JEE MAIN ONLINE PAPER 2021

Held on February 25, 2021 (Morning)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section-A

Q.1 Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

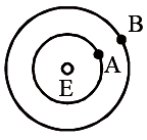
(4) Both A and R are true and R is the correct explanation of A

Q.2 A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is:

(1) 13 cm (2) 16.6 cm

(3) 18.4 cm (4) 14.8 cm

Q.3 Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

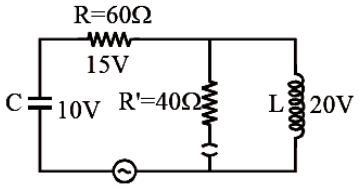


[Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg]

(1) 1.33×10^3 s (2) 3.33×10^2 s

(3) 4.24×10^3 s (4) 4.24×10^2 s

Q.4 The angular frequency of alternating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(1) 0.8 H and 150 μF (2) 0.8 H and 250 μF
(3) 1.33 H and 250 μF (4) 1.33 H and 150 μF

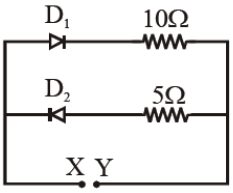
Q.5 A proton, a deuteron and an a particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ____ and their speed is ____ in the ratio.
(1) 1 : 2 : 4 and 2 : 1 : 1
(2) 2 : 1 : 1 and 4 : 2 : 1
(3) 4 : 2 : 1 and 2 : 1 : 1
(4) 1 : 2 : 4 and 1 : 1 : 2

Q.6 Given below are two statement :
Statement-I: A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The band width requirement for the signal is 4 kHz.
Statement-II : The side band frequencies are 1002 kHz. and 998 kHz.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Q.7 If the time period of a two meter long simple pendulum is 2s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :
(1) $\pi^2 ms^{-2}$ (2) 9.8 ms^{-2}
(3) $2\pi^2 ms^{-2}$ (4) 16 m/s^2

Q.8 The pitch of the screw gauge is 1mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is
(1) 1.64 mm (2) 0.82 mm
(3) 1.80 mm (4) 0.90 mm

Q.9 A 5V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



(1) ~ 0.5 A (2) ~ 1.5 A
(3) ~ 0.86 A (4) ~ 0.43 A

Q.10 An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is
(1) 3.8 (2) 8
(3) 7.8 (4) 2.8

Q.11 A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_v = \frac{5}{2}R$ is heated at constant pressure. The ratio dU : dQ : dW :
(1) 5 : 7 : 3 (2) 5 : 7 : 2
(3) 3 : 7 : 2 (4) 3 : 5 : 2

Q.12 An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is:
(1) $\sqrt{\frac{v^2+u^2}{2}}$ (2) $\frac{v-u}{2}$
(3) $\frac{u+v}{2}$ (4) $\sqrt{\frac{v^2-u^2}{2}}$

Q.13 Match List-I with List-II :

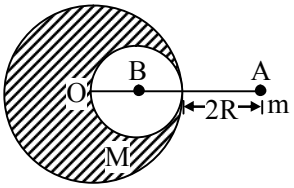
List-I	**List-II**
(a) h (Planck's constant)	(i) $[M L T^{-1}]$
(b) E (kinetic energy)	(ii) $[M L^2 T^{-1}]$
(c) V (electric potential)	(iii) $[M L^2 T^{-2}]$
(d) P (linear momentum)	(iv)$[M L^2 I^{-1} T^{-3}]$

Choose the correct answer from the options given below :
(1) (a)→(iii), (b)→(iv), (c)→(ii), (d)→(i)
(2) (a)→(ii), (b)→(iii), (c)→(iv), (d)→(i)
(3) (a)→(i), (b)→(ii), (c)→(iv), (d)→(iii)
(4) (a)→(iii), (b)→(ii), (c)→(iv), (d)→(i)

Q.14 Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is ____.

(1) 0.2 m (2) 0.1 m
(3) 0.15 m (4) 1.0 m

Q.15 A solid sphere of radius R gravitationally attracts a particle placed at 3R form its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :

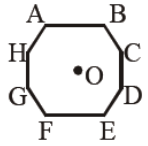


(1) 25 : 36 (2) 36 : 25
(3) 50 : 41 (4) 41 : 50

Q.16 Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to :

(1) $\frac{1}{8}$ (2) $\frac{3}{1}$
(3) $\frac{8}{1}$ (4) $\frac{1}{3}$

Q.17 In an octagon ABCDEFGH of equal side, what is the sum of
$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$ if,
$\overrightarrow{AO}= 2\hat{i}+3\hat{j}-4\hat{k}$

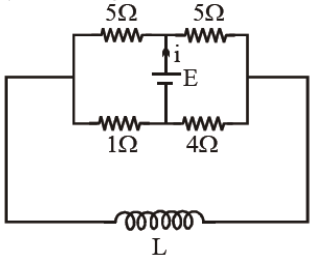


(1) $-16\hat{i}-24\hat{j}+32\hat{k}$ (2) $16\hat{i}+24\hat{j}-32\hat{k}$
(3) $16\hat{i}+24\hat{j}+32\hat{k}$ (4) $16\hat{i}-24\hat{j}+32\hat{k}$

Q.18 Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.
Reason R : The product of their mass and radius must be same, $M_1R_1 = M_2R_2$
In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) A is not correct but R is correct

Q.19 The current (i) at time t = 0 and t = ∞ respectively for the given circuit is :



(1) $\frac{18E}{55},\frac{5E}{18}$ (2) $\frac{10E}{33},\frac{5E}{18}$
(3) $\frac{5E}{18},\frac{18E}{55}$ (4) $\frac{5E}{18},\frac{10E}{33}$

Q.20 Two coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be :

(1) $\frac{2\sqrt{2x}}{x+1}$ (2) $\frac{\sqrt{2x}}{2x+1}$
(3) $\frac{\sqrt{2x}}{x+1}$ (4) $\frac{2\sqrt{2x}}{2x+1}$

Section-B

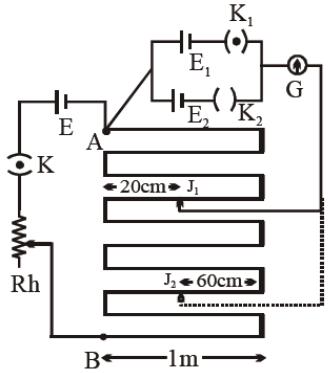
Q.21 A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is ____ × 10^{-8} H.

Q.22 The electric field in a region is given $\vec{E}=\left(\frac{3}{5}E_0\hat{i}+\frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y – z plane) to that of the surface of area 0.3 m^2 (parallel to x – z plane) is a : b, where a = _____.
[Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Q.23 In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be ___ nR, where n denotes number of moles of a gas.

Q.24 A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the height position is _____ m/s. (Take g = 10 m/s^2)

Q.25 In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = _____



Q.26 The same size images are formed by a convex lens when the object is placed at 20cm or at 10cm from the lens. The focal length of convex lens is _____ cm.

Q.27 512 identical drops of mercury are charged to a potential of 2V each. The drops are joined to form a single drop. The potential of this drop is ____ V.

Q.28 A coil of inductance 2H having negligible resistance is connected to a source of supply whose voltage is given by V = 3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4s is _____ J.

Q.29 A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is ___.

Q.30 The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U=\frac{\alpha}{r^{10}}-\frac{\beta}{r^2}-3$ where, a and b are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{a/b}$, where a = ____.

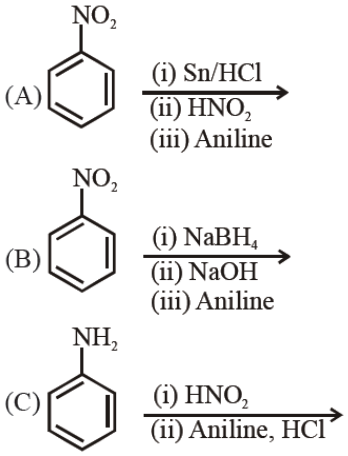
CHEMISTRY

Section-A

Q.31 Given below are two statements:
Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.
Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but statement II is true
(2) Statement I is true but statement II is false
(3) Both statement I and statement II are true
(4) Both statement I and statement II are false

Q.32 According to molecular theory, the species among the following that does not exist is:
(1) He_2^+ (2) He_2^-
(3) Be_2 (4) O_2^{2-}

Q.33 Which of the following reaction/s will not give p-aminoazobenzene?

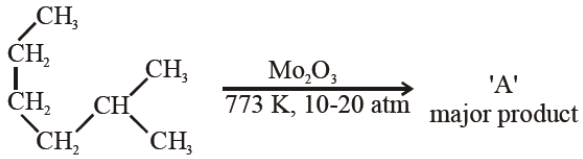


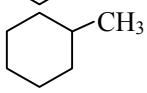
(1) A only (2) B only
(3) C only (4) A and B

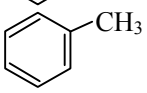
Q.34 Which of the following equation depicts the oxidizing nature of H_2O_2 ?

(1) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
(2) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
(3) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$
(4) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$

Q.35 Identify A in the given chemical reaction.



(1) 
(2) 
(3) 

(4) 

Q.36 Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is:

(1) 51.63 (2) 63.53
(3) 53.33 (4) 50.33

Q.37 Which one of the following reactions will not form acetaldehyde?

(1) $CH_3CH_2OH \xrightarrow[573K]{Cu}$

(2) $CH_3CN \xrightarrow[(ii)\,H_2O]{(i)\,DIBAL-H}$

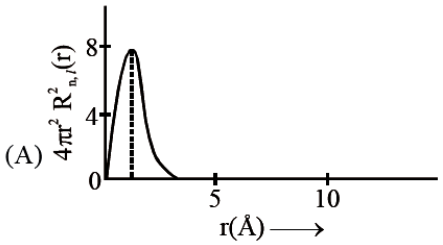
(3) $CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

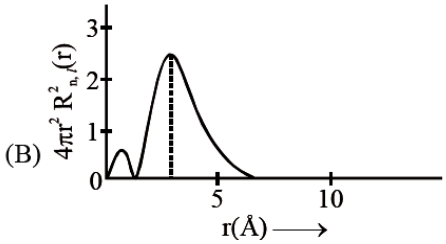
(4) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$

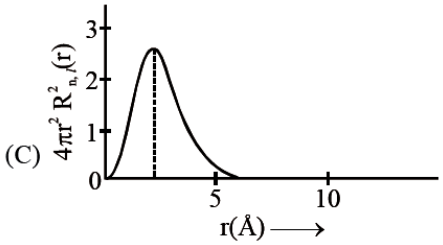
Q.38 The correct statement about B_2H_6 is:

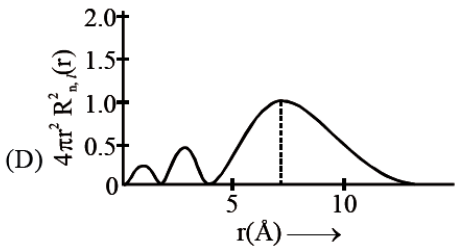
(1) Terminal B–H bonds have less p-character when compared to bridging bonds.
(2) The two B–H–B bonds are not of same length
(3) All B–H–B angles are of 120°
(4) Its fragment, BH_3, behaves as a Lewis base

Q.39 The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below:

(A) 

(B) 

(C) 

(D) 

The correct plot for 3s orbital is:

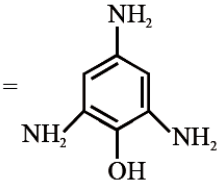
(1) (B) (2) (A)
(3) (D) (4) (C)

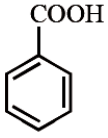
Q.40 Given below are two statements:
Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.
Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog. In the light of the above statements, choose the correct answer from the options given below:
(1) Both statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Statement I is false but Statement II is true

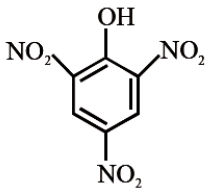
Q.41 In which of the following pairs, the outer most electronic configuration will be the same?
(1) Cr^{+} and Mn^{2+} (2) Ni^{2+} and Cu^{+}
(3) Fe^{2+} and Co^{+} (4) V^{2+} and Cr^{+}

Q.42 Which of the glycosidic linkage between galactose and glucose is present in lactose?
(1) C-1 of galactose and C-4 of glucose
(2) C-1 of glucose and C-6 of galactose
(3) C-1 of glucose and C-4 of galactose
(4) C-1 of galactose and C-6 of glucose

Q.43 Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are:

A = 

B = 

C = 

(1) B only (2) C only
(3) B and C only (4) A and B only

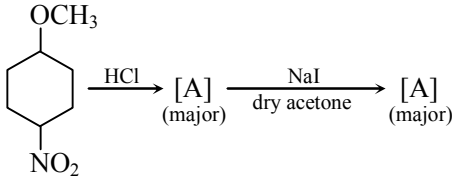
Q.44 The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are:
(1) d^2sp^3 and diamagnetic
(2) sp^3d^2 and diamagnetic
(3) d^2sp^3 and paramagnetic
(4) sp^3d^2 and paramagnetic

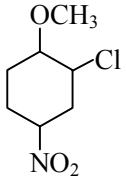
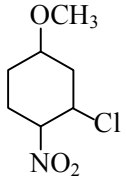
Q.45 Ellingham diagram is a graphical representation of:
(1) ΔH vs T (2) ΔG vs T
(3) ΔG vs P (4) $(\Delta G - T\Delta S)$ vs T

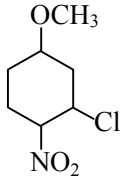
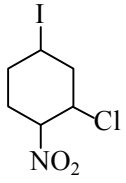
Q.46 The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is:
[Assume : No cyano complex is formed;
$K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and
$K_a(HCN) = 6.2 \times 10^{-10}$]
(1) 0.625×10^{-6} (2) 1.9×10^{-5}
(3) 2.2×10^{-16} (4) 1.6×10^{-6}

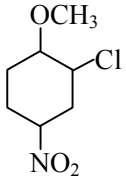
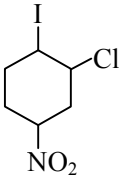
Q.47 In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is
(1) zero (2) $\frac{1}{n}$
(3) 1 (4) ∞

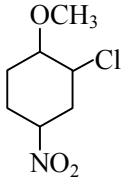
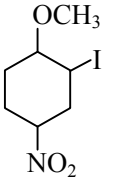
Q.48 Identify A and B in the chemical reaction.



(1) A=  B= 

(2) A=  B= 

(3) A=  B= 

(4) A=  B= 

Q.49 Which statement is correct ?
(1) Synthesis of Buna-S needs nascent oxygen.
(2) Neoprene is an addition copolymer used in plastic bucket manufacturing.
(3) Buna-S is a synthetic and linear thermosetting polymer.
(4) Buna-N is a natural polymer.

Q.50 The major product of the following chemical reaction is :

$$CH_3CH_2CN \xrightarrow[(3)\,Pd/BaSO_4,H_2]{(1)\,H_3O^+,\Delta \;(2)\,SOCl_2} ?$$

(1) $CH_3CH_2CH_3$ (2) $CH_3CH_2CH_2OH$
(3) $(CH_3CH_2CO)_2O$ (4) CH_3CH_2CHO

Section-B

Q.51 Among the following, the number of halide(s) which is/are inert to hydrolysis is _____.
(A) BF_3 (B) $SiCl_4$
(C) PCl_5 (D) SF_6

Q.52 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _____ K. (Rounded-off to the nearest integer)
[Given Kb for (H_2O) = 0.52 K kg mol^{-1}]

Q.53 In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _____ mL. (Rounded-off to the nearest integer)

Q.54 A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is _____. (Rounded-off to the nearest integer)

Q.55 The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be –742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CN_{(s)} + \frac{3}{2}O_2(g) \rightarrow N_2(g) + O_2(g) + H_2O(l)$$

is _____ kJ. (Rounded off to the nearest integer)
[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

Q.56 Using the provided information in the following paper chromatogram :

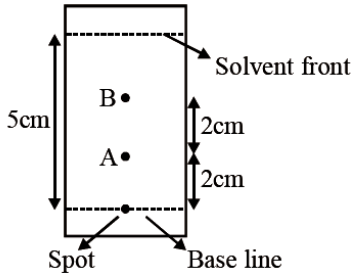
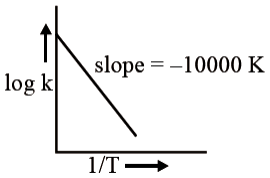


Figure : Paper chromatography for compounds A and B.

the calculate R_f value of A _____ $\times 10^{-1}$.

Q.57 For the reaction, aA + bB → cC + dD, the plot of log k vs $\frac{1}{T}$ is given below :



The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is _____ K.
(Rounded-off to the nearest integer)
[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K.]

Q.58 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After his methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is_____. (Rounded-off to the nearest integer)

Q.59 Consider the following chemical reaction.

$$CH{\equiv}CH \xrightarrow[(2)\,CO,\,HCl,\,AlCl_3]{(1)\,Red\ hot\ Fe\ tube,\,873K} Product$$

The number of sp^2 hybridized carbon atom(s) present in the product is _____.

Q.60 The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is –325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is –728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (–) _____ $\times 10^{-1}$ kJ mol^{-1}.

MATHEMATICS

Section-A

Q.61 When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{4}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

Q.62 If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta . \sin^{2n}\phi$ then :

(1) $xy - z = (x + y)\, z$ (2) $xy + yz + zx = z$
(3) $xyz = 4$ (4) $xy + z = (x + y)z$

Q.63 Let $f, g : N \to N$ such that $f(n + 1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function.
Which of the following statements is NOT true?
(1) If fog is one-one, then g is one-one
(2) If f is onto, then $f(n) = n\ \forall\ n \in N$
(3) f is one-one
(4) If g is onto, then fog is one-one

Q.64 The equation of the line through the point (0,1,2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is :

(1) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$ (2) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$
(3) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$ (4) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Q.65 Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$
(3) $\frac{5}{8}$ (4) $\frac{1}{2}$

Q.66 The value of the integral

$$\int \sin\theta \cdot \frac{\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$$

is :
(where c is a constant of integration)

(1) $\frac{1}{18}[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta]^{3/2} + c$

(2) $\frac{1}{18}[9 - 2\cos^6\theta + 3\cos^4\theta - 6\sin^2\theta]^{3/2} + c$

(3) $\frac{1}{18}[9 - 2\sin^6\theta + 3\cos^4\theta - 6\sin^2\theta]^{3/2} + c$

(4) $\frac{1}{18}[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta]^{3/2} + c$

Q.67 The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\le$ t, is :

(1) $\frac{e-1}{3e}$ (2) $\frac{e+1}{3}$
(3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

Q.68 A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

(1) 10 (2) $10\sqrt{3}$
(3) $10(\sqrt{3} + 1)$ (4) $10(\sqrt{3} - 1)$

Q.69 A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

(1) (–6, 0) (2) (4, 5)
(3) (5, 4) (4) (0, 3)

Q.70 All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(3) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

Q.71 Let the lines $(2 - i)z = (2 + i)\ \bar{z}$ and $(2 + i)z + (i - 2)\ \bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C.
If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is:

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $3\sqrt{2}$ (4) $\frac{3}{2\sqrt{2}}$

Q.72 The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 2)^2 = 12$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 4)^2 = 4$

Q.73 If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(1) $a + b = c + d$ (2) $a - b = c - d$

(3) $a - c = b + d$ (4) $ab = \frac{c+d}{a+b}$

Q.74 $\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)$ is equal to

(1) $\frac{1}{2}$ (2) 0

(3) $\frac{1}{e}$ (4) 1

Q.75 The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

(1) $\frac{1}{72}$ (2) $\frac{5}{216}$

(3) $\frac{1}{36}$ (4) $\frac{1}{54}$

Q.76 The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

(1) 36 (2) 24

(3) 45 (4) 30

Q.17 The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

(1) 3 (2) 2

(3) 0 (4) 4

Q.18 If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

(1) (5, 4) (2) (4, 5)

(3) (4, 4) (4) (5, 5)

Q.79 The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

(1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \rightarrow B)$

(3) $A \rightarrow (A \leftrightarrow B)$ (4) $A \rightarrow (A \vee B)$

Q.80 If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right)=0$, then ordered pair (a, b) is equal to :

(1) (5, 8) (2) (–5, 8)

(3) (5, – 8) (4) (–5, –8)

Section-B

Q.81 Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1 and x = 1. If $\lim\limits_{x\to0}\frac{f(x)}{x^4}=1$, then 5.f(2) is equal to _____.

Q.82 The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is _____.

Q.83 The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _____.

Q.84 Let $A_1, A_2, A_3, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of An is less than one, is _____.

Q.85 Let $A = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is_____.

Q.86 If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _____ .

Q.87 The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

Q.88 Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to _____.

Q.89 If the system of equations
$kx + y + 2z = 1$
$3x - y - 2z = 2$
$-2x - 2y - 4z = 3$
has infinitely many solutions, then k is equal to _____.

Q.90 The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is _____.

JEE MAIN ONLINE PAPER 2021

Held on February 25, 2021 (Morning)

Hints & Solutions

PHYSICS

1.[1] A and R are true but R is not the correct explanation of A.

2.[4] d = 6cm, f = 504, v = 336 m/s
e = 0.3d

$l + e = \frac{\lambda}{4} = \frac{V}{4f}$

$l = 16.66 - 0.3 \times 6$

$l = 14.866$ cm

$l = 14.8$ cm

3.[1] $T = 2\pi\sqrt{\frac{r^2}{GM}}$

$T_A = 2\pi\sqrt{\frac{(6400+600)\times10^3}{GM}}$

$T_A = 2\pi \times 10^9\sqrt{\frac{7^3}{GM}}$

$T_B = 2\pi \times 10^9\sqrt{\frac{8^3}{GM}}$

$T_B - T_A = \frac{2\pi10^9}{\sqrt{GM}}\left[8\sqrt{8} - 7\sqrt{7}\right]$

$= 314 \times 4.107$

$= 1289.64$

$= 1.289 \times 10^3$ s

4.[2] Current through 60Ω resistance $\frac{15}{60} = \frac{1}{4}$ A

thus capacitor current = $\frac{1}{4}$ A

$\because V_C = I X_C$

$10 = \frac{1}{4} \times \frac{1}{\omega C}$

$\therefore C = \frac{1}{40\omega} = \frac{1}{4000} = 250\ \mu F$

Now,

current through 40W resistance = $\frac{20}{40} = \frac{1}{2}$ A

thus current through inductor = $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ A

$V_L = IX_L = \frac{1}{4} \times \omega L$

$20 = \frac{1}{4} \times 100 \times L$

$\Rightarrow L = 0.8$ H

5.[2] $F = q(\vec{v} \times \vec{B}) = \frac{q}{m}(\vec{P} \times \vec{B})$

$\Rightarrow F \propto \frac{q}{m}$

thus $F_1 : F_2 : F_3 = \frac{q_1}{m_1} : \frac{q_2}{m_2} : \frac{q_3}{m_3}$

$= \frac{e}{m_p} : \frac{e}{2m_p} : \frac{2e}{4m_p}$

$= \frac{1}{1} : \frac{1}{2} : \frac{2}{4}$

$= 2 : 1 : 1$

Now for speed calculation

P = constant $\Rightarrow v \propto \frac{1}{m}$

thus $v_1 : v_2 : v_3 = \frac{1}{m_p} : \frac{1}{2m_p} : \frac{1}{4m_p}$

$= \frac{1}{1} : \frac{1}{2} : \frac{1}{4} = 4 : 2 : 1$

6.[3] $f_m = 2$kHz
$f_c = 1$MHz $= 1000$ kHz
Band width $= 2\ f_m = 4$ kHz
$\therefore$ Side frequencies will be
$= f_c \pm f_m$
$= (1000 \pm 2)$ kHz
$= 998$ kHz & 1002 kHz
So statement-I & statement-II both are correct.

7.[3] $T = 2\pi\sqrt{\frac{\ell}{g}}$ g

$2 = 2\pi\sqrt{\frac{2}{g}}$

$\Rightarrow g = 2\pi^2$

8.[2] Least count = $\frac{1\text{mm}}{100} = 0.01\text{mm}$

zero error = + 8 × LC = + 0.08 mm

True reading (Diameter)

= (1 mm + 72 × LC) – (Zero error)

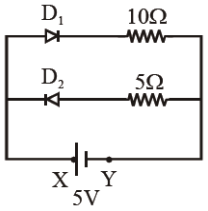
= (1mm +72 × 0.01 mm) – 0.08 mm

= 1.72 mm – 0.08 mm

= 1.64 mm

therefore, radius = $\frac{1.64}{2}$ = 0.82 mm.

9.[4]



Here only D_1 will work and we know for silicon diode, potential drop on D_1 will be 0.7V

$I = \frac{5-0.7}{10} = 0.43$ A

10.[4] $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p \times 2e}{m_p \times e}} = \sqrt{8} = 2\sqrt{2}$

$= 2\sqrt{2}$

$\frac{\lambda_p}{\lambda_\alpha} = 2 \times 1.4 = 2.8$

11.[2] $dU = nC_v dT$

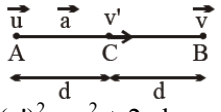
$dQ = nC_p dT$

$dW = PdV = nRdT$ (isobaric process)

$dU : dQ : dW : C_v : C_p : R$

$= \frac{5R}{2} : \frac{7R}{2} : R = 5 : 7 : 2$

12.[1]



$(v')^2 = u^2 + 2ad$

$v^2 = (v')^2 + 2ad$

solving, we get

$v' = \sqrt{\frac{v^2 + u^2}{2}}$

13.[2] By dimensional analysis.

14.[2] We know, the magnetic field on the axis of a current carrying circular ring is given by

$B = \frac{\mu_0}{4\pi} \frac{2NIA}{(R^2 + x^2)^{3/2}}$

$\therefore \frac{B_1}{B_2} = \frac{8}{1} = \left[\frac{R^2 + (0.2)^2}{R^2 + (0.05)^2}\right]^{3/2}$

$4[R^2 + (0.05)^2] = [R^2 + (0.2)^2]$

$4R^2 - R^2 = (0.2)^2 - 4 \times (0.05)^2$

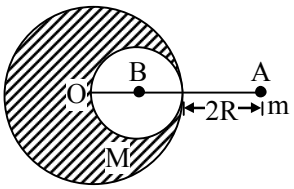
$4R^2 - R^2 = (0.2)^2 - (0.1)^2$

$3R^2 = 0.3 \times 0.1$

$R^2 = (0.1)^2 \Rightarrow R = 0.1$

15.[3] Let initial mass of sphere is m'. Hence mass of removed portion will be m'/8

$F_1 = \text{m.E.} = \frac{\text{m.Gm'}}{9R^2}$



$F_2 = m\left[\frac{G.m'}{(3R)^2} - \frac{G.m'/8}{(5R/2)^2}\right]$

$= \frac{Gm'}{9R^2} - \frac{Gm' \times 4}{8 \times 25}$

$= \left(\frac{1}{9} - \frac{1}{50}\right)\frac{Gm'}{R^2}$

$F_2 = \frac{41}{50 \times 9} \cdot \frac{Gm'}{R^2}$

$\frac{F_1}{F_2} = \frac{1}{9} \times \frac{50 \times 9}{41} = \frac{50}{41}$

16.[3] $T_x = t$; $T_y = 2t$

$3T_y = 6t$,

$N_1' = N_2'$

$N_1 e^{-\lambda_1 6t} = N_2 e^{-\lambda_2 6t}$

$\frac{N_1}{N_2} = e^{(\lambda_1 - \lambda_2)6t} = e^{\ell n 2\left(\frac{1}{t} - \frac{1}{2t}\right) \times 6t}$

$= e^{(\ell n 2) \times 3} = e^{\ell n 8} = 8$

$\frac{N_1}{N_2} = \frac{8}{1}$

17.[2] We know,

$\because \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}=\vec{0}$

By triangle law of vector addition, we can write

$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$; $\overrightarrow{AC}=\overrightarrow{AO}+\overrightarrow{OC}$

$\overrightarrow{AD}=\overrightarrow{AO}+\overrightarrow{OD}$; $\overrightarrow{AE}=\overrightarrow{AO}+\overrightarrow{OE}$

$\overrightarrow{AF}=\overrightarrow{AO}+\overrightarrow{OF}$; $\overrightarrow{AG}=\overrightarrow{AO}+\overrightarrow{OG}$

$\overrightarrow{AH}=\overrightarrow{AO}+\overrightarrow{OH}$

Now

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$

$=(7\overrightarrow{AO}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}$

$=(7\overrightarrow{AO})+\vec{0}-\overrightarrow{OA}$

$=(7\overrightarrow{AO})+\overrightarrow{OA}$

$=8\overrightarrow{AO}=8(2\hat{i}+3\hat{j}-4\hat{k})$

$=16\hat{i}+24\hat{j}-32\hat{k}$

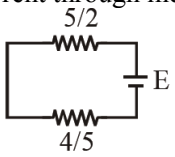
18.[2] $V_e=\sqrt{\frac{2GM}{R}}$

$\frac{M_1}{R_1}=\frac{M_2}{R_2}$

$M_1R_2 = M_2R_1$

Hence reason R is not correct.

19.[4] At t = 0, current through inductor is zero,



hence $R_{eq} = (5+1) \parallel (5+4) = \frac{18}{5}$

$i_1 = \frac{E}{18/5}=\frac{5E}{18}$

At t = ∞, inductor becomes a simple wire and now the circuit will be as shown in figure

hence Req = (5||5) + (4||1) = $\frac{33}{10}$; (|| ⇒ parallel)

$i_2 = \frac{E}{33/10}=\frac{10E}{33}$

20.[4] Given that, $\frac{I_1}{I_2}=2x$

We know,

$I_{max}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ & $I_{min}=\left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

$\therefore \frac{I_{max}-I_{min}}{I_{max}+I_{min}}=\frac{2\sqrt{I_1I_2}}{I_1+I_2}=\frac{2\sqrt{I_1I_2}}{\frac{I_1}{I_2}+1}=\frac{2\sqrt{2x}}{2x+1}$

21.[10] λ = 960 m

C = 2.56 μF = 2.56 × 10^{-6} F

c = 3 × 10^8 m/s

L = ?

Now at resonance, $\omega_0=\frac{1}{\sqrt{LC}}$

[Resoant frequency]

$2\pi f_0=\frac{1}{\sqrt{LC}}$

On substituting $f_0=\frac{c}{\lambda}$, we have $2\pi\frac{c}{\lambda}=\frac{1}{\sqrt{LC}}$

Squaring both sides : $4\pi^2\frac{c^2}{\lambda^2}=\frac{1}{LC}$

$=\frac{4\times10\times(3\times10^8)^2}{(960)^2}=\frac{1}{L\times2.56\times10^{-6}}$

$\Rightarrow \frac{1}{L}=\frac{4\times10\times9\times10^{16}\times2.56\times10^{-6}}{960\times960}$

$\Rightarrow L = 10 \times 10^{-8}$ H

22.[1] $\vec{E}=\left(\frac{3}{5}E_0\hat{i}+\frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$

$A_1 = 0.2$ m^2 [parallel to y – z plane]

$= \vec{A}_1 = 0.2\ m^2\,\hat{i}$

$A_2 = 0.3$ m^2 [parallel to x – z plane]

$\vec{A}_2 = 0.3m\,\hat{j}$

Now

$\phi_a=\left[\frac{3}{5}E_0\hat{i}+\frac{4}{5}E_0\hat{j}\right].\left[0.2\hat{i}\right]=\frac{3\times0.2}{5}E_0$

& $\phi_b=\left[\frac{3}{5}E_0\hat{i}+\frac{4}{5}E_0\hat{j}\right].\left[0.3\hat{i}\right]=\frac{4\times0.3}{5}E_0$

Now $\frac{\phi_a}{\phi_b}=\frac{0.6}{1.2}=\frac{1}{2}=\frac{a}{b}$

$\Rightarrow$ a : b = 1 : 2

$\Rightarrow$ a = 1

23.[50] $P = kV^3$

$T_i = 100°C$ & $T_f = 300°C$

$\Delta T = 300 - 100$

$\Delta T = 200°C$

$P = kV^3$ now PV = nRT

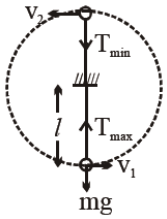
$\therefore kV^4 = nRT$ now $4kV^3\,dV = nRdT$

$\therefore PdV = nRdT/4$

$\therefore$ Work = $\int PdV=\int\frac{nRdT}{4}=\frac{nR}{4}\Delta T$

$=\frac{200}{4}\times nR = 50nR$

24.[5] Let the speed of bob at lowest position be v_1 and at the highest position be v_2. Maximum tension is at lowest position and minimum tension is at the highest position. Now, using, conservation of mechanical energy,



$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mg2\ell$$

$$\Rightarrow v_1^2 = v_2^2 + 4gl \ldots (1)$$

Now $T_{max} - mg = \frac{mv_1^2}{\ell}$

$$\Rightarrow T_{max} = mg + \frac{mv_1^2}{\ell} \ \& \ T_{min} = \frac{mv_2^2}{\ell} - mg$$

$$\frac{T_{max}}{T_{min}} = \frac{5}{1}$$

$$\Rightarrow \frac{mg + \frac{mv_1^2}{\ell}}{\frac{mv_2^2}{\ell} - mg} = \frac{5}{1}$$

$$\Rightarrow mg + \frac{mv_1^2}{\ell} = \left[\frac{mv_2^2}{\ell} = mg\right]5$$

$$\Rightarrow mg + \frac{m}{\ell}\left[v_2^2 + 4g\ell\right] = \frac{5mv_2^2}{\ell} - 5mg$$

$$\Rightarrow mg + \frac{mv_2^2}{\ell} + 4mg = \frac{5mv_2^2}{\ell} - 5mg$$

$$\Rightarrow 10\ mg = \frac{4mv_2^2}{\ell}$$

$$v_2^2 = \frac{10 \times 10 \times 1}{4}$$

$\Rightarrow v_2^2 = 25 \Rightarrow v_2 = 5$ m/s

Thus, velocity of bob at highest position is 5 m/s.

25.[1] Length of AB = 10 m

For battery E_1, balancing length is ℓ_1

$\ell_1 = 380$ cm [from end A]

For battery E_2, balancing length is ℓ_2

$\ell_2 = 760$ cm [from end A]

Now, we know that $\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$

$$\Rightarrow \frac{E_1}{E_2} = \frac{380}{760} = \frac{1}{2} = \frac{a}{b}$$

$\therefore$ a = 1 & b = 2.

a = 1

26.[15] $m = \frac{f}{u+f}$

$$+m = \frac{f}{-10+f} \quad \ldots.(1)$$

$$-m = \frac{f}{-20+f} \quad \ldots.(2)$$

(1)/(2)

$$-1 = \frac{f-20}{f-10}$$

$10 - f = f - 20$

$30 = 2f$

f = 15 cm

27.[128] Q = 512q

$Volume_i = Volume_f$

$$512 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$2^9 r^3 = R^3$

R = 8r

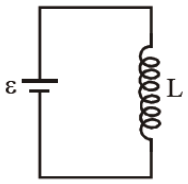
$$2 = \frac{kq}{r}$$

$$v = \frac{kQ}{r} = \frac{k512q}{8r}$$

v = 128.

28.[144] $\varepsilon = \frac{Ldl}{dt}$

$$3\int_0^4 tdt = 2\int_0^4 dI$$



$$\frac{3}{2} \times 16 = 2I$$

I = 12

$$V = \frac{1}{2}LI^2 = \frac{1}{2} \times 2(12)^2 = 144\ J$$

29.[3600] Given that mass of gas is 4u hence its molar mass M is 4g/mol

$$\therefore \frac{1}{2}mv^2 = nC, \Delta T$$

$$\frac{1}{2}m\times(30)^2 = \frac{m}{M}\times\frac{3R}{2}\times\Delta T$$

$$\therefore \Delta T = \frac{3600}{3R}$$

30.[1] For equilibrium

$$\frac{dU}{dr} = 0$$

$$\frac{-10\alpha}{r^{11}} + \frac{5\beta}{r^6} = 0$$

$$\frac{5\beta}{r^6} = \frac{10\alpha}{r^{11}}$$

$$r^5 = \frac{2\alpha}{\beta}$$

$$r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$$

a = 1

CHEMISTRY

31.[3] The +3 oxidation state of lanthanide is most stable and therefore lanthanide in +4 oxidation state has strong tendency to gain e– and converted into +3 and therefore act as strong oxidizing agent.

eg Ce^{+4}

And therefore CeO_2 is used to oxidized alcohol aldehyde and ketones.

Lanthanide in +2 oxidation state has strong tendency to loss e– and converted into +3 oxidation state therefore act as strong reducing agent.

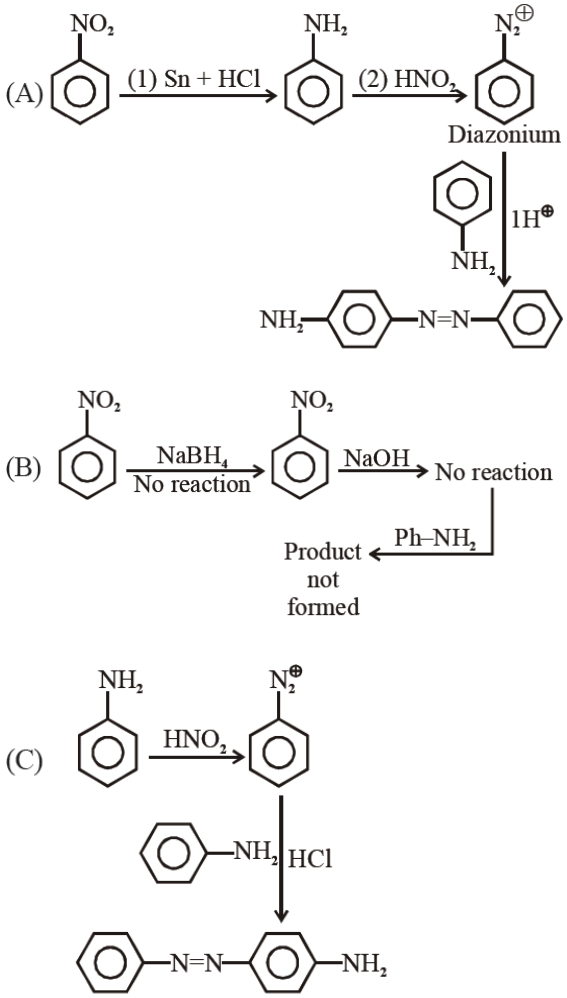
$\therefore$ $EuSO_4$ act as strong reducing agent.

32.[3]

Chemical Species	Bond Order
He_2^+	0.5
He_2^-	0.5
Be_2	0
O_2^{2-}	1

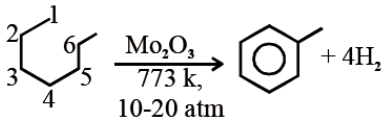
According to M.O.T. If bond order of chemical species is zero then that chemical species does not exist.

33.[2] In basic or neutral medium N–N coupling favourable while in slightly acidic medium C–N coupling favourable.



34.[2] I^- is oxidised to I_2 by H_2O_2

Hence answer is (2)

35.[4] 

Mo_2O_3 at 773 K temperature and 10-20-atm pressure is aromatising agent.

36.[3] $n_c = n_{CO_2} = \frac{2.64}{44} = 0.06$

$$n_H = 2 \times n_{H_2O} = \frac{1.08}{18}\times 2 = 0.12$$

$$m_0 = 1.80 - 12 \times \frac{2.64}{44} - \frac{1.08}{18} \times 2$$

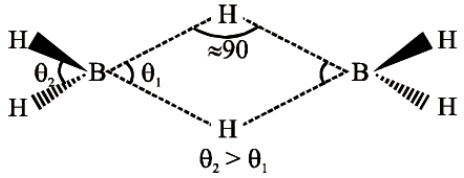
$= 1.80 - 0.72 - 0.12 = 0.96$ gm

$$\% 0 = \frac{0.96}{1.80} \times 100 = 53.33\%$$

Hence answer is (3)

37.[4] $CH_3–CH_2–OH \xrightarrow[\text{Strong oxidising agent}]{CrO_3 \cdot H_2SO_4} CH_3–\overset{\overset{O}{\|}}{C}–OH$ (Carboxylic acid is formed by complete oxidation)

38.[1]



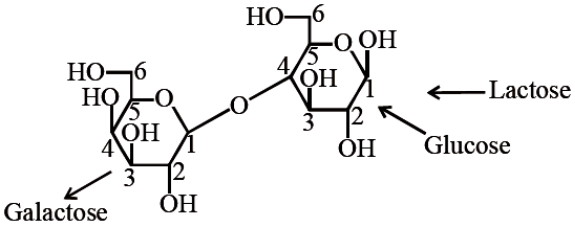
$\theta_2 > \theta_1$, ∴ B–H (terminal) having less p-character as compare to bridge bond.
Both B–H–B bridge bond having same bond length.
B–H–B bond angle is ≈ 90°
BH_3 is e^- deficient species and therefore act as lewis acid

39.[3] Number of radial nodes = $n - \ell - 1 = 3 - 0 - 1 = 2$
Therefore corresponding graph is (D)
Hence answer is (3)

40.[1] Reducing smog is a mixture of smoke, fog and sulphur dioxide. Tropospheric pollutants such as hydrocarbon and nitrogen oxide contribute to the formation of photochemical smog.

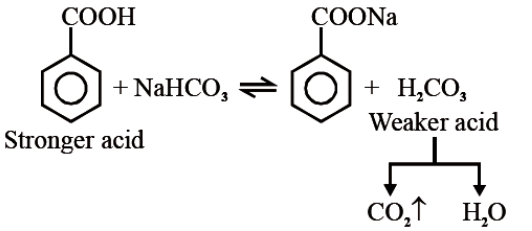
41.[1] Option-1 Mn^{+2} $[Ar]3d^5$,Cr^+ $[Ar]3d^5$
Option-2 Ni^{+2} $[Ar]3d^8$,Cu^+ $[Ar]3d^{10}$
Option-3 Fe^{+2} $[Ar]3d^6$,Co^+ $[Ar]3d^7$ $4s^1$
Option-4 V^{+2} $[Ar]3d^3$,Cr^+ $[Ar]3d^5$

42.[1]

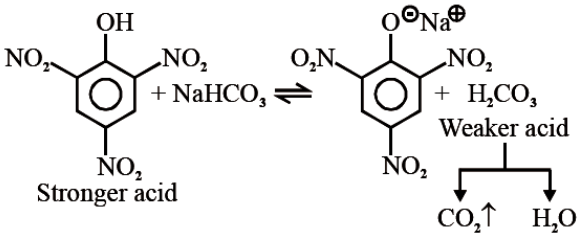


In lactose linkage is formed between C_1 of galactose and C_4 of gluocse.

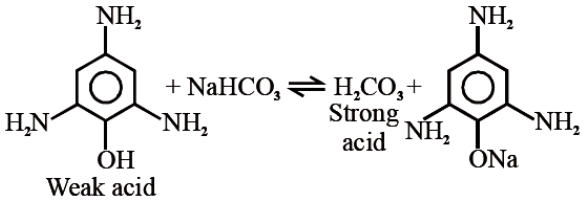
43.[3]



equilibrium favours forward direction and CO^2 ↑ is librated.

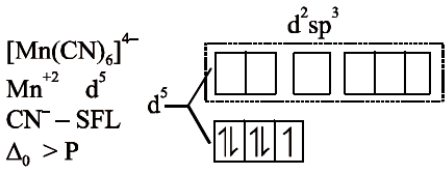


Equilibrium favours forward direction and CO_2 ↑ is librated.

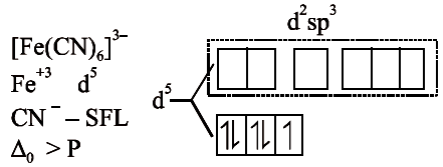


Equilibrium favours back word direction and CO_2 ↑ is not librated.

44.[3]



∴ hybridisation is d^2sp^3 and due to presence of unpaired e^- complex is paramagnetic in nature



∴ hybridisation is d^2sp^3 and due to presence of unpaired e^- complex paramagnetic in nature

45.[2] Ellingham diagram is a graphical representation of ΔG vs T when metal heated with oxygen to form metal oxide

46.[2] $\frac{K_{sp}}{Ka} = \frac{s^2}{(H^+)}$; $s = \sqrt{\frac{K_{sp}}{K_a}(H^+)}$

$s = \sqrt{\frac{2.2\times10^{-16}}{6.2\times10^{-10}}\times10^{-3}}$

$s = 1.9 \times 10^{-5}$

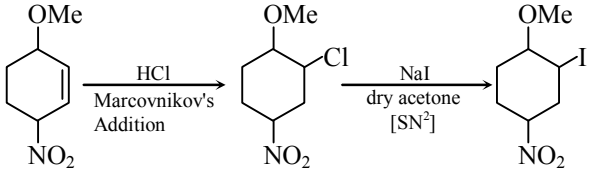
Hence answer is (2)

47.[2] As per Freundlich adsorption isotherm

$\left(\frac{x}{m}\right) = KP^{1/n} \rightarrow x = \frac{1}{n}$

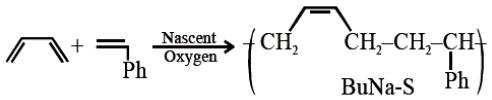
Hence answer is (2)

48.[4]

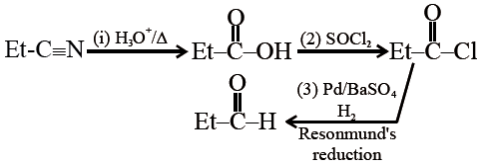


$\Rightarrow$ Ist reaction marcovnikov's addition of HCl on double bond while 2nd reaction is halide substitution by finkelstein reaction.

49.[1]



50.[4]



Final product of reaction is propanaldehyde.

51.[1] SF_6 is inert towards hydrolysis

$\therefore$ answer is (1)

52.[375] $\Delta T_b = iK_bm = (1 + 4\alpha) \times 0.52 \times 1$

$= 3.4 \times 0.52 \times 1 = 1.768$

$T_b = 1.768 + 313.15 = 374.918$ K = 375K

Hence answer is (375)

53.[173] $\overset{+6}{Cr}O_4^{2-} + \overset{+6}{S}_2O_3^{2-} \rightarrow \overset{+6}{S}O_4^{2-} + \overset{+3}{Cr}(OH)_4^-$

gm equi. of $CrO_4^{2-} = S_2O_3^{2-}$

$0.154 \times 3 \times v = 0.25 \times 40 \times 8$

$v = 173.16 = 173$ ml

Hence answer is (173)

54.[70] $P \propto T$

$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow \frac{40}{35} = \frac{T_2}{300}$

$T_2 = 342.854$ K = 69.70°C ; 70°C

Hence answer is (70)

55.[741] $\Delta H = \Delta U + \Delta n_g RT$

$= -742.24 + \frac{1}{2} \times \frac{8.314}{1000} \times 298$

$= -741$ kJ/mol

Hence answer is (741)

56.[4] $Rf = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$

on chromatogram distance travelled by compound is $\rightarrow$ 2 cm

Distance travelled by solvent = 5 cm

So $R_f = \frac{2}{5} = 4 \times 10^{-1} = 0.4$

57.[526] $\log K = \log A - \frac{Ea}{2.303RT}$

$|\text{Slope}| = \frac{Ea}{2.303R} = 10{,}000$

$\log\left(\frac{K_2}{K_1}\right) = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log\left(\frac{10^{-4}}{10^{-5}}\right) = 10{,}000\left[\frac{1}{500} - \frac{1}{T_2}\right]$

$T_2 = 526.31 \approx 526$K

Hence answer is (526)

58.[4] Upto first end point

gm equi. of $(NaOH + Na_2CO_3) = HCl$

$x + y \times 1 = \frac{1}{10} \times 17.5$

$x + y = 1.75$ …(1)

Upto second end point

$NaOH + Na_2CO_3 \equiv HCl$

$x + y \times 2 = \frac{1}{10} \times 19$

$x + 2y = 1.9$ …(2)

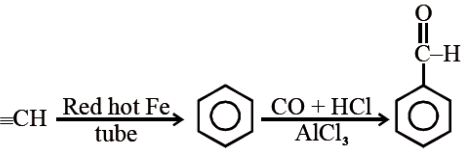
$y = 0.15$

$$\%Na_2CO_3 = \frac{0.15\times10^{-3}\times106}{0.4}\times100$$

$= 3.975\% = 4\%$

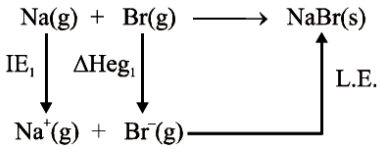
Hence answer is (4)

59.[7]



In benzaldehyde total number of sp^2 'C' are 7.

60.[5576]



$\Delta H_{formation} = IE_1 + \Delta Heg_1 + LE$

$= 495.8 + (-325) + (-728.4)$

$= -557.6$

$= -5576 \times 10^{-1}$ KJ/mol.

Note: The above calculation is not for $\Delta H_{formation}$ but for $\Delta H_{Reaction}$.

But on the basis of given data it is the best ans.

61.[3] Required probability $= \left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

62.[4] $x = \frac{1}{1-\cos^2\theta} \Rightarrow \sin^2\theta = \frac{1}{x}$

Also, $\cos^2\theta = \frac{1}{y}$ & $1 - \sin^2\theta\cos^2\theta = \frac{1}{z}$

So, $1-\frac{1}{x}\times\frac{1}{y} = \frac{1}{z} \Rightarrow z(xy-1) = xy$ ....(1)

Also, $\frac{1}{x}\times\frac{1}{y} = 1 \Rightarrow x + y = xy$ .....(2)

From (i) and (ii)

$xy + z = xyz = (x + y)\,z$

63.[4] $f(n + 1) - f(n) = f(1)$

$\Rightarrow f(n) = nf(1)$

$\Rightarrow$ f is one-one

Now, Let $f(g(x_2)) = f(g(x_1))$

$\Rightarrow g(x_2) = g(x_1)$ (as f is one-one)

$\Rightarrow x_1 = x_2$ (as fog is one-one)

$\Rightarrow$ g is one-one

Now, $f(g(n)) = g(n)\,f(1)$

may be many-one if

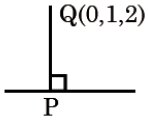
g(n) is many-one

64.[4] $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = r$

$\Rightarrow P(x, y, z) = (2r + 1, 3r - 1, -2r + 1)$

Since, $\overline{QP} \perp (2\hat{i} + 3\hat{j} - 2\hat{k})$

$\Rightarrow 4r + 2 + 9r - 6 + 4r + 2 = 0$



$\Rightarrow r = \frac{2}{17}$

$\Rightarrow P\left(\frac{21}{17}, \frac{-11}{17}, \frac{13}{17}\right)$

$\Rightarrow \overrightarrow{PQ} = \frac{21\hat{i} - 28\hat{j} - 21\hat{k}}{17}$

So, $\overleftrightarrow{QP} : \frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

65.[3] $n = \ell + m$

Now, $\ell + m^2 = n^2 = (\ell + m)^2$

$\Rightarrow 2\ell m = 0$

If $\ell = 0 \Rightarrow m = n = \pm\frac{1}{\sqrt{2}}$

And, If $m = 0 \Rightarrow n = \ell = \pm\frac{1}{\sqrt{2}}$

So, direction cosines of two lines are

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Thus, $\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

66.[4]

$$I = \int \frac{\sin\theta.\sin2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1-\cos2\theta}d\theta$$

$$\Rightarrow I = \int \frac{\sin\theta.2\sin\theta\cos\theta.\sin^2\theta(\sin^4\theta + \sin^2\theta + 1)(2\sin^4\theta + 3\sin^2\theta + 6)^{1/2}}{2\sin^2\theta}d\theta$$

$$= \int \sin^2\theta.\cos\theta(\sin^4\theta + \sin^2\theta + 1)(2\sin^4\theta + 3\sin^2\theta + 6)^{1/2}d\theta$$

Let $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$$\therefore I = \int t^2(t^4 + t^2 + 1)(2t^4 + 3t^2 + 6)^{1/2}dt$$

$$= \int (t^5 + t^3 + t)t(2t^4 + 3t^2 + 6)^{1/2}dt$$

$= \int (t^5 + t^3 + t)(t^2)^{1/2}(2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t)(2t^6 + 3t^4 + 6t^2)^{1/2} dt$

Let $2t^6 + 3t^4 + 6t^2 = u^2$

$\Rightarrow 12(t^5 + t^3 + t)\, dt = 2u\, du$

$\therefore I = \int (u^2)^{1/2} . \frac{2u\, du}{12} = \int \frac{u^2}{6} du = \frac{u^3}{18} + C$

$= \frac{(2t^6 + 3t^4 + 6t^2)^{3/2}}{18} + C$

when $t = \sin\theta$

and $t^2 = 1 . \cos^2\theta$ will give option (4)

67.[3] $I = \int_{-1}^{1} x^2 e^{[x^3]} dx$

$= \int_{-1}^{0} x^2 e^{[x^3]} dx + \int_{0}^{1} x^2 e^{[x^3]} dx$

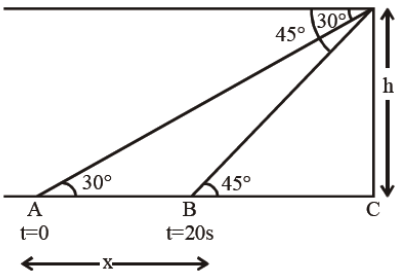
$= \int_{-1}^{0} x^2 e^{-1} dx + \int_{0}^{1} x^2 e^{0} dx$

$= \frac{1}{e} \times \frac{x^3}{3}\Big|_{-1}^{0} + \frac{x^3}{3}\Big|_{0}^{1}$

$= \frac{1}{e} \times \left(0 - \left(\frac{-1}{3}\right)\right) + \frac{1}{3}$

$= \frac{1}{3e} + \frac{1}{3} = \frac{1+e}{3e}$

68.[3]



Let speed of boat is u m/s and height of tower is h meter & distance AB = x metre

$\therefore x = h \cot 30° - h \cot 45°$

$\Rightarrow x = h(\sqrt{3} - 1)$

$\therefore u = \frac{x}{20} = \frac{h(\sqrt{3}-1)}{20}$ m/s

$\therefore$ Time taken to travel from B to C (Distance = h meter)

$= \frac{h}{u} = \frac{h}{h\frac{(\sqrt{3}-1)}{20}} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1)$ sec

69.[3] Slope of tangent = $m_T = m$

So, $m(-2) = -1 \Rightarrow m = \frac{1}{2}$

Equation : $y = mx + \frac{a}{m}$

$\Rightarrow y = \frac{1}{2}x + \frac{3}{2 \times \frac{1}{2}} \left(a = \frac{6}{4} = \frac{3}{2}\right)$

$\Rightarrow y = \frac{x}{2} + 3$

$\Rightarrow 2y = x + 6$

Point (5, 4) will not lie on it

70.[4] $\sin 2\theta + \tan 2\theta > 0$

$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$

$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta (2\cos^2\theta) > 0$

Note : $\cos 2\theta \neq 0$

$\Rightarrow 1 - 2\sin^2\theta \neq 0 \Rightarrow \sin\theta \neq \pm \frac{1}{\sqrt{2}}$

Now, $\tan 2\theta(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

As $\sin\theta \neq \pm \frac{1}{\sqrt{2}}$; which has been already considered

71.[4] (i) $(2 - i)z = (2 + i)\bar{z}$

$\boxed{y = \frac{x}{2}}$

(ii) $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$\boxed{x + 2y = 2}$

(iii) $iz + \bar{z} + 1 + i = 0$

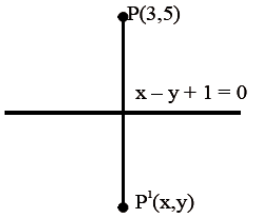
Eq^n of tangent $\boxed{x - y + 1 = 0}$

Solving (i) and (ii)

$x = 1, y = \frac{1}{2}$

Now, $p = r \Rightarrow \left|\frac{1 - \frac{1}{2} + 1}{\sqrt{2}}\right| = r \Rightarrow r = \frac{3}{2\sqrt{2}}$

72.[4]



$$\frac{x-3}{1}=\frac{y-5}{-1}=-2\left(\frac{3-5+1}{1+1}\right)$$

So, x = 4, y = 4

Hence, $(x-2)^2+(y-4)^2=4$

73.[2] For orthogonal curves a – c = b – d

$\Rightarrow$ a – b = c – d

74.(4) Given limit is of 1^∞ form

So, $\ell = \exp\left(\lim\limits_{n\to\infty}\dfrac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)$

Now,

$$0 \le 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n} \le 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+.....+\frac{1}{\sqrt{n}}$$

$$\le 2\sqrt{n}-1$$

So, ℓ = exp (0) (from sandwich theorem)

= 1

75.[2] $ax^2 + bx + c = 0$

For equal roots D = 0

$\Rightarrow b^2 = 4ac$

Case I : ac = 1

(a, b, c) = (1, 2, 1)

Case II : ac = 4

(a, b, c) = (1, 4, 4)

or (4, 4, 1)

or (2, 4, 2)

Case III : ac = 9

(a, b, c) = (3, 6, 3)

Required probability = $\dfrac{5}{216}$

76.[4] $xyz = 2^3 \times 3^1$

Let $x = 2^{\alpha_1} \times 3^{\beta_1}$

$y = 2^{\alpha_2} \times 3^{\beta_2}$

$z = 2^{\alpha_3} \times 3^{\beta_2}$

Now $\alpha_1 + \alpha_2 + \alpha_3 = 3$

No. of non-negative integral sol = ${}^5C_2 = 10$

& $\beta_1 + \beta_2 + \beta_3 = 1$

No. of non-negative integral sol^n = ${}^3C_2 = 3$

Total ways = 10 × 3 = 30.

17.[1] $x^2 - 2(3K-1)x + 8K^2 - 7 > 0$

Now, D < 0

$\Rightarrow 4(3K-1)^2 - 4 \times 1 \times (8K^2-7) < 0$

$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$

$\Rightarrow K^2 - 6K + 8 < 0$

$\Rightarrow (K-4)(K-2) < 0$

$\Rightarrow K \in (2,4)$

18.[4] Given

y (0) = 0

& $\dfrac{dy}{dx}=\dfrac{(x-2)^2+y+4}{x-2}$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x-2}=(x-2)+\frac{4}{x-2}$$

$$\Rightarrow \text{I.F.} = e^{-\int\frac{1}{x-2}dx}=\frac{1}{x-2}$$

Solution of L.D.E.

$$\Rightarrow y\frac{1}{x-2}=\int\frac{1}{x-2}\left((x-2)+\frac{4}{x-2}\right).dx$$

$$\Rightarrow \frac{y}{x-2}=x-\frac{4}{x-2}+C$$

Now, at x = 0, y = 0 $\Rightarrow$ C = –2

$y = x(x-2) - 4 - 2(x-2)$

$\Rightarrow y = x^2 - 4x$

This curve passes through (5, 5)

79.[4] $A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim A \vee A) \vee \sim B$

$\equiv T \vee \sim B \equiv T$

$\therefore T \vee B = T$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \to (A \vee B)$

80.[1] f(1) = f(2)

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow 3a - b = 7$ .....(1)

Also $f'\left(\dfrac{4}{3}\right) = 0$ (given)

$\Rightarrow (3x^2 - 2ax + b)_{x=4/3} = 0$

$\Rightarrow \frac{16}{3} - \frac{8a}{3} + b = 0$

$\Rightarrow 8a - 3b - 16 = 0$ .....(2)

Solving (1) and (2)

$a = 5, b = 8$

81.[144] Let $f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

as $\lim_{x\to 0} \frac{f(x)}{x^3} = 1$ non-zero finite

So, $d = e = f = 0$

and $f(x) = x^3(x^3 + ax^2 + bx + c)$

Hence, $\lim_{x\to 0} \frac{f(x)}{x^3} = c = 1$

Now, as $f(x) = x^6 + ax^5 + bx^4 + x^3$

and $f'(x) = 0$ at $x = 1$ and $x = -1$

i.e., $f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

$f'(1) = 0$

$\Rightarrow 6 + 5a + 4b + 3 = 0$

$\Rightarrow 5a + 4b = -9$

& $f'(-1) = 0$

$\Rightarrow -6 + 5a - 4b + 3 = 0$

$\Rightarrow 5a - 4b = 3$

Solving both we get,

$a = \frac{-6}{10} = \frac{-3}{5}; \quad b = \frac{-3}{2}$

$\therefore f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$\therefore 5f(2) = 5\left[64 - \frac{3}{5}.32 - \frac{3}{2}.16 + 8\right]$

$= 320 - 96 - 120 + 40$

$= 144$

82.[2] $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |x + 2|\,|x - 1|$

$= |2x + 1| + |x + 2|\,(|x - 1| - 3)$

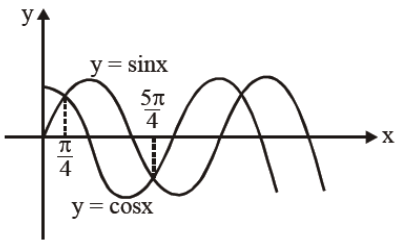
Critical points are $x = \frac{-1}{2}, -2, -1$

but $x = -2$ is making a zero.

twice in product so, points of non differentability are $x = \frac{-1}{2}$ and $x = -1$

$\therefore$ Number of points of non-differentiability = 2

83[64]



$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx$

$= (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$

$= \left(-\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right)\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$

$\Rightarrow A = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow A^4 = (2\sqrt{2})^4$

$= 16 \times 4 = 64$

84.[9] Let a_n be the side length of A_n.

So, $a_n = \sqrt{2a}_{n+1}$, $a_1 = 12$

$\Rightarrow a_n = 12 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$

Now,

$(a_n)^2 < 1 \Rightarrow \frac{144}{2^{(n-1)}} < 1$

$\Rightarrow 2^{(n-1)} > 144$

$\Rightarrow n - 1 \geq 8$

$\Rightarrow n \geq 9$

85.[7] $A^2 = I$

$\Rightarrow AA' = I$ (as $A' = A$)

$\Rightarrow A$ is orthogonal

So, $x^2 + y^2 + z^2 = 1$ and $xy + yz + zx = 0$

$\Rightarrow (x + y + z)^2 = 1 + 2 \times 0$

$\Rightarrow x + y + z = 1$

Thus,

$x^3 + y^3 + z^3 = 3 \times 2 + 1 \times (1 - 0) = 7$

86.[13] $a^2 + b^2 = |I_2 + A||I_2 - A|^{-1}$

$= \sec^2\frac{\theta}{2} \times \cos^2\frac{\theta}{2} = 1$

87.[32] We need three digits numbers.
Since $1 + 2 + 3 + 4 + 5 = 15$
So, number of possible triplets for multiple of 15 is $1 \times 2 \times 2$
so Ans. $= 4 \times \lfloor 3 + 4 \times 3 - 1 \times 2 \times \lfloor 2 = 32$

88.[12] $(\vec{r} - \vec{c}) \times \vec{a} = 0$
$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$
Now, $0 = \vec{b} \cdot \vec{c} + \lambda\vec{a} \cdot \vec{b}$
$\Rightarrow \lambda = \dfrac{-\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = -\dfrac{2}{-1} = 2$
So, $\vec{r} \cdot \vec{a} = \vec{a} \cdot \vec{c} + 2a^2 = 12$

89.[21] We observe $5P_2 \,.\, P_1 = 3P_3$
So, $15 - K = -6$
$\Rightarrow K = 21$

90.[2] $K = \dfrac{4\sqrt{3}}{\sqrt{3}x + y} = \dfrac{\sqrt{3}x - y}{4\sqrt{3}}$
$\Rightarrow 3x^2 - y^2 = 48$
$\Rightarrow \dfrac{x^2}{16} = \dfrac{y^2}{48} = 1$
Now, $48 = 16(e^2 - 1)$
$\Rightarrow e = \sqrt{4} = 2$