sides are equal and all angles are right angles.

(vii) True

All sides are equal and diagonals bisect each other.

(viii) True

Opposite sides and opposite angles are equal.

(ix) False

Rectangle forms right angle between adjacent sides but it is not necessary for every parallelogram.

(x) True

Opposite sides and opposite angles are equal.

Exercise 16B

1. Question

The two diagonals are not necessarily equal in a

A. rectangle

B. square

C. rhombus

D. isosceles trapezium

Answer

All sides of Rhombus are equal in length but in case of angle it is not necessary to be equal.

If all the angles are equal then it will become a square. That's why diagonals of rhombus are not necessary to be equal in length.

2. Question

The lengths of the diagonals of a rhombus are 16 cm and 12 cm. The length of each side of the rhombus is

A. 8 cm

B. 9 cm

C. 10 cm

D. 12 cm

Answer

Let ABCD be a rhombus with diagonals AC and BD.

AC and BD bisect at O.

So, AO = $\frac{16}{2}$ = 8 cm

And BO = $\frac{12}{2}$ = 6 cm

In right angled triangle AOB,

$AB^2 = AO^2 + OB^2$ (According to Pythagoras theorem)

$AB^2 = 8^2 + 6^2$

$AB^2 = 64 + 36$

AB = $\sqrt{100}$

= 10 cm.

So, Length of Rhombus of each side is 10 cm.

3. Question

Two adjacent angles of a parallelogram are $(2x + 25)°$ and $(3x - 5)°$. The value of x is

A. 28

B. 32

C. 36

D. 42

Answer

Let $\angle A = (2x + 25)°$, $\angle B = (3x - 5)°$

Since the sum of any two adjacent angles of a parallelogram is 180°,

$\angle A + \angle B = 180°$

$(2x + 25)° + (3x - 5)° = 180°$

$5x + 20° = 180°$

$5x = 180° - 20°$

$5x = 160°$

$X = 160/5 = 32°$

So, Value of x is 32°.

4. Question

The diagonals do not necessarily intersect at right angles in a

A. parallelogram

B. rectangle

C. rhombus

D. kite

Answer

The diagonals do not necessarily intersect at right angles in a parallelogram. Only opposite sides, opposite angles are equal and diagonal bisects each other in parallelogram. If diagonals intersect each other at right angle then it would be square or rhombus.

5. Question

The length and breadth of a rectangle are in the ratio 4 : 3. If the diagonal measures 25 cm then the perimeter of the rectangle is

A. 56 cm

B. 60 cm

C. 70 cm

D. 80 cm

Answer

Let x be the common multiple.

Length = 4x

Breadth = 3x

According to Pythagoras theorem,

$$\left(4x\right)^2 + \left(3x\right)^2 = \left(25\right)^2$$

$$16x^2 + 9x^2 = 625$$

$$25x^2 = 625$$

$$x^2 = \frac{625}{25}$$

X = 5

So,

Length = 4x = 20 cm

Breadth = 3x = 15 cm

Perimeter = $2(l+w)$

= 2 (20 + 15)

= 70 cm

So, perimeter of rectangle is 70 cm.

6. Question

The bisectors of any two adjacent angles of a parallelogram intersect at

A. 30°

B. 45°

C. 60°

D. 90°

Answer

Let ABCD is a parallelogram.
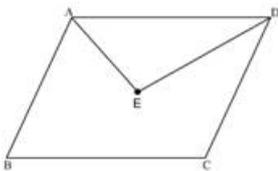
AE and AD is the bisector angles of adjacent angles of ∠A and ∠D.

As we know that,

∠A + ∠D = 180⁰ (Sum of interior angles on the same side of traversal is 180⁰)

$\frac{1}{2}$ ∠A + $\frac{1}{2}$ ∠D = $\frac{1}{2} \times 180°$

= 90⁰ --------------(i)

Now, in triangle AOD,

∠AED + $\frac{1}{2}$ ∠A + $\frac{1}{2}$ ∠D = 180⁰ (AE and AD is the angle bisector of ∠A and ∠D.)

∠AED + 90⁰ = 180⁰ (From eq (i))

∠AED = 180⁰ - 90⁰ = 90⁰

So, the bisectors of any two adjacent angles of a parallelogram intersect at 90⁰

7. Question

If an angle of a parallelogram is two-thirds of its adjacent angle, the smallest angle of the parallelogram is

A. 54°

B. 72°

C. 81°

D. 108°

Answer

Let ∠A = x°, ∠B = $\frac{2x°}{3}$

Since the sum of any two adjacent angles of a parallelogram is 180°,

∠A + ∠B = 180°

x° + $\frac{2x°}{3}$ = 180°

$\frac{5x°}{3}$ = 180°

$5x°$ = 540°

x° = 108°

∠A = 108°

∠B = 72°

Also, ∠B + ∠C = 180° [Since, ∠B and ∠C are adjacent angles]

72° + ∠C = 180°

∠C = (180° - 72°) = 108°

Further, ∠C + ∠D = 180° [Since, ∠C and ∠D are adjacent angles]

108° + ∠D = 180°

∠D = (180° - 108°) = 72°

Therefore, smallest angle of the parallelogram is 72°.

8. Question

The diagonals do not necessarily bisect the interior angles at the vertices in a

A. rectangle

B. square

C. rhombus

D. all of these

Answer

In rectangle, only opposite sides are equal which makes diagonals are not to be perpendicular to each other. As diagonals are not perpendicular to each other, they will not bisect the interior angles.

9. Question

In a square ABCD, AB = (2x + 3) cm and BC = (3x - 5) cm. Then, the value of x is

A. 4

B. 5

C. 6

D. 8

Answer

In square all sides are equal.

So, AB = BC

2x + 3 = 3x -5

3x -2x = 5 + 3

X = 8.

10. Question

If one angle of a parallelogram is 24° less than twice the smallest angle then the largest angle of the parallelogram is

A. 68°

B. 102°

C. 112°

D. 176°

Answer

Let $\angle A = x°$,

$\angle B = (2x - 24)°$

$\angle C = x°$ (Opposite angles are equal.)

$\angle D = (2x - 24)°$ (Opposite angles are equal.)

Since the sum of angles of a parallelogram is 360°,

$\angle A + \angle B + \angle C + \angle D = 360°$

$x° + (2x - 24)° + x° + (2x - 24)° = 360°$

$6x° - 48 = 360°$

$6x° = 408°$

$x° = 68°$

$\angle A = 68°$

$\angle B = (2x - 24)° = 112°$

Therefore, largest angle of the parallelogram is 112°.