

## 5. Congruence of Triangles and Inequalities in a Triangle

### Exercise 5A

#### 1. Question

In a  $\triangle ABC$ , if  $AB=AC$  and  $\angle A=70^\circ$ , find  $\angle B$  and  $\angle C$ .

#### Answer

Given that

$$AB = AC \text{ and } \angle A = 70^\circ$$

To find:  $\angle B$  and  $\angle C$

$$AB = AC \text{ and also } \angle A = 70^\circ$$

As two sides of triangle are equal, we say that  $\triangle ABC$  is isosceles triangle.

Hence by the property of isosceles triangle, we know that base angles are also equal.

ie. we state that  $\angle B = \angle C$ . ...(1)

Now,

Sum of all angles in any triangle =  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

Hence,

$$70^\circ + \angle B + \angle C = 180^\circ$$

$$2 \angle B = 180^\circ - 70^\circ \text{ ...from (1)}$$

$$\therefore 2\angle B = 110^\circ$$

$$\angle B = 55^\circ$$

Therefore, our base angles,  $\angle B$  and  $\angle C$ , are  $55^\circ$  each.

#### 2. Question

The vertical angle of an isosceles triangle is  $100^\circ$ . Find its base angles.

#### Answer

Given: The given triangle is isosceles triangle. Also vertex angle is  $100^\circ$

To find: Measure of base angles.

It is given that triangle is isosceles.

So let our given triangle be  $\triangle ABC$ .

And let  $\angle A$  be the vertex angle, which is given as  $\angle A = 100^\circ$

By the property of isosceles triangle, we know that base angles are equal.

So,

$$\angle B = \angle C \dots(1)$$

We know that,

Sum of all angles in any triangle =  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$100^\circ + 2\angle B = 180^\circ \dots \text{from (1)}$$

$$\therefore 2\angle B = 180^\circ - 100^\circ$$

$$2\angle B = 80^\circ$$

$$\therefore \angle B = 40^\circ$$

Therefore, our base angles,  $\angle B$  and  $\angle C$ , are  $40^\circ$  each.

### 3. Question

In a  $\triangle ABC$ , if  $AB=AC$  and  $\angle B=65^\circ$ , find  $\angle C$  and  $\angle A$ .

### Answer

Given: In  $\triangle ABC$ ,

$$AB=AC \text{ and } \angle B=65^\circ$$

To find :  $\angle A$  and  $\angle C$

It is given that  $AB=AC$  and  $\angle B=65^\circ$

As two sides of the triangle are equal, we say that triangle is isosceles triangle, with vertex angle A.

Hence by the property of isosceles triangle we know that base angles are equal.

$$\therefore \angle B = \angle C$$

$$\therefore \angle C = \angle B = 65^\circ$$

Also, We know that,

Sum of all angles in any triangle =  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 65^\circ + 65^\circ = 180^\circ$$

$$\angle A + 130^\circ = 180^\circ$$

$$\therefore \angle A = 180^\circ - 130^\circ$$

$$\angle A = 50^\circ$$

Hence,  $\angle C = 65^\circ$  and  $\angle A = 50^\circ$

#### 4. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

#### Answer

Given: Our given triangle is isosceles triangle. Also, the vertex angle is twice the sum of the base angles

To find: Measures of angles of triangle.

It is given that that given triangle is isosceles triangle.

Let vertex angle be  $y$  and base angles be  $x$  each.

So by given condition,

$$y = 2(x + x)$$

$$\therefore y = 4x$$

Also, We know that,

Sum of all angles in any triangle =  $180^\circ$

$$\therefore y + x + x = 180^\circ$$

$$y + 2x = 180^\circ$$

$$4x + 2x = 180^\circ$$

$$\therefore 6x = 180^\circ$$

$$x = 30^\circ$$

$$\therefore y = 4 \times 30^\circ$$

$$y = 120^\circ$$

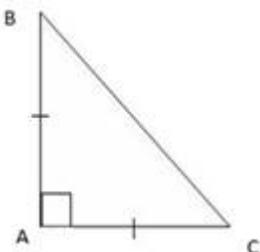
Hence, vertex angle is  $120^\circ$  and base angles are  $30^\circ$  each.

#### 5. Question

What is the measure of each of the equal angles of a right-angled isosceles triangle?

#### Answer

Here given triangle is isosceles right angled triangle.



So let our triangle be  $\triangle ABC$ , right angled at A.

$$\therefore \angle A = 90^\circ$$

Here,  $AB = AC$ , as our given triangle is isosceles triangle.

Hence, base angles,  $\angle B$  and  $\angle C$  are equal.

Also, We know that,

Sum of all angles in any triangle =  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$90^\circ + 2 \angle B = 180^\circ$$

$$2\angle B = 90^\circ$$

$$\angle B = 45^\circ$$

Hence the measure of each of the equal angles of a right-angled isosceles triangle is  $45^\circ$

### 6. Question

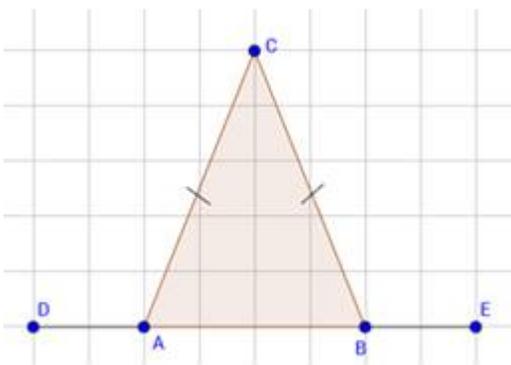
If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

### Answer

Given:  $\triangle ABC$  is isosceles triangle.

To prove:  $\angle CAD = \angle CBE$

Let  $\triangle ABC$  be our isosceles triangle as shown in the figure.



We know that base angles of the isosceles triangle are equal.

$$\text{Here, } \angle CAB = \angle CBA \dots(1)$$

Also here,  $\angle CAD$  and  $\angle CBE$  are exterior angles of the triangle.

So, we know that,

$$\angle CAB + \angle CAD = 180^\circ \dots \text{ exterior angle theorem}$$

$$\text{And } \angle CBA + \angle CBE = 180^\circ \dots \text{ exterior angle theorem}$$

So from (1) and above statement, we conclude that,

$$\angle CAB + \angle CAD = 180^\circ$$

$$\text{And } \angle CAB + \angle CBE = 180^\circ$$

Which implies that,

$$\angle CAD = 180^\circ - \angle CAB$$

$$\text{And } \angle CBE = 180^\circ - \angle CAB$$

Hence we say that  $\angle CAD = \angle CBE$

$\therefore$  For the isosceles triangle, the exterior angles so formed are equal to each other.

## 7. Question

Find the measure of each exterior angle of an equilateral triangle.

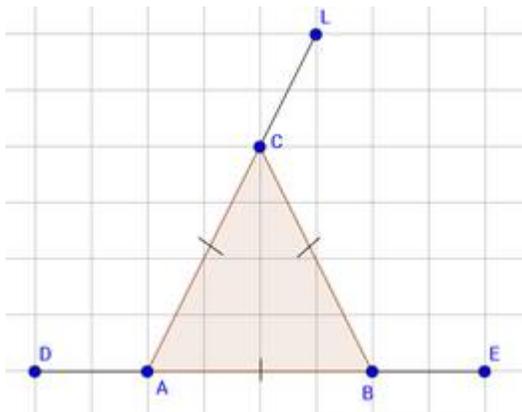
### Answer

Given:  $\triangle ABC$  is equilateral triangle.

To prove:  $\angle CAD = \angle CBE = \angle BCL$

Proof:

Let our triangle be  $\triangle ABC$ , which is equilateral triangle as shown in the figure.



Hence all angles are equal and measure  $60^\circ$  each.

$$\therefore \angle CAB = \angle CBA = \angle BCA = 60^\circ \dots(1)$$

Also here,  $\angle CAD$  and  $\angle CBE$  are exterior angles of the triangle.

So, we know that,

$$\angle CAB + \angle CAD = 180^\circ \dots \text{exterior angle theorem}$$

$$\angle CBA + \angle CBE = 180^\circ \dots \text{exterior angle theorem}$$

$$\angle BCA + \angle BCL = 180^\circ \dots \text{exterior angle theorem}$$

From (1) and above statements, we state that,

$$60^\circ + \angle CAD = 180^\circ$$

$$60^\circ + \angle CBE = 180^\circ$$

$$60^\circ + \angle BCL = 180^\circ$$

Simplifying above statements,

$$\angle CAD = 180^\circ - 60^\circ = 120^\circ$$

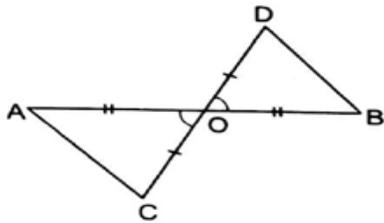
$$\angle CBE = 180^\circ - 60^\circ = 120^\circ$$

$$\angle BCL = 180^\circ - 60^\circ = 120^\circ$$

Hence, the measure of each exterior angle of an equilateral triangle is  $120^\circ$

### 8. Question

In the given figure, O is the midpoint of each of the line segments AB and CD. Prove that  $AC=BD$  and  $AC \parallel BD$ .



### Answer

Given:  $AO = OB$  ,  $DO = OC$

To prove:  $AC=BD$  and  $AC \parallel BD$

Proof:

It is given that, O is the midpoint of each of the line segments AB and CD.

This implies that  $AO = OB$  and  $DO = OC$

Here line segments AB and CD are concurrent.

So,

$\angle AOC = \angle BOD$  .... As they are vertically opposite angles.

Now in  $\Delta AOC$  and  $\Delta BOD$ ,

$AO = OB$ ,

$OC = OD$

Also,  $\angle AOC = \angle BOD$

Hence,  $\Delta AOC \cong \Delta BOD$  ... by SAS property of congruency

So,

$AC = BD$  ... by cpct

$\therefore \angle ACO = \angle BDO$  ... by cpct

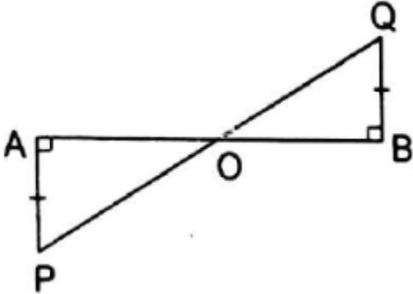
But  $\angle ACO$  and  $\angle BDO$  are alternate angles.

$\therefore$  We conclude that AC is parallel to BD.

Hence we proved that  $AC=BD$  and  $AC \parallel BD$

### 9. Question

In the adjoining figure,  $PA \perp AB$ ,  $QB \perp AB$  and  $PA=QB$ . If PQ intersects AB at O, show that O is the midpoint of AB as well as that of PQ.



### Answer

Given:  $PA \perp AB$ ,  $QB \perp AB$  and  $PA=QB$

To prove:  $AO = OB$  and  $PO = OQ$

It is given that  $PA \perp AB$  and  $QB \perp AB$ .

This means that  $\triangle PAO$  and  $\triangle QBO$  are right angled triangles.

It is also given that  $PA=QB$

Now in  $\triangle PAO$  and  $\triangle QBO$ ,

$$\angle OAP = \angle OBQ = 90^\circ$$

$$PA = QB$$

Hence by hypotenuse-leg congruency,

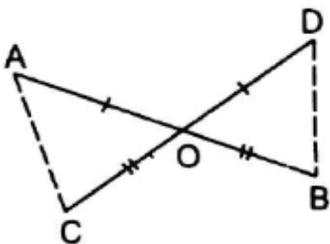
$$\triangle PAO \cong \triangle QBO$$

$\therefore AO = OB$  and  $PO = OQ$  ....by cpct

Hence proved that  $AO = OB$  and  $PO = OQ$

### 10. Question

Let the line segments AB and CD intersect at O in such a way that  $OA=OD$  and  $OB=OC$ . Prove that  $AC=BD$  but AC may not be parallel to BD.



### Answer

Given:  $AO = OD$  and  $CO = OB$

To prove:  $AC = BD$

Proof :

It is given that  $AO = OD$  and  $CO = OB$

Here line segments  $AB$  and  $CD$  are concurrent.

So,

$\angle AOC = \angle BOD$  ... As they are vertically opposite angles.

Now in  $\triangle AOC$  and  $\triangle DOB$ ,

$AO = OD$ ,

$CO = OD$

Also,  $\angle AOC = \angle BOD$

Hence,  $\triangle AOC \cong \triangle DOB$  ... by SAS property of congruency

So,

$AC = BD$  ... by cpct

Here,

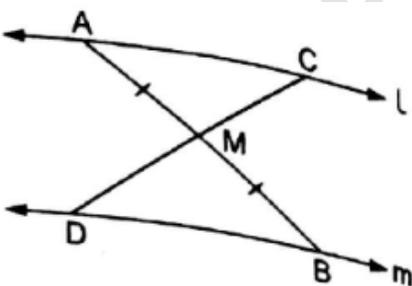
$\angle ACO \neq \angle BDO$  or  $\angle OAC \neq \angle OBD$

Hence there are no alternate angles, unless both triangles are isosceles triangle.

Hence proved that  $AC=BD$  but  $AC$  may not be parallel to  $BD$ .

### 11. Question

In the given figure,  $l \parallel m$  and  $M$  is the midpoint of  $AB$ . Prove that  $M$  is also the midpoint of any line segment  $CD$  having its end points at  $l$  and  $m$  respectively.



### Answer

Here it is given that  $l \parallel m$  ie.  $AC \parallel DB$ .

Also given that  $AM = MB$

Now in  $\triangle AMC$  and  $\triangle BMD$ ,

$\angle CAM = \angle DBM$  ... Alternate angles

$$AM = MB$$

$\angle AMC = \angle BMD$  ... vertically opposite angles

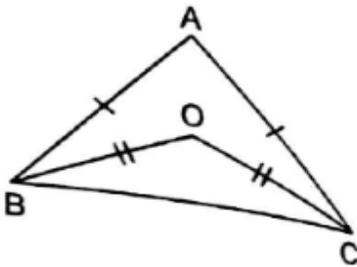
Hence,  $\triangle AMC \cong \triangle BMD$  ... by ASA property of congruency

$$\therefore CM = MD \text{ ...cpct}$$

Hence proved that M is also the midpoint of any line segment CD having its end points at l and m respectively.

## 12. Question

In the given figure,  $AB=AC$  and  $OB=OC$ . Prove that  $\angle ABO = \angle ACO$ . Give that  $AB=AC$  and  $OB=OC$ .



## Answer

$\triangle ABC$  and  $\triangle OBC$  are isosceles triangle.

$$\therefore \angle ABC = \angle ACB \text{ and } \angle OBC = \angle OCB \text{ ....(1)}$$

Also,

$$\angle ABC = \angle ABO + \angle OBC$$

$$\text{And } \angle ACB = \angle ACO + \angle OCB$$

From 1 and above equations, we state that,

$$\angle ABC = \angle ABO + \angle OBC$$

$$\text{And } \angle ACB = \angle ACO + \angle OCB$$

This implies that,

$$\angle ABO = \angle ABC - \angle OBC$$

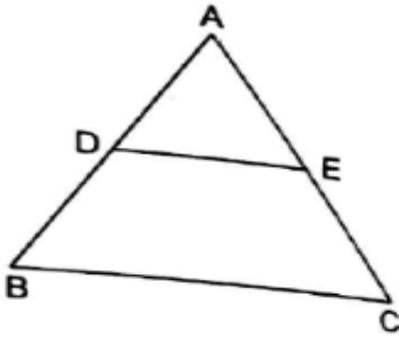
$$\text{And } \angle ACO = \angle ACB - \angle OCB$$

Hence,

$$\angle ABO = \angle ACO = \angle ABC - \angle OBC$$

## 13. Question

In the given figure,  $ABC$  is a triangle in which  $AB=AC$  and  $D$  is a point on  $AB$ . Through  $D$ , a line  $DE$  is drawn parallel to  $BC$  and meeting  $AC$  at  $E$ . Prove that  $AD=AE$ .



**Answer**

Given that  $AB = AC$  and also  $DE \parallel BC$ .

So by Basic proportionality theorem or Thales theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore \frac{DB}{AD} = \frac{EC}{AE}$$

Now adding 1 on both sides,

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

$$\frac{DB+AD}{AD} = \frac{EC+AE}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE} \dots \text{as } AB = AD + DE \text{ and } AC = AE + EC$$

But is given that  $AB = AC$ ,

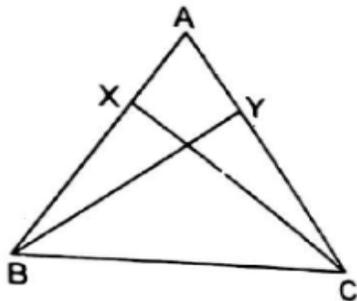
$$\therefore \frac{AB}{AD} = \frac{AB}{AE}$$

Hence,

$$AD = AE.$$

**14. Question**

In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of  $\Delta ABC$  such that  $AX=AY$ . Prove that  $CX=BY$ .



**Answer**

Here it is given that  $AX = AY$ .

Now in  $\Delta CXA$  and  $\Delta BYA$ ,

$$AX = AY$$

$\angle XAC = \angle YAB$  ... Same angle or common angle.

$AC = AB$  ... given condition Hence by SAS property of congruency,

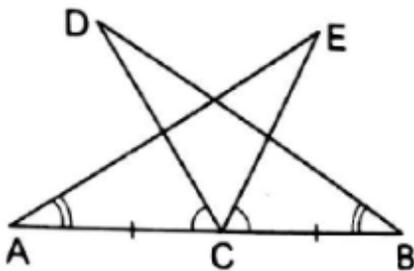
$$\Delta CXA \cong \Delta BYA$$

Hence by cpct, we conclude that,

$$CX = BY$$

### 15. Question

In the given figure, C is the midpoint of AB. If  $\angle DCA = \angle ECB$  and  $\angle DBC = \angle EAC$ , prove that  $DC = EC$ .



### Answer

It is given that  $AC = BC$ ,  $\angle DCA = \angle ECB$  and  $\angle DBC = \angle EAC$ .

Adding angle  $\angle ECD$  both sides in  $\angle DCA = \angle ECB$ , we get,

$$\angle DCA + \angle ECD = \angle ECB + \angle ECD$$

$$\therefore \angle ECA = \angle DCB \text{ ...addition property}$$

Now in  $\Delta DBC$  and  $\Delta EAC$ ,

$$\angle ECA = \angle DCB$$

$$BC = AC$$

$$\angle DBC = \angle EAC$$

Hence by ASA postulate, we conclude,

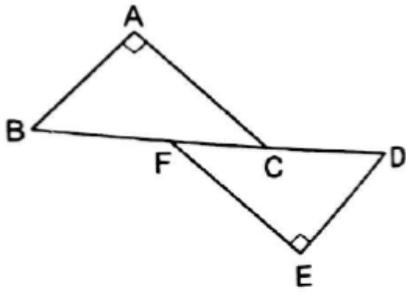
$$\Delta DBC \cong \Delta EAC$$

Hence, by cpct, we get,

$$DC = EC$$

### 16. Question

In the given figure,  $BA \perp AC$  and  $DE \perp EF$  such that  $BA = DE$  and  $BF = DC$ . Prove that  $AC = EF$ .



**Answer**

Given :  $BA \perp AC$  and  $DE \perp EF$  such that  $BA=DE$  and  $BF=DC$

To prove:  $AC = EF$

Proof:

In  $\triangle ABC$ , we have,

$$BC = BF + FC$$

And , in  $\triangle DEF$ ,

$$FD = FC + CD$$

$$\text{But, } BF = CD$$

$$\text{So, } BC = BF + FC$$

$$\text{And, } FD = FC + BF$$

$$\therefore BC = FD$$

So, in  $\triangle ABC$  and  $\triangle DEF$ , we have,

$$\angle BAC = \angle DEF \dots \text{given}$$

$$BC = FD$$

$$AB = DE \dots \text{given}$$

Thus by Right angle - Hypotenuse- Side property of congruence, we have,

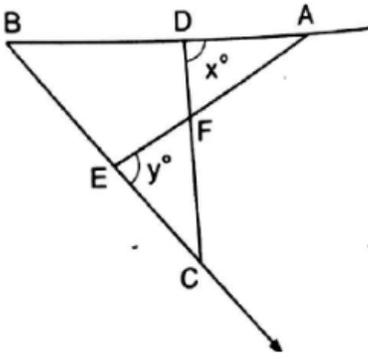
$$\triangle ABC \cong \triangle DEF$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore AC = EF$$

**17. Question**

In the given figure, if  $x=y$  and  $AB=CB$ , then prove that  $AE=CD$ .



### Answer

Given:  $x=y$  and  $AB=CB$

To prove:  $AE = CD$

Proof:

In  $\triangle ABE$ , we have,

$\angle AEC = \angle EBA + \angle BAE$  ...Exterior angle theorem

$$y^\circ = \angle EBA + \angle BAE$$

Now in  $\triangle BCD$ , we have,

$$x^\circ = \angle CBA + \angle BCD$$

Since, given that,

$$x = y ,$$

$$\angle CBA + \angle BCD = \angle EBA + \angle BAE$$

$\therefore \angle BCD = \angle BAE$  ... as  $\angle CBA$  and  $\angle EBA$  and same angles.

Hence in  $\triangle BCD$  and  $\triangle BAE$ ,

$$\angle B = \angle B$$

$$BC = AB \text{ ...given}$$

$$\angle BCD = \angle BAE$$

Thus by ASA property of congruence, we have,

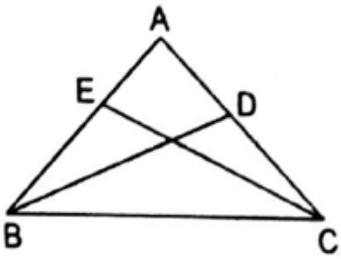
$$\triangle BCD \cong \triangle BAE$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore CD = AE$$

### 18. Question

ABC is a triangle in which  $AB=AC$ . If the bisectors of  $\angle B$  and  $\angle C$  meet AC and AB in D and E respectively, prove that  $BD=CE$ .



**Answer**

Given:  $AB=AC$  and  $BD$  and  $AB$  are angle bisectors of  $\angle B$  and  $\angle C$

To prove:  $BD = CE$

Proof:

In  $\triangle ABD$  and  $\triangle ACE$ ,

$$\angle ABD = \frac{1}{2} \angle B$$

$$\text{And } \angle ACE = \frac{1}{2} \angle C$$

But  $\angle B = \angle C$  as  $AB = AC$  ... As in isosceles triangle, base angles are equal

$$\angle ABD = \angle ACE$$

$$AB = AC$$

$$\angle A = \angle A$$

Thus by ASA property of congruence,

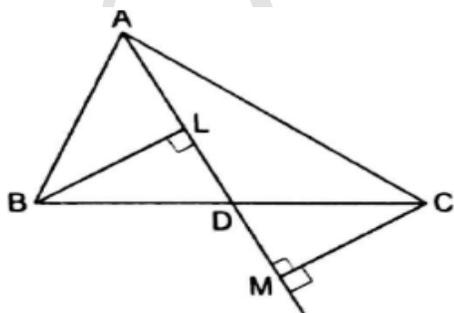
$$\triangle ABD \cong \triangle ACE$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore BD = CE$$

**19. Question**

In the adjoining figure,  $AD$  is a median of  $\triangle ABC$ . If  $BL$  and  $CM$  are drawn perpendiculars on  $AD$  and  $AD$  produced, prove that  $BL=CM$



**Answer**

Given:  $BD = DC$  and  $BL \perp AD$  and  $DM \perp CM$

To prove:  $BL=CM$

Proof:

In  $\triangle BLD$  and  $\triangle CMD$ ,

$\angle BLD = \angle CMD = 90^\circ$  ... given

$\angle BLD = \angle MDC$  ... vertically opposite angles

$BD = DC$  ... given

Thus by AAS property of congruence,

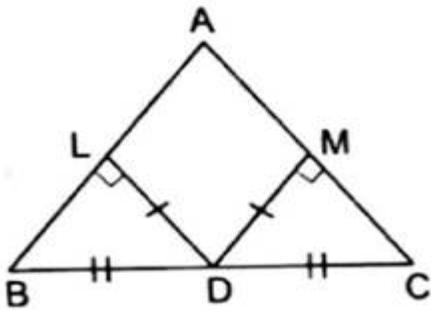
$\triangle BLD \cong \triangle CMD$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore BL = CM$

## 20. Question

In  $\triangle ABC$ ,  $D$  is the midpoint of  $BC$ . If  $DL \perp AB$  and  $DM \perp AC$  such that  $DL=DM$ , prove that  $AB=AC$ .



## Answer

Given:  $BD = DC$  and  $DL \perp AB$  and  $DM \perp AC$  such that  $DL=DM$

To prove:  $AB = AC$

Proof:

In right angled triangles  $\triangle BLD$  and  $\triangle CMD$ ,

$\angle BLD = \angle CMD = 90^\circ$

$BD = CD$  ... given

$DL = DM$  ... given

Thus by right angled hypotenuse side property of congruence,

$\triangle BLD \cong \triangle CMD$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle ABD = \angle ACD$

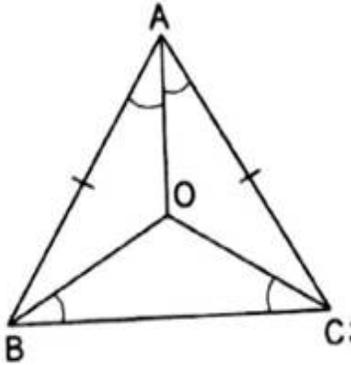
In  $\triangle ABC$ , we have,

$$\angle ABD = \angle ACD$$

$\therefore AB = AC$  ... Sides opposite to equal angles are equal

### 21. Question

In  $\triangle ABC$ ,  $AB = AC$  and the bisectors of  $\angle B$  and  $\angle C$  meet at a point  $O$ . prove that  $BO = CO$  and the ray  $AO$  is the bisector of  $\angle A$ .



### Answer

Given: In  $\triangle ABC$ ,  $AB = AC$  and the bisectors of  $\angle B$  and  $\angle C$  meet at a point  $O$ .

To prove:  $BO = CO$  and  $\angle BAO = \angle CAO$

Proof:

In  $\triangle ABC$  we have,

$$\angle OBC = \frac{1}{2} \angle B$$

$$\angle OCB = \frac{1}{2} \angle C$$

But  $\angle B = \angle C$  ... given

So,  $\angle OBC = \angle OCB$

Since the base angles are equal, sides are equal

$\therefore OC = OB$  ...(1)

Since  $OB$  and  $OC$  are bisectors of angles  $\angle B$  and  $\angle C$  respectively, we have

$$\angle ABO = \frac{1}{2} \angle B$$

$$\angle ACO = \frac{1}{2} \angle C$$

$\therefore \angle ABO = \angle ACO$  ...(2)

Now in  $\triangle ABO$  and  $\triangle ACO$

$AB = AC$  ... given

$\angle ABO = \angle ACO$  ... from 2

BO = OC ... from 1

Thus by SAS property of congruence,

$$\Delta ABO \cong \Delta ACO$$

Hence, we know that, corresponding parts of the congruent triangles are equal

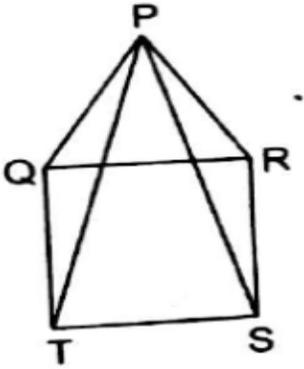
$$\angle BAO = \angle CAO$$

ie. AO bisects  $\angle A$

## 22. Question

In the given figure, PQR is an equilateral triangle and QRST is a square. Prove that

(i)  $PT=PS$ , (ii)  $\angle PSR=15^\circ$ .



## Answer

Given: PQR is an equilateral triangle and QRST is a square

To prove:  $PT=PS$  and  $\angle PSR=15^\circ$ .

Proof:

Since  $\Delta PQR$  is equilateral triangle,

$$\angle PQR = \angle PRQ = 60^\circ$$

Since QRTS is a square,

$$\angle RQT = \angle QRS = 90^\circ$$

In  $\Delta PQT$ ,

$$\angle PQT = \angle PQR + \angle RQT$$

$$= 60^\circ + 90^\circ$$

$$= 150^\circ$$

In  $\Delta PRS$ ,

$$\angle PRS = \angle PRQ + \angle QRS$$

$$= 60^\circ + 90^\circ$$

$$= 150^\circ$$

$$\therefore \angle PQT = \angle PRS$$

Thus in  $\Delta PQT$  and  $\Delta PRS$ ,

$PQ = PR$  ... sides of equilateral triangle

$$\angle PQT = \angle PRS$$

$QT = RS$  ... side of square

Thus by SAS property of congruence,

$$\Delta PQT \cong \Delta PRS$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore PT = PS$$

Now in  $\Delta PRS$ , we have,

$$PR = RS$$

$$\therefore \angle PRS = \angle PSR$$

$$\text{But } \angle PRS = 150^\circ$$

SO, by angle sum property,

$$\angle PRS + \angle PSR + \angle SPR = 180^\circ$$

$$150^\circ + \angle PSR + \angle SPR = 180^\circ$$

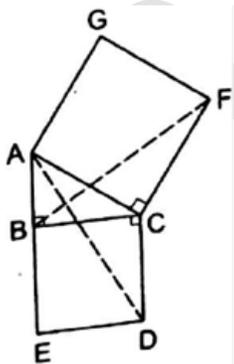
$$2\angle PSR = 180^\circ - 150^\circ$$

$$2\angle PSR = 30^\circ$$

$$\angle PSR = 15^\circ$$

### 23. Question

In the given figure,  $ABC$  is a triangle, right angled at  $B$ . If  $BCDE$  is a square on side  $BC$  and  $ACFG$  is a square on  $AC$ , prove that  $AD=BF$ .



### Answer

Given:  $\angle ABC = 90^\circ$ ,  $BCDE$  is a square on side  $BC$  and  $ACFG$  is a square on  $AC$

To prove:  $AD = EF$

Proof:

Since BCDE is square,

$$\angle BCD = 90^\circ \dots(1)$$

In  $\triangle ACD$ ,

$$\angle ACD = \angle ACB + \angle BCD$$

$$= \angle ACB + 90^\circ \dots(2)$$

In  $\triangle BCF$ ,

$$\angle BCF = \angle BCA + \angle ACF$$

Since ACFG is square,

$$\angle ACF = 90^\circ \dots(3)$$

From 2 and 3, we have,

$$\angle ACD = \angle BCF \dots(4)$$

Thus in  $\triangle ACD$  and  $\triangle BCF$ , we have,

$$AC = CF \dots \text{sides of square}$$

$$\angle ACD = \angle BCF \dots \text{from 4}$$

$$CD = BC \dots \text{sides of square}$$

Thus by SAS property of congruence,

$$\triangle ACD \cong \triangle BCF$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore AD = BF$$

#### **24. Question**

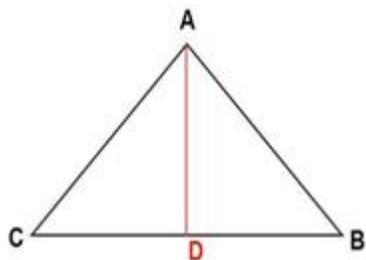
Prove that median from the vertex of an isosceles triangle is the bisector of the vertical angle.

#### **Answer**

Given:  $\triangle ABC$  is isosceles triangle where  $AB = AC$  and  $BD = DC$

To prove:  $\angle BAD = \angle DAC$

Proof:



In  $\triangle ABD$  and  $\triangle ADC$

$AB = AC$  ...given

$BD = DC$  ...given

$AD = AD$  ... common side

Thus by SSS property of congruence,

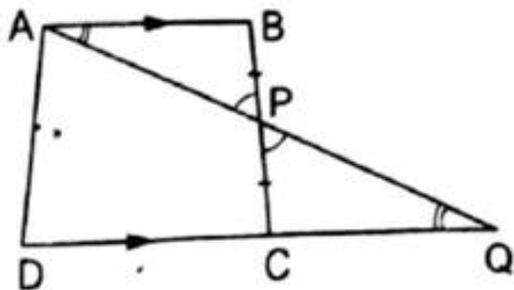
$\triangle ABD \cong \triangle ADC$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle BAD = \angle DAC$

## 25. Question

In the given figure, ABCD is a quadrilateral in which  $AB \parallel DC$  and P is the midpoint of BC. On producing, AP and DC meet at Q. prove that (i)  $AB = CQ$ , (ii)  $DQ = DC + AB$ .



## Answer

Given: ABCD is a quadrilateral in which  $AB \parallel DC$  and  $BP = PC$

To prove:  $AB = CQ$  and  $DQ = DC + AB$

Proof:

In  $\triangle ABP$  and  $\triangle PCQ$  we have,

$\angle PAB = \angle PQC$  ...alternate angles

$\angle APB = \angle CPQ$  ... vertically opposite angles

$BP = PC$  ... given

Thus by AAS property of congruence,

$\triangle ABP \cong \triangle PCQ$

Hence, we know that, corresponding parts of the congruent triangles are equal

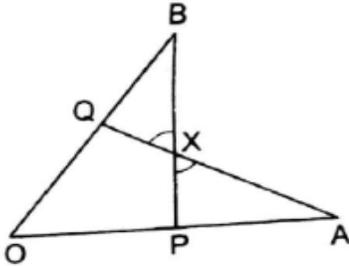
$$\therefore AB = CQ \dots(1)$$

$$\text{But, } DQ = DC + CQ$$

$$= DC + AB \dots \text{from 1}$$

## 26. Question

In the given figure,  $OA=OB$  and  $OP=OQ$ . Prove that (i)  $PX=QX$ , (ii)  $AX=BX$ .



## Answer

Given:  $OA=OB$  and  $OP=OQ$

To prove:  $PX=QX$  and  $AX=BX$

Proof:

In  $\triangle OAQ$  and  $\triangle OPB$ , we have

$$OA = OB \dots \text{given}$$

$$\angle O = \angle O \dots \text{common angle}$$

$$OQ = OP \dots \text{given}$$

Thus by SAS property of congruence,

$$\triangle OAP \cong \triangle OPB$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\angle OBP = \angle OAQ \dots(1)$$

Thus, in  $\triangle BXQ$  and  $\triangle PXA$ , we have,

$$BQ = OB - OQ$$

$$\text{And } PA = OA - OP$$

$$\text{But } OP = OQ$$

$$\text{And } OA = OB \dots \text{given}$$

$$\text{Hence, we have, } BQ = PA \dots(2)$$

Now consider  $\triangle BXQ$  and  $\triangle PXA$ ,

$$\angle BXQ = \angle PXA \dots \text{vertically opposite angles}$$

$\angle OBP = \angle OAQ$  ...from 1

$BQ = PA$  ... from 2

Thus by AAS property of congruence,

$\Delta BXQ \cong \Delta PXA$

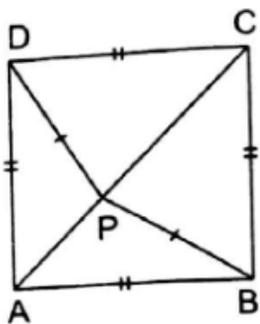
Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore PX = QX$

And  $AX = BX$

## 27. Question

In the given figure, ABCD is a square and P is a point inside it such that  $PB=PD$ . Prove that CPA is a straight line.



## Answer

Given: ABCD is a square and  $PB=PD$

To prove: CPA is a straight line

Proof:

$\Delta APD$  and  $\Delta APB$ ,

$DA = AB$  ...as ABCD is square

$AP = AP$  ... common side

$PB = PD$  ... given

Thus by SSS property of congruence,

$\Delta APD \cong \Delta APB$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle APD = \angle APB$  ...(1)

Now consider  $\Delta CPD$  and  $\Delta CPB$ ,

$CD = CB$  ... ABCD is square

$CP = CP$  ... common side

PB = PD ... given

Thus by SSS property of congruence,

$$\triangle CPD \cong \triangle CPB$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\angle CPD = \angle CPB \dots (2)$$

Now,

Adding both sides of 1 and 2,

$$\angle CPD + \angle APD = \angle APB + \angle CPB \dots (3)$$

Angles around the point P add upto  $360^\circ$

$$\therefore \angle CPD + \angle APD + \angle APB + \angle CPB = 360^\circ$$

From 4,

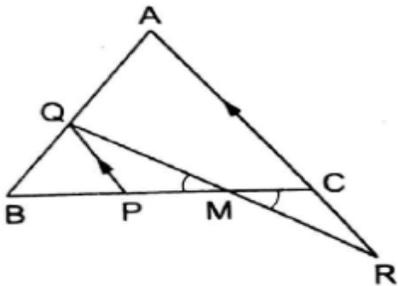
$$2(\angle CPD + \angle APD) = 360^\circ$$

$$\angle CPD + \angle APD = \frac{360^\circ}{2} = 180^\circ$$

This proves that CPA is a straight line.

## 28. Question

In the given figure, ABC is an equilateral triangle, PQ  $\parallel$  AC and AC is produced to R such that CR=BP. Prove that QR bisects PC.



## Answer

Given: ABC is an equilateral triangle, PQ  $\parallel$  AC and CR=BP

To prove: QR bisects PC or PM = MC

Proof:

Since,  $\triangle ABC$  is equilateral triangle,

$$\angle A = \angle ACB = 60^\circ$$

Since, PQ  $\parallel$  AC and corresponding angles are equal,

$$\angle BPQ = \angle ACB = 60^\circ$$

In  $\triangle BPQ$ ,

$$\angle B = \angle ACB = 60^\circ$$

$$\angle BPQ = 60^\circ$$

Hence,  $\triangle BPQ$  is an equilateral triangle.

$$\therefore PQ = BP = BQ$$

Since we have  $BP = CR$ ,

$$\text{We say that } PQ = CR \dots(1)$$

Consider the triangles  $\triangle PMQ$  and  $\triangle CMR$ ,

$$\angle PQM = \angle CRM \dots \text{alternate angles}$$

$$\angle PMQ = \angle CMR \dots \text{vertically opposite angles}$$

$$PQ = CR \dots \text{from 1}$$

Thus by AAS property of congruence,

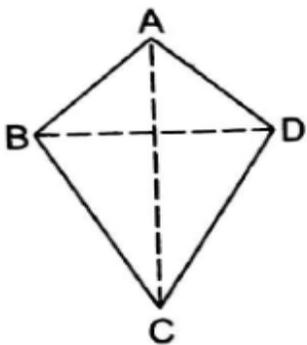
$$\triangle PMQ \cong \triangle CMR$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore PM = MC$$

### 29. Question

In the given figure,  $ABCD$  is a quadrilateral in which  $AB=AD$  and  $BC=DC$ . Prove that (i)  $AC$  bisects  $\angle A$  and  $\angle C$ , (ii)  $AC$  is the perpendicular bisector of  $BD$ .



### Answer

Given:  $ABCD$  is a quadrilateral in which  $AB=AD$  and  $BC=DC$

To prove:  $AC$  bisects  $\angle A$  and  $\angle C$ , and  $AC$  is the perpendicular bisector of  $BD$

Proof:

In  $\triangle ABC$  and  $\triangle ADC$ , we have

$$AB = AD \dots \text{given}$$

$$BC = DC \dots \text{given}$$

$AC = AC$  ... common side

Thus by SSS property of congruence,

$$\triangle ABC \cong \triangle ADC$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\angle BAC = \angle DAC$$

$$\therefore \angle BAO = \angle DAO \dots(1)$$

It means that AC bisects  $\angle BAD$  ie  $\angle A$

$$\text{Also, } \angle BCA = \angle DCA \dots \text{cpct}$$

It means that AC bisects  $\angle BCD$ , ie  $\angle C$

Now in  $\triangle ABO$  and  $\triangle ADO$

$$AB = AD \dots \text{given}$$

$$\angle BAO = \angle DAO \dots \text{from 1}$$

$$AO = AO \dots \text{common side}$$

Thus by SAS property of congruence,

$$\triangle ABO \cong \triangle ADO$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\angle BOA = \angle DOA$$

$$\text{But } \angle BOA + \angle DOA = 180^\circ$$

$$2\angle BOA = 180^\circ$$

$$\therefore \angle BOA = \frac{180^\circ}{2} = 90^\circ$$

$$\text{Also } \triangle ABO \cong \triangle ADO$$

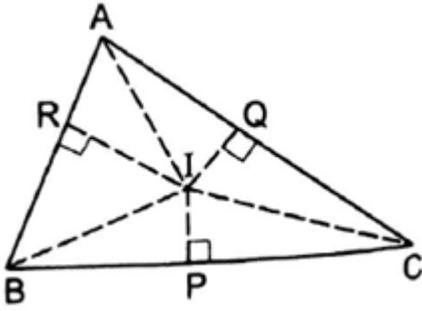
$$\text{So, } BO = OD$$

Which means that  $AC = BD$

### 30. Question

In the given figure, the bisectors of  $\angle B$  and  $\angle C$  of  $\triangle ABC$  meet at I. If  $IP \perp BC$ ,  $IQ \perp CA$  and  $IR \perp AB$ , prove that

(i)  $IP=IQ=IR$ , (ii) IA bisects  $\angle A$ .



### Answer

Given:  $IP \perp BC$ ,  $IQ \perp CA$  and  $IR \perp AB$  and the bisectors of  $\angle B$  and  $\angle C$  of  $\triangle ABC$  meet at  $I$

To prove:  $IP=IQ=IR$  and  $IA$  bisects  $\angle A$

Proof:

In  $\triangle BIP$  and  $\triangle BIR$  we have,

$\angle PBI = \angle RBI$  ...given

$\angle IRB = \angle IPB = 90^\circ$  ...Given

$IB = IB$  ...common side

Thus by AAS property of congruence,

$\triangle BIP \cong \triangle BIR$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore IP = IR$

Similarly,

$IP = IQ$

Hence,  $IP = IQ = IR$

Now in  $\triangle AIR$  and  $\triangle AIQ$

$IR = IQ$  ...proved above

$IA = IA$  ... Common side

$\angle IRA = \angle IQA = 90^\circ$

Thus by SAS property of congruence,

$\triangle AIR \cong \triangle AIQ$

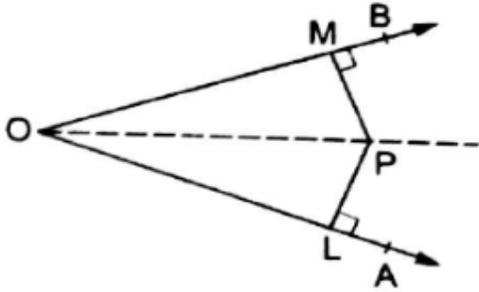
Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore \angle IAR = \angle IAQ$

This means that  $IA$  bisects  $\angle A$

### 31. Question

In the adjoining figure, P is a point in the interior of  $\angle AOB$ . If  $PL \perp OA$  and  $PM \perp OB$  such that  $PL=PM$ , show that OP is the bisector of  $\angle AOB$



**Answer**

Given: P is a point in the interior of  $\angle AOB$  and  $PL \perp OA$  and  $PM \perp OB$  such that  $PL=PM$

To prove:  $\angle POL = \angle POM$

Proof:

In  $\triangle OPL$  and  $\triangle OPM$ , we have

$\angle OPM = \angle OPL = 90^\circ$  ...given

$OP = OP$  ...common side

$PL = PM$  ... given

Thus by Right angle hypotenuse side property of congruence,

$\triangle OPL \cong \triangle OPM$

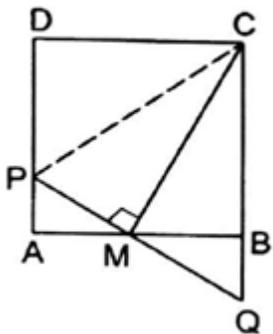
Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore \angle POL = \angle POM$

Ie. OP is the bisector of  $\angle AOB$

**32. Question**

In the given figure, ABCD is a square, M is the midpoint of AB and  $PQ \perp CM$  meets AD at P and CB produced at Q. prove that (i)  $PA=BQ$ , (ii)  $CP=AB+PA$ .



**Answer**

Given: ABCD is a square,  $AM = MB$  and  $PQ \perp CM$

To prove:  $PA=BQ$  and  $CP=AB+PA$

Proof:

In  $\triangle AMP$  and  $\triangle BMQ$ , we have

$\angle AMP = \angle BMQ$  ...vertically opposite angle

$\angle PAM = \angle MBQ = 90^\circ$  ...as ABCD is square

$AM = MB$  ...given

Thus by AAS property of congruence,

$\triangle AMP \cong \triangle BMQ$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore PA = BQ$  and  $MP = MQ$  ...(1)

Now in  $\triangle PCM$  and  $\triangle QCM$

$PM = QM$  ... from 1

$\angle PMC = \angle QMC$  ... given

$CM = CM$  ... common side

Thus by AAS property of congruence,

$\triangle PCM \cong \triangle QCM$

Hence, we know that, corresponding parts of the congruent triangles are equal

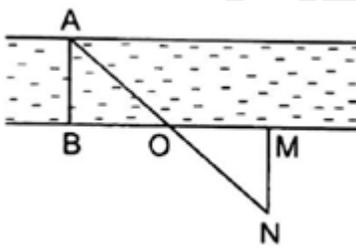
$\therefore PC = QC$

$PC = QB + CB$

$PC = AB + PA$  ...as  $AB = CB$  and  $PA = QB$

### 33. Question

In the adjoining figure, explain how one can find the breadth of the river without crossing it.



### Answer

Given:  $AB \perp BO$  and  $NM \perp OM$

In  $\triangle ABO$  and  $\triangle NMO$ ,

$\angle OBA = \angle OMN$

$OB = OM$  ...O is mid point of BM

$\angle BOA = \angle MON$  ...vertically opposite angles

Thus by AAS property of congruence,

$$\triangle ABO \cong \triangle NMO$$

Hence, we know that, corresponding parts of the congruent triangles are equal

$$\therefore AB = MN$$

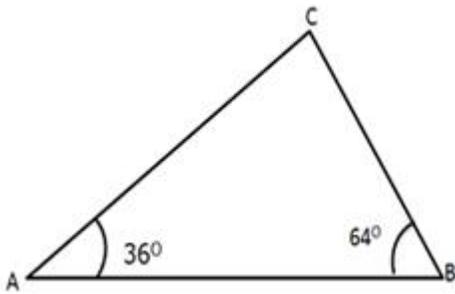
Hence, we can calculate the width of the river by calculating MN

### 34. Question

In  $\triangle ABC$ , if  $\angle A = 36^\circ$  and  $\angle B = 64^\circ$ , name the longest and shortest sides of the triangle.

### Answer

Given:  $\angle A = 36^\circ$  and  $\angle B = 64^\circ$



To find: The longest and shortest sides of the triangle

Given that  $\angle A = 36^\circ$  and  $\angle B = 64^\circ$

Hence, by the angle sum property in  $\triangle ABC$ , we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$36^\circ + 64^\circ + \angle C = 180^\circ$$

$$100^\circ + \angle C = 180^\circ$$

$$\angle C = 80^\circ$$

So, we have  $\angle A = 36^\circ$ ,  $\angle B = 64^\circ$  and  $\angle C = 80^\circ$

$\therefore \angle C$  is largest and  $\angle A$  is shortest

Hence,

Side opposite to  $\angle C$  is longest.

$\therefore AB$  is longest

Side opposite to  $\angle A$  is shortest.

$\therefore BC$  is shortest

### 35. Question

In  $\triangle ABC$ , if  $\angle A = 90^\circ$ , which is the longest side?

#### Answer

It is given that  $\angle A = 90^\circ$ .

In right angled triangle at  $90^\circ$

Sum of all angles in triangle is  $180^\circ$ , so other two angles must be less than  $90^\circ$

So, other angles are smaller than  $\angle A$ .

Hence  $\angle A$  is largest angle.

We know that side opposite to largest angle is largest.

$\therefore BC$  is longest side, which is opposite to  $\angle A$ .

### 36. Question

In  $\triangle ABC$ , if  $\angle A = \angle B = 45^\circ$ , name the longest side.

#### Answer

In  $\triangle ABC$  given that  $\angle A = \angle B = 45^\circ$

So, by the angle sum property in  $\triangle ABC$ , we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$45^\circ + 45^\circ + \angle C = 180^\circ$$

$$90^\circ + \angle C = 180^\circ$$

$$\therefore \angle C = 180^\circ - 90^\circ$$

$$\angle C = 90^\circ$$

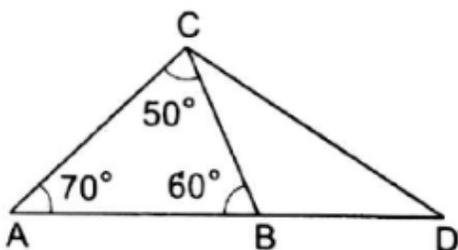
Hence, largest angle is  $\angle C$

We know that side opposite to largest angle is longest, which is  $AB$

Hence our longest side is  $AB$

### 37. Question

In  $\triangle ABC$ , side  $AB$  is produced to  $D$  such that  $BD = BC$ . If  $\angle B = 60^\circ$  and  $\angle A = 70^\circ$ , prove that (i)  $AD > CD$  and (ii)  $AD > AC$ .



## Answer

Given: In  $\triangle ABC$ ,  $BD=BC$  and  $\angle B=60^\circ$  and  $\angle A=70^\circ$

To prove:  $AD > CD$  and  $AD > AC$

Proof:

In  $\triangle ABC$ , by the angle sum property, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$70^\circ + 60^\circ + \angle C = 180^\circ$$

$$130^\circ + \angle C = 180^\circ$$

$$\angle C = 50^\circ$$

Now in  $\triangle BCD$  we have,

$$\angle CBD = \angle DAC + \angle ACB \dots \text{as } \angle CBD \text{ is the exterior angle of } \triangle ABC$$

$$= 70^\circ + 50^\circ$$

Since  $BC = BD$  ...given

$$\text{So, } \angle BCD = \angle BDC$$

$$\therefore \angle BCD + \angle BDC = 180^\circ - \angle CBD$$

$$= 180^\circ - 120^\circ = 60^\circ$$

$$2\angle BCD = 60^\circ$$

$$\angle BCD = \angle BDC = 30^\circ$$

Now in  $\triangle ACD$  we have

$$\angle A = 70^\circ, \angle D = 30^\circ$$

$$\text{And } \angle ACD = \angle ACB + \angle BCD$$

$$= 50^\circ + 30^\circ = 80^\circ$$

$\therefore \angle ACD$  is greatest angle

So, the side opposite to largest angle is longest, ie  $AD$  is longest side.

$$\therefore AD > CD$$

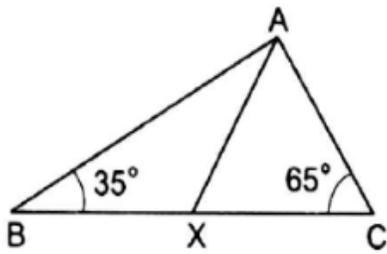
Since,  $\angle BDC$  is smallest angle,

The side opposite to  $\angle BDC$ , ie  $AC$ , is the shortest side in  $\triangle ACD$ .

$$\therefore AD > AC$$

## 38. Question

In  $\triangle ABC$ ,  $\angle B=35^\circ, \angle C=65^\circ$  and the bisector of  $\angle BAC$  meets  $BC$  in  $X$ . Arrange  $AX, BX$  and  $CX$  in descending order.



**Answer**

Given: In  $\triangle ABC$ ,  $\angle B=35^\circ, \angle C=65^\circ$  and  $\angle BAX = \angle XAC$

To find: Relation between AX, BX and CX in descending order.

In  $\triangle ABC$ , by the angle sum property, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 35^\circ + 65^\circ = 180^\circ$$

$$\angle A + 100^\circ = 180^\circ$$

$$\therefore \angle A = 80^\circ$$

$$\text{But } \angle BAX = \frac{1}{2} \angle A$$

$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

Now in  $\triangle ABX$ ,

$$\angle B = 35^\circ$$

$$\angle BAX = 40^\circ$$

$$\begin{aligned} \text{And } \angle BXA &= 180^\circ - 35^\circ - 40^\circ \\ &= 105^\circ \end{aligned}$$

So, in  $\triangle ABX$ ,

$\angle B$  is smallest, so the side opposite is smallest, ie AX is smallest side.

$$\therefore AX < BX \dots(1)$$

Now consider  $\triangle AXC$ ,

$$\angle CAX = \frac{1}{2} \times \angle A$$

$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

$$\begin{aligned} \angle AXC &= 180^\circ - 40^\circ - 65^\circ \\ &= 180^\circ - 105^\circ = 75^\circ \end{aligned}$$

Hence, in  $\triangle AXC$  we have,

$$\angle CAX = 40^\circ, \angle C = 65^\circ, \angle AXC = 75^\circ$$

$\therefore \angle CAX$  is smallest in  $\triangle AXC$

So the side opposite to  $\angle CAX$  is shortest

I.e.  $CX$  is shortest

$$\therefore CX < AX \dots (2)$$

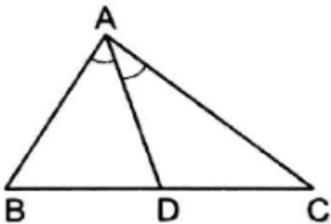
From 1 and 2 ,

$$BX > AX > CX$$

This is required descending order

### 39. Question

In  $\triangle ABC$ , if  $AD$  is the bisector of  $\angle A$ , show that  $AB > BD$  and  $AC > DC$



### Answer

Given:  $\angle BAD = \angle DAC$

To prove:  $AB > BD$  and  $AC > DC$

Proof:

In  $\triangle ACD$ ,

$$\angle ADB = \angle DAC + \angle ACD \dots \text{exterior angle theorem}$$

$$= \angle BAD + \angle ACD \dots \text{given that } \angle BAD = \angle DAC$$

$$\angle ADB > \angle BAD$$

The side opposite to angle  $\angle ADB$  is the longest side in  $\triangle ADB$

So,  $AB > BD$

Similarly in  $\triangle ABD$

$$\angle ADC = \angle ABD + \angle BAD \dots \text{exterior angle theorem}$$

$$= \angle ABD + \angle CAD \dots \text{given that } \angle BAD = \angle DAC$$

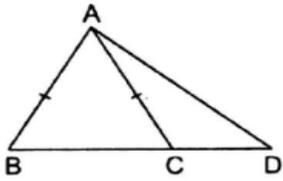
$$\angle ADC > \angle CAD$$

The side opposite to angle  $\angle ADC$  is the longest side in  $\triangle ACD$

So,  $AC > DC$

#### 40. Question

In the given figure, ABC is a triangle in which  $AB=AC$ . If D be a point on BC produced, prove that  $AD>AC$ .



#### Answer

Given:  $AB=AC$

To prove:  $AD>AC$

Proof:

In  $\triangle ABC$ ,

$$\angle ACD = \angle B + \angle BAC$$

$$= \angle ACB + \angle BAC \dots \text{as } \angle C = \angle B \text{ as } AB = AC$$

$$= \angle CAD + \angle CDA + \angle BAC \dots \text{as } \angle ACB = \angle CAD + \angle CDA$$

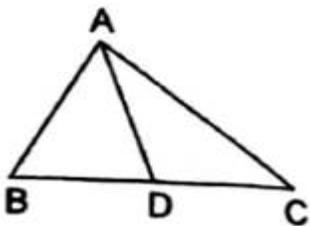
$$\therefore \angle ACD > \angle CDA$$

So the side opposite to  $\angle ACD$  is the longest

$$\therefore AD > AC$$

#### 41. Question

In the adjoining figure,  $AC>AB$  and AD is the bisector of  $\angle A$ . show that  $\angle ADC>\angle ADB$ .



#### Answer

Given:  $AC>AB$  and  $\angle BAD = \angle DAC$

To prove:  $\angle ADC>\angle ADB$

Proof:

Since  $AC > AB$

$$\angle ABC > \angle ACB$$

Adding  $\frac{1}{2} \angle A$  on both sides

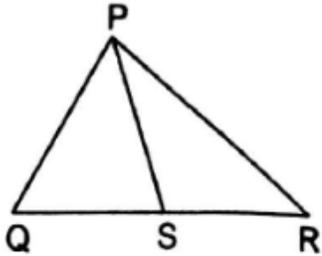
$$\angle ABC + \frac{1}{2} \angle A > \angle ACB + \frac{1}{2} \angle A$$

$\angle ABC + \angle BAD > \angle ACB + \angle DAC$  ... As AD is a bisector of  $\angle A$

$\therefore \angle ADC > \angle ADB$

#### 42. Question

In  $\triangle PQR$ , if S is any point on the side QR, show that  $PQ + QR + RP > 2PS$ .



#### Answer

Given: S is any point on the side QR

To prove:  $PQ + QR + RP > 2PS$ .

Proof:

Since in a triangle, sum of any two sides is always greater than the third side.

So in  $\triangle PQS$ , we have,

$$PQ + QS > PS \dots(1)$$

Similarly,  $\triangle PSR$ , we have,

$$PR + SR > PS \dots(2)$$

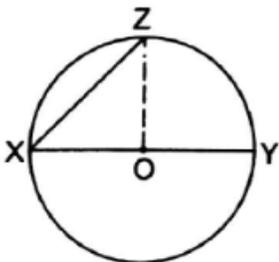
Adding 1 and 2

$$PQ + QS + PR + SR > 2PS$$

$$PQ + PR + QR > 2PS \dots \text{as } QR = QS + SR$$

#### 43. Question

In the given figure, O is the center of the circle and XOY is a diameter. If XZ is any other chord of the circle, show that  $XY > XZ$ .



#### Answer

Given: XOY is a diameter and XZ is any chord of the circle.

To prove:  $XY > XZ$

Proof:

In  $\Delta XOZ$ ,

$OX + OZ > XZ$  ... sum of any sides in a triangle is a greater than its third side

$\therefore OX + OY > XZ$  ... As  $OZ = OY$ , radius of circle

Hence,  $XY > XZ$  ...As  $OX + OY = XY$

#### 44. Question

If O is a point within  $\Delta ABC$ , show that:

(i)  $AB + AC > OB + OC$

(ii)  $AB + BC + CA > OA + OB + OC$

(iii)  $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

#### Answer

Given: O is a point within  $\Delta ABC$

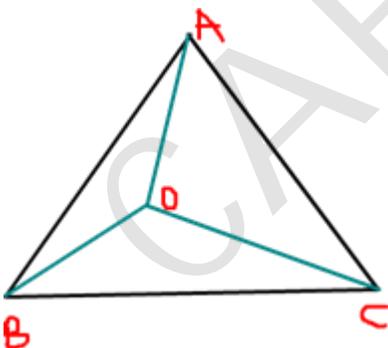
To prove:

(i)  $AB + AC > OB + OC$

(ii)  $AB + BC + CA > OA + OB + OC$

(iii)  $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Proof:



In  $\Delta ABC$ ,

$AB + AC > BC$  ... (1)

And in  $\Delta OBC$ ,

$OB + OC > BC$  ... (2)

Subtracting 1 from 2 we get,

$$(AB + AC) - (OB + OC) > (BC - BC)$$

$$\text{I.e. } AB + AC > OB + OC$$

From 1,  $AB + AC > OB + OC$

Similarly,  $AB + BC > OA + OC$

And  $AC + BC > OA + OB$

Adding both sides of these three inequalities, we get,

$$(AB + AC) + (AB + BC) + (AC + BC) > (OB + OC) + (OA + OC) + (OA + OB)$$

$$\text{I.e. } 2(AB + BC + AC) > 2(OA + OB + OC)$$

$$\therefore AB + BC + OA > OA + OB + OC$$

In  $\triangle OAB$ ,

$$OA + OB > AB \dots(1)$$

In  $\triangle OBC$ ,

$$OB + OC > BC \dots(2)$$

In  $\triangle OCA$

$$OC + OA > CA \dots(3)$$

Adding 1,2 and 3,

$$(OA + OB) + (OB + OC) + (OC + OA) > AB + BC + CA$$

$$\text{I.e. } 2(OA + OB + OC) > AB + BC + CA \therefore OA + OB + OC > \frac{1}{2} (AB + BC + CA)$$

#### 45. Question

Can we draw a triangle ABC with  $AB=3\text{cm}$ ,  $BC=3.5\text{cm}$  and  $CA=6.5\text{cm}$ ? Why?

#### Answer

Our given lengths are  $AB=3\text{cm}$ ,  $BC=3.5\text{cm}$  and  $CA=6.5\text{cm}$ .

$$\therefore AB + BC = 3 + 3.5 = 6.5 \text{ cm}$$

But  $CA = 6.5 \text{ cm}$

So,  $AB + BC = CA$

A triangle can be drawn only when the sum of two sides is greater than the third side

So, a triangle cannot be drawn with such lengths

#### CCE Questions

##### 1. Question

Which of the following is not a criterion for congruence of triangles?

- A. SSA
- B. SAS
- C. ASA
- D. SSS

**Answer**

From the above given four options, SSA is not a criterion for the congruence of triangles

∴ Option (A) is correct

**2. Question**

If  $AB=QR$ ,  $BC=RP$  and  $CA=PQ$ , then which of the following holds?

- A.  $\triangle ABC \cong \triangle PQR$
- B.  $\triangle CBA \cong \triangle PQR$
- B.  $\triangle CAB \cong \triangle PQR$
- D.  $\triangle BCA \cong \triangle PQR$

**Answer**

It is given in the question that,

$$AB = QR$$

$$BC = RP$$

$$\text{And, } CA = PQ$$

∴ By SSS congruence criterion

$$\triangle CBA \cong \triangle PQR$$

Hence, option (B) is correct

**3. Question**

If  $\triangle ABC \cong \triangle PQR$  AND  $\triangle ABC$  is not congruent to  $\triangle RPQ$ , then which of the following is not true?

- A.  $BC=PQ$
- B.  $AC=PR$
- C.  $BC=QR$
- D.  $AB=PQ$

**Answer**

According to the condition given in the question,

If  $\triangle ABC \cong \triangle PQR$  and  $\triangle ABC$  is not congruent to  $\triangle RPQ$

Then, clearly  $BC \neq PQ$

$\therefore$  It is false

Hence, option (A) is correct

#### 4. Question

It is given that  $\triangle ABC \cong \triangle FDE$  in which  $AB=5\text{cm}$ ,  $\angle B=40^\circ$ ,  $\angle A=80^\circ$  and  $FD=5\text{cm}$ . Then, which of the following is true?

A.  $\angle D=60^\circ$

B.  $\angle E=60^\circ$

C.  $\angle F=60^\circ$

D.  $\angle D=80^\circ$

#### Answer

It is given in the question that,

$\triangle ABC \cong \triangle FDE$  where,

$AB = 5 \text{ cm}$

$FD = 5 \text{ cm}$

$\angle B = 40^\circ$

$\angle A = 80^\circ$

We know that sum of all angles of a triangle is equal to  $180^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$80^\circ + 40^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 120^\circ$

$= 60^\circ$

As, Angle C = Angle E

$\therefore$  Angle E =  $60^\circ$

Hence, option (B) is correct

#### 5. Question

In  $\triangle ABC$ ,  $AB=2.5\text{cm}$  and  $BC=6\text{cm}$ . Then, the length of AC cannot be

A. 3.4

B. 4 cm

C. 3.8 cm

D. 3.6 cm

**Answer**

It is given in the question that,

In  $\triangle ABC$

$AB = 2.5$  cm

$BC = 6$  cm

We know that, the length of a side must be less than the sum of the other two sides

Let us assume the side of AC be  $x$  cm

$$\therefore x < 2.5 + 6$$

$$x < 8.5$$

Also, we know that the length of a side must be greater than the difference between the other two sides

$$\therefore x > 6 - 2.5$$

$$x > 3.5$$

Hence, the limits of the value of  $x$  is

$$3.5 < x < 8.5$$

$\therefore$  It is clear the length of AC cannot be 3.4 cm

Hence, option (A) is correct

**6. Question**

In  $\triangle ABC$ ,  $\angle A = 40^\circ$  and  $\angle B = 60^\circ$ , Then, the longest side of  $\triangle ABC$  is

A. BC

B. AC

C. AB

D. cannot be determined

**Answer**

It is given in the question that,

In  $\triangle ABC$ ,  $\angle A = 40^\circ$

$$\angle B = 60^\circ$$

We know that, sum of all angles of a triangle is equal to  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$60^\circ + 40^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 100^\circ$$

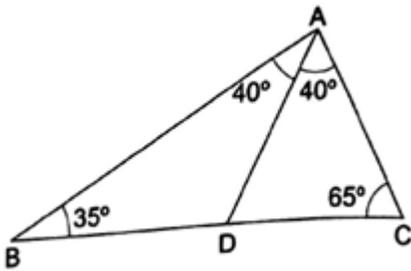
$$\angle C = 80^\circ$$

Hence, the side which is opposite to  $\angle C$  is the longest side of the triangle

$\therefore$  Option (C) is correct

### 7. Question

In  $\triangle ABC$ ,  $\angle B=35^\circ$ ,  $\angle C=65^\circ$  and the bisector AD of  $\angle BAC$  meets BC at D. Then, which of the following is true?



A.  $AD > BD > CD$

B.  $BD > AD > CD$

C.  $AD > CD > BD$

D. None of these

### Answer

It is given in the question that,

In  $\triangle ABC$ , we have

$$\angle B = 35^\circ$$

$$\angle C = 65^\circ$$

Also the bisector AD of  $\angle BAC$  meets at D

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 35^\circ + 65^\circ = 180^\circ$$

$$\angle A = 180^\circ - 100^\circ$$

$$\angle A = 80^\circ$$

As, AD is the bisector of  $\angle BAC$

$$\therefore \angle BAD = \angle CAD = 40^\circ$$

In  $\triangle ABD$ , we have

$$\angle BAD > \angle ABD$$

$$BD > AD$$

Also, in  $\triangle ACD$

$$\angle ACD > \angle CAD$$

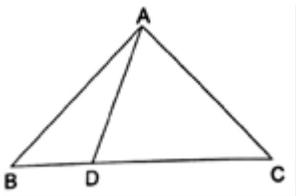
$$AD > CD$$

Hence,  $BD > AD > CD$

$\therefore$  Option (B) is correct

### 8. Question

In the given figure,  $AB > AC$ . Then, which of the following is true?



- A.  $AB < AD$
- B.  $AB = AD$
- C.  $AB > AD$
- D. cannot be determined

### Answer

From the given figure, we have

$$AB > AC$$

$$\therefore \angle ACB > \angle ABC$$

Also,  $\angle ADB > \angle ACD$

$$\angle ADB > \angle ACB > \angle ABC$$

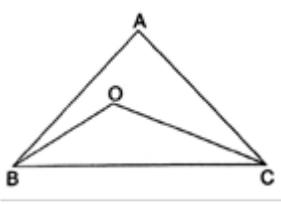
$$\angle ADB > \angle ABD$$

$$\therefore AB > AD$$

Hence, option (C) is correct

### 9. Question

In the given figure,  $AB > AC$ . If BO and CO are the bisectors of  $\angle B$  and  $\angle C$  respectively, then



- A.  $OB=OC$
- B.  $OB>OC$
- C.  $OB<OC$

**Answer**

From the given figure, we have

$$AB > AC$$

Also,  $\angle C > \angle B$

$$\frac{1}{2}\angle C > \frac{1}{2}\angle B$$

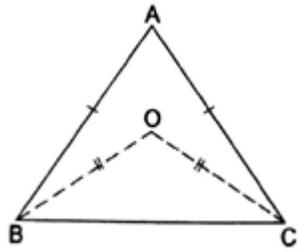
$\angle OCB > \angle OBC$  (Given)

$$\therefore OB > OC$$

Hence, option (C) is correct

**10. Question**

In the given figure,  $AB=AC$  and  $OB=OC$ . Then,  $\angle ABO: \angle ACO=?$



- A. 1:1
- B. 2:1
- C. 1:2
- D. None of these

**Answer**

It is given in the question that,

In  $\triangle OAB$  and  $\triangle OAC$ , we have

$$AB = AC$$

$$OB = OC$$

$OA = OA$  (Common)

$\therefore$  By SSS congruence criterion

$$\triangle OAB \cong \triangle OAC$$

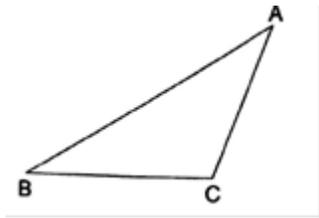
$$\therefore \angle ABO = \angle ACO$$

So,  $\angle ABO : \angle ACO = 1 : 1$

Hence, option (A) is correct

### 11. Question

In  $\triangle ABC$ , IF  $\angle C > \angle B$ , then



A.  $BC > AC$

B.  $AB > AC$

C.  $AB < AC$

D.  $BC < A$

### Answer

It is given in the question that,

In  $\triangle ABC$ , we have

$$\angle C > \angle B$$

We know that, side opposite to the greater angle is larger

$$\therefore AB > AC$$

Hence, option (B) is correct

### 12. Question

O is any point in the interior of  $\triangle ABC$ . Then, which of the following is true?

A.  $(OA+OB+OC) > (AB+BC+CA)$

B.  $(OA+OB+OC) > \frac{1}{2} (AB+BC+CA)$

C.  $(OA+OB+OC) < \frac{1}{2} (AB+BC+CA)$

D. None of these

**Answer**

From the given question, we have

In  $\triangle OAB$ ,  $\triangle OBC$  and  $\triangle OCA$  we have:

$$OA + OB > AB$$

$$OB + OC > BC$$

$$\text{And, } OC + OA > AC$$

Adding all these, we get:

$$2(OA + OB + OC) > (AB + BC + CA)$$

$$(OA + OB + OC) > \frac{1}{2}(AB + BC + CA)$$

$\therefore$  Option (C) is correct

**13. Question**

If the altitudes from two vertices of a triangle to the opposite sides are equal, then the triangle is

- A. Equilateral
- B. isosceles
- C. Scalene
- D. right-angled

**Answer**

It is given in the question that,

In  $\triangle ABC$ ,  $BL$  is parallel to  $AC$

Also,  $CM$  is parallel  $AB$  such that  $BL = CM$

We have to prove that:  $AB = AC$

Now, in  $\triangle ABL$  and  $\triangle ACM$  we have:

$$BL = CM \text{ (Given)}$$

$$\angle BAL = \angle CAM \text{ (Common)}$$

$$\angle ALB = \angle AMC \text{ (Each angle equal to } 90^\circ)$$

$\therefore$  By AAS congruence criterion

$$\triangle ABL \cong \triangle ACM$$

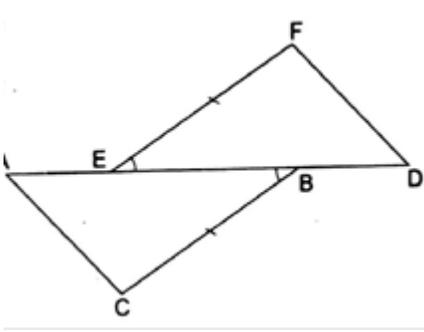
$$AB = AC \text{ (By Congruent parts of congruent triangles)}$$

As opposite sides of the triangle are equal, so it is an isosceles triangle

Hence, option (B) is correct

#### 14. Question

In the given figure,  $AE = DB$ ,  $CB = EF$  And  $\angle ABC = \angle FED$ . Then, which of the following is true?



- A.  $\triangle ABC \cong \triangle DEF$
- B.  $\triangle ABC \cong \triangle EFD$
- C.  $\triangle ABC \cong \triangle FED$
- D.  $\triangle ABC \cong \triangle EDF$

#### Answer

From the given figure, we have

$$AE = DB$$

$$\text{And, } CB = EF$$

$$\text{Now, } AB = (AD - DB)$$

$$= (AD - AE)$$

$$DE = (AD - AE)$$

Now, in  $\triangle ABC$  and  $\triangle DEF$  we have:

$$AB = DE$$

$$CB = EF$$

$$\angle ABC = \angle FED$$

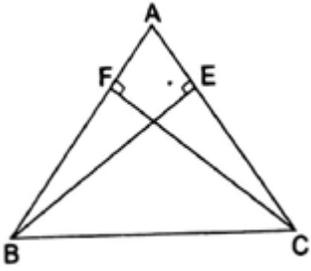
$\therefore$  By SAS congruence criterion

$$\triangle ABC \cong \triangle DEF$$

Hence, option (A) is correct

#### 15. Question

In the given figure,  $BE \perp CA$  and  $CF \perp BA$  such that  $BE = CF$ . Then, which of the following is true?



- A.  $\triangle ABE \cong \triangle ACF$
- B.  $\triangle ABE \cong \triangle AFC$
- C.  $\triangle ABE \cong \triangle CAF$
- D.  $\triangle ABE \cong \triangle FAC$

**Answer**

From the given figure, we have

BE is perpendicular to CA

Also, CF is perpendicular to BA

And, BE = CF

Now, in  $\triangle ABE$  and  $\triangle ACF$  we have:

BE = CF (Given)

$\angle BEA = \angle CFA = 90^\circ$

$\angle A = \angle A$  (Common)

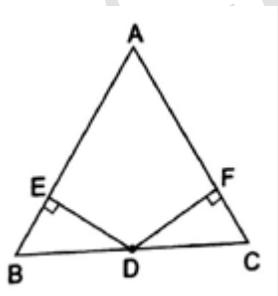
$\therefore$  By AAS congruence criterion

$\triangle ABE \cong \triangle ACF$

Hence, option (A) is correct

**16. Question**

In the given figure, D is the midpoint of BC,  $DE \perp AB$  and  $DF \perp AC$  such that  $DE=DF$ . Then, which of the following is true?



- A.  $AB=AC$
- B.  $AC=BC$

C.  $AB=BC$

D. None of these

**Answer**

From the given figure, we have

D is the mid-point of BC

Also, DE is perpendicular to AB

DF is perpendicular to AC

And,  $DE = DF$

Now, in  $\triangle BED$  and  $\triangle CFD$  we have:

$DE = DF$

$BD = CD$

$\angle E = \angle F = 90^\circ$

$\therefore$  By RHS congruence rule

$\triangle BED \cong \triangle CFD$

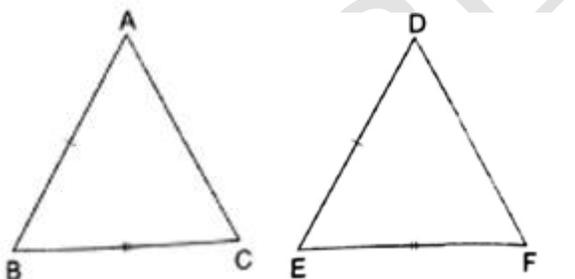
Thus,  $\angle B = \angle C$

$AC = AB$

Hence, option (A) is correct

**17. Question**

In  $\triangle ABC$  and  $\triangle DEF$ , it is given that  $AB=DE$  and  $BC=EF$ . In order that  $\triangle ABC \cong \triangle DEF$ , we must have



A.  $\angle A = \angle D$

B.  $\angle B = \angle E$

C.  $\angle C = \angle F$

D. none of these

**Answer**

From the question, we have:

In  $\triangle ABC$  and  $\triangle DEF$

$AB = DE$  (Given)

$BC = EF$  (Given)

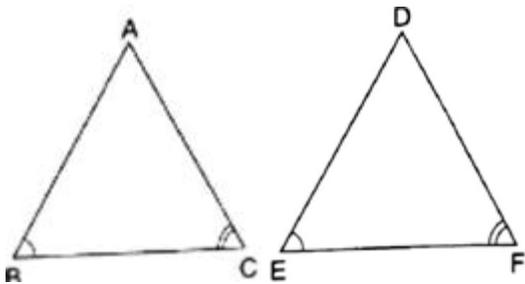
So, in order to have  $\triangle ABC \cong \triangle DEF$

$\angle B$  must be equal to  $\angle E$

$\therefore$  Option (B) is correct

### 18. Question

In  $\triangle ABC$  and  $\triangle DEF$ , it is given that  $\angle B = \angle E$  and  $\angle C = \angle F$ . In order that  $\triangle ABC \cong \triangle DEF$ , we must have



A.  $AB = DF$

B.  $AC = DE$

C.  $BC = EF$

D.  $\angle A = \angle D$

### Answer

From the question, we have:

In  $\triangle ABC$  and  $\triangle DEF$

$\angle B = \angle E$  (Given)

$\angle C = \angle F$  (Given)

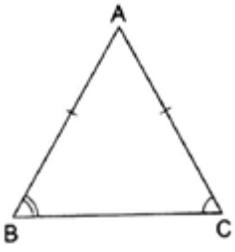
So, in order to have  $\triangle ABC \cong \triangle DEF$

$BC$  must be equal to  $EF$

$\therefore$  Option (C) is correct

### 19. Question

In  $\triangle ABC$  and  $\triangle PQR$ , it is given that  $AB = AC$ ,  $\angle C = \angle P$  and  $\angle P = \angle Q$ . Then, the two triangles are



- A. Isosceles but not congruent
- B. Isosceles and congruent
- C. Congruent but not isosceles
- D. Neither congruent not isosceles

**Answer**

It is given in the question that,

In  $\triangle ABC$  and  $\triangle PQR$ , we have

$$AB = AC$$

$$\text{Also, } \angle C = \angle B$$

$$\text{As, } \angle C = \angle P \text{ and, } \angle B = \angle Q$$

$$\therefore \angle P = \angle Q$$

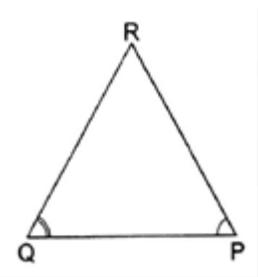
So, both triangles are isosceles but not congruent

Hence, option (A) is correct

**20. Question**

Which is true?

- A. A triangle can have two right angles.



- B. A triangle can have two obtuse angles.
- C. A triangle can have two acute angles.
- D. An exterior angle of a triangle is less than either of the interior opposite angles.

**Answer**

We know that,

Sum of all angles of a triangle is equal to  $180^\circ$

$\therefore$  A triangle can have two acute angles because sum of two acute angles of a triangle is always less than  $180^\circ$

Thus, it satisfies the angle sum property of a triangle

Hence, option (C) is correct

### 21. Question

Three statements are given below:

(I) In a  $\Delta ABC$  in which  $AB=AC$ , the altitude  $AD$  bisects  $BC$ .

(II) If the altitudes  $AD$ ,  $BE$  and  $CF$  of  $\Delta ABC$  are equal, then  $\Delta ABC$  is equilateral.

(III) If  $D$  is the midpoint of the hypotenuse  $AC$  of a right  $\Delta ABC$ , then  $BD=AC$ .

Which is true?

A. I only

B. II only

C. I and II

D. II and III

### Answer

Here we can clearly see that the true statements are as follows:

(I) In a  $\Delta ABC$  in which  $AB=AC$ , the altitude  $AD$  bisects  $BC$ .

(II) If the altitudes  $AD$ ,  $BE$  and  $CF$  of  $\Delta ABC$  are equal, then  $\Delta ABC$  is equilateral.

$\therefore$  Option C is correct

### 22. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
If $AD$ is a median of $\Delta ABC$ , then $AB+AC>2AD$ .	The angles opposite to equal sides of a triangle are equal.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

According to the question,

In  $\triangle ABD$  and  $\triangle ACD$ ,

Since, sum of any two sides of a triangle is greater than the third side.

$$AB + DB > AD \text{ (i)}$$

$$AC + DC > AD \text{ (ii)}$$

Adding (i) and (ii)

$$AB + AC + DB + DC > 2AD$$

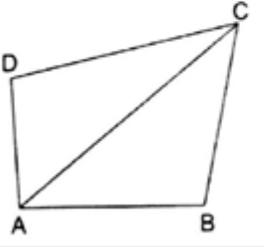
$$AB + AC + BC > 2AD$$

Hence, the assertion and the reason are both true, but Reason does not explain the assertion.

$\therefore$  Option B is correct

**23. Question**

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
<p>In a quadrilateral ABCD, we have <math>(AB+BC+CD+DA) &gt; 2AC</math>.</p> 	<p>The sum of any two sides of a triangle is greater than the third side.</p>

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

Since, sum of two sides is greater than the third side

$$\therefore AB + BC > AC \text{ (i)}$$

$$CD + DA > AC \text{ (ii)}$$

Adding (i) and (ii),

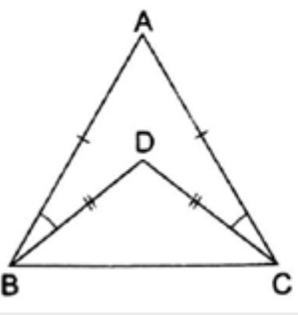
$$AB + BC + CD + DA > 2AC$$

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

$\therefore$  Option A is correct

**24. Question**

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
<p><math>\triangle ABC</math> and <math>\triangle DBC</math> are two isosceles triangles on the same base BC. Then, <math>\angle ABD = \angle ACD</math>.</p> 	<p>The angles opposite to equal sides of a triangle are equal.</p>

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

Since, angles opposite to equal sides are equal

$$AB = AC$$

$$\angle ABC = \angle ACB \text{ (i)}$$

$$DB = DC$$

$$\angle DBC = \angle DCB \text{ (ii)}$$

Subtracting (ii) from (i),

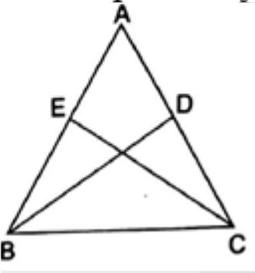
$$\angle ABC - \angle DBC = \angle ACB - \angle DCB$$

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

$\therefore$  Option A is correct

**25. Question**

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
<p>It is always possible to draw a triangle whose sides measure 4 cm, 5cm and 10cm respectively.</p>	<p>In an isosceles <math>\Delta ABC</math> with <math>AB=AC</math>, if <math>BD</math> and <math>CE</math> are bisectors of <math>\angle B</math> and <math>\angle C</math> respectively, then <math>BD=CE</math>.</p> 

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

**Answer**

In  $\Delta BDC$  and  $\Delta CEB$ ,

$$\angle DCB = \angle ECB \text{ (Given)}$$

$$BC = CB \text{ (Common)}$$

$$\angle B = \angle C \text{ (AC = AB)}$$

$$\frac{1}{2} \angle B = \frac{1}{2} \angle C$$

$$\angle CEB = \angle BCE$$

$$\therefore \Delta BDC \cong \Delta CEB$$

$$BD = CE \text{ (By c.p.c.t.)}$$

And, we know that the sum of two sides is always greater than the third side in any triangle.

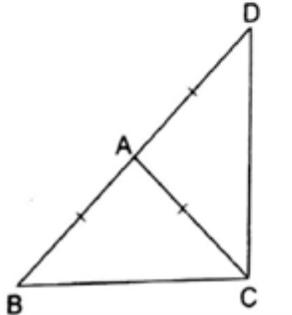
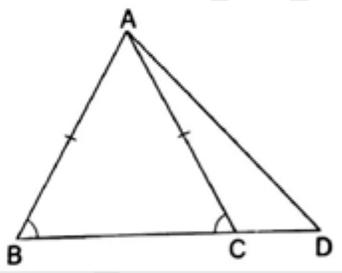
$$\text{But, } (5 + 4) < 10$$

Hence, the reason is true, but the assertion is false.

∴ Option D is true

## 26. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
<p>In the given figure, <math>\triangle ABC</math> is given with <math>AB=AC</math> and <math>BA</math> is produced to <math>D</math>, such that <math>AB=AD</math>.</p> <p>Then, <math>\angle BCD=90^\circ</math>.</p> 	<p>In the given figure <math>AB=AC</math> and <math>D</math> is a point on <math>BC</math> produced. Then, <math>AB&gt;AD</math>.</p> 

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

### Answer

According to the question,

$$AB = AC$$

$$\angle ACB = \angle ABC \text{ (i)}$$

Now,  $\angle ACD > \angle ACB = \angle ABC$  (Side BC is produced to D)

And, In  $\triangle ADC$ , side DC is produced to B

$\angle ACB > \angle ADC$  (ii)

$\angle ABC > \angle ADC$

Now, using (i) and (ii),

$AD > AB$

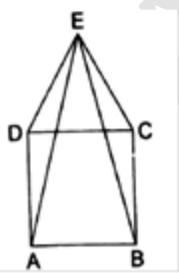
Hence, the reason is wrong but the assertion is true.

$\therefore$  Option C is correct

### **27. Question**

Match the following columns.

CAREER POINT

Column I	Column II
(a) In $\Delta ABC$ , if $AB=AC$ and $\angle A=50^\circ$ , then $\angle C=.....$	(p) its perimeter
(b) The vertical angle of an isosceles triangle is $130^\circ$ . Then, each base angle is.....	(q) $15^\circ$
(c) The sum of three altitudes of a $\Delta ABC$ is less than.....	(r) $65^\circ$
<p>(d) In the given figure, ABCD is a square and <math>\Delta EDC</math> is an equilateral triangle. Then, <math>\angle EBC</math> is.....</p> 	(s) $25^\circ$

The correct answer is:

(A)-....., (B)-....., (C)-....., (D)-.....,

**Answer**

The parts of the question are solved below:

a. Given: In  $\triangle ABC$ ,  $AB = AC$  and  $\angle A = 50^\circ$

Thus,  $\angle B = \angle C$

Now,  $\angle A + \angle B + \angle C = 180^\circ$  (The angle sum property of triangle)

$$50 + 2\angle B = 180^\circ$$

$$2\angle B = 130^\circ$$

$$\angle C = \angle B = 65^\circ$$

b. As per the question,

Let the vertical angle be  $A$  and  $\angle B = \angle C$

Now,  $\angle A + \angle B + \angle C = 180^\circ$  (The angle sum property of triangle)

$$130 + 2\angle B = 180^\circ$$

$$2\angle B = 50^\circ$$

$$\angle C = \angle B = 25^\circ$$

c. We know that, the sum of three altitudes of a triangle  $ABC$  is less than its perimeter.

d. Here,  $ABCD$  is a square and  $EDC$  is an equilateral triangle.

$$\therefore ED = CD = AB = BC = AD = EC$$

In  $\triangle ECB$ ,

$$EC = BC$$

$$\angle C = \angle B = x$$

$$\angle ECD = 60^\circ \text{ and } \angle DCB = 90^\circ$$

$$\angle ECB = 60^\circ + 90^\circ$$

$$= 150^\circ$$

$$\text{Now, } x + x + 150^\circ = 180^\circ$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

$$\therefore \angle EBC = 15^\circ$$

$$\therefore a = r, b = s, c = p, d = q$$

## 28. Question

Fill in the blanks with  $<$  or  $>$ .

(A) (Sum of any two sides of a triangle)..... (the third side)

(B) (Difference of any two sides of a triangle)..... (the third side)

- (C) (Sum of three altitudes of a triangle)..... (sum of its three sides)
- (D) (Sum of any two sides of a triangle)..... (twice the median to the 3<sup>rd</sup> side)
- (E) (Perimeter of a triangle)..... (Sum of its three medians)

**Answer**

- a) Sum of any two sides of a triangle > the third side
- b) Difference of any two sides of a triangle < the third side
- c) Sum of three altitudes of a triangle < sum of its three side
- d) Sum of any two sides of a triangle > twice the median to the 3<sup>rd</sup> side
- e) Perimeter of a triangle > sum of its three medians

**29. Question**

Fill in the blanks:

- (A) Each angle of an equilateral triangle measures.....
- (B) Medians of an equilateral triangle are.....
- (C) In a right triangle the hypotenuse is the .....side.
- (D) Drawing a  $\Delta ABC$  with  $AB=3\text{cm}$ ,  $BC=4\text{cm}$  and  $CA=7\text{cm}$  is.....

**Answer**

- a) Each angle of an equilateral triangle measures  **$60^\circ$**
- b) Medians of an equilateral triangle are **equal**
- c) In a right triangle, the hypotenuse is the **longest** side
- d) Drawing a  $\Delta ABC$  with  $AB = 3\text{cm}$ ,  $BC = 4\text{cm}$  and  $CA = 7\text{cm}$  is **not possible**.

**Formative Assessment (Unit Test)**

**1. Question**

In an equilateral  $\Delta ABC$ , find  $\angle A$ .

**Answer**

We know that,

In any equilateral triangle all the angles are equal

Let the three angles of the triangle  $\angle A$ ,  $\angle B$  and  $\angle C$  be  $x$

$$\therefore x + x + x = 180^\circ$$

$$3x = 180^\circ$$

$$x = \frac{180}{3}$$

$$x = 60$$

Hence,  $\angle A = 60^\circ$

## 2. Question

In a  $\triangle ABC$ , if  $AB=AC$  and  $\angle B=65^\circ$ , find  $\angle A$ .

### Answer

It is given in the question that,

In triangle  $ABC$ ,  $AB = AC$

$$\angle B = 65^\circ$$

As  $ABC$  is an isosceles triangle

$$\therefore \angle C = \angle B$$

$$\angle C = 65^\circ$$

Now, we know that sum of all angles of a triangle is  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 65^\circ + 65^\circ = 180^\circ$$

$$\angle A + 130^\circ = 180^\circ$$

$$\angle A = 180^\circ - 130^\circ$$

$$\angle A = 50^\circ$$

## 3. Question

In a right  $\triangle ABC$ ,  $\angle B=90^\circ$ . Find the longest side.

### Answer

It is given in the question that,

In right triangle  $ABC$ ,

$$\angle B = 90^\circ$$

$$\text{So, } \angle A + \angle C = 90^\circ$$

$$\therefore \angle A, \angle C < \angle B$$

Hence, the side opposite to  $\angle B$  is longest

Thus,  $AC$  is the longest side

## 4. Question

In a  $\triangle ABC$ ,  $\angle B > \angle C$ . Which of  $AC$  and  $AB$  is longer?

### Answer





























