JEE Main Online Exam 2019

Questions & Solutions

12th January 2019 | Shift - II

PHYSICS

Q.1 An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

(1) 0.5×10^{-8} s (2) 4×10^{-8} s

(3) 3×10^{-6} s (4) 2×10^{-7} s

Ans. [2]

$$t = \frac{1}{\sqrt{2}n\pi d^2 v_{avg}}$$

$$\tau \propto \frac{1}{n.v_{avg}}$$ n = no. of molecules per unit volume

$$\tau \propto \frac{v}{\sqrt{T}}$$ & Pv = nRT

$$t \propto \frac{\sqrt{T}}{v}$$ $$v \propto \frac{T}{P}$$

$$\frac{\tau_1}{t_2} = \sqrt{\frac{T_1}{T_2}} \cdot \frac{P_2}{P_1}$$

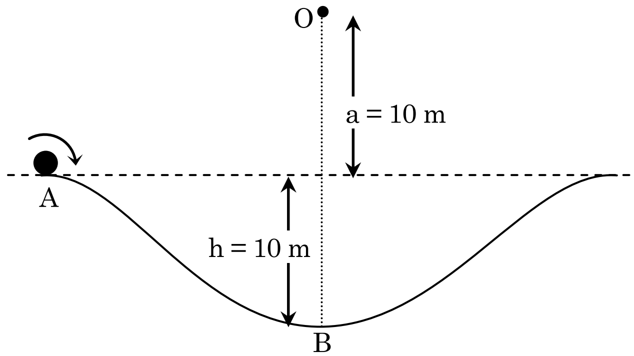
$$\tau_2 = \frac{P_1}{P_2} \times \sqrt{\frac{T_2}{T_1}}\ \tau_1$$

$$= \frac{P}{2P}\sqrt{\frac{500}{300}} \cdot 6 \times 10^{-8}$$

$$= 3\sqrt{\frac{5}{3}} \times 6 \times 10^{-8}$$

$$= 4 \times 10^{-8}\text{s}$$

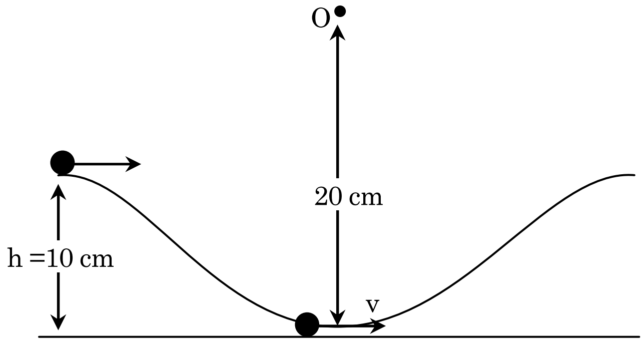
Q.2 A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. when the particle reaches point b, its angular momentum about O will be :
(Take g = 10 m/s^2)



(1) 6 kg-m^2/s (2) 8 kg-m^2/s (3) 2 kg-m^2/s (4) 3 kg-m^2/s

Ans. **[1]**

Sol.



conservation energy

$k_i + v_i = k_f + v_f$

$\frac{1}{2}$ m/s^2 + mg ·10 = $\frac{1}{2}$ mv^2 + 6

$25 + 200 = v^2$

$\Rightarrow v = 15$ m/s

angular moment about 'O' is L = mvr

$= 20 \times 10^{-3} \times 15 \cdot 20$

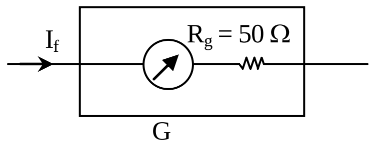
$= 6$ kg – m^2/s

Q.3 A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of :

(1) 200 ohm (2) 250 ohm (3) 6200 ohm (4) 6250 ohm

Ans. **[1]**

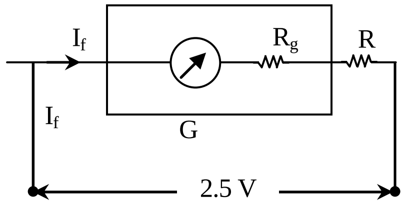
Sol.



$I_{full} = 25 \times 4 \times 10^{-4}$

$= 0.01$ Amp

Let 'R' Resistance is connected is series



V = iR

$2.5 = I_f (R_g + R) \Rightarrow R_g + 50 = \frac{2.50}{0.01}$

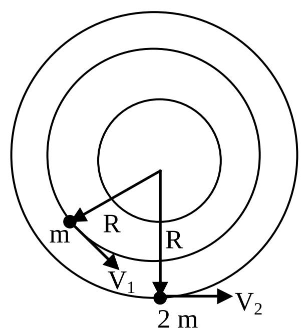
$\Rightarrow R = 200\ \Omega$

Q.4 Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is ;

(1) 2 (2) $\frac{1}{2}$ (3) $\sqrt{\frac{1}{2}}$ (4) 1

Ans. **[4]**

Sol.



$K_1 = \frac{GMem}{2R}$

$K_2 = \frac{GMe(2m)}{2(2R)} = \frac{GMem}{2R}$

$\therefore \frac{K_1}{K_2} = \frac{1}{1}$

Q.5 Let ℓ, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be :

(1) $[A^{-1}]$ (2) [LTA] (3) $[LA^{-2}]$ (4) $[LT^2]$

Ans. **[1]**

Sol. $\frac{\ell}{rc\nu} = \frac{\ell}{Rc - iR} = \frac{\ell}{R(Rc) \times i}$

$\frac{\ell}{R}$ = t (time)

R = t (time)

i = current

$\therefore$ dimensional formula = $\frac{[T]}{[T][A]} = [A^{-1}]$

Q.6 A simple harmonic motion is represented by :

y = 5 (sin 3 π t + $\sqrt{3}$ cos 3 πt) cm

The amplitude and time period of the motion are :

(1) 10 cm, $\frac{3}{2}$ s (2) 5cm, $\frac{2}{3}$ s, (3) 5 cm, $\frac{3}{2}$ s (4) 10 cm, $\frac{2}{3}$ s

Ans. **[4]**

Sol. $y = 2 \times 5 \left[\frac{\sin 3\pi t}{2} + \frac{\sqrt{3}\cos 3\pi t}{2}\right]$

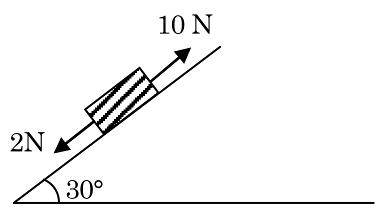
y = 10 [sin (3πt + π/3)]

$\therefore$ Amplitude A' = 10 cm.

$\frac{2\pi}{T} = \omega = 3\pi$

T = $\frac{2}{3}$ s

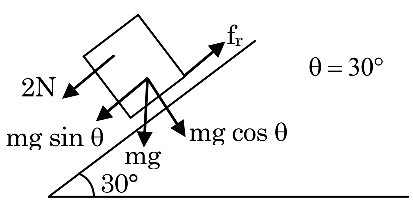
Q.7 A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is : [Take g = 10 m/s^2]



(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{2}{3}$

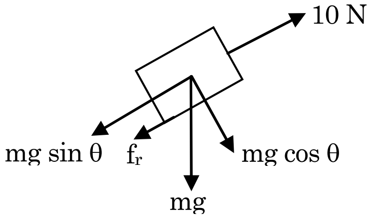
Ans. **[3]**

Sol.



$\theta = 30°$

$\therefore\ 2 + mg \sin\theta = f_r = \mu mg \cos\theta$ .....(i)



$f_r + mg \sin\theta = 10$

$mg\cos\theta + mg\sin\theta = 10$ .....(ii)

(i) + (ii) equation

$2 + 2\,mg\sin\theta = 10$

$2\,mg \cdot \frac{1}{2} = 8$

$m \times 10 = 8$

$m = 0.8$ kg

from (1)

$2 + 0.8 \times 10 \times \frac{1}{2} = \mu \times 0.8 \times 10 \times \frac{\sqrt{3}}{2}$

$6 = \mu \times \sqrt{3}$

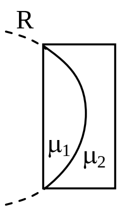
$\mu = \frac{\sqrt{3}}{2}$

Q.8 A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be :

(1) $f_1 + f_2$ (2) $f_1 - f_2$ (3) $\frac{R}{\mu_2 - \mu_1}$ (4) $\frac{2f_1f_2}{f_1 + f_2}$

Ans. **[3]**

Sol.



$P_2 = P_1 + P_2$

$\frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_2}$

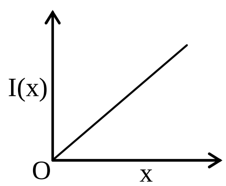
$= (\mu_2 - 1)\left[\frac{1}{\infty} - \frac{1}{-R}\right] + \mu_k - 1\left[\frac{1}{-R} - \frac{1}{\infty}\right]$

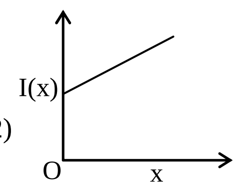
$= (\mu_2 - 1)\,\frac{1}{R} - (\mu_1 - 1)\,\frac{1}{R}$

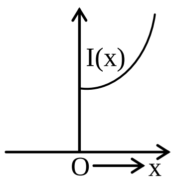
$$\frac{1}{f_2} = \frac{(\mu_2 - \mu_1)}{R}$$

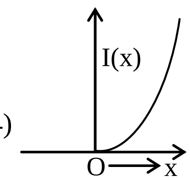
$$f_w = \left(\frac{R}{\mu_2 - \mu_1}\right)$$

Q.9 The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly ?

(1)


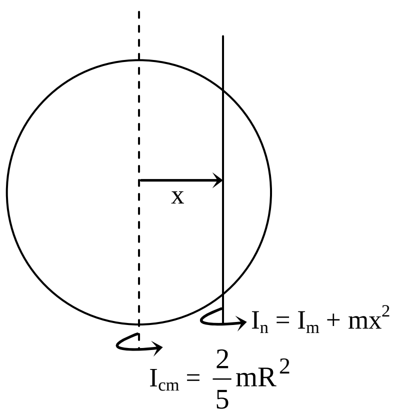
(2)


(3)


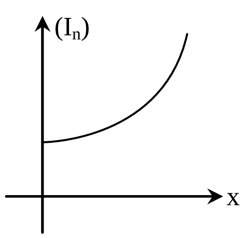
(4)


Ans. **[3]**

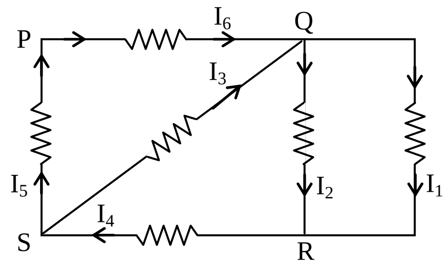
Sol.



$I_n = \frac{2}{8}mR^2 + mx^2$ [Parabola]



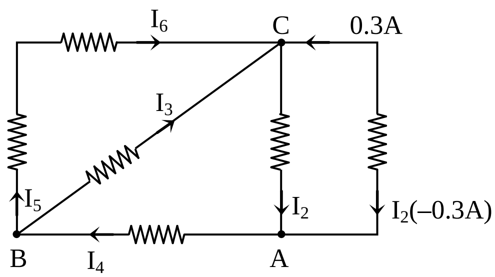
Q.10 In the given circuit diagram, the currents, $I_1 = -0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are :



(1) 1.1 A, – 0.4 A, 0.4 A (2) – 0.4 A, 0.4 A, 1.1 A
(3) 0.4 A, 1.1 A, 0.4 A (4) 1.1 A, 0.4 A, 0.4 A

Ans. **[4]**

Sol.



(Actual direction is reverse)

$I_1 = 0.3A$ $I_4 = 0.8A$ $I_5 = 0.9A$

At node ‘A’ by KCL

$I_2 = I_Q + 0.3$

$I_2 = 0.8 + 0.3$

$I_2 = 1.1A$

At node ‘B’ by K.C.L.

$I_4 = I_5 + I_3$

$0 - 8 = 0.4 + I_3$

$\boxed{I_3 = 0.4A}$

At node ‘C’ by K.C.L.

$I_6 + I_3 + 0.3 = I_2$

$I_6 + 0.4 + 0.3 = 1.1$

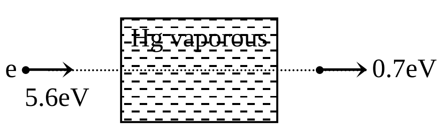
$I_6 = 0.4$ A

Q.11 In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to :

(1) 2020 nm (2) 250 nm
(3) 1700 nm (4) 220 nm

Ans. **[2]**

Sol.



$\therefore$ Energy absorbed by Hg atom = 5.6 – 0.7 = 4.4 eV.

$$\lambda_{min.} = \frac{1240}{4.9} \text{ nm}$$

$\simeq$ 250 nm

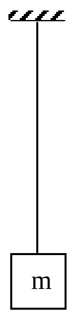
Q.12 A load of mass M kg is suspended from a steel wire of length 2m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8.

The new value of increase in length of the steel wire is:

(1) 5.0 mm (2) zero (3) 3.0 mm (4) 4.0 mm

Ans. **[3]**

Sol.



$$\therefore y = \frac{\text{stree}}{\text{strain}}$$

$$\Rightarrow \quad \frac{mg}{A} = y \,.\, \frac{\Delta\ell}{\ell}$$

$$\Rightarrow \quad mg \propto \Delta\ell \qquad \text{.......(1)}$$

If it is dipped in a liquid

Stress = mg – B

Stress = mg – m'g

$$mg - m'g \propto \Delta\ell' \qquad \text{.....(2)}$$

(2)/(1)

$$\frac{m - m'}{m} = \frac{\Delta\ell'}{\Delta\ell}$$

$$\Rightarrow \Delta\ell' = \Delta\ell' = \left(1 - \frac{m'}{m}\right)$$

$$= 4\text{mm}\left(1 - \frac{2}{8}\right)$$

$\Delta\ell'$ = 3mm

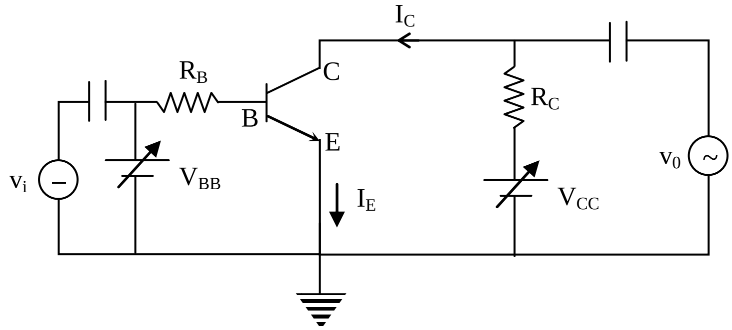
Q.13 When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is :

(1) 2ν (2) $\frac{4}{3}\nu$ (3) $\frac{3\nu}{2}$ (4) $\frac{5\nu}{3}$

Ans. **[Bonus]**

Sol. Question is not correct

Q.14 In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, V_{CC} = 5V, β_{dc} = 200, R_B = 100 kΩ, R_C = 1 kΩ and V_{BE} = 1.0 V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively :



(1) 20 μA and 2.8 V (2) 25 μA and 2.8 V

(3) 20 μA and 3.5 V (4) 25 μA and 3.5 V

Ans. **[4]**

Sol. For out put loop

$V_{CC} - I_C R_C - V_{CE} = 0$

For saturation $V_{CE} \approx 0.2$ V

$\therefore I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{5 - 0.2}{1 \times 10^3} = 4.8$ mA

$\therefore I_C = BI_B \Rightarrow I_B = \frac{I_C}{B} = \frac{4.8\text{mA}}{200} = 2.4 \times 10^{-5}$

$I_B = 24\ \mu A \approx 25\ \mu A$

For i/p loop

$vi - i_B R_B - V_{BE} = 0$

$vi - i_B R_B + V_{BE}$

$= 25 \times 10^{-6} \times 100 \times 10^3 + 1.0$

$= 2.5 - 11$

$vi = 3.5$ V

Q.15 A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is :

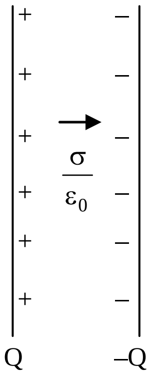
(Take $\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N - m^2}$)

(1) 9.85×10^{-10} C (2) 8.85×10^{-10} C

(3) 6.85×10^{-10} C (4) 7.85×10^{-10} C

Ans. [2]

Sol.



$\therefore \frac{\sigma}{\varepsilon_0} = 100$

$\frac{q}{A\varepsilon_0} = 100$

$q = (1\ m^2) \times (8.85 \times 10^{-12}) \times 100$

$q = 8.85 \times 10^{-10}$ C

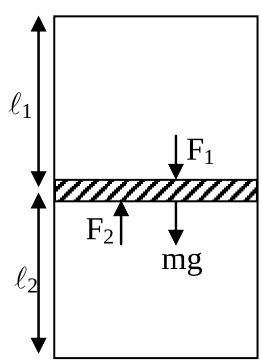
Q.16 A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is ℓ_1, and that below the piston is ℓ_2, such that $\ell_1 > \ell_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by :

(R is universal gas constant and g is the acceleration due to gravity)

(1) $\frac{nRT}{g}\left[\frac{\ell_1 - \ell_2}{\ell_1\ell_2}\right]$ (2) $\frac{RT}{g}\left[\frac{2\ell_1 + \ell_2}{\ell_1\ell_2}\right]$ (3) $\frac{nRT}{g}\left[\frac{1}{\ell_2} + \frac{1}{\ell_1}\right]$ (4) $\frac{RT}{ng}\left[\frac{\ell_1 - 3\ell_2}{\ell_1\ell_2}\right]$

Ans. [1]

Sol.



$Mg + F_1 = F_2$

$Mg + p_1A = P_2A$ $PV = nRT$

Mg = (p2 – p1) A

$= nRT\left[\frac{1}{\ell_2} - \frac{1}{\ell_1}\right]$

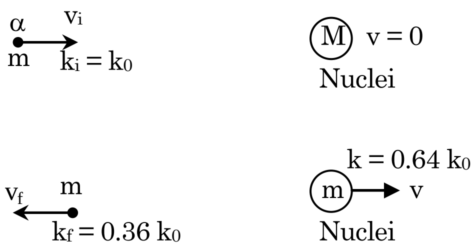
$\Rightarrow m = \frac{nRT}{g}\left[\frac{\ell_1 - \ell_2}{\ell_1\ell_2}\right]$

Q.17 An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :

(1) 2 m (2) 4 m (3) 1.5 m (4) 3.5 m

Ans. **[2]**

Sol.



Conservation of momentum

$mv_i = MV - mv_f$.

$k = \frac{1}{2}mv^2$

$\Rightarrow p = \sqrt{2km}$

$\Rightarrow \sqrt{2k_0m} = \sqrt{2(0.64k_0)M} - \sqrt{2(0.36k_0)m}$

$\Rightarrow \sqrt{m}\left(1+\frac{6}{10}\right) = \frac{8}{10} \times \sqrt{m}$

$16 = 8\sqrt{m}$

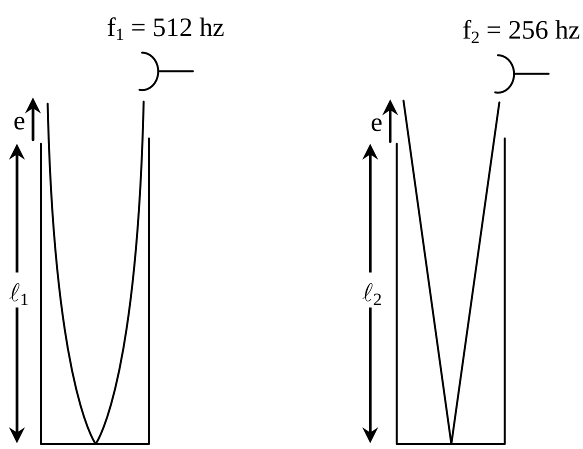
$\Rightarrow M = 4m$

Q.18 A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to :

(1) 335 ms^{-1} (2) 328 ms^{-1} (3) 341 ms^{-1} (4) 322 ms^{-1}

Ans. **[2]**

Sol.



$\frac{\lambda}{4} = \ell_1 + e$ $\qquad$ $\frac{\lambda}{4} = \ell_2 + e$

$\lambda = a(\lambda + e)$

$\therefore f_1 = \frac{v}{4(\ell_1 + e)}$ $\qquad$ $f_2 = \frac{v}{4(\ell_2 + e)}$

$\ell_1 + e = \frac{v}{4f_1}$ .....(i)

$\ell_2 + e = \frac{v}{4f_2}$ .....(ii)

(ii) – (i)

$\ell_2 - \ell_1 = \frac{v}{4}\left[\frac{1}{f_2} - \frac{1}{f_1}\right]$

$0.16 = \frac{v}{4}\left[\frac{1}{256} - \frac{1}{512}\right]$

$\Rightarrow$ v = 328 m/s

Q.19 The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closet to :

(1) 10^2T (2) 10^{-2} T (3) 10^{-4} T (4) 1T

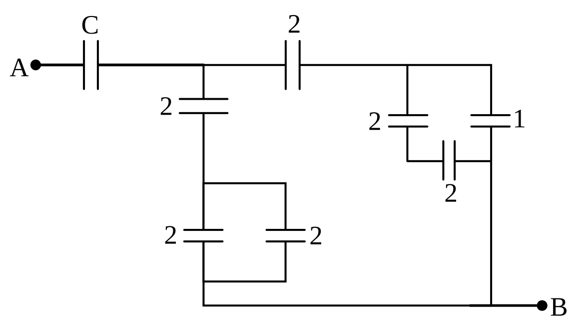
Ans. **[3]**

Sol. $I = \frac{B_0^2}{2\mu_0} \cdot c$

$B_0 = \sqrt{\frac{2\mu_0 I}{c}}$

$B_0 \approx 10^{-4}$T

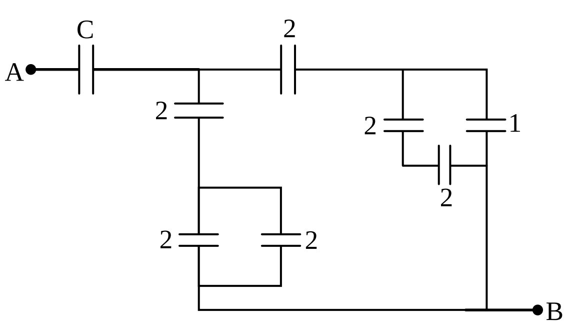
Q.20 In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.



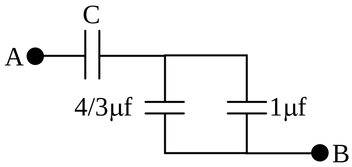
(1) $\frac{6}{5}$μF (2) $\frac{7}{11}$μF (3) 4μF (4) $\frac{7}{10}$μF

Ans. **[2]**

Sol.



$C_{AB} = 0.5\ \mu F$



$$C_{AB} = 0.1\ \mu f = \frac{C.\frac{7}{3}}{C + \frac{7}{3}}$$

$$\frac{1}{2} = \frac{7C}{3C+7}$$

$3C + 7 = 14\ C$

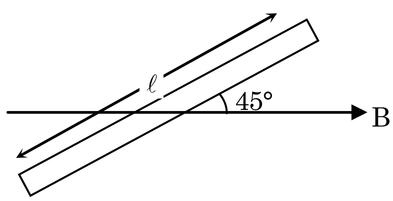
$$C = \frac{7}{11}\mu F$$

Q.21 A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is :

(1) 0.3×10^{-3} V (2) 2.5×10^{-3} V (3) 1.5×10^{-3} V (4) 1.1×10^{-3} V

Ans. **[4]**

Sol.



Emf = $Bv\ell \sin 45°$

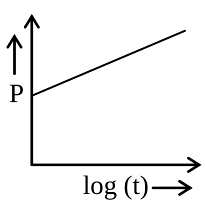
$$= 0.3 \times 10^{-4} \times 5 \times 10 \times \frac{1}{\sqrt{2}}$$

$$= 1.07 \times 10^{-3}\ V$$

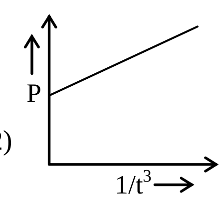
Emf $\approx$ 1.1 mV

Q.22 A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by :

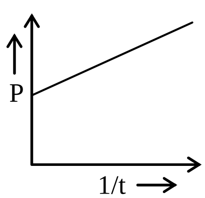
(1)



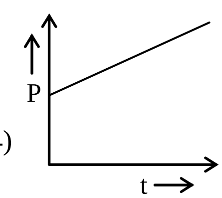
(2)



(3)



(4)



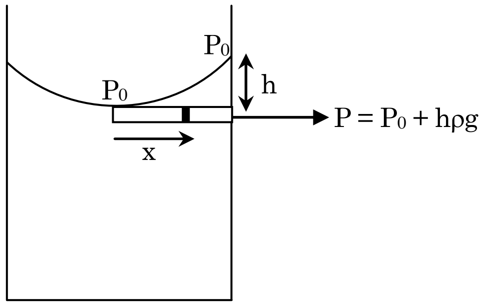
Ans. **[3]**

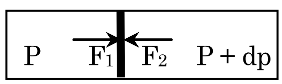
Q.23 A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be :

(1) 2.0 (2) 1.2 (3) 0.1 (4) 0.4

Ans. **[1]**

Sol.





$F_{net} = dF = dp.\ A$

$dmx\omega^2 = dpA.$

$x\rho A dx\omega^2 = dpA$

$$\Rightarrow \int_{p_0}^{p} dp = \rho\omega^2 \int_0^x x dx$$

$$p - p_0 = \frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow p = p_0 + \frac{\rho\omega^2 x^2}{2}$$

But $p = p_0 + h\rho g$ if h is the height rised

$$\therefore p_0 + h\rho g = p_0 + \frac{\rho\omega^2 x^2}{2}$$

$\Rightarrow$ at x = R

$$h\rho g = \frac{\rho\omega^2 R^2}{2}$$

$$h = \frac{\omega^2 R^2}{2g} = \frac{(2.2\pi)^2 \times (5\times 10^{-2})^2}{2.10}$$

$$= \frac{16\cdot\pi^2 25 \times 10^{-4}}{2\cdot 10}$$

$$= \frac{4000\times 10^{-4}}{2}$$

$= 200 \times 10^{-4}$

h = 2 cm

Q.24 In a radioactive decay chain, the initial nucleus is $^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by :

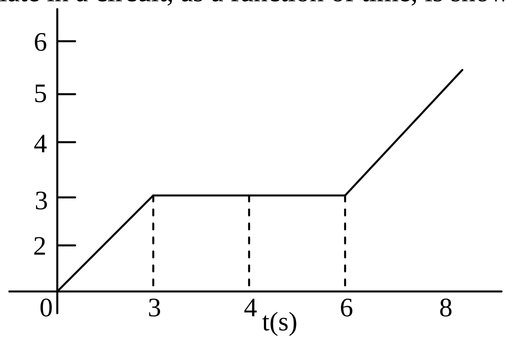
(1) A = 208 ; Z = 80 (2) A = 208 ; Z = 82

(3) A = 200 ; Z = 81 (4) A = 202 ; Z = 80

Ans. **[2]**

Sol. $_{90}Th^{232} \xrightarrow{-6\alpha} {}_{78}Y^{208} \xrightarrow{-4\beta} {}_{82}X^{208}$

Q.25 The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure :

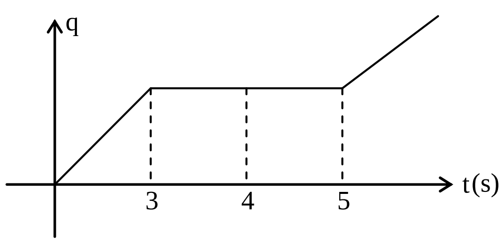


When is the value of current at t = 4 s ?

(1) zero (2) 1.5 μA (3) 2 μA (4) 3 μA

Ans. **[1]**

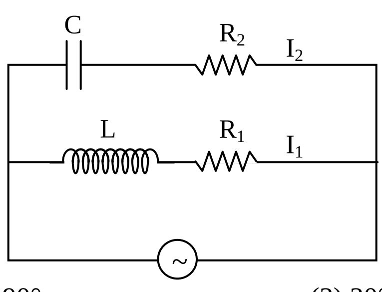
Sol.



$i = \frac{dq}{dt}$ at t = 4s $\frac{dq}{dt}$ = 0 (slope)

$\therefore$ i = 0

Q.26 In the above circuit, $C = \frac{\sqrt{3}}{2}$ μF, $R_2 = 20\ \Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\ \Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C. source is given by, $V = 200\sqrt{2}\sin(100\ t)$ volts. The phase difference between I_1 and I_2 is :



(1) 60° (2) 90° (3) 30° (4) 0°

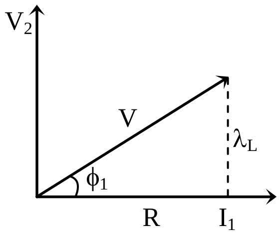
Ans. **[Bonus]**

Sol. [Data not correct]

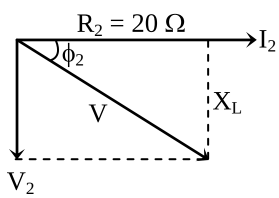
R_2 is not given correctly.

R_2 should be = 20 kΩ

if R_2 is 20 kΩ



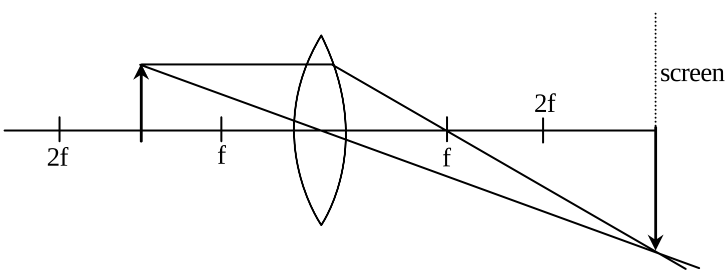
$$\tan\phi_1 = \frac{X_L}{L} = \frac{\omega L}{R_1} = \frac{100\sqrt{3}}{10\times10} = \sqrt{3}$$



$$\tan\phi_2 = \frac{X_C}{R_2} = \frac{1}{\omega CR_2} = \frac{1}{100\times\frac{\sqrt{3}}{2}\times10^{-6}\times20} = \frac{1000}{\sqrt{3}}$$

Further it can not be simplified.

Q.27 Formation of real image using a biconvex lens is shown below :



If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen ?

(1) Image disappears

(2) Magnified image

(3) Erect real image

(4) No change

Ans. **[1]**

Sol. $\frac{1}{f'} = \left(\frac{\mu_1}{\mu_2}-1\right)\left[\frac{2}{R}\right]$ | $\frac{1}{f} = (\mu-1)\left[\frac{2}{R}\right]$

$$f' = \frac{\mu_2 R}{2(\mu_1 - \mu_2)}$$

$$f' = \frac{\frac{4}{3}R}{2\left[\frac{3}{2} - \frac{4}{3}\right]}$$

$$= \frac{4R}{3.2\frac{1}{7.2}}$$

$$f' = 4R$$

$$f' = \frac{4.2f}{3}$$

$$\boxed{f' = \frac{8}{3}f} > 2$$

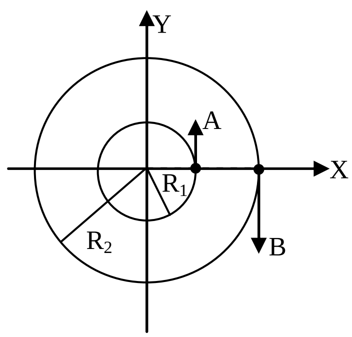
$$\frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{2}{R}\right)$$

$$f = \frac{3R}{2}$$

Therefore objective will lie between f' and optical center.

$\therefore$ image is virtual and would not be obtained on screen.

Q.28 Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure. :



The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by :

(1) $-\omega(R_1 + R_2)\,\hat{i}$ (2) $\omega(R_2 - R_1)\,\hat{i}$ (3) $\omega(R_1 + R_2)\,\hat{i}$ (4) $\omega(R_1 - R_1)\,\hat{i}$

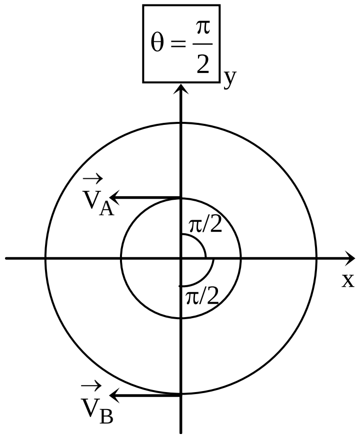
Ans. **[2]**

Sol. $7 = \frac{2\pi}{\omega}$

$\theta = \omega t$

$= \omega \cdot \frac{2\pi}{2\omega}$

At $t = \frac{\pi}{2\omega}$



$\vec{V}_A = -R_1\omega\,\hat{i}$

$\vec{V}_B = R_2\omega\,\hat{i}$

$\therefore\ \vec{V}_A - \vec{V}_B = \omega(R_2 - R_1)\ \hat{i}$

Q.29 A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is :

(1) 3.726×10^{-4} (2) 2.672×10^{-4} (3) 3.672×10^{-4} (4) 3.267×10^{-4}

Ans. [4]

Sol. $\chi_m \propto \frac{1}{T}$

$\therefore\ \chi_m = \frac{T_1}{T_2}\chi$

$\chi_m = \frac{280}{300} \times 2.8 \times 10^{-4}$

$\chi_m = 3.267 \times 10^{-4}$

Q.30 To double the covering range of a TV transmittion tower, its height should be multiplied by :

(1) 4 (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Ans. [1]

Sol. $d = \sqrt{2hR_e}$

$2d = \sqrt{2h'R_e}$

$\frac{1}{2} = \sqrt{\frac{h}{h'}}$

$h' = 4h$

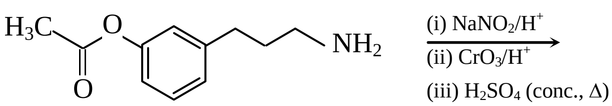
JEE Main Online Exam 2019

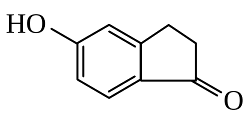
Questions & Solution

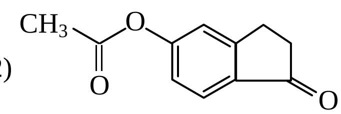
12th January 2019 | Shift - II

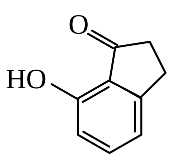
CHEMISTRY

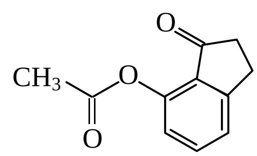
Q.1 The major product of the following reaction is –



(1) 

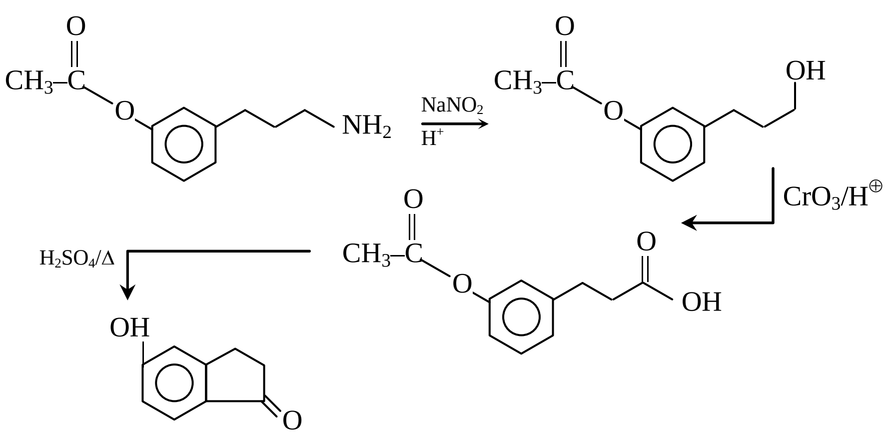
(2) 

(3) 

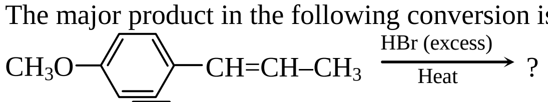
(4) 

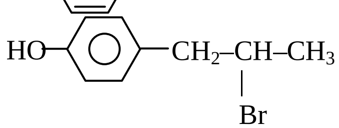
Ans. **[1]**

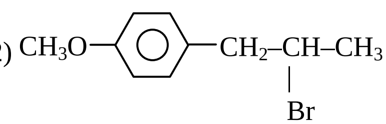
Sol.

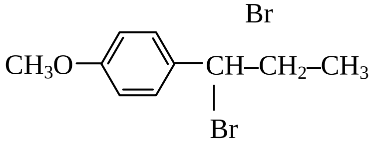


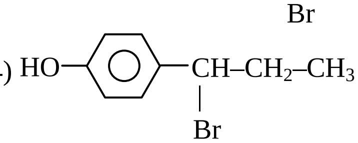
Q.2 The major product in the following conversion is –



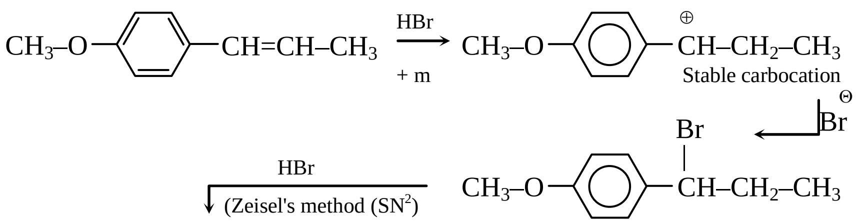
(1) 

(2) 

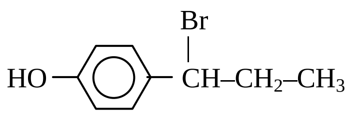
(3) 

(4) 

Ans. **[4]**



Sol.



Q.3 $\overset{\circ}{\wedge}_m$ for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^2 mol^{-1}, respectively. If the conductivity of 0.001 M HA is-
5×10^{-5} S cm^{-1}, degree of dissociation of HA is -

(1) 0.50 (2) 0.125 (3) 0.25 (4) 0.75

Ans. **[2]**

Sol. $\overset{\circ}{\wedge}_m (HA) = \overset{\circ}{\wedge}_m (HCl) + \overset{\circ}{\wedge}_m (NaA) - \overset{\circ}{\wedge}_m (NaCl)$

$= 425.9 + 100.5 - 126.4$

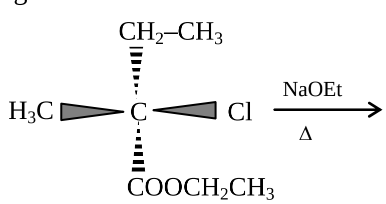
$= 400s \text{ cm}^2 \text{ mol}^{-1}$

$$\overset{\circ}{\wedge}_m = \frac{1000K}{M} = \frac{1000 \times 5 \times 10^{-5}}{10^{-3}}$$

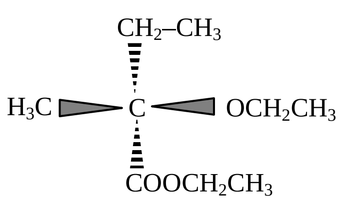
$= 50s \text{ cm}^2 \text{ mol}^{-1}$

$$\lambda = \frac{\wedge_m}{\overset{\circ}{\wedge}_m} = \frac{50}{400} = 0.125$$

Q.4 The major product of the following reaction is –

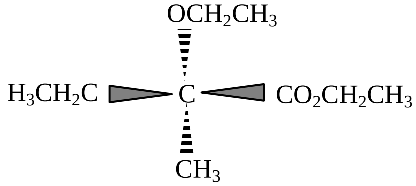


(1)



(2) CO_2CH_2–CH_3
|
CH_3C=$CHCH_3$

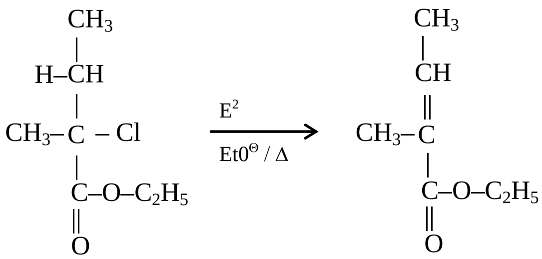
(3)



(4) CH_3CH_2C=CH_2
|
$CO_2CH_2CH_3$

Ans. **[2]**

Sol. Elimination occur due to the presence of strong base and heating



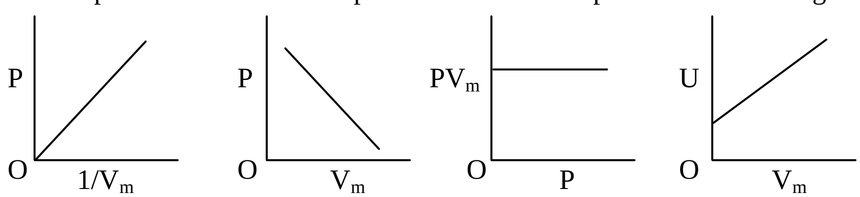
Q.5 The compound that is NOT a common component of photochemical smog is

(1) O_3 (2) $H_3C-\underset{\underset{O}{||}}{C}-OONO_2$ (3) $CH_2 = CHCHO$ (4) CF_2Cl_2

Ans. **[4]**

Sol. Photochemical smog contain O_3, peroxy acetyl nitrate acrolien but $CFCl_2$ is not present

Q.6 The combination of plots which does not represent isothermal expansion of an ideal gas is –



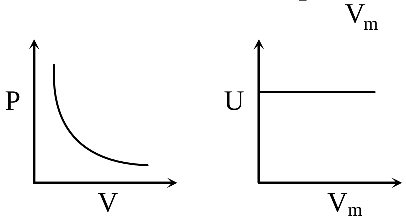
(A) (B) (C) (D)

(1) A and D (2) B and D (3) B and C (4) A and C

Ans. **[2]**

Sol. Isothermal expansion, $PV_m = \text{const}(K)$

$$P = \frac{K}{V_m}$$



Q.7 Molecules of benzoic acid (C_6H_5COOH) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2K. If the percentage association of the acid to form dimmer in the solution is 80, then w is – (Its given that K_f = 5 K kg mol–1, Molar mass of benzoic acid = 122 g mol^{-1})

(1) 1.5 g (2) 1.8 g (3) 1.0 g (4) 2.4 g

Ans. **[4]**

Sol. I = 1 – 0.8 + 0.4

= 0.6

$\Delta T_f = iK_fM$

$$2 = 0.6 \times 5 \times \frac{W \times 1000}{122 \times 30}$$

W = 2.44 gm

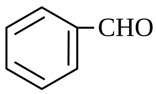
Q.8 The element that does NOT show catenation is -

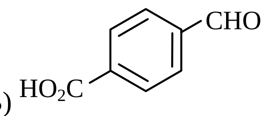
(1) Ge (2) Si (3) Sn (4) Pb

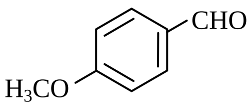
Ans. **[4]**

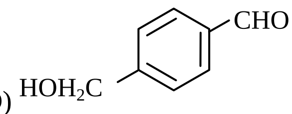
Sol. Catanation is not shown by Pb.

Q.9 The aldehydes which will not form Grignard product with one equivalent Grignard reagents are -

(A) 

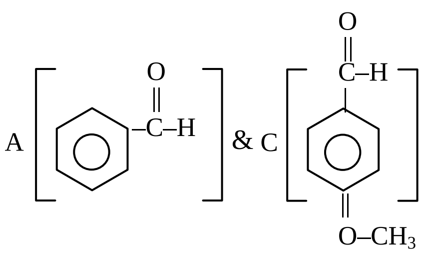
(B) 

(C) 

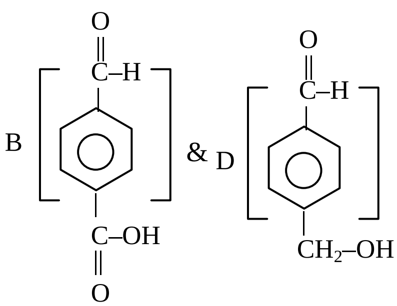
(D) 

(1) B and D (2) C and D (3) B, C and D (4) B and C

Ans. **[1]**

Sol. Only  required

Only one equation of GR while

 required

more no. of GR eq. for GR product

Q.10 If the de Broglie wavelength of the electron in n^{th} Bohr orbit in a hydrogenic atom is equal to $1.5\ \pi a_0$ (a_0 is Bohr radius), then the value of n/z is -

(1) 0.75 (2) 0.40 (3) 1.50 (4) 1.0

Ans. **[1]**

Sol. According to de-broglies' hypothesis

$$2\pi\ r = n\lambda$$

$$\Rightarrow \quad 2\pi r = n \times 1.5\ \pi\ a_0$$

$$\Rightarrow \quad 2\pi a_0 = \frac{n^2}{z} = n \times 1.5 \pi a_0$$

$$\Rightarrow \quad \frac{n}{z} = 0.75$$

Q.11 Given

(i) $C(graphite) + O_2(g) \rightarrow CO_2(g); \Delta rH^{\ominus} = x \text{ kJ mol}^{-1}$

(ii) $C(graphite) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \Delta rH^{\ominus} = y \text{ kJ mol}^{-1}$

(iii) $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \Delta rH^{\ominus} = z \text{ kJ mol}^{-1}$

Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct ?

(1) z = x + y (2) x = y + z (3) x = y – z (4) y = 2z – x

Ans. **[2]**

Sol. $C(graphite) + O_2(g) \rightarrow CO_2(g); \ \Delta H_f^{\circ} = x \text{ kJ / mole}$ …. (i)

$C(graphite) + \frac{1}{2}O_2(g) \rightarrow CO(g); \Delta H_f^{\circ} = y \text{ kJ mole}$ …. (ii)

$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \ \Delta H_f^{\circ} = z \text{ kJ mole}$ …. (iii)

(i) = (ii) + (iii)

x = y + z

But in question: equation → (ii) is wrong

There fore, this question should be

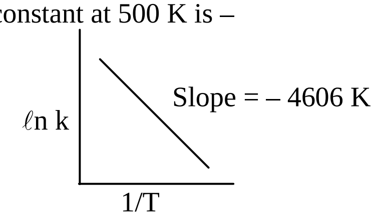
Q.12 The upper stratosphere consisting of the ozone layer protects us from sun's radiation that falls in the wavelength region of -

(1) 200-315 nm (2) 400-550 nm (3) 0.8-1.5 nm (4) 600-750 nm

Ans. **[1]**

Sol. Upper stratosphere consist ozone layer which protect us from sun radiation (ultraviolet) which have range 10nm to 400 nm

Q.13 For a reaction consider the plot of ℓn k versus 1/T given in the figure. If the rate constant of this reaction at 400 K is 10^{-5} s^{-1}, then the rate constant at 500 K is –



(1) 10^{-4} s^{-1} (2) 4×10^{-4} s^{-1} (3) 10^{-6} s^{-1} (4) 2×10^{-4} s^{-1}

Ans. **[1]**

Sol. $\ell nk = \ell nA - \frac{E_a}{RT}$

$\frac{E_a}{R} = -4606$

$\ell n\left(\frac{k_2}{k_1}\right) = \frac{Ea}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \Rightarrow \ell n\frac{K_2}{10^{-5}} = 4606\left(\frac{1}{400} - \frac{1}{500}\right)$

$\ell n\frac{K_2}{10^{-5}} = 2.303$

$\frac{K_2}{10^{-5}} = 10$

$K_2 = 10^{-4}\ 5^{-1}$

Q.14 8 g of NaOH is dissolved in 18g of H_2O. Mole fraction of NaOH in solution and molality (in mol kg^{-1}) of the solution respectively are -

(1) 0.2, 11.11 (2) 0.167, 22.20 (3) 0.167, 11.11 (4) 0.2, 22.20

Ans. **[3]**

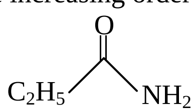
Sol. $X_{NaOH} = \dfrac{8/40}{\dfrac{8}{40}+\dfrac{18}{18}}$

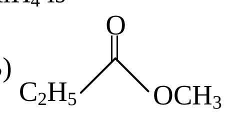
$= \dfrac{\frac{1}{5}}{\frac{1}{5}+1}$

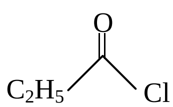
$= \dfrac{1}{6} = 0.167$

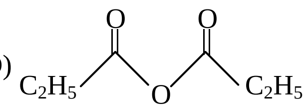
$m = \dfrac{8/40}{18} \times 1000 = 11.11$

Q.15 The increasing order of the reactivity of the following with $LiAlH_4$ is

(A) 

(B) 

(C) 

(D) 

(1) A < B < C < D (2) B < A < C < D (3) B < A < D < C (4) A < B < D < C

Ans. **[4]**

Sol. Reactivity of acid derivaties toward $LiAlH_4$ is based upon Nucleo philic substitution via addition Elimination process [SNAE] So depends upon LGA (leaving group ability) which is in following order

$$\overset{\ominus}{C\ell} > \overset{\ominus}{O}\text{–}\overset{O}{\overset{\|}{C}}\text{–}CH_3 > \overset{\ominus}{O}\text{–}CH_3 > \overset{\ominus}{N}H_2$$

Good LG Poor LG

Order towards LAH = $C_2H_5\text{–}\overset{O}{\overset{\|}{C}}\text{–}Cl > (C_2H_5\text{–}\overset{O}{\overset{\|}{C}})_2O > C_2H_5\text{–}\overset{O}{\overset{\|}{C}}\text{–}OCH_3 > C_2H_5\text{–}\overset{O}{\overset{\|}{C}}\text{–}NH_2$

Q.16 If K_{sp} of Ag_2CO_3 is 8×10^{-12}, the molar solubility of Ag_2CO_3 in 0.1 M $AgNO_3$ is -

(1) 8×10^{-12} M (2) 8×10^{-10} M (3) 8×10^{-13} M (4) 8×10^{-11} M

Ans. **[2]**

Sol. $Ag_2CO_3(s) \rightleftharpoons \underset{2s_1+0.1}{2Ag^+_{(aq)}} + \underset{S_1}{CO_3^{2-}}$

$AgNO_3 \longrightarrow \underset{2s_1+0.1M}{Ag^+_{(aq)}} + \underset{0.1\,M}{NO^-_{3(aq)}}$

$s_1 (2s_1 + 0.1)^2 = 8 \times 10^{-12}$

$2s_1 << 0.1$

$\therefore s_1 (0.1)^2 = 8 \times 10^{-12}$

$\therefore s_1 = 8 \times 10^{-10}$

Q.17 An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) it has escaped from the vessel assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is -

(1) 750 K (2) 500 °C (3) 750 °C (4) 500 K

Ans. **[4]**

Sol. $\frac{n_1}{n_2} = \frac{T_2}{T_1}$

$\frac{n_1}{\frac{3}{5}n_1} = \frac{T_2}{300}$

$T_2 = 300 \times \frac{5}{3}$

= 500 K

Q.18 The two monomers for the synthesis of Nylon 6, 6 are

(1) $HOOC(CH_2)_6COOH, H_2N(CH_2)_4NH_2$ (2) $HOOC(CH_2)_4COOH, H_2N(CH_2)_6NH_2$

(3) $HOOC(CH_2)_4COOH, H_2N(CH_2)_4NH_2$ (4) $HOOC(CH_2)_6COOH, H_2N(CH_2)_6NH_2$

Ans. **[2]**

Sol. Nylon 6, 6 is Polyamide polymer which is obtained by condensation reaction between Hexamethylene diamine & adipic acid

$$n\ 1\!\left[NH{-}(CH_2)_6\ NH{-}\right]\!H + n\ HO\!\left]\overset{\overset{\displaystyle O}{\|}}{C}{-}(CH_2)_4{-}\overset{\overset{\displaystyle O}{\|}}{C}\right[\!OH$$

$$\Big\downarrow -n\ H_2O$$

$$\left[NH{-}(CH_2)_6{-}\ NH{-}\overset{\overset{\displaystyle O}{\|}}{C}{-}(CH_2)_4{-}\overset{\overset{\displaystyle O}{\|}}{C}\right]_n$$

Nylon-6, 6

Q.19 The volume strength of 1M H_2O_2 is (Molar mass of H_2O_2 = 34 g mol^{-1})

(1) 5.6 (2) 22.4 (3) 11.35 (4) 16.8

Ans. **[3]**

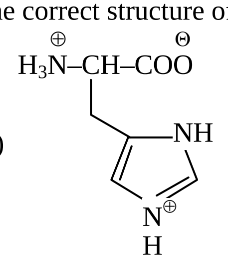
Sol. Volume strength of H_2O_2 = molarity × 11.2

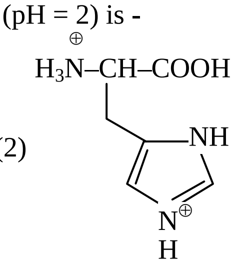
= 1 × 11.2

= 11.2

≈ 11.35

Q.20 The correct structure of histidine in a strongly acidic solution (pH = 2) is -

(1)


(2)


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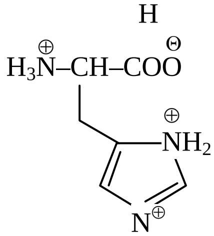
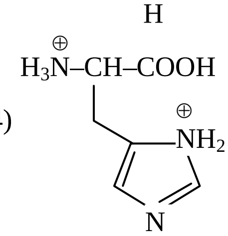
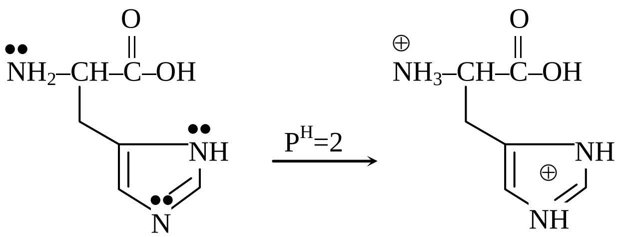


(4)


Ans. **[2]**

Sol. In strongly acidic medium protonation occurs at two nitrogen



Q.21 The element that shows greater ability to form pπ-pπ multiple bonds, is -

(1) Si (2) Ge (3) C (4) Sn

Ans. **[3]**

Sol. Due to small size of carbon it shows powerful pπ-pπ multiple bond

Q.22 The pair that does NOT require calcination is -

(1) Fe_2O_3 and $CaCO_3 \cdot MgCO_3$ (2) $ZnCO_3$ and CaO

(3) ZnO and $Fe_2O_3 \cdot xH_2O$ (4) ZnO and MgO

Ans. **[4]**

Sol. Calcination means heating without oxygen. It is not required if ore is already in oxide are (anhydrous)

Q.23 Chlorine on reaction with hot and concentrated sodium hydroxide give -

(1) $C\ell O_3^-$ and $C\ell O_2^-$ (2) $C\ell^-$ and $C\ell O^-$ (3) $C\ell^-$ and $C\ell O_2^-$ (4) $C\ell^-$ and $C\ell O_3^-$

Ans. **[4]**

Sol.

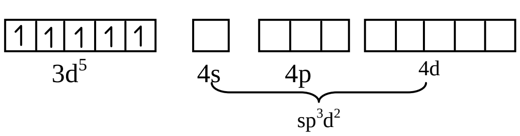
$$Cl_2 + \underset{\substack{\text{Hot}\\ \text{Conc.}}}{NaOH} \rightarrow \overset{-1}{NaCl} + \overset{+5}{NaClO_3}$$

Q.24 The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 BM. The suitable ligand for this complex is -

(1) CN^- (2) ethylenediamine (3) NCS^- (4) CO

Ans. **[3]**

Sol.

$Mn^{2+} = 3d^5$ 

To have 5.92 BM magnetic moment ligand should be weak ligand like $SCN^{\ominus}$

Q.25 The correct order of atomic radii is -

(1) Ho > N > Eu > Ce (2) Ce > Eu > Ho > N (3) N > Ce > Eu > Ho (4) Eu > Ce > Ho > N

Ans. **[4]**

Sol. The correct order of Atomic Radi

Europium > Cerium > Holmium > Nitrogen

$$\begin{array}{cccc} \text{Europium} & \text{Cerium} & \text{Holmium} & \text{Nitrogen} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 199\text{ PM} & 183\text{ PM} & 176\text{ PM} & 65\text{ PM} \end{array}$$

Q.26 The major product of the following reaction is –

$$\underset{\displaystyle\quad\quad\ \ \mathrm{Br}\ \ \ \mathrm{Br}}{CH_3CH_2CH{-}CH_2} \xrightarrow[\text{(ii) } NaNH_2 \text{ in liq. } NH_3]{\text{(i) KOH alc}}$$

(1) $CH_3CH_2C{\equiv}CH$ (2) $CH_3CH{=}C{=}CH_2$

(3) $\underset{\displaystyle\quad\quad NH_2\ NH_2}{CH_3CH_2CH{-}CH_2}$ (4) $CH_3CH{=}CHCH_2NH_2$

Ans. **[1]**

Sol. Reaction based upon cross elimination

$$\underset{(\alpha\text{-butylene dibromide})}{\underset{\displaystyle\quad\quad\ \ \mathrm{Br}\ \ \ \mathrm{Br}}{CH_3{-}CH_2{-}CH{-}CH_2}} \xrightarrow[\text{(ii) } NaNH_2 \text{ in liq. } NH_3]{\text{(i) KOH alc}} \underset{\text{(1-butyne)}}{CH_3{-}CH_2{-}C{\equiv}CH}$$

Q.27 Among the following the false statement is -

(1) Lyophilic sol can be coagulated by adding an electrote

(2) Tyndall effect can be used to distinguish between a colloidal solution and a true solution

(3) It is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane

(4) Latex is a colloidal solution of rubber particles which are positively charged

Ans. **[4]**

Sol. 1, 2, 3 are true fact

Q.28 The correct statement (s) among I to III with respect to potassium ions that are abundant within the cell fluid is /are

I. They activate many enzymes

II. They participate in the oxidation of glucose to produce ATP

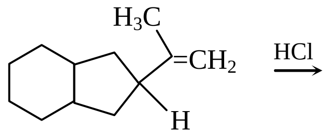
III. Along with sodium ions they are responsible for the transmission of nerve signals

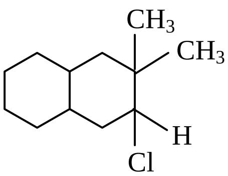
(1) III only (2) I and II only (3) I and III only (4) I, II and III

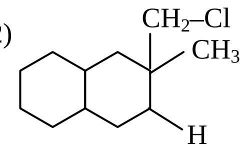
Ans. **[4]**

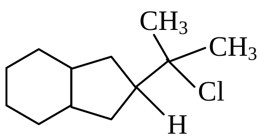
Sol. Facts

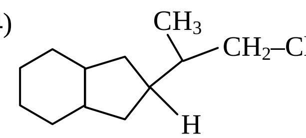
Q.29 The major product of the following reaction is –



(1) 

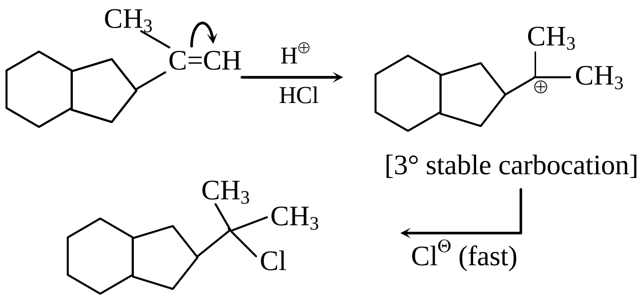
(2) 

(3) 

(4) 

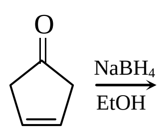
Ans. **[3]**

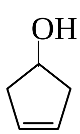
Sol.

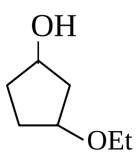


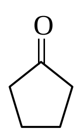
Reaction according to markowinkiff's rule

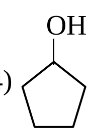
Q.30 The major product of the following reaction is –



(1) 

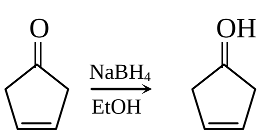
(2) 

(3) 

(4) 

Ans. **[1]**

Sol. $NaBH_4$ do not reduces C = C bond



JEE Main Online Exam 2019

Questions & Solutions

12th January 2019 | Shift - II

MATHEMATICS

Q.1 $\lim_{x\to 1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to :

(1) $\sqrt{\frac{2}{\pi}}$ (2) $\frac{1}{\sqrt{2\pi}}$ (3) $\sqrt{\frac{\pi}{2}}$ (4) $\sqrt{\pi}$

Ans. **[1]**

Sol. $\lim_{h\to 0} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{1-(1-h)}}$

$= \lim_{h\to 0} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}}$

$= \lim_{h\to 0} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}} \times \frac{\sqrt{\pi}+\sqrt{2\sin^{-1}(1-h)}}{\sqrt{\pi}+\sqrt{2\sin^{-1}(1-h)}}$

$= \lim_{h\to 0} \frac{(\pi)-2\sin^{-1}(1-h)}{\sqrt{h}\left(\sqrt{\pi}+\sqrt{2\sin^{-1}(1-h)}\right)}$

$= 2\lim_{h\to 0} \frac{\left(\frac{\pi}{2}-\sin^{-1}(1-h)\right)}{\sqrt{h}\left(\sqrt{\pi}+\sqrt{2\sin^{-1}(1-h)}\right)}$

$= 2\lim_{h\to 0} \frac{\cos^{-1}(1-h)}{\sqrt{h}\left(\sqrt{\pi}+\sqrt{2.\frac{\pi}{2}}\right)}$

$= 2\lim_{h\to 0} \frac{\cos^{-1}(1-h)}{\sqrt{h}(2\sqrt{\pi})}$

$= \frac{1}{\sqrt{\pi}}\lim_{h\to 0} \frac{\cos^{-1}(1-h)}{\sqrt{h}}$

Using L' hospital rule

$= \sqrt{\frac{2}{\pi}}$

Q.2 The expression $\sim(\sim p \rightarrow q)$ is logically equivalent to :

(1) $p \wedge q$ (2) $\sim p \wedge \sim q$ (3) $p \wedge \sim q$ (4) $\sim p \wedge q$

Ans. **[2]**

Sol.

p	q	$\sim p$	$\sim p \rightarrow q$	$\sim(\sim p \rightarrow p)$	$(\sim p \wedge \sim q)$
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

Q.3 The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive directions of x-axis, is :

(1) $x = y \cot\theta - 2\tan\theta$ (2) $y = x\tan\theta + 2\cot\theta$

(3) $x = y\cot\theta + 2\tan\theta$ (4) $y = x\tan\theta - 2\cot\theta$

Ans. **[3]**

Sol. $x^2 = 8y$

Equation of tangent

$y = mx - am^2$

$y = \tan\theta\, x - 2\tan^2\theta$

divide by $\tan\theta$

$\cot\theta\, y = x - 2\tan\theta$

$x = y\cot\theta + 2\tan\theta$

Q.4 If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to :

(1) 9 (2) 108 (3) 27 (4) 54

Ans. **[3]**

Sol. Given $\left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3$ .... 15 terms = 225 k

$= \left(\frac{3}{4}\right)^3 [1^3 + 2^3 + 3^3 \text{ ..... 15 terms}] = 225\text{ k}$

$= \frac{27}{64} \times (15 \times 8)^2 = 225\text{ k} \Rightarrow \text{k} = 27$

Q.5 There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 12 (2) 9 (3) 7 (4) 11

Ans. **[1]**

Sol. Let m–men, 2 women

${}^mC_2 \times 2 = {}^mC_1 \,.\, {}^2C_1 \,.\, 2 + 84$

$m^2 - 5m - 84 = 0$

$(m - 12)(m + 7) = 0$

$m = 12$

Q.6 In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

(1) $\frac{400}{3}$ loss (2) 0 (3) $\frac{400}{9}$ loss (4) $\frac{400}{3}$ gain

Ans. **[2]**

Sol.

x	0	50	100	-150
P(x)	$\frac{4}{27}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{8}{27}$

$$\sum P(x) = \frac{4}{27} + \frac{2}{9} + \frac{1}{3} + \frac{8}{27}$$

$$= \frac{4+6+9+8}{27}$$

$$\sum P(x) = 1$$

$$\text{Req. } E(x) = \sum_{i=1}^{n} x_i P(x_i)$$

$$= 0 + \frac{50\times 2}{9} + \frac{100}{3} - \frac{150\times 8}{27}$$

$$= 0$$

Q.7 If a curve passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :

(1) (–1, 2) (2) $(-\sqrt{2}, 1)$ (3) $(\sqrt{3}, 0)$ (4) (3, 0)

Ans. **[3]**

Sol.

$$\frac{dy}{dx} = \frac{x^2-2y}{x}$$

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y.x^2 = \int x^2.x\ dx + C$$

$$y.x^2 = \frac{x^4}{4} + C \qquad \dots (i)$$

curve passes through (1, –2)

$$-2\times 1^2 = \frac{1}{4} + c$$

$$c = -2 - \frac{1}{4} = \frac{-9}{4}$$

$$y.x^2 = \frac{x^4}{4} - \frac{9}{4}$$

$$4x^2y = x^4 - 9$$

which satisfy $(\sqrt{3}, 0)$

Q.8 The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x \, dx$ is equal to :

(1) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$ (2) $\frac{3}{2}-e-\frac{1}{2e^2}$ (3) $\frac{1}{2}-e-\frac{1}{e^2}$ (4) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$

Ans. **[2]**

Sol. $\int_1^e \left(\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right) \log x \cdot dx$

I II

$= \int_1^e \left(\frac{x}{e}\right)^{2x} \log_e x \cdot dx - \int_1^e \left(\frac{e}{x}\right)^{x} \log_e x \cdot dx$

put $\left(\frac{x}{e}\right)^{2x} = u$; put $\left(\frac{e}{x}\right)^{x} = v$

$= \frac{1}{2}\int_{(1/e)^2}^{1} du + \int_e^1 dv$

$= \frac{1}{2}(u)_{1/e^2}^{1} + (v)_e^1$

$= \frac{1}{2}\left(1-\frac{1}{e^2}\right)+1-e$

$= \frac{3}{2}-\frac{1}{2e^2}-e$

Q.9 If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\ \sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to :

(1) $-\sqrt{2}$ (2) 0 (3) –1 (4) $\sqrt{2}$

Ans. **[1]**

Sol. A.M. ≥ G.M.

$\frac{\sin^4\alpha + 4\cos^4\theta + 1 + 1}{4} \ge (\sin^4\alpha \cdot 4\cos^4\beta \cdot 1.1)^{1/4}$

$\sin^4\alpha + 4\cos^4\beta + 2 \ge 4\sqrt{2}\ \sin\alpha\cos\beta$

Note it is given that $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\ \sin\alpha\cos\beta$

Hence A.M. = G.M.

Can be possible when $\sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1$	$\cos\beta = \pm\frac{1}{\sqrt{2}}$
$\alpha = \frac{\pi}{2}$	$\sin\beta = \frac{1}{\sqrt{2}}$, $\beta \in [0, \pi]$

Now, $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

$= -2 \times 1 \times \frac{1}{\sqrt{2}} = -\sqrt{2}$

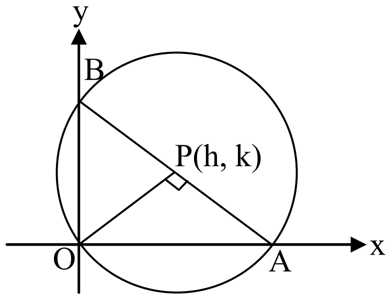
Q.10 If a circle of radius R passes through the origin O and intersects the coordinates axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $(x^2 + y^2)^2 = 4R^2x^2y^2$ (2) $(x^2 + y^2)(x + y) = R^2xy$

(3) $(x^2 + y^2)^2 = 4Rx^2y^2$ (4) $(x^2 + y^2)^3 = 4R^2x^2y^2$

Ans. **[4]**

Sol.



Equation of OP is

$$y - k = -\frac{h}{k}(x - h)$$

$$hx + ky = h^2 + k^2$$

$$OA = \frac{h^2 + k^2}{h}, \quad OB = \frac{h^2 + k^2}{k}$$

Now, $OA^2 + OB^2 = AB^2$

$$\frac{(h^2 + k^2)^2}{h^2} + \frac{(h^2 + k^2)^2}{k^2} = 4R^2$$

$$(x^2 + y^2)^3 = 4R^2x^2y^2$$

Q.11 The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to : (where C is a constant of integration)

(1) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$ (2) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

(3) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$ (4) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

Ans. **[1]**

Sol. $$\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$$

$$= \int \frac{(3x^{13} + 2x^{11})}{(x^4)^4\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4} dx$$

$$= \int \frac{3x^{-3} + 2x^{-5}}{(2 + 3x^{-2} + x^{-4})^4} dx$$

$$= -\frac{1}{2}\int \frac{dt}{t^4} \qquad \text{Put} \quad t = 2 + 3x^{-2} + x^{-4}$$

$$\frac{dt}{dx} = 0 - 6x^{-3} - 4x^{-5}$$

$$\frac{dt}{-2} = (3x^{-3} + 2x^{-5})dx$$

$$= -\frac{1}{2}\int t^{-4}dt$$

$$= -\frac{1}{2}\frac{t^{-3}}{-3} + C$$

$$= \frac{1}{6t^3}$$

$$= \frac{1}{6\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3} + C$$

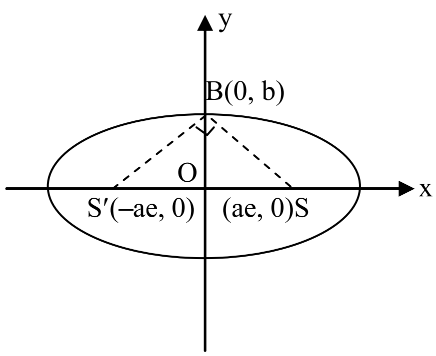
$$= \frac{x^{12}}{6(2x^4+3x^2+1)^3} + C$$

Q.12 Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If ΔS'BS is a right angled triangle with right angle at B and area (ΔS'BS) = 8 sq. units, then the length of a latus rectum of the ellipse is :

(1) 2 (2) $4\sqrt{2}$ (3) 4 (4) $2\sqrt{2}$

Ans. **[3]**

Sol.



$\because \angle SBS' = 90^\circ$

$\therefore \angle BS'O = 45^\circ$

$\tan 45^\circ = \frac{b}{ae}$

$b = ae$

$\therefore b^2 = a^2 - a^2e^2$

$a^2e^2 = a^2 - a^2e^2$

$e = \frac{1}{\sqrt{2}}$

Now area ΔSS′B = 8

$\frac{1}{2} \times 2ae \,.\, b = 8$

$(ae)^2 = 8$

$ae = 2\sqrt{2}$

$a = 4$

$\therefore b = 2\sqrt{2}$

Length of L.R. $= \frac{2b^2}{a} = \frac{2\times 8}{4} = 4$

Q.13 Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is :

(1) 2^{12} (2) 2^{18} (3) 2^{10} (4) 2^{15}

Ans. **[4]**

Sol. $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$(x + 2)(x^2 - 5x + 6) = 0$

$(x + 2)(x - 2)(x - 3) = 0$

$x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$

$-3 < 2x - 1 < 9$

$-2 < 2x < 10$

$-1 < x < 5$

$B = \{0, 1, 2, 3, 4\}$

A × B has 15 elements so number of subset of A × B is 2^{15}

Q.14 Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to :

(1) {1, –1} (2) {3, –3} (3) $\{\sqrt{3}\}$ (4) $\{\sqrt{3}, -\sqrt{3}\}$

Ans. **[4]**

Sol. All four points are coplanar so

$A(-\lambda^2, 1, 1)$ $B(1, -\lambda^2, 1)$ $C(1, 1, -\lambda^2)$ $D(-1, -1, 1)$

$[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} 1+\lambda^2 & 1+\lambda^2 & 0 \\ 1+\lambda^2 & 0 & -(\lambda^2+1) \\ \lambda^2-1 & -2 & 0 \end{vmatrix} = 0$$

$-(1 + \lambda^2) \cdot 2(1 + \lambda^2) + (1 + \lambda^2)(\lambda^2 + 1)(\lambda^2 - 1) = 0$

$(1 + \lambda^2)^2(-2 + \lambda^2 - 1) = 0$

$(1 + \lambda^2)^2(\lambda^2 - 3) = 0$

$\lambda = \pm\sqrt{3}$

Q.15 If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0\ (x \neq 1)$ is :

(1) –7 (2) 5 (3) 7 (4) 6

Ans. **[3]**

Sol. $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$

$f(x) = 3x^2 - 6(a - 2)x + 3a \geq 0, \quad x \in [0, 1]$

also $f'(x) \leq 0, \quad x \in [1, 5]$

hence $f'(x) = 0$ at $x = 1$

$f'(x) = 3x^2 - 6(a - 2)x + 3a$

$f'(1) = 3 - 6(a - 2) + 3a = 0$

$= 1 - 2a + 4 + a = 0$

$\Rightarrow$ a = 5

at a = 5

$f(x) = x^3 - 9x^2 + 15x + 7$

$f(x) - 14 = x^3 - 9x^2 + 15x - 7$

$= (x - 1)^2 (x - 7)$

$$\frac{f(x)-14}{(x-1)^2} = x - 7$$

Root of equation x = 7

Q.16 $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+...+\frac{1}{5n}\right)$ is equal to :

(1) $\tan^{-1}(2)$ (2) $\tan^{-1}(3)$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. **[1]**

Sol. $\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$

$$= \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2\left(1+\frac{r^2}{n^2}\right)}$$

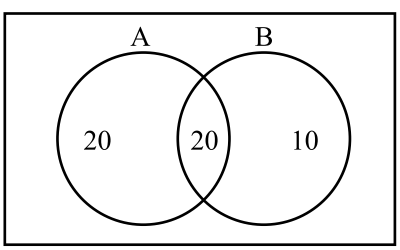
$$= \int_0^2 \frac{dx}{(1+x^2)} = [\tan^{-1}x]_0^2 = \tan^{-1}2$$

Q.17 In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the students selected has opted neither for NCC nor for NSS is :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{2}{3}$ (4) $\frac{5}{6}$

Ans. **[2]**

Sol.



A opted NCC

B opted NSS

$$P(\text{neither A nor B}) = \frac{10}{60} = \frac{1}{6}$$

Q.18 If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det (A) lies in the interval :

(1) $\left(\frac{3}{2}, 3\right]$ (2) $\left(0, \frac{3}{2}\right]$ (3) $\left[\frac{5}{2}, 4\right)$ (4) $\left(1, \frac{5}{2}\right]$

Ans. **[1]**

Sol. $|A| = 2(1 + \sin^2\theta)$; $\theta \in \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

$\sin^2\theta \in \left[0, \frac{1}{2}\right]$

$2\left(1+\left[0, \frac{1}{2}\right]\right)$

$2\left[1, \frac{3}{2}\right]$

det A $\in$ [2, 3]

Q.19 Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is :

(1) 0 (2) 1 (3) 2 (4) $\sqrt{2}$

Ans. **[1]**

Sol. $|z_1| = 9$

$x^2 + y^2 = 81$

$|z_2 - 3i - 4| = 4$

$(x - 3)^2 + (y - 4)^2 = 16$

$C_1C_2 = \sqrt{3^2 + 4^2} = 5$

$r_1 = 9$

$r_2 = 4$

$C_1C_2 = r_1 - r_2$

Internal touching.

Q.20 If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be :

(1) 11 (2) 12 (3) 9 (4) 14

Ans. **[4]**

Sol. $2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow \frac{2 \times n!}{(n-1)!\ 5!} = \frac{n!}{4!\ (n-4)!} + \frac{n!}{6!\ (n-6)!}$

$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$

$\Rightarrow n^2 - 21n + 98 = 0$

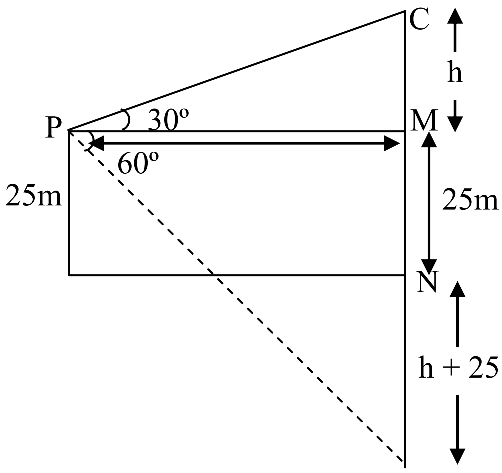
$\Rightarrow n = 14$ and $n = 7$

Q.21 If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is :

(1) 45 (2) 42 (3) 50 (4) 60

Ans. **[3]**

Sol. $\tan 30^o = \dfrac{h}{x}$



$x = \sqrt{3}\,h$ ...(1)

$\tan 60^o = \dfrac{h+50}{x}$

$\sqrt{3}\,x = h + 5$

$\sqrt{3}\,(\sqrt{3}\,h) = h + 50$

$h = 25$

Q.22 Let f be a differentiable function such that f(1) = 2 and f '(x) = f(x) for all x ∈ R. If h(x) = f(f(x)), then h'(1) is equal to :

(1) 4e (2) $2e^2$ (3) $4e^2$ (4) 2e

Ans. **[1]**

Sol. $f'(x) = f(x)$

$\ln(f(x)) = x + c$

$f(x) = e^{x+c}$

$f(x) = ke^x$

$f(1) = k.e^1$

$2 = k.e$

$k = 2/e$

$f(x) = ke^x$

$f'(x) = k.\,e^x$

$h'(x) = f'(f(x))\,.\,f'(x)$

$h'(1) = f'(f(11))\,.\,f'(1)$

$= f'(2)\,.\left(\dfrac{2}{e}\times e\right)$

$= \dfrac{2}{e}.e^2\,.\,2 = 4e$

Q.23 If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, x – 2y – kz = 3 is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is :

(1) $\sqrt{\frac{3}{5}}$ (2) $-\frac{5}{3}$ (3) $-\frac{3}{5}$ (4) $\sqrt{\frac{5}{3}}$

Ans. **[4]**

Sol. Direction ratio of line are 2, 1, – 2

Normal vector of plane is $\hat{i}-2\hat{j}-k\hat{k}$

$$\sin\alpha=\frac{(2\hat{i}-\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-k.\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha=\frac{2k}{3\sqrt{k^2+5}} \qquad \ldots (1)$$

$$\cos\alpha=\frac{2\sqrt{2}}{3} \qquad \ldots (2)$$

Squaring and add

$$1=\frac{4k^2}{9(k^2+5)}+\frac{4\times 2}{9}$$

$9k^2 + 45 = 4k^2 + 8\ (k^2 + 5)$

$$k^2=\frac{5}{3}$$

$$k=\sqrt{\frac{5}{3}}$$

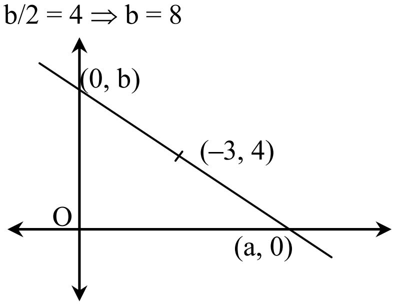
Q.24 If a straight line passing through the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

(1) x – y + 7 = 0 (2) 4x – 3y + 24 = 0 (3) 4x + 3y = 0 (4) 3x – 4y + 25 = 0

Ans. **[2]**

Sol. a/2 = –3 ⇒ a = –6

b/2 = 4 ⇒ b = 8



$$\frac{x}{-6}+\frac{y}{8}=1$$

$$\frac{-4x+3y}{24}=1$$

–4x + 3y = 24

4x – 3y = –24

4x – 3y + 24 = 0

Q.25 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to :

(1) 90°　(2) 30°　(3) 45°　(4) 60°

Ans. **[2]**

Sol. $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{1}{2}\vec{b}$

$\left(\vec{a}.\vec{c}-\frac{1}{2}\right)\vec{b}-(\vec{a}.\vec{b})\vec{c}=0$

$\vec{a}.\vec{c}=\frac{1}{2}$

$\cos\theta = -\frac{1}{2}$

$\theta = \pi/3$

$\vec{a}.\vec{b} = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

$|\alpha-\beta| = \left|\frac{\pi}{2}-\frac{\pi}{3}\right| = \frac{\pi}{6}$

Q.26 The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4 ; then the absolute value of the difference of the other two observations, is :

(1) 1　(2) 7　(3) 3　(4) 5

Ans. **[2]**

Sol. $\frac{3+4+4+x+y}{5} = 4$

$11 + x + y = 20$

$x + y = 9$

$\frac{1}{5}(9 + 16 + 16 + x^2 + y^2) - 16 = 5.2$

$41 + x^2 + y^2 = 106$

$x^2 + y^2 = 65$

$(x + y)^2 - 2xy = 65$

$81 - 65 = 2xy$

$xy = 8$

$(x - y)^2 = x^2 + y^2 - 2xy$

$= 65 - 2 \times 8 = 49$

$\therefore |x - y| = 7$

Q.27 The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point :

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(-\frac{1}{8}, 7\right)$

(3) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (4) $\left(\frac{1}{8}, -7\right)$

Ans. [4]

$$\frac{dy}{dx} = 2x - 5\Big|_{x=x_1} = 2$$

$$2x_1 = 7 \Rightarrow x_1 = \frac{7}{2}$$

$$y_1 = \frac{49}{4} - \frac{35}{2} + 5$$

$$= \frac{49 - 70 + 20}{4} = -\frac{1}{4}$$

$$y + \frac{1}{4} = 2\left(x - \frac{7}{2}\right) \qquad \left(\frac{7}{2}, -\frac{1}{4}\right)$$

$$4y + 1 = 8x - 28 \Rightarrow 8x - 4y - 29 = 0$$

Now check options $x = \frac{1}{8}, y = -7$

Q.28 The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solutions :

(1) is an empty set (2) contains more than two elements

(3) is a singleton (4) contains exactly two elements

Ans. [3]

Sol.
$$\begin{vmatrix} \lambda - 1 & 2 & 2 \\ 1 & 2 - \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$(\lambda - 1)(2\lambda - \lambda^3 - 1) - 2(\lambda - 1) + 2(1 - 2 + \lambda) = 0$

$= (\lambda - 1)^3 = 0$

$\lambda - 1$ is a singelnton

Q.29 The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is :

(1) 7 (2) 8 (3) 3 (4) 6

Ans. **[1]**

Sol. $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$ is always positive, hence

$2m + 1 > 0$ and $D < 0$

$m > -1/2$ $4(1 + 3m)^2 - 4(1 + 2m)\,4(1 + m) < 0$

$m^2 - 6m - 3 < 0$

$3 - \sqrt{12} < m < 3 + \sqrt{12}$

Integral value of m = {0, 1, 2, 3, 4, 5, 6}

Q.30 The total number of irrational terms in the binomial expansion of $(7^{1/5} - 3^{1/10})^{60}$ is :

(1) 54 (2) 55 (3) 49 (4) 48

Ans. **[1]**

Sol. General term $T_{r+1} = {}^{60}C_r . 7^{\frac{60-r}{5}} . 3^{\frac{r}{10}}$

For rational term

r = 0, 10, 20, 30, 40, 50, 60

No. of rational terms = 7

Number of irrational terms = 54