JEE MAIN ONLINE PAPER 2021

Held on February 25, 2021 (Evening)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 For extrinsic semiconductors; when doping level is increased;

(1) Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward.

(2) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.

(3) Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature.

(4) Fermi-level of p and n-type semiconductors will not be affected.

Q.2 In a ferromagnetic material, below the curie temperature, a domain is defined as :

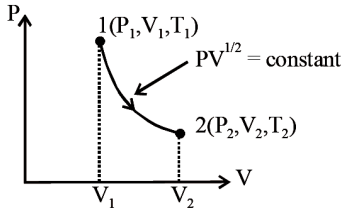
(1) a macroscopic region with zero magnetization.

(2) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.

(3) a macroscopic region with randomly oriented magnetic dipoles.

(4) a macroscopic region with saturation magnetization.

Q.3 Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is :



(1) $\frac{1}{2}$ (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Q.4 A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is :

(1) 35 m (2) 45m (3) 50 m (4) 25m

Q.5 Given below are two statements :

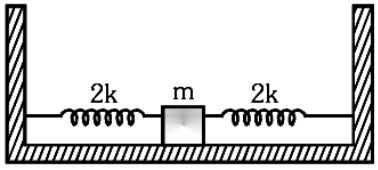
Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Both Statement I and Statement II are true.
(4) Statement I is true but Statement II is false.

Q.6 Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure).When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is :



(1) $2\pi\sqrt{\frac{m}{k}}$ (2) $\pi\sqrt{\frac{m}{2k}}$

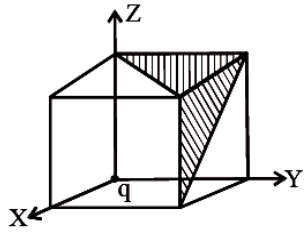
(3) $2\pi\sqrt{\frac{m}{2k}}$ (4) $\pi\sqrt{\frac{m}{k}}$

Q.7 If a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency 'f_c' and radiated through an antenna, the wavelength of the corresponding signal in air is :

(1) $\frac{c}{f_c - f_m}$ (2) $\frac{c}{f_m}$

(3) $\frac{c}{f_c + f_m}$ (4) $\frac{c}{f_c}$

Q.8 A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ through the shaded area is :



(1) $\frac{q}{4\varepsilon_0}$ (2) $\frac{q}{24\varepsilon_0}$

(3) $\frac{q}{48\varepsilon_0}$ (4) $\frac{q}{8\varepsilon_0}$

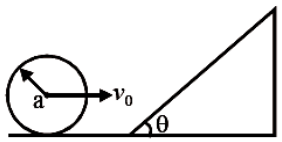
Q.9 The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is :

(1) 194.8 nm (2) 913.3 nm
(3) 490.7 nm (4) 121.8 nm

Q.10 An LCR circuit contains resistance of 110 Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be :

(1) 1A (2) 2.5 A
(3) 1.5 A (4) 2A

Q.11 A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel ?



(1) $\frac{10v_0^2}{7g\sin\theta}$ (2) $\frac{v_0^2}{5g\sin\theta}$

(3) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$ (4) $\frac{v_0^2}{2g\sin\theta}$

Q.12 An electron of mass m_e and a proton of mass $m_p = 1836\ m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be :

(1) 1836 (2) 1

(3) 918 (4) $\frac{1}{1836}$

Q.13 $Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{2\pi}{3}$

Q.14 If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :

(1) $[M^0 L^0 T^0]$ (2) $[L C^{-1}]$

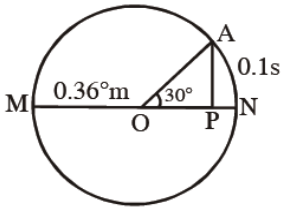
(3) $[M L T^{-1}]$ (4) $[M L T^0]$

Q.15 An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be :

(1) $\frac{\sin^2\beta}{\cos^2\alpha}$ (2) $\frac{\cos^2\beta}{\cos^2\alpha}$

(3) $\frac{\cos\beta}{\cos\alpha}$ (4) $\frac{\cos\beta}{\sin\alpha}$

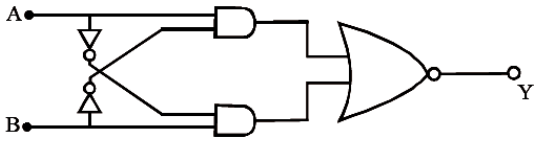
Q.16 The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be :



(1) 100 N (2) 0.49 N

(3) 50 N (4) 9.87 N

Q.17 The truth table for the following logic circuit is :



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Q.18 The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is :

(1) 329 nm (2) 309 nm

(3) 382 nm (4) 400 nm

Q.19 Match List I with List II.

	List I		List II
(a)	Rectifier	(i)	Used either for stepping up or stepping down the a.c. voltage
(b)	Stabilizer	(ii)	Used to convert a.c. voltage into d.c. voltage
(c)	Transformer	(iii)	Used to remove any ripple in the rectified output voltage
(d)	Filter	(iv)	Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below :

(1) (a) – (ii), (b) – (iv), (c) – (i), (d) – (iii)
(2) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)
(3) (a) – (ii), (b) – (i), (c) – (iv), (d) – (iii)
(4) (a) – (ii), (b) – (i), (c) –(iii), (d) – (iv)

Q.20 Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that :

(1) its size decreases, and intensity decreases
(2) its size increases, and intensity increases
(3) its size increases, but intensity decreases
(4) its size decreases, but intensity increases

Section –B

Q.21 The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}}\ \frac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of x is_____.

Q.22 Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21}\times 10^{-8}$C. The value of 'a' will be _____.
[Given g = 10 ms^{-2}]

Q.23 The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be _____.

Q.24 The wavelength of an X-ray beam is 10Å. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}$h kg. The value of x is_____.
(h = Planck's constant)

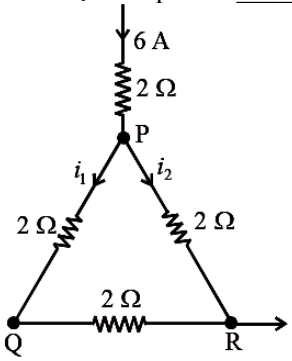
Q.25 A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K, its efficiency is doubled. The temperature in Kelvin of the source will be _____.

Q.26 The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be _____%.

Q.27 If $\vec{P}\times\vec{Q} = \vec{Q}\times\vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is θ ($0° < \theta < 360°$). The value of 'θ' will be _____°.

Q.28 Two identical conducting spheres with negligible volume have 2.1 nC and –0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is_____$\times 10^{-9}$ N.
[Given : $4\pi\varepsilon_0 = \frac{1}{9\times 10^9}$ SI unit]

Q.29 A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The currents i_1 in ampere is _____.



Q.30 Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n : 2. The value of n will be _____.

CHEMISTRY

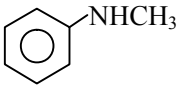
Section -A

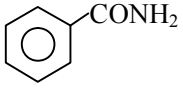
Q.31 Which among the following species has unequal bond lengths ?

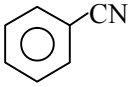
(1) BF_4^- (2) XeF_4

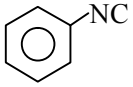
(3) SF_4 (4) SiF_4

Q.32 Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline?

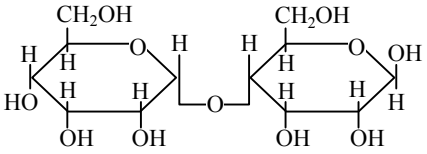
(1) 

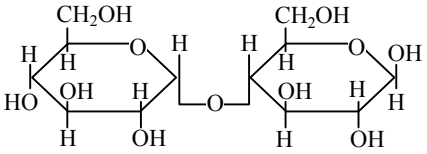
(2) 

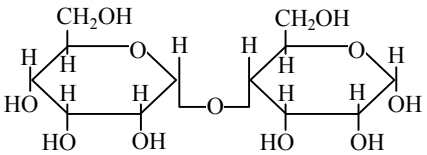
(3) 

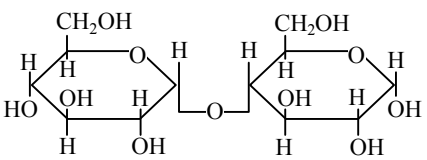
(4) 

Q.33 Which of the following is correct structure of α-anomer of maltose ?

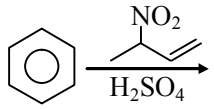
(1) 

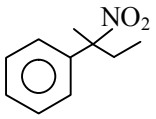
(2) 

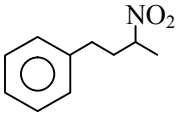
(3) 

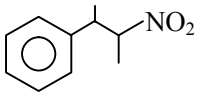
(4) 

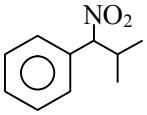
Q.34 The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

Q.35 The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is :

(1) HNO_3/H_2SO_4, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl

(2) $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, HNO_3/H_2SO_4, Zn/HCl

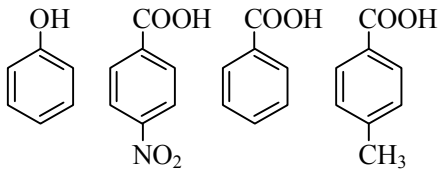
(3) $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, HNO_3/H_2SO_4, Zn/HCl

(4) $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, HNO_3/H_2SO_4

Q.36 Water does not produce CO on reacting with:

(1) CO_2 (2) C (3) CH_4 (4) C_3H_8

Q.37 The correct order of acid character of the following compounds is :

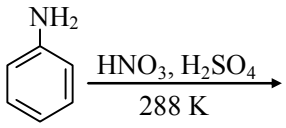


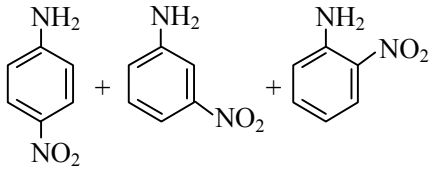
I **II** **III** **IV**

Options:

(1) III > II > I > IV (2) IV > III > II > I

(3) I > II > III > IV (4) II > III > IV > I

Q.38 



(A) (B) (C)

Correct statement about the given chemical reaction is :

••

(1) $-NH_2$ group is *ortho* and *para* directive, so product (B) is not possible.
(2) Reaction is possible and compound (B) will be the major product.
(3) The reaction will form sulphonated product instead of nitration.
(4) Reaction is possible and compound (A) will be major product.

Q.39 The correct order of bond dissociation enthalpy of halogens is :
(1) $Cl_2 > F_2 > Br_2 > I_2$ (2) $I_2 > Br_2 > Cl_2 > F_2$
(3) $Cl_2 > Br_2 > F_2 > I_2$ (4) $F_2 > Cl_2 > Br_2 > I_2$

Q.40 Given below are two statements :
Statement I :
The pH of rain water is normally ~5.6.
Statement II :
If the pH of rain water drops below 5.6, it is called acid rain.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is true but Statement II is false.
(2) Both Statement I and Statement II are false.
(3) Statement I is false but Statement II is true.
(4) Both Statement I and Statement II are true.

Q.41 The major components of German Silver are :
(1) Ge, Cu and Ag (2) Zn, Ni and Ag
(3) Cu, Zn and Ni (4) Cu, Zn and Ag

Q.42 In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment ?
(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$
(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$
(1) (i) > (iii) > (iv) > (ii)
(2) (ii) > (iii) > (i) > (iv)
(3) (iii) > (iv) > (ii) > (i)
(4) (ii) > (i) > (iii) > (iv)

Q.43 Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens?
(1) Nitric acid
(2) Ammonia
(3) Hydrochloric acid
(4) Sodium hydroxide

Q.44 Which one of the following statements is FALSE for hydrophilic sols ?
(1) Their viscosity is of the order of that of H_2O.
(2) The sols cannot be easily coagulated.
(3) They do not require electrolytes for stability.
(4) These sols are reversible in nature.

Q.45 The solubility of $Ca(OH)_2$ in water is :
[Given : The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}]
(1) 1.77×10^{-6} (2) 1.11×10^{-6}
(3) 1.11×10^{-2} (4) 1.77×10^{-2}

Q.46 Given below are two statements :
Statement I :
The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH.
Statement II :
The dimethyl glyoxime is a bidentate neutral ligand.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are true.

Q.47 The major product of the following reaction is:

$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

(1) $CH_3CH_2CH{=}CH{-}CHO$
(2) $CH_3CH_2\underset{\displaystyle CHO}{\underset{|}{C}}{=}CH_2$
(3) $CH_3CH_2CH_2CH_2CHO$
(4) $CH_3CH_2CH_2CHO$

Q.48 The method used for the purification of Indium is :
(1) van Arkel method (2) liquation
(3) zone refining (4) vapour phase refining

Q.49 What is 'X' in the given reaction?

$$\begin{matrix}CH_2OH\\|\\CH_2OH\end{matrix} + \text{oxalic acid} \xrightarrow{210^oC} \underset{\text{(major product)}}{X}$$

(1) $\begin{matrix}CH_2\\\|\\CH_2\end{matrix}$ (2) $\begin{matrix}CH{-}OH\\\|\\CH_2\end{matrix}$
(3) $\begin{matrix}CHO\\|\\CHO\end{matrix}$ (4) $\begin{matrix}CH_2OH\\|\\CHO\end{matrix}$

Q.50 Given below are two statements :

Statement-I : α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement-II : At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true.

(2) Both Statement I and Statement II are true.

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are false.

Section -B

Q.51 If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is _____ molal.
(Rounded-off to the nearest integer)
[$K_b = 0.52K$ kg mol^{-1}]

Q.52 The number of compound/s given below which contain/s –COOH group is _____.

(A) Sulphanilic acid

(B) Picric acid

(C) Aspirin

(D) Ascorbic acid

Q.53 The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol^{-1} is _____ (Rounded-off to the nearest integer)
[$R = 8.314$ J K^{-1} mol^{-1}]

Q.54 Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____ (Integer answer)

(A) Li (B) Na

(C) Rb (D) Cs

Q.55 The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is _____ BM.

Q.56 Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is _____. (Rounded-off to the nearest integer)
[$h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^{8}$ ms^{-1},
$N_A = 6.02 \times 10^{23}$ mol^{-1}]

Q.57 Consider titration of NaOH solution versus 1.25M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 mL (ii) 4.5 mL

(iii) 4.4 mL (iv) 4.4 mL

(v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is _____M. (Rounded-off to the nearest integer)

Q.58 Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is _____ kJ mol^{-1}. (Rounded-off to the nearest integer) [Use $R = 8.314$ J $mol^{-1}K^{-1}$]

Q.59 Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is _____. (Rounded-off to the nearest integer)
[Given, $E^\circ_{Cu^{2+}/Cu} = 0.34$ V, $E^\circ_{NO_3^-/NO} = 0.96$ V,

$E^\circ_{NO_3^-/NO_2} = 0.79$ V and at 298 K,

$\frac{RT}{F}(2.303) = 0.059$]

Q.60 The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is _____. [Molar mass of Cu : 63.54 g ; Avogadro Number = 6.022×10^{23}]

MATHEMATICS

Section -A

Q.61 Let A be a 3×3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to :
(1) 16 (2) 80 (3) 128 (4) 64

Q.62 The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, $x > 0$, is equal to :
(where c is a constant of integration)
(1) $\log_e |x^2 + 5x - 7| + c$
(2) $4\log_e |x^2 + 5x - 7| + c$
(3) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$
(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Q.63 The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :
(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0

Q.64 If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :
(1) –3 (2) –7 (3) 7 (4) 3

Q.65 A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is :
(1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Q.66 If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :
(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$
(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

Q.67 A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length :
(1) $\sqrt{\frac{2}{7}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

Q.68 In a group of 400 people, 160 are smokers and non- vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :
(1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

Q.69 $\operatorname{cosec}\left[2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :
(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

Q.70 If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is :
(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$
(3) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Q.71 The contrapositive of the statement "If you will work, you will earn money" is :
(1) You will earn money, if you will not work
(2) If you will earn money, you will work
(3) If you will not earn money, you will not work
(4) To earn money, you need to work

Q.72 A function f(x) is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots\ldots f\left(\frac{39}{20}\right)$ is equal to :
(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{29}{2}$ (4) $\frac{39}{2}$

Q.73 If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is :

(1) 4 (2) 2 (3) 3 (4) 1

Q.74 The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to :

(1) 2a (2) $2\sqrt{a}$

(3) $a + \frac{1}{a}$ (4) a + 1

Q.75 If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$, then :

(1) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in G.P.

(2) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.

(3) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.

(4) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in A.P.

Q.76 $\lim\limits_{n \to \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + ... + \frac{n}{(2n-1)^2} \right]$ is equal to :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Q.77 Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{2}{9}$ (2) $\frac{122}{297}$

(3) $\frac{97}{297}$ (4) $\frac{1}{5}$

Q.78 Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is :

(1) 2 (2) 1 (3) 4 (4) 3

Q.79 Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then :

(1) y = 273x (2) 2y = 91x

(3) y = 91x (4) 2y = 273x

Q.80 The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has infinitely many solutions

(3) does not have any solution

(4) has a unique solution

Section -B

Q.81 The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.

Q.82 A function f is defined on [–3, 3] as

$$f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \leq x \leq 2 \\ [|x|], & 2 < |x| \leq 3 \end{cases}$$

where [x] denotes the greatest integer ≤ x. The number of points, where f is not differentiable in (–3, 3) is _____.

Q.83 Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____:

Q.84 If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _____.

Q.85 If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

Q.86 A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to _____.

Q.87 If $\lim\limits_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is _____.

Q.88 If the curve, y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, 2x – 3y = 1 and 3x + 2y = 8, then |y(1)| is equal to _____.

Q.89 The value of $\int_{-2}^{2}|3x^2-3x-6|dx$ is _____.

Q.90 A line 'l' passing through origin is perpendicular to the lines

$l_1 : \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$l_2 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then 18(a + b + c) is equal to _____.

JEE MAIN ONLINE PAPER 2021

Held on February 25, 2021 (Evening)

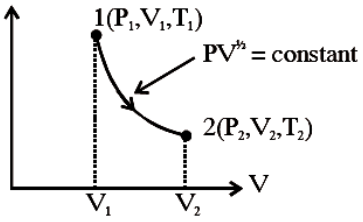
Hints & Solutions

PHYSICS

1.[2] Conceptual

2.[4] Conceptual

3.[3]



$PV^{1/2} = c$

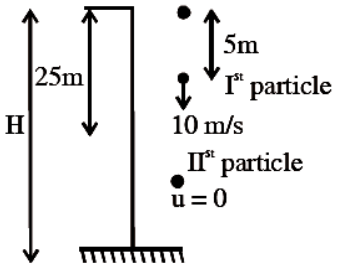
$\frac{nRT}{V}V^{1/2} = c$

$T = c^1 V^{1/2}$

$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1/2} = \left(\frac{2V_1}{V_1}\right)^{1/2}$

$\frac{T_2}{T_1} = \sqrt{2}$

4.[2]



Time for particle to meet = $t' = \frac{S_{rel}}{S_{rel}} = \frac{20}{10} = 2$ sec

Time taken by Ist particle to reach ground = 3sec

$H = \frac{1}{2}g\ (3)^2 = 45$ m

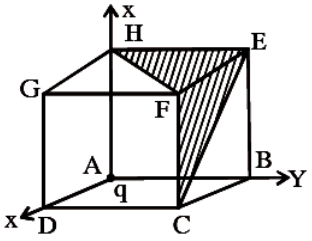
5.[4] Translational degree of freedom = 3
Rotational degree of freedom = 2

6.[4] For parallel combination $k_{eq} = k_1 + k_2$
$k_{eq} = 4k$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

7.[4] $\lambda = \frac{v}{f} = \frac{c}{f_c}$

8.[2]



flux through cube = $\frac{q}{8\in_0}$

flux through surfaces ABEH, ADGH, ABCD will be zero

$\phi(EFGH) = \phi(DCFG) = \phi(EBCF) = \frac{1}{3}\left(\frac{q}{8\in_0}\right)$

$= \frac{q}{24\in_0}$

9.[4] $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda = 121.8$ nm.

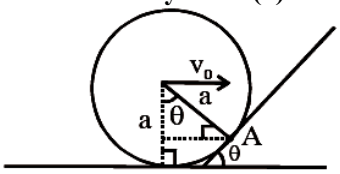
10.[4] $\tan 45^\circ = \frac{1}{\omega CR} = \frac{\omega L}{R} \Rightarrow X_L = X_C$

$\Rightarrow$ resonance

$i = \frac{V}{R} = \frac{220}{110} = 2A$

11.[Bonus]

Official Ans. by NTA (1)



Angular momentum conservation about A

$mv_0\, a\cos\theta + \frac{2}{5}ma^2\omega = mva + \frac{2}{5}ma^2\omega'$

$mv_0 a\left[\frac{2}{5} + \cos\theta\right] = \frac{7}{5}mva$

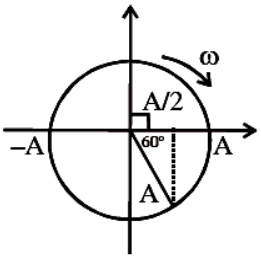
$$v = \frac{5}{7}v_0\left[\frac{2}{5} + \cos\theta\right]$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 = mgh$$

No option Matching

12.[1] $\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}} = 1836$

13.[3]



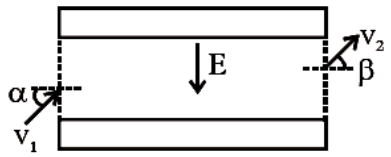
initial phase $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

14.[1] $F = \frac{1}{4\pi\in_0}\frac{e^2}{r^2}$

$E = \frac{hc}{\lambda}$

$$\left[\frac{e^2}{4\pi\varepsilon_0}\times\frac{1}{hc}\right] = \frac{Fr^2}{E\lambda} = \left(M^0L^0T^0\right)$$

15.[2]

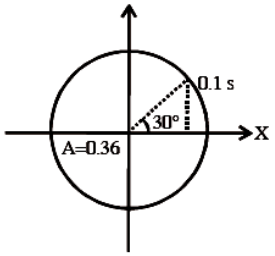


velocity along the plate will not change.

$\therefore v_1\cos\alpha = v_2\cos\beta$

$$\frac{K_1}{K_2} \Rightarrow \frac{v_1^2}{v_2^2} = \frac{\cos^2\beta}{\cos^2\alpha}$$

16.[4]



$30° \rightarrow 0.1$ s

$360° \rightarrow 1.2$ s = T

$\omega = \frac{2\pi}{T} = \frac{5\pi}{3}$

At M, $F = m\omega^2A \Rightarrow \frac{F}{m} = \omega^2A$

17.[2] $y = \overline{(A\bar{B} + \bar{A}B)}$

$y = \overline{A\bar{B}\cdot\bar{A}B}$

$y = (\bar{A} + B)\cdot(A + \bar{B})$

$y = \bar{A}.A + \bar{A}\bar{B} + A.B + \bar{B}\bar{B}$

$y = AB + \bar{A}\bar{B}$

A	B	$Y = AB + \bar{A}\bar{B}$
0	0	1
0	1	0
1	0	0
1	1	1

18.[3] $\frac{hc}{\lambda} = \phi + eV_s$

$\frac{1240}{\lambda} = \phi + 0.71$ … (1)

$\frac{1240}{\lambda} = \phi + 1.43$ … (2)

19.[1] (a) Rectifier → AC to DC

(b) Stabilizer → used for constant output voltage even when input voltage or current change.

(c) Transformer → Step - up or step - down ac voltage.

(d) Filter → used to remove any ripple in the rectified output voltage.

20.[4] $\sin\theta = \frac{m\lambda}{a}$

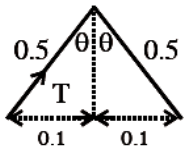
when a increases, θ decreases, width decreases width decreases so intensity will increases

21.[2] $I = \frac{1}{2}c\in_0 E_0^2$

$$\frac{8}{4\pi\times10^2}\times\frac{1}{2} = \frac{1}{4}\times c\times\frac{1}{\mu_0c^2}\times E_0^2$$

$E_0 = \frac{2}{10}\times\sqrt{\frac{\mu_0 c}{\pi}} \Rightarrow x = 2$

22.[20]



$T\cos\theta = mg = 10 \times 10^{-6} \times 10 = 10^{-4}$

$T\sin\theta = \dfrac{9\times10^9\times q^2}{0.04} = F$

$\tan\theta = \dfrac{0.1}{\sqrt{0.24}} = \dfrac{F}{mg}$

$q = \dfrac{2\sqrt{10}}{3\sqrt{\sqrt{24}}}\times10^{-8}$

$0.95 \times 10^{-8} = \dfrac{a}{21}\times10^{-8} \Rightarrow a = 20$

23.[10] $\dfrac{-GMm}{11R} = \dfrac{-GMm}{R} + \dfrac{1}{2}mv^2$

$v = \sqrt{\dfrac{20GM}{11R}}$

24.[10] $\dfrac{hc}{\lambda} = mc^2 \Rightarrow m = \dfrac{h}{c\lambda}$

25.[208] $\eta = \dfrac{1}{4} = 1 - \dfrac{T_2}{T_1}$

$\dfrac{T_2}{T_1} = \dfrac{3}{4}$

$\dfrac{T_2 - 52}{T_1} = \dfrac{1}{2}$

26.[2] $v = \sqrt{\dfrac{T}{\mu}}$

$\dfrac{\Delta V}{V} = \dfrac{1}{2}\dfrac{\Delta T}{T}$

27.[180] $-PQ\sin\theta = PQ\sin\theta \Rightarrow \theta = 180°$

28.[36] $q = \dfrac{(2.1-0.1)}{2}nC = 1nC$

$f = \dfrac{9\times10^9\times10^{-18}}{(0.5)^2} = 36 \times 10^{-9}$

29.[2] For parallel combination current devides in the inverse ratio of resistance.

$i_{PQ} = \dfrac{2}{6}\times 6\ A$

30.[1] $\dfrac{p_1^2}{2\times4} = \dfrac{p_2^2}{2\times16}$

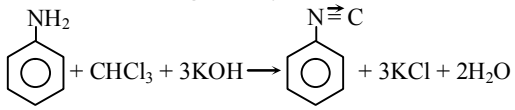
$\dfrac{p_1}{p_2} = \dfrac{1}{2}$

CHEMISTRY

31.[3]

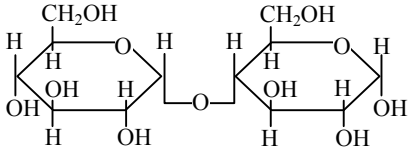
Species	Hybridisation	Bond length
$BF_4^{\ominus}$	sp^3 (Tetrahedral)	All bond lengths equal
XeF_4	sp^3d^2 (sq. planar)	All bond lengths equal
SF_4	sp^3d(see saw)	axial bond length > equitorial length
SiF_4	sp^3 (Tetrahedral)	all bond lengths equal

32.[4] CARBYL amine given by 1° amine

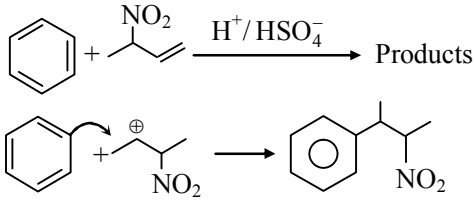


33.[4] α-ANOMER OF MALTOSE

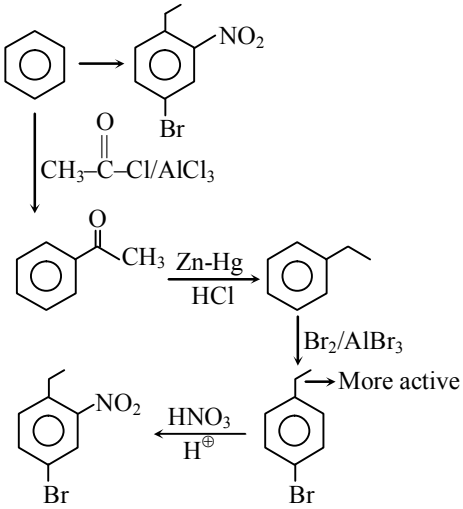
maltose is disaccharides of α-D-glucopyranose by C_1–C_4 glycosidic linkage



34.[3]



35.[4]



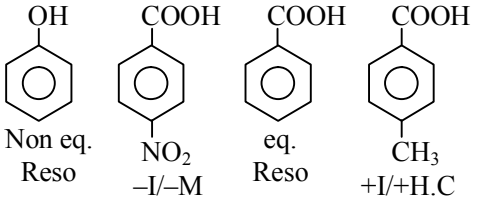
36.[1] $CO_2 + H_2O \rightarrow H_2CO_3$

$C + H_2O(steam) \rightarrow CO + H_2$

$CH_4 + H_2O \rightarrow CO + 3H_2$
$C_3H_8 + H_2O \rightarrow 3CO + H_2$] both reactions are carried out at 1270K temp. with Ni catalyst

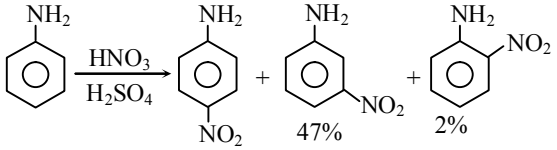
Thus CO_2 does not produce CO.

37.[4]



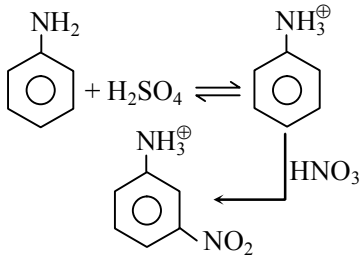
$2 > 3 > 4 > 1$

38.[4]



51%

Due to formation of anilinium ion by acid base reaction m-product is form as considerable amount.



39.[3] Correct order of bond dissociation enthalpy of halogens is $Cl_2 > Br_2 > F_2 > I_2$.

Due to inter electronic repulsions F–F bond becomes weak and easily broken.

40.[4] Both statements are correct.

Normally rain water has pH of 5.6 due to the presence of H^+ ions formed by the reaction of rain water with carbon dioxide present in the atmosphere.

41.[3] Major components of German silver are:

Cu,	Zn,	Ni
(50%)	(30%)	(20%)

42.[1] Complex

(i) $[FeF_6]^{3-}$ $Fe^{3+} \Rightarrow [Ar]3d^5$, F^- is WFL

Fe^{3+} | 1 | 1 | 1 | 1 | 1 |

3d

$n = 5$, $\mu = \sqrt{35}$ B.M.

(ii) $[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+}[Ar]3d^6$, NH_3 is SFL

Co^{3+} | 1↓ | 1↓ | 1↓ | | |

3d

$n = 0$, $\mu = 0$

(iii) $[NiCl_4]^{2-}$ $Ni^{2+} \Rightarrow [Ar]3d^8$, $Cl^- \rightarrow$ WFL

Ni^{2+} | 1↓ | 1↓ | 1↓ | 1 | 1 |

3d

$n = 2$, $\mu = \sqrt{8}$ B.M.

(iv) $[Cu(NH_3)_4]^{2+}$ $Cu^{2+} \Rightarrow [Ar]\ 3d^9$, $NH_3 \rightarrow$ SFL

Cu^{2+} | 1↓ | 1↓ | 1↓ | | | | | | 1 |

3d 4s 4p

$n = 1$, $\mu = \sqrt{3}$ B.M.

Thus correct order of spin only magnetic moment is (i) > (iii) > (iv) > (ii)

43.[1] For testing of halogens, Nitric acid is added to the sodium extract because if CN^- or S^{2-} are present then they will be oxidised and removed before the test of halides.

44.[1]
- → Viscosity of hydrophilic sol > viscosity of H_2O
- → Hydrophilic sol is more stable so can't be easily coagulated.
- → Hydrophilic sols are reversible sols.
- → No electrolytes are required to stabilise hydrophilic sol.

45.[3] $Ca(OH)_2 \rightleftharpoons Ca^{2+}(aq) + 2OH^-(aq)$

s 2s

$k_{sp} = s(2s)^2 \Rightarrow 5.5 \times 10^{-6} = 4S^3$

$\Rightarrow s = \left(\frac{5.5}{4}\right)^{\frac{1}{3}} \times 10^{-2} = 1.11 \times 10^{-2}$

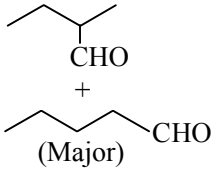
46.[3] Neutral dimethyl glyoxime does not act as ligand.

When Ni^{2+} reacts with dimethyl glyoxime in presence of NH_4OH, it produce dimethyl glyoximate then it form rozy red ppt.

$Ni^{2+}_{(aq)} + 2dmg^- \rightarrow \underset{\text{Rosy red ppt.}}{[Ni(dmg)_2]}$

47.[3] OXO PROCESS (Hydroformylation) :

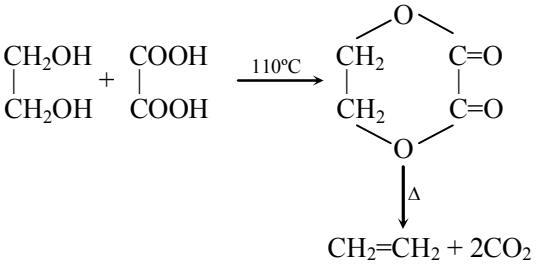
$CH_3–CH_2–CH=CH_2 + CO + H_2 \xrightarrow[\text{catalyst}]{Rh}$



(Major)

48.[3] Zone refining is used for the purification of indium.

49.[1]



50.[3] α-sulphur $\underset{< 369\ K}{\overset{> 369\ K}{\rightleftharpoons}}$ β-sulphur

at room temperature α-sulphur (Rhombic) is most stable form.

51.[3] $\alpha = 0.75$, $n = 2$

$i = 1 - \alpha + n\alpha = 1 - 0.75 + 2 \times 0.75 = 1.75$

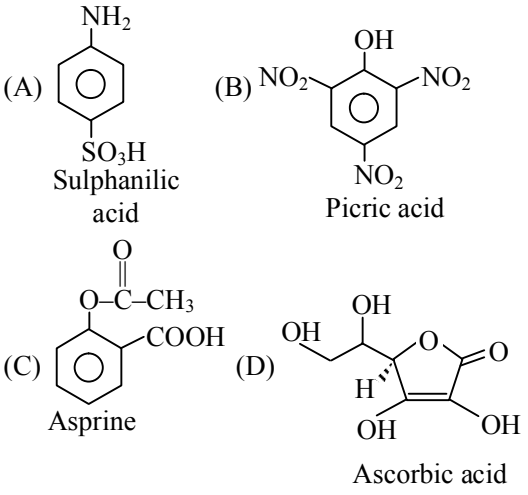
$\Delta T_b = ik_bm$

or, $2.5 = 1.75 \times 0.52 \times m$

or, $m = \dfrac{2.5}{1.75 \times 0.52} = 2.74$

$\therefore$ nearest integer answer will be 3

52.[1] $\longrightarrow$ COOH group present in Asprine



53.[52] $T_1 = 300K$, $T_2 = 325K$, $K_2 = 5K$,

$$\ln\frac{K_2}{K_1} = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

or, $\ln 5 = \dfrac{Ea}{8.314}\left[\dfrac{1}{300} - \dfrac{1}{325}\right]$

or, $Ea = 0.7 \times 2.303 \times 8.314 \times 12 \times 325$

$= 52271\ J = 52.271\ kJ$

Nearest integer answer will be 52 kJ

54.[1] Cs is used as electrodes in the photoelectric cell.

55.[2] $Z = 29$ (Cu)

Cu^{2+} form $[Cu(H_2O)_4]^{2+}$ complex ion with H_2O.

$[Cu(H_2O)_4]^{2+} \Rightarrow Cu^{2+}\ [Ar]3d^9$, $H_2O \rightarrow$ WFL

Cu^{2+} $\boxed{\uparrow\downarrow}\ \boxed{\uparrow\downarrow}\ \boxed{\uparrow\downarrow}\ \boxed{\uparrow\downarrow}\ \boxed{1}$

3d

number of unpaired $e^- = 1$

$\mu = \sqrt{1(1+2)}$ B.M.

$\mu = \sqrt{3}$

$\Rightarrow 1.73$ B.M.

$\Rightarrow$ round off ans.

$\Rightarrow 2$

56.[180]

$$E = \frac{hc}{\lambda} \times \frac{N_A}{1000}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{663 \times 10^{-9} \times 1000}$$

$= = 3 \times 6.02 \times 10\ kJ$

$= 180.6\ kJ$

57.[6] $V_{NaOH} = 4.4$ ml

eq. of NaOH = eq. of $H_2C_2O_4$

or, $M \times 4.4 \times 1 = 1.25 \times 10 \times 2$

or, $M = 5.68$ M

$\therefore$ Nearest integer answer is 6

58.[3] **Official Ans. by NTA (15)**

$n = 5$, $T = 293K$ = const, $\Delta U = 0$,

$P_1 = 2.1$ MPa, $P_2 = 1.3$ MPa

$P_{ex}t = 4.3$ MPa = const.

$$W = -P_{ext}(V_2 - V_1) = -P_{ext}\left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right)$$

or, $W = -P_{ext}\ nRT\left(\dfrac{1}{P_2} - \dfrac{1}{P_1}\right)$

$$= -4.3 \times 5 \times 8.314 \times 293\left(\frac{1}{1.3} - \frac{1}{2.1}\right)$$

$$= -4.3 \times 5 \times 8.314 \times 293\left(\frac{2.1 - 1.3}{1.3 \times 2.1}\right)$$

$= -15347.7J$

or, $W = -15.35$ kJ

$\Delta U^0 = q + W$

$\therefore q = -W$

or, $q = 15.35$ kJ (for 5 moles)

$\therefore$ q/mole $= \dfrac{15.35}{5} = 3\ kJ\ mol^{-1}$

59.[4] Official Ans. by NTA (1)

If the partial pressure of NO and NO_2 gas is taken as 1 bar, then Answer is 4, else the question is bonus.

$NO_3^- + 4H^+ + 3e^- \longrightarrow NO + 2H_2O$

$E^\circ_{NO_3^-/NO} = 0.96$

$NO_3^- + 2H^+ + e^- \longrightarrow NO_2 + H_2O$

$E^\circ_{NO_3^-/NO_2} = 0.79$

Let $[HNO_3] = y \Rightarrow [H^+] = y$ and $[NO_3^-] = y$ for same thermodynamic tendency

or, $E^\circ_{NO_3^-/NO} = E^\circ_{NO_3^-/NO_2}$

or, $E^\circ_{NO_3^-/NO} = -\frac{0.059}{3}\log\frac{P_{NO}}{y \times y^4}$

$= E^\circ_{NO_3^-/NO_2} - \frac{0.059}{1}\log\frac{P_{NO_2}}{y \times y^2}$

or, $0.96 - \frac{0.059}{3}\log\frac{P_{NO}}{y^5} = 0.79 - \frac{0.059}{1}\log\frac{P_{NO_2}}{y^3}$

or, $0.17 = \frac{0.059}{1}\log\frac{P_{NO_2}}{y^3} + \frac{0.0591}{3}\log\frac{P_{NO}}{y^5}$

$0.17 = -\frac{0.059}{1}\log\frac{P_{NO_2}}{y^3} + \frac{0.0591}{3}\log\frac{P_{NO}}{y^5}$

$0.17 = -\frac{0.059}{3}\log\frac{P^3_{NO_2}}{y^9} + \frac{0.0591}{3}\log\frac{P_{NO}}{y^5}$

$0.17 = \frac{0.059}{3}\left[\log\frac{P_{NO}}{y^5} + \log\frac{P^3_{NO_2}}{y^9}\right]$

$0.17 = \frac{0.059}{3}\left[\log\left(\frac{P_{NO}}{y^5} \times \frac{y^9}{P^3_{NO_2}}\right)\right]$

Assume $P_{NO} \simeq P_{NO_2} = 1$ bar

$\frac{0.17 \times 3}{0.059} = \log y^4 = 8.644$

$\log y = \frac{8.644}{4}$

$\log y = 2.161$

$y = 10^{2.16}$

$\therefore 2x = 2 \times 2.161$

$= 4.322$

60.[9077] FCC,

$$d = \frac{Z \times M}{N_A \times a^3} = \frac{4 \times 63.54}{1000 \times 6.022 \times 10^{23} \times (3.596 \times 10^{-10})^3}$$

$= 9076 \text{ kg/m}^3$

MATHEMATICS

61.[4] $|A| = 4$

$\Rightarrow |2A| = 2^3 \times 4 = 32$

$\because$ B is obtained by $R_2 \to 2R_2 + 5R_3$

$\Rightarrow |B| = 2 \times 32 = 64$

option (4)

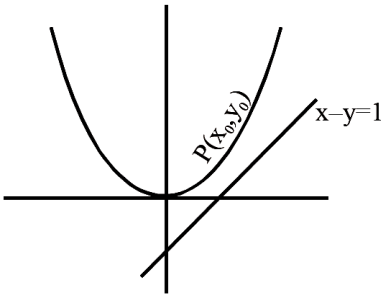
62.[2] $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, $x > 0$

$= \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{4x^2(2x+5)}{x^2(x^2+5x-7)} dx$

$= 4\int \frac{d(x^2+5x-7)}{(x^2+5x-7)} = 4\log_e|x^2 + 5x - 7| + c$

Option (2)

63.[2]



Shortest distance between curves is always along common normal.

$\left.\frac{dy}{dx}\right|_P$ = slope of line = 1

$\Rightarrow x_0 = 1$ $\quad\quad$ $\therefore y_0 = \frac{1}{2}$

$\Rightarrow P\left(1, \frac{1}{2}\right)$

$\therefore$ Shortest distance $= \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1^2+1^2}}\right| = \frac{1}{2\sqrt{2}}$

Option (2)

64.[2] $\because \alpha, \beta \in R \Rightarrow$ other root is $1 + 2i$

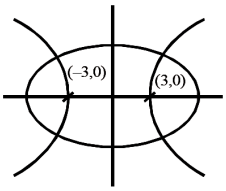
$\alpha = -$(sum of roots) $= -(1 - 2i + 1 + 2i) = -2$

β = product of roots $= (1 - 2i)(1 + 2i) = 5$

$\therefore \alpha - \beta = -7$

Option (2)

65.[2]



For ellipse $e_1 = \sqrt{1-\frac{b^2}{a^2}} = \frac{3}{5}$

For hyperbola $e_2 = \frac{5}{3}$

Let hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\because$ it passes through (3, 0) $\Rightarrow \frac{9}{a_2} = 1$

$\Rightarrow a^2 = 9$

$\Rightarrow b^2 = a^2(e^2 - 1)$

$= 9\left(\frac{25}{9} - 1\right) = 16$

$\therefore$ Hyperbola is

$\frac{x^2}{9} - \frac{y^2}{16} = 1$ … Option (2).

66.[2] $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$

$\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$

$+ \frac{1}{4}.\cos^2\left(\frac{x-y}{2}\right) + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$

and $\cos\left(\frac{x+y}{2}\right) = \frac{1}{2}\cos\left(\frac{x-y}{2}\right)$

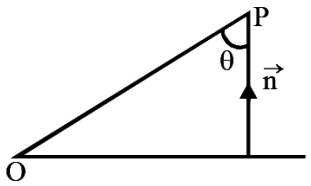
$\Rightarrow x = y$ and $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$

Option (2)

67.[3]



Normal to plane $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k}$

$\cos\theta = \frac{6 \mp 1 + 1}{\sqrt{6}\sqrt{11}} = \frac{8}{\sqrt{66}} \Rightarrow \sin\theta = \sqrt{\frac{2}{66}}$

$\therefore$ Projection of $\overrightarrow{OP}$ on plane = $\left|\overrightarrow{OP}\right| \sin\theta$

$= \sqrt{\frac{2}{11}}$

Option (3)

68.[3] Consider following events

A : Person chosen is a smoker and non vegetarian.

B : Person chosen is a smoker and vegetarian.

C : Person chosen is a non-smoker and vegetarian.

E : Person chosen has a chest disorder

Given

$P(A) = \frac{160}{400}$ $P(B) = \frac{100}{400}$ $P(C) = \frac{140}{400}$

$P\left(\frac{E}{A}\right) = \frac{35}{100}$ $P\left(\frac{E}{B}\right) = \frac{20}{100}$ $P\left(\frac{E}{C}\right) = \frac{10}{100}$

To find

$$P\left(\frac{A}{E}\right) = \frac{P(A)P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{160}{400} \times \frac{35}{100}}{\frac{160}{400} \times \frac{35}{100} + \frac{100}{400} \times \frac{20}{100} + \frac{140}{100} \times \frac{10}{100}}$$

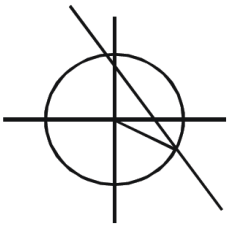
$= \frac{28}{45}$ Option (3)

69.[2] $\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$

$\operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$

$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{56}{33}\right)\right] = \frac{65}{56}$ Option (2)

70.[4]



Homogenising

$x^2 + 2y^2 - 2(x+y)^2 = 0$

$\Rightarrow -x^2 - 4xy = 0 \Rightarrow x^2 + 4xy = 0$

Lines are $x = 0$ and $y = -\frac{x}{4}$

$\therefore$ Angle between lines $\frac{\pi}{2} + \tan^{-1}\frac{1}{4}$

71.[3] Constrapositive of $p \to q$ is $\sim q \to \sim p$

$\Rightarrow$ If you will not earn money, you will not work.

72.[4] $f(x) = \frac{5^x}{5^x+5}$ $\qquad f(2-x) = \frac{5}{5^x+5}$

$f(x) + f(2-x) = 1$

$\Rightarrow f\left(\frac{1}{20}\right) + + f\left(\frac{2}{20}\right) + \ldots + f\left(\frac{39}{20}\right)$

$= \left(f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right) + \ldots + \left(f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right) + f\left(\frac{20}{20}\right)\right)$

$= 19 + \frac{1}{2} = \frac{39}{2}$

73.[4] $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$ $\qquad AA^T = I_2$

$= \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 0$ & $\beta^2 = 1$

$\therefore \alpha^4 + \beta^4 = 1$

74.[2] A.M. $\geq$ G.M.

$f(x) = a^{a^x} + a^{1-a^x} = a^{a^x} + \frac{a}{a^{a^x}} \geq 2\sqrt{a}$

75.[4] $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx = \int_{\pi/4}^{\pi/2} \cot^{n-2} x(\operatorname{cosec}^2 x - 1)\,dx$

$= -\frac{\cot^{n-1} x}{n-1}\Big]_{\pi/4}^{\pi/2} - I_{n-2}$

$= \frac{1}{n-1} - I_{n-2}$

$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$

$\Rightarrow I_2 + I_4 = \frac{1}{3}$

$I_3 + I_5 = \frac{1}{4}$

$I_4 + I_6 = \frac{1}{5}$

$\therefore \frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

76.[1] $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \ldots + \frac{n}{(2n-1)^2}\right]$

$= \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2} = \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{n^2+2nr+r^2}$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{(r/n)^2+2(r/n)+1}$

$= \int_0^1 \frac{dx}{(x+1)^2} = \left[\frac{-1}{(x+1)}\right]_0^1 = \frac{1}{2}$

77.[3] n(s) = n(when 7 appears on thousands place) + n(7 does not appear on thousands place)

$= 9\times9\times9 + 8\times9\times3$

$= 33\times9\times9$

n(E) = n(last digit 7 & 7 appears once) + n(last digit 2 when 7 appears once)

$= 8\times9\times9 + (9\times9 + 8\times9\times2)$

$\therefore P(E) = \frac{8\times9\times9+9\times25}{33\times9\times9} = \frac{97}{297}$

78.[1] $\alpha^2 - 6\alpha - 2 = 0$

$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

Similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a^{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \dfrac{a_{10} - 2a_8}{3a_9} = 2$

79.[2] $x = {}^5C_3 \times 3! = 60$

$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13 = 30 \times 91$

$\therefore 2y = 91x$

80.[4] The following system of linear equations

$2x + 3y + 2z = 9$ ...(1)

$3x + 2y + 2z = 9$ ...(2)

$x - y + 4z = 8$ ...(3)

$(1) - (2) \Rightarrow -x + y = 0 \Rightarrow x - y = 0$

from (3) $4z = 8 \Rightarrow z = 2$

from (1) $2x + 3y = 5$

$\Rightarrow x = y = 1$

$\therefore$ system has unique solution

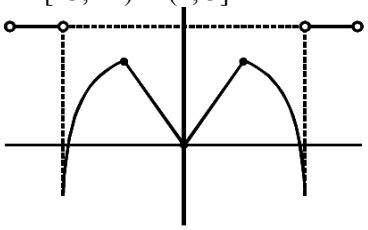
81.[45] for $3^n + 7^n$ to be divisible by 10

n can be any odd number

$\therefore$ Number of odd two digit numbers = 45

82.[5] $f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \le x \le 2 \\ [|x|], & 2 < |x| \le 3 \end{cases}$

$\Rightarrow x \in [-3, -2) \cup (2, 3]$



Number of points of non-differentiability in $(-3, 3) = 5$

83.[2] $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

are of parallelogram $= |\vec{a} \times \vec{b}| = 8\sqrt{3}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = \hat{i}(4\alpha) - \hat{j}(-8) + \hat{k}(-4\alpha)$$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{64 + 32\alpha^2} = 8\sqrt{3}$

$\Rightarrow 2 + \alpha^2 = 6 \Rightarrow \alpha^2 = 4$

$\therefore \vec{a}.\vec{b} = 3 - \alpha^2 + 3 = 2$

84.[1] $x = 4k + 3$

$\therefore (2020 + x)^{2022} = (2020 + 4k + 3)^{2022}$

$= (4(505 + k) + 3)^{2022}$

$= (4\lambda + 3)^{2022} = (16\lambda^2 + 24\lambda + 9)^{1011}$

$= (8(2\lambda^2 + 3\lambda + 1) + 1)^{1011}$

$= (8p + 1)^{1011}$

$\therefore$ Remainder when divided by 8 = 1

85.[4] $x = y^4$ $xy = k$

for intersection $y^5 = k$ ...(1)

Also $x = y^4$

$\Rightarrow 1 = 4y^3 \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4y^3}$

for $xy = k \Rightarrow x = \dfrac{k}{y}$

$\Rightarrow 1 = \dfrac{-k}{y^2}.\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-y^2}{k}$

$\because$ Curve cut orthogonally

$\Rightarrow \dfrac{1}{4y^3} \times \left(\dfrac{-y^2}{k}\right) = -1$

$\therefore y = \dfrac{1}{4k}$

$\therefore$ from (1) $y^5 = k$

$\Rightarrow \dfrac{1}{(4k)^2} = k$

$\Rightarrow 4 = (4k)^6$

86.[9] Let coordinate of point $A(t^2, 2t)$ ($\because a = 1$)

equation of tangent at point A

$yt = x + t^2$

$x - ty + t^2 = 0$

centre of circle (3, 0)

Now PD = radius

$\left|\dfrac{|3 - 0 + t^2|}{\sqrt{1 + t^2}}\right| = 3$

$(3 + t^2)^2 = 9(1 + t^2)$

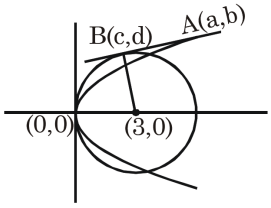
$9 + t^4 + 6t^2 = 9 + 9t^2$

$t = 0, -\sqrt{3}, \sqrt{3}$

So point $A(3, 2\sqrt{3})$

$\Rightarrow a = 3, b = 2\sqrt{3}$

(Since it lies in first quadrant)



For point B which is foot of perpendicular from centre (3, 0) to the tangent $x - \sqrt{3}y + 3 = 0$

$\dfrac{c - 3}{1} = \dfrac{d - 0}{-\sqrt{3}} = \dfrac{-(3 - 0 + 3)}{4}$

$\Rightarrow c = \frac{3}{2} \quad d = \frac{3\sqrt{3}}{2}$

$\Rightarrow 2\left(\frac{3}{2}+3\right) = 9$

87.[5] $\lim\limits_{x\to 0} \frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)} \quad \left(\frac{0}{0}\right)$

$= \lim\limits_{x\to 0} \frac{ax-(e^{4x}-1)}{ax.4x}$ Use $\lim\limits_{x\to 0} \frac{e^{4x}-1}{4x} = 1$

Apply L'Hospital Rule

$= \lim\limits_{x\to 0} \frac{a-4e^{4x}}{8ax} \quad \left(\frac{a-4}{0} \text{ form}\right)$

Limit exists only when $a - 4 = 0 \Rightarrow a = 4$

$= \lim\limits_{x\to 0} \frac{4-4e^{4x}}{32x}$

$= \lim\limits_{x\to 0} \frac{1-e^{4x}}{8x} \quad \left(\frac{0}{0}\right)$

$= \lim\limits_{x\to 0} \frac{-e^{4x}.4}{8} = -\frac{1}{2} \quad \Rightarrow b = -\frac{1}{2}$

$a - 2b = 4 - 2\left(-\frac{1}{2}\right) = 5$

88.[1] $(2xy^2 - y)dx + xdy = 0$

$2xy^2\,dx - y\,dx + x\,dy = 0$

$2x\,dx = \frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$

Now integrate

$x^2 = \frac{x}{y} + c$

Now point of intersection of lines are (2, 1)

$4 = \frac{2}{1} + c \quad \Rightarrow c = 2$

$x^2 = \frac{x}{y} + 2$

Now $y(1) = -1$

$\Rightarrow |y(1)| = 1$

89.[19] $\int_{-2}^{2} 3|x^2 - x - 2|dx$

$= 3\int_{-2}^{2} |x^2 - x - 2|dx$

$= 3\left[\int_{-2}^{-1} (x^2 - x - 2)dx + \int_{-1}^{2} -(x^2 - x - 2)dx\right]$

$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)\Big|_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-1}^{2}\right]$

$= 3\left[7 - \frac{2}{3}\right] = 19$

90.[44] $\ell_1 : \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$\ell_2 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (4 + s)\hat{k}$

DR of $\ell_1 \equiv (1, 2, 2)$

DR of $\ell_2 \equiv (2, 2, 1)$

DR of ℓ (line $\perp$ to ℓ_1 & ℓ_2)

$= (-2, 3, -2)$

$\therefore \ell : \vec{r} = -2\mu\hat{i} + 3\mu\hat{j} - 2\mu\hat{k}$

for intersection of ℓ & $\ell 1$

$3 + t = -2\mu$

$-1 + 2t = 3\mu$

$4 + 2t = -2\mu$

$\Rightarrow t = -1$ & $\lambda = -1$

$\therefore$ Point of intersection $P \equiv (2, -3, 2)$

Let point on ℓ_2 be Q $(3 + 2s, 3 + 2s, 2 + s)$

Given $PQ = \sqrt{17} \quad \Rightarrow (PQ)^2 = 17$

$\Rightarrow (2s + 1)^2 + (6 + 2s)^2 + (s)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0$

$\Rightarrow s = -2, -\frac{10}{9}$

$s \neq -2$ as point lies on 1st octant.

$\therefore a = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$

$b = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$

$c = 2 + \left(-\frac{10}{9}\right) = \frac{8}{9}$

$\therefore 18(a + b + c) = 18\left(\frac{22}{9}\right) = 44$