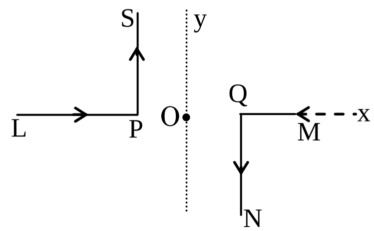
JEE Main Online Exam 2019

Questions & Solution

12th January 2019 | Shift - I

PHYSICS

Q.1 As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) :



(1) 40 A, perpendicular into the page

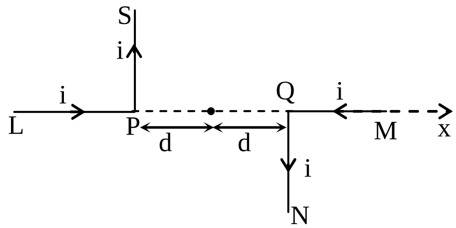
(2) 40 A, perpendicular out of the page

(3) 20 A, perpendicular into the page

(4) 40 A, perpendicular out of the page

Ans. **[3]**

Sol.



$\vec{B}$ due LP & MQ are zero.

$d = 4 \times 10^{-2}$ m.

$B = 2B'$

$$10^{-4} = 2 \times \frac{\mu_0 i}{4\pi d}$$

$$10^{-4} = 2 \times 10^{-7} \frac{i}{4 \times 10^{-2}}$$

$\Rightarrow i = 20$ A

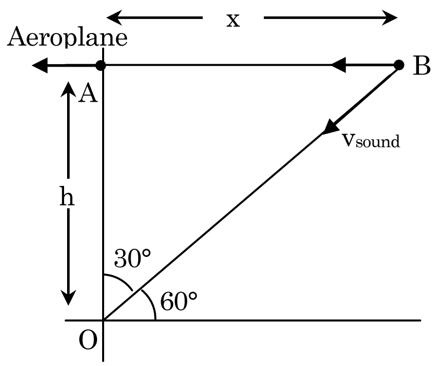
into the page

Q.2 A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is :

(1) $\frac{\sqrt{3}}{2}v$ (2) $\frac{2v}{\sqrt{3}}$ (3) v (4) $\frac{v}{2}$

Ans. [4]

Sol.



$$OB = \sqrt{h^2 + \left(\frac{h}{\sqrt{3}}\right)^2} = \frac{2h}{\sqrt{3}}$$

$$\tan 30° = \frac{x}{h}$$

$$x = \frac{h}{\sqrt{3}}$$

Let it take time from B to A

$\therefore$ AB = x = $v_{plane}t$ & OB = $v_{sound} \cdot t$

$$\Rightarrow t = \frac{x}{v_{plane}} = \frac{OB}{v_{sound}}$$

$$\Rightarrow v_{plane} = v_{sound} \cdot \frac{x}{OB}$$

$$= v \cdot \frac{h/\sqrt{3}}{2h/\sqrt{3}}$$

$$\boxed{v_{plane} = \frac{v}{2}}$$

Q.3 An ideal gas occupies a volume of $2m^3$ at a pressure of 3×10^6 Pa. The energy of the gas is :

(1) 6×10^4 J (2) 9×10^6 J (3) 3×10^2 J (4) 10^8 J

Ans. [2]

Sol. $V = 2m^3$, $P = 3 \times 10^6$ Pa

$$E = \frac{3}{2} PV$$

$$= \frac{3}{2} \times 3 \times 10^6 \times 2$$

$E = 9 \times 10^6$ joule

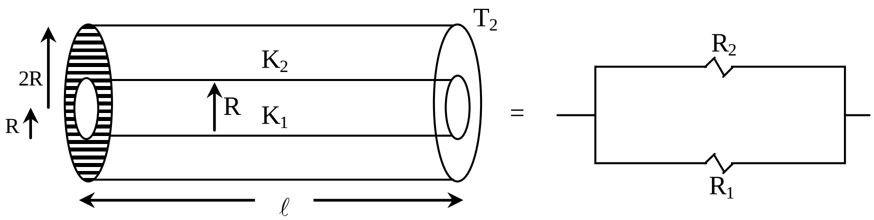
Gas is monatomic

Q.4 A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and the of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is :

(1) $K_1 + K_2$ (2) $\frac{K_1 + 3K_2}{4}$ (3) $\frac{K_1 + K_2}{2}$ (4) $\frac{2K_1 + 3K_2}{5}$

Ans. **[2]**

Sol.



$$\frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eq}4\pi R^2}{\ell} = \frac{K_1\pi R^2}{\ell} + \frac{K_2 \times 3\pi R^2}{\ell}$$

$4\ K_{eq} = K_1 + 3K_2$

$$K_{eq} = \frac{K_1 + 3K_2}{4}$$

Q.5 A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass ' 4 m' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to :

(1) 4.47 (2) 10.00 (3) 14.14 (4) 0.07

Ans. **[3]**

Sol. $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mk}}$

de Broglie wave length $\lambda = \frac{h}{\sqrt{2mq\Delta V}}$

$$\therefore \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{m_B \Delta v_B}{m_A \Delta v_A}} = \sqrt{\frac{4m \times 2500}{m \times 50}} = \sqrt{200} = 10\sqrt{2}$$

$$\frac{\lambda_A}{\lambda_B} = 14.14$$

Q.6 A light wave is incident normally on a glass slab of refractive index 1.5. If 4 % of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propagating in the glass medium will be :

(1) 6 V/m (2) 10 V/m (3) 30 V/m (4) 24 V/m

Ans. **[4]**

Sol. $I_r = 0.96\ I_i$

$$\frac{1}{2}\ \varepsilon_r\varepsilon_0\ E_r^2\ v = 0.96 \times \frac{1}{2}\ \varepsilon_0\, E_i^2\ c.$$

$$\Rightarrow \varepsilon_r\ E_r^2\ \frac{c}{n} = 0.96\ E_i^2 \times c$$

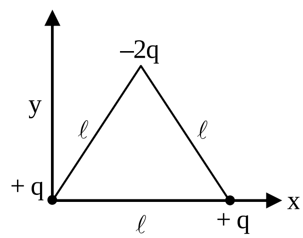
$$\varepsilon_r\ \frac{E_r^2}{\sqrt{\mu_r\varepsilon_r}} = 0.96\ \ E_i^2\ \ [\because \mu_r = 1]$$

$$\Rightarrow E_r = \sqrt{\frac{0.96}{\sqrt{\varepsilon_r}}}\ \ E_i \qquad\qquad n = \sqrt{\mu_r\varepsilon_r}$$

$$E_r = \sqrt{\frac{0.96}{1.5}} \times 30 \qquad\qquad n = \sqrt{\varepsilon_r} = 1.5$$

$E_r = 24$ V/m.

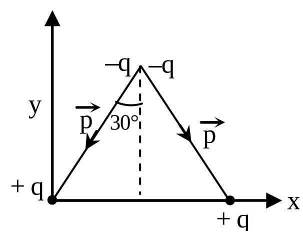
Q.7 Determine the electric dipole moment of the system of the three charges, placed on the vertices of an equilateral triangle, as shown in the figure :



(1) $2q\ell\ \hat{j}$ (2) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (3) $\sqrt{3}\ q\ell\ \frac{\hat{j}-\hat{i}}{\sqrt{2}}$ (4) $-\sqrt{3}\ q\ell\ \hat{j}$

Ans. **[4]**

Sol.



$$\vec{P}_{net} = -\,2p\cos 30°\ \hat{j}$$

$$= -\,2q \times\ \ell \times \frac{\sqrt{3}}{2}\ \hat{j}$$

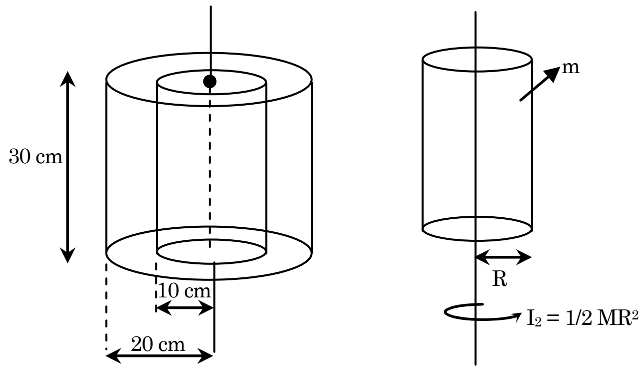
$$\vec{P}_{net} = -\,q\ell\sqrt{3}\ \ \hat{j}$$

Q.8 Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is :

(1) 16 cm (2) 12 cm (3) 14 cm (4) 18 cm

Ans. **[1]**

Sol.



Let density of material is ρ.

$I_1 = \frac{1}{2}[\rho.m(0.2)^2(0.3)](0.2)^2 - \frac{1}{2}[\rho\ \pi(0.1)^2 0.3](0.1)^2$

$\because I_1 = I_2 \Rightarrow \frac{1}{2}\rho\pi\left[(0.2)^4 0.3 - (0.1)^4\cdot 0.3)\right] = \rho(\pi\, 0.2^2 - \pi\, 0.1^2)\, 0.3\, R^2$

$\Rightarrow \left[(0.2)^2 - (0.1)^2\right]\left[0.2^2 + 0.1^2\right] = \left[0.2^2 - 0.1^2\right]2R^2$

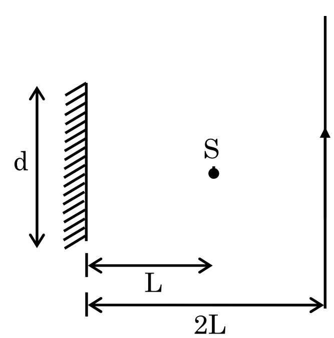
$\Rightarrow 2R = 0.04 + 0.01$

$R^2 = \frac{5}{200}$

$R = \frac{100}{\sqrt{10}}$ cm

$R \approx 16$ cm

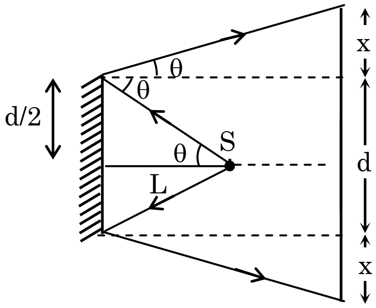
Q.9 A point source of light, S is placed at distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2L as shown below. The distance over which the man can see the image of the light source in the mirror is



(1) d (2) d/2 (3) 3d (4) 2d

Ans. [3]

Sol.



$\tan\theta = \dfrac{d}{2L}$

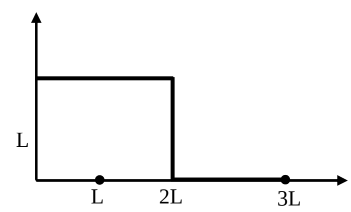
$\tan\theta = \dfrac{d}{2L} = \dfrac{x}{2L}$

$x = d$

The distance = x + d + x

= 3d

Q.10 The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is :

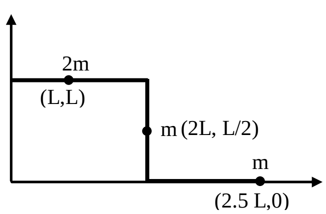


(1) $\vec{r}_{cm} = \dfrac{11}{8}L\,\hat{x} + \dfrac{3}{8}L\,\hat{y}$ (2) $\vec{r}_{cm} = \dfrac{5}{8}L\,\hat{x} + \dfrac{13}{8}L\,\hat{y}$

(3) $\vec{r}_{cm} = \dfrac{13}{8}L\,\hat{x} + \dfrac{5}{8}L\,\hat{y}$ (4) $\vec{r}_{cm} = \dfrac{3}{8}L\,\hat{x} + \dfrac{11}{8}L\,\hat{y}$

Ans. [3]

Sol.



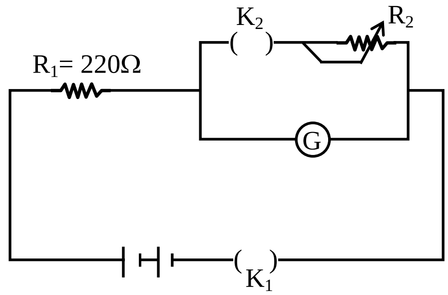
$\therefore\ \vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j}$

$x_{cm} = \dfrac{2m\times L + m\times 2L + m\times 2.5L}{4m} = \dfrac{6.5L}{4} = \dfrac{13L}{8}$

$y_{cm} = \dfrac{2m.L + m\dfrac{L}{2} + m.0}{4m} = \dfrac{2.5}{4}L = \dfrac{5}{8}L$

$\therefore\ \vec{r}_{cm} = \dfrac{13}{8}L\hat{i} + \dfrac{5}{8}L\hat{j}$

Q.11 The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery] :



(1) 5 Ω (2) 25 Ω (3) 12 Ω (4) 22 Ω

Ans. **[4]**

Sol. Let Resistance of Galvanometer is R_g

When K_1 is closed

$I_1 \propto \theta_0$

$\Rightarrow \frac{v}{220+R_g} \propto \theta_0$ ......(1)

When K_1 & K_2 both are closed $I_2 \propto \frac{\theta_0}{5}$

$$\frac{R_2}{R_2+R_g} \times \frac{v}{220+\frac{R_2R_g}{R_2+R_g}} \propto \frac{\theta_0}{5} \qquad .....(2)$$

equation 1/2

$$\frac{v}{(220+R_g)\times\frac{R_2 v}{220(R_2+R_g)+R_2R_g}} = \frac{\theta_0}{\frac{\theta_0}{5}} = 5$$

$$\frac{220(5+R_g)+5R_g}{5(220+R_g)} = 5$$

$\Rightarrow 220 + 45\ R_g = 1100 + 5\ R_g$

$40\ R_g = 880$

$R_g = 22\ \Omega$

Q.12 A travelling harmonic wave is represented by the equation $y(x,t) = 10^{-3}\sin(50t + 2x)$, where, x and y are in mater and t is in seconds. Which of the following is a correct statement about the wave ?

(1) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(2) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

(3) The wave is propagating along the negative x-axis with speed 25 ms^{-1}

(4) The wave is propagating along the negative x-axis with speed 100 ms^{-1}

Ans. **[3]**

Sol. $y(x,t) = 10^{-3} \sin(50t + 2x)$

wave is propagating in – ve x axis

compare with $\omega t + kx$

$\omega = 80$ & $k = 2$

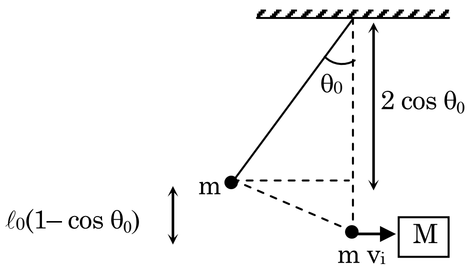
$v = \frac{\omega}{k} = \frac{50}{2} = 25$ m/s

Q.13 A simple pendulum, made of a string of length ℓ and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by :

(1) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (3) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (4) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

Ans. **[3]**

Sol.



$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$ conservation of energy $mgh = \frac{1}{2} mv^2 \Rightarrow v = \sqrt{2gh}$

$v_f = \frac{m - M}{m + M} v_i$

$\sqrt{2g\ell(1 - \cos\theta_1)} = \left|\frac{m - M}{m + M}\right| \sqrt{2g\ell(1 - \cos\theta_0)}$

$-\sin\frac{\theta_1}{2} = \frac{m - M}{m + M} \sin\frac{v_0}{2}$

$-\frac{\theta_1}{\theta_0} = \frac{m - M}{m + M} \Rightarrow \frac{\theta_1}{\theta_0} = \frac{M - m}{M + m}$

$\frac{\theta_1 + \theta_0}{\theta_1 - \theta_0} = \frac{M}{-m}$

$\Rightarrow M = m\left[\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right]$

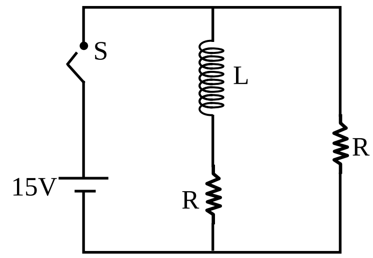
Q.14 The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is :

(1) 500 (2) 100 (3) 200 (4) 50

Ans. **[3]**

Number of division = $\frac{1\text{mm}}{5\mu\text{m}}$ = 200.

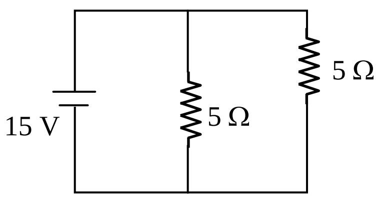
Q.15 In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed ?



(1) 6 A (2) 7.5 A (3) 3 A (4) 5.5 A

Ans. **[1]**

Sol. After long time inductor is short circuited.



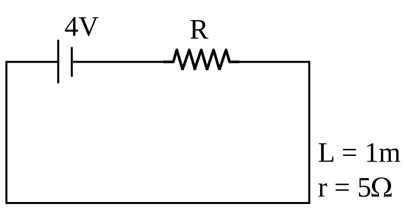
Current through battery I = $\dfrac{15}{\frac{5}{2}}$ = 6A

Q.16 An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is :

(1) 480 Ω (2) 495 Ω (3) 490 Ω (4) 395 Ω

Ans. **[4]**

Sol.



Voltage across potentiometer wire = 5 × 10 mV = 50 mV

$50 \times 10^{-3} = \dfrac{5}{5+R} \times 4$

400 = 5 + R

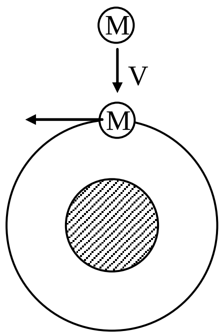
R = 395 Ω

Q.17 A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be :

(1) in the same circular orbit of radius R
(2) such that it escapes to infinity
(3) in a circular orbit of a different radius
(4) in an elliptical orbit

Ans. **[4]**

Sol.



Momentum after collision = $\sqrt{2}\,MV$

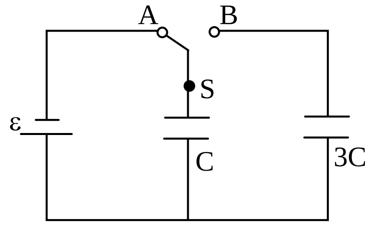
If speed of combined mass is x

$2mx = \sqrt{2}\,MV$

$x = \dfrac{V}{\sqrt{2}}$

Combined mass will move in elliptical orbit.

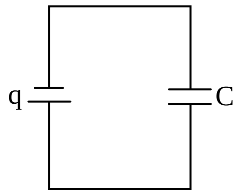
Q.18 In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is :



(1) $\frac{1}{8}\frac{Q^2}{C}$ (2) $\frac{5}{8}\frac{Q^2}{C}$ (3) $\frac{3}{4}\frac{Q^2}{C}$ (4) $\frac{3}{8}\frac{Q^2}{C}$

Ans. **[4]**

Sol.

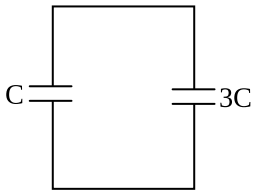


Charge on capacitance

$$Q = C\varepsilon$$

Initially energy = $\dfrac{Q^2}{2C}$

When switch is changed to position B

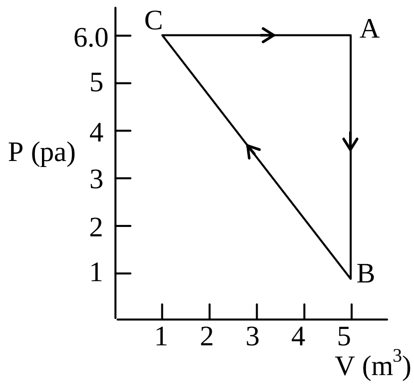


Total energy $E_2 = \dfrac{Q^2}{2(4C)} = \dfrac{Q^2}{8C}$

Change in energy = $E_1 - E_2$

$$\frac{Q^2}{2C} - \frac{Q^2}{8C} = \frac{3Q^2}{8C}$$

Q.19 For the given cyclic process CAB as shown for a gas, the work done is :



(1) 1 J (2) 10 J (3) 5 J (4) 30 J

Ans. **[2]**

Sol. Work done = Area of loop

$$= \frac{1}{2} \times 4 \times 5 = 10 \text{ J}$$

Q.20 A straight rod of length L extends from x = a to x = L + a. The gravitational force it exerts on a point mass 'm' at x = 0, if the mass per unit length of the rod is $A + Bx^2$, is given by :

(1) $Gm\left[A\left(\dfrac{1}{a} - \dfrac{1}{a+L}\right) - BL\right]$

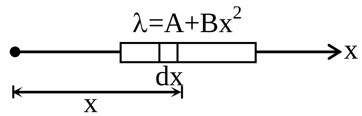
(2) $Gm\left[A\left(\dfrac{1}{a} - \dfrac{1}{a+L}\right) + BL\right]$

(3) $Gm\left[A\left(\dfrac{1}{a+L} - \dfrac{1}{a}\right) + BL\right]$

(4) $Gm\left[A\left(\dfrac{1}{a+L} - \dfrac{1}{a}\right) - BL\right]$

Ans. [2]

Sol.



Force on m due to mass of small element dx is

$$df = \frac{Gm\,d\,dx}{x^2}$$

$$\text{total force} = f = \int \frac{Gm(A + Bx^2)}{x^2}dx$$

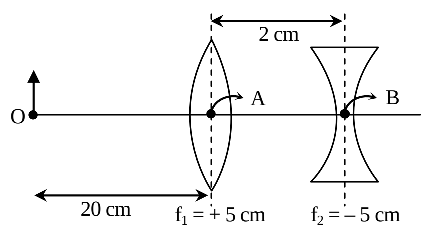
$$f = Gm\int_a^{L+a} \frac{(A + Bx^2)}{x^2}dx$$

$$f = Gm\left[-\frac{A}{x} + Bx\right]_a^{L+a}$$

$$f = Gm\left[\frac{A}{a} - \frac{A}{(L+a)} + B(L + a - a)\right]$$

$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{(L+a)}\right) + BL\right]$$

Q.21 What is the position and nature of image formed by lens combination shown in figure ? (f_1, f_2 are focal lengths)



(1) $\frac{20}{3}$ cm from point B at right, real

(2) 70 cm from point B at right ; real

(3) 40 cm from point B at right ; real

(4) 70 cm from point B at left ; virtual

Ans. [2]

Sol.

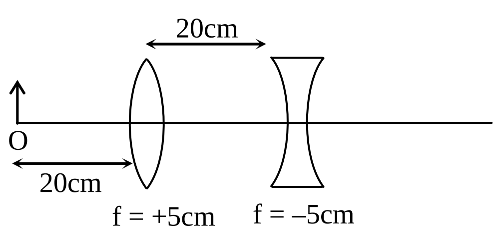


Image formed by convex lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{20} = \frac{1}{5}$$

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20} = \frac{3}{20}$$

Image formed by A is object for lens B

$$u = +\left[\frac{20}{3} - 2\right] = +\left[\frac{14}{3}\right]$$

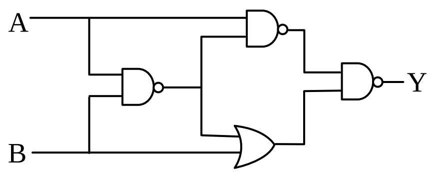
$$\frac{1}{v} - \frac{3}{14} = -\frac{1}{5}$$

$$\frac{1}{v} = -\frac{1}{5} + \frac{3}{14} = \frac{-14+15}{14\times 5}$$

v = 70 cm

70 cm from B at right and real.

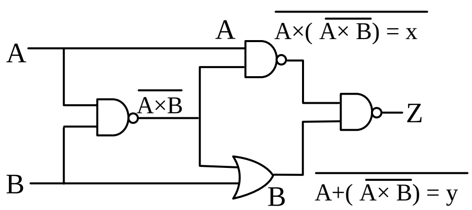
Q.22 The output of the given logic circuit is :



(1) $\overline{A}B$ (2) $AB + \overline{AB}$ (3) $A\overline{B} + \overline{A}B$ (4) $A\overline{B}$

Ans. **[4]**

Sol.



$$y = B + \overline{A} + \overline{B} = \overline{A} + 1 = 1$$

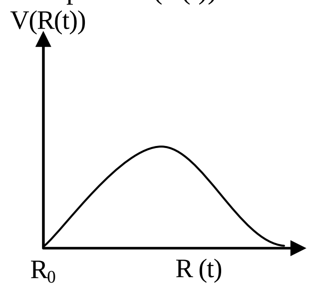
$$x = \overline{A} + \overline{\overline{(A\times B)}} = \overline{A} + (A\times B)$$

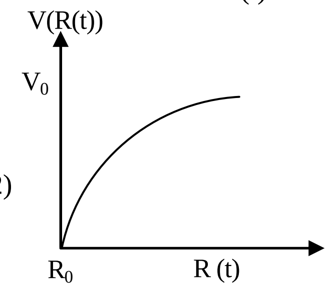
$$z = \overline{(\overline{A} + (A\times B)\times 1)} = A \times \overline{(A\times B)}$$

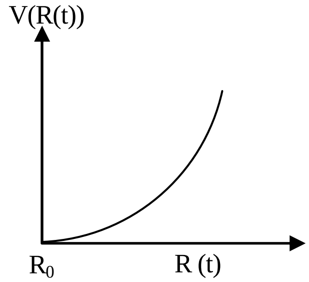
$$= A \times (\overline{A} + \overline{B})$$

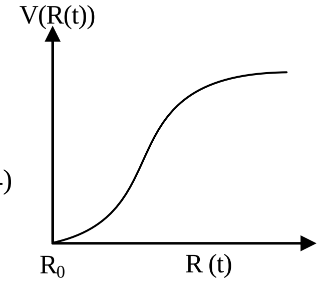
$$z = A \times \overline{A} + A \times \overline{B} = A\overline{B}$$

Q.23 There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V (R(t)) of the distribution as a function of its instantaneous radius R (t) is :

(1)


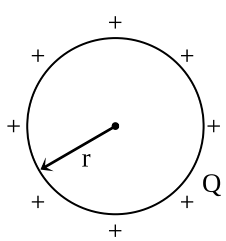
(2)


(3)


(4)


Ans. **[2]**

Sol.



$U = \dfrac{kq}{2r}$

$\therefore$ W.F.T.

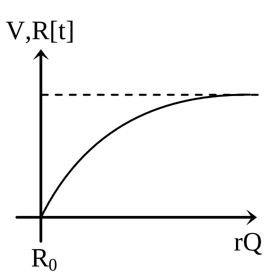
Work done be field = DK = $K_f - \cancel{K_i}$

$-(U_f - V_i) = \dfrac{1}{2}mV^2$

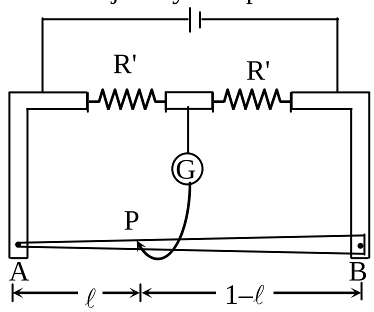
$\dfrac{1}{2}mV^2 = V_1 - V_f = \dfrac{kq}{2R_0} - \dfrac{kq}{2r}$

$\Rightarrow \quad 1v = \sqrt{\dfrac{k_{eq}}{m}\left(\dfrac{1}{R_0} - \dfrac{1}{r}\right)}$

At t = R_0 $V(R_0) = 0$; At t = ∞ $V(\infty) = \sqrt{\dfrac{ka}{mR_0}}$



Q.24 In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP ?



(1) 0.3 m (2) 0.25 m (3) 0.35 m (4) 0.2 m

Ans. **[2]**

Sol. $\frac{dR}{d\ell} = \frac{k}{\sqrt{\ell}}$

$R = k2\sqrt{\ell}$

$\boxed{R = 2k\sqrt{\ell}}$

$\therefore \quad R_{AP} = 2k\sqrt{\ell}$

$\therefore \quad R_{AB} = 2k\sqrt{\ell} = 2k$

$\therefore \quad R_{PB} = 2k\sqrt{\ell} - 2\sqrt{\ell}$

$= 2k(1 - \sqrt{\ell})$

$\therefore \quad \frac{R'}{R_{AP}} = \frac{R'}{R_{PB}}$

$\Rightarrow \quad 2K(1 - \sqrt{2}) = 2k\sqrt{\ell}$

$1 = 2\sqrt{\ell}$

$\Rightarrow \quad l = \frac{1}{4} = 0.025$ m

Q.25 A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index ?

(1) 0.5 (2) 0.6 (3) 0.4 (4) 0.3

Ans. **[2]**

Sol. $A_C = 100$ V

$A_C - A_m = 40$ V

$A_m = 60$ V

$\mu = \frac{A_m}{A_C} = \frac{60}{100} = 0.6$

Q.26 A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be ;

(1) $1 : \sqrt{3}$ (2) 1 : 3 (3) $1 : \sqrt{2}$ (4) 1 : 2

Ans. **[3]**

Sol. Radius of curvature $R = \dfrac{mV}{qB} = \dfrac{\sqrt{2mqV}}{qB}$

$$R \propto \sqrt{\frac{m}{q}}$$

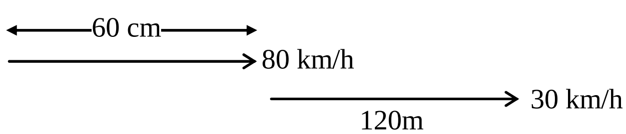
$$\frac{R_P}{R_\alpha} = \sqrt{\frac{m(2q)}{q(4m)}} = 1 : \sqrt{2}$$

Q.27 A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction , and (ii) in the opposite direction is :

(1) $\dfrac{25}{11}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{11}{5}$ (4) $\dfrac{5}{2}$

Ans. **[3]**

Sol. Trains are moving in same direction



relative speed of passenger train = 50 km/h

$$T_1 = \frac{180}{50}$$

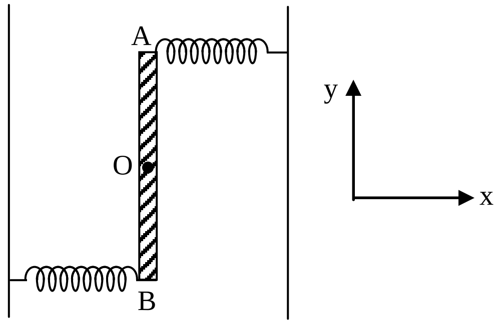
Train are moving in opposite direction

Relative speed of train = 110 km/h

$$T_2 = \frac{180}{110}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{110}{50} = \frac{11}{5}$$

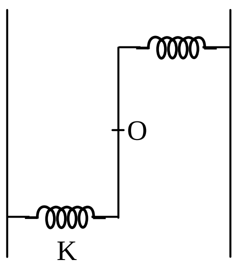
Q.28 Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is :

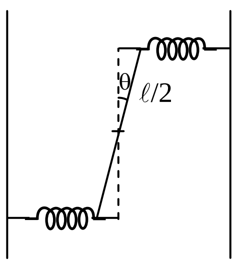


(1) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

Ans. **[2]**

Sol.





Compression | Expansion in spring = $x = \frac{\ell}{2}\theta$

Torque on rod = $(2kx) \times \frac{\ell}{2}$

$z = 2k\left(\frac{\ell}{2}\theta\right)\times\frac{\ell}{2} = \frac{k\ell^2}{2}\theta$

$\frac{k\ell^2}{2} = I\omega^2$

$\frac{k\ell^2}{2} = \frac{1}{12}m\ell^2\omega^2$

$$\omega = \sqrt{\frac{6k}{m}}$$

$$2\pi f = \sqrt{\frac{6k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

Q.29 Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then :

(1) $P_1 = 4W, P_2 = 16$ W

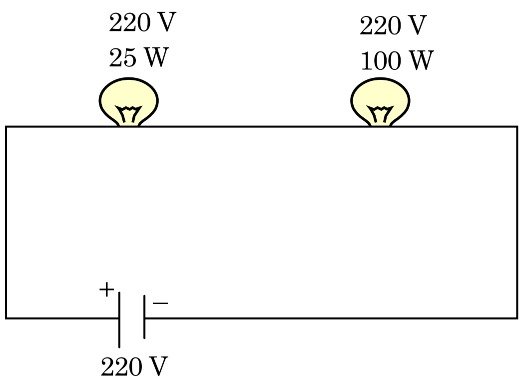
(2) $P_1 = 16W, P_2 = 4$ W

(3) $P_1 = 9W, P_2 = 16$ W

(4) $P_1 = 16W, P_2 = 9$ W

Ans. **[2]**

Sol.



Voltage across 25 W bulb

$$\mathbf{V} = \frac{100}{125} \times 220 = 176 \text{ volt}$$

voltage across 100 W bulb

$$V = \frac{25}{125} \times 220 = 44 \text{ volt}$$

Power dissipation in 25 W bulb

$$P_1 = \frac{(176)^2}{(220)^2} \times 25 = 16 \text{ W}$$

Power dissipation in 100 W bulb

$$P_2 = \left(\frac{44}{220}\right)^2 \times 100 = 4 \text{ W}$$

Q.30 A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr 's quantization conditions are applied, radii of possible orbitls and energy levels vary with quantum number n as :

(1) $r_n \propto \sqrt{n}$, $E_n \propto n$

(2) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$

(3) $r_n \propto n$, $E_n \propto n$

(4) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

Ans. **[1]**

Sol. $U = \frac{1}{2}kr^2$

Centripetal force $F = -\frac{dU}{dr} = -\frac{1}{2}k(2r) = -kr$

$F = kr = \frac{mV^2}{r}$

$mV^2 = kr^2$

$mVr = \frac{nh}{2\pi} \Rightarrow V = \frac{nh}{2\pi(mr)}$

$\frac{m(n^2h^2)}{4\pi^2m^2r^2} = kr^2$

$r^4 \propto n^2 \Rightarrow r \propto \sqrt{n}$

Energy $\propto r^2$

$E \propto r^2$

$E \propto n$

CAREER POINT

JEE Main Online Exam 2019

Question & Solutions

12th January 2019 | Shift - I

CHEMISTRY

Q.1 In the following reaction : Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

Aldehyde	Alcohol
HCHO	tBuOH
CH_3CHO	MeOH

The best combination is

(1) CH_3CHO and MeOH   (2) HCHO and MeOH

(3) CH_3CHO and tBuOH   (4) HCHO and tBuOH

Ans. **[2]**

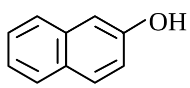
Sol. Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

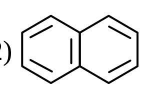
This is nucleophillic addition reaction in which less steric hindrance should be present.

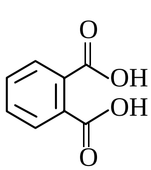
So best combination is

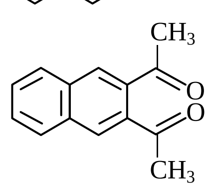
$\underset{\overset{\|}{O}}{H\text{–}C\text{–}H} + CH_3 – OH$

Q.2 Among the following four aromatic compounds, which one will have the lowest melting point ?

(1) 

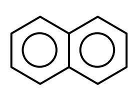
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. Non polar compound have weak vanderwaal attraction force, so their melting point is lowest



Q.3 The element with Z = 120 (not yet discovered) will be an/a -

(1) Alkaline earth metal   (2) Alkali metal

(3) Transition metal   (4) Inner transition metal

Ans. **[1]**

Sol. Z = 120

Its general electronic configuration can be represented as $[Noble]ns^2$

Q.4 What is the work function of the metal if the light of wavelength 4000Å generates photoelectrons of velocity 6×10^5 ms^{-1} from it ?
(Mass of electron = 9×10^{-31} kg; Velocity of light = 3×10^8 ms^{-1}
Plank's constant = 6.626×10^{-34} Js; Charge of electron = 1.6×10^{-19} JeV^{-1})
(1) 4.0 eV (2) 0.9 eV (3) 2.1 eV (4) 3.1 eV

Ans. **[3]**

Sol. $\phi = h\nu - \frac{1}{2}mv^2$

$$\phi = \frac{6.626\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}} - \frac{1}{2}\times9\times10^{-31}(6\times10^{5})^2$$

$\phi = 3.35 \times 10^{-19}$ J

$\phi \approx 2.1$ eV

Q.5 The pair of metal ions that can give a spin only magnetic moment of 3.9 BM for the complex $[M(H_2O)_6]Cl_2$, is -
(1) V^{2+} and Fe^{2+} (2) V^{2+} and Co^{2+} (3) Co^{2+} and Fe^{2+} (4) Cr^{2+} and Mn^{2+}

Ans. **[2]**

Sol. V^{2+} and Co^2

V^{+2} = [Ar] $3d^3$

$= \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\ }\boxed{\ }$

3 unpaired electron

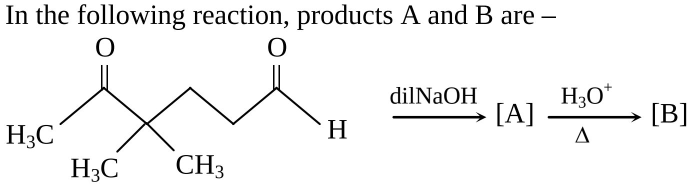
spin only magnetic moment = 3.89 B.M.

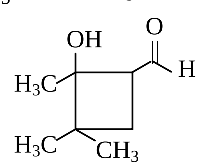
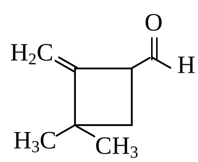
Co^{+2} = [Ar] $3d^7$

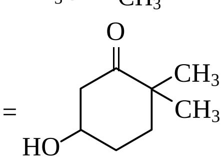
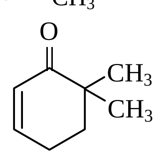
$= \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

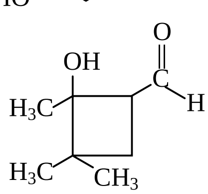
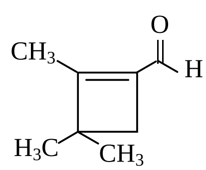
3 unpaired e $\Rightarrow \mu = 3.89$ B.M.

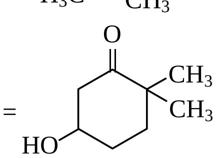
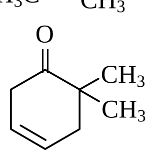
Q.6 In the following reaction, products A and B are –



(1) A =  B = 

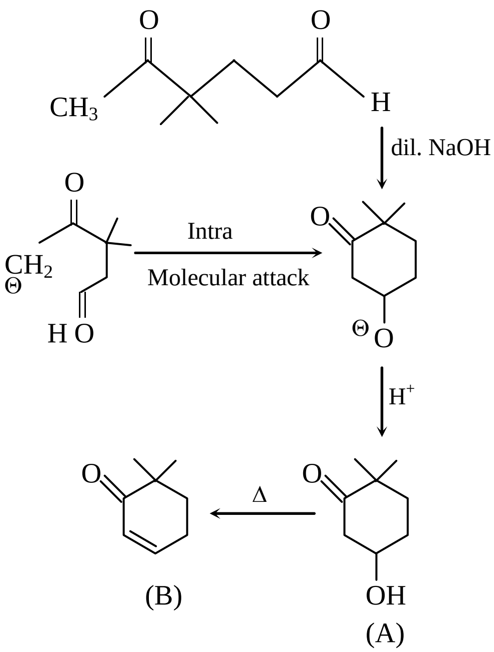
(2) A =  B = 

(3) A =  B = 

(4) A =  B = 

Ans. [2]

Sol.



Q.7 Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is -

(1) 3 (2) 1 (3) 7 (4) 5

Ans. [4]

Sol. $I_2 + 10\ HNO_3 \longrightarrow 2HIO_3 + 10NO_2 + 4H_2O$

Q.8 The correct order for acid strength of compounds : $CH \equiv CH$, $CH_3{-}C \equiv CH$ and $CH_2 = CH_2$ is as follows

(1) $CH \equiv CH > CH_2 = CH_2 > CH_3 - C \equiv CH$ (2) $CH_3 - C \equiv CH > CH_2 = CH_2 > HC \equiv CH$

(3) $HC \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$ (4) $CH_3 - CH \equiv CH > CH \equiv CH > CH_2 = CH_2$

Ans. [3]

Sol. Acidic strength order

$$\underset{sp}{CH{\equiv}CH} > \underset{\underset{+I}{\downarrow}}{CH_3}{-}\underset{sp}{C{\equiv}C{-}H} > \underset{sp^2}{CH_2 = CH_2}$$

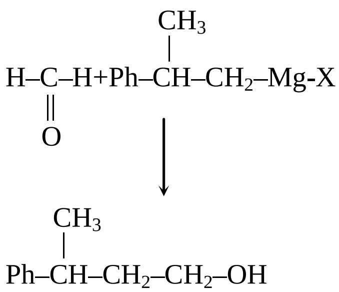
Q.9 $CH_3 - CH_2 - \underset{\underset{Ph}{|}}{\overset{\overset{OH}{|}}{C}} - CH_3$ can not be prepared by -

(1) $CH_3CH_2COCH_3 + PhMgX$ (2) $PhCOCH_3 + CH_3CH_2MgX$

(3) $PhCOCH_2CH_3 + CH_3MgX$ (4) $HCHO + PhCH(CH_3)CH_2MgX$

Ans. [4]

Sol.



Q.10 50 mL of 0.5 M oxalic acid is needed to neutralize 25 mL of sodium hydroxide solution. The amount of NaOH in 50 mL of the given sodium hydroxide solution is -

(1) 20 g (2) 40 g (3) 80 g (4) 10 g

Ans. **[]**

Sol. **[DROP]**

number of equivalent of oxalic acid = number of equivalent of NaOH

$[n_f \times M \times V]_{\text{(Oxalic acid)}} = [n_f \times M \times V]_{\text{(NaOH)}}$

$2 \times 0.5 \times 50 \times 10^{-3} = 1 \times M \times 25 \times 10^{-3}$

$M_{NaOH} = 2\ M$

n_{NaOH} in 50 ml $= 50 \times 10^{-3} \times 2$

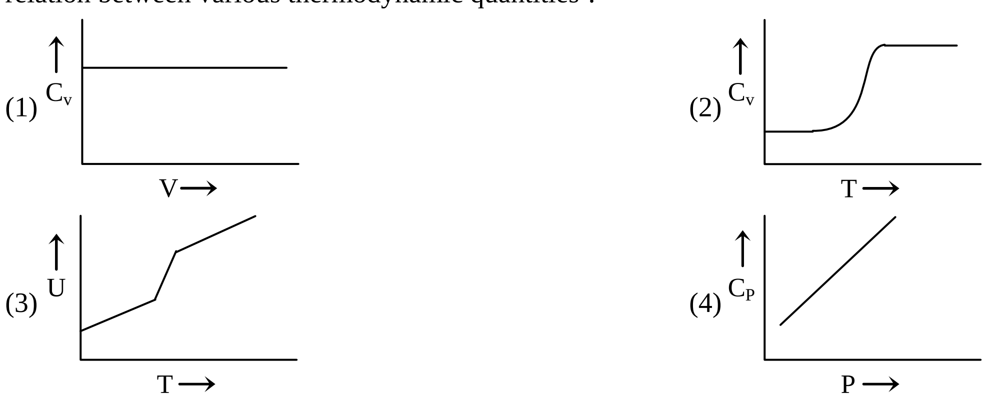
$= 0.1$ moles

$\therefore$ mass of NaOH = 0.1 × 40

= 4 gm

Ans. is not given in options So, bonus

Q.11 For diatomic ideal gas in a closed system, which of the following plots does not correctly describe the relation between various thermodynamic quantities ?



Ans. **[4]**

Sol. $C_P = \frac{7}{2}R$ which is independent of P at high temp.

$\therefore$ graph number 4 is incorrect

Q.12 Given

Gas	H_2	CH_4	CO_2	SO_2
Critical Temperature / K	33	190	304	630

On the basis of data given above, predict which of the following gases shows least adsorption on a definite amount of charcoal ?

(1) SO_2 (2) CH_4 (3) H_2 (4) CO_2

Ans. **[3]**

Sol. Smaller is the value of critical temperature of gas, lesser is the extent of a adsorption. So least adsorbed gas is H_2.

Q.13 Freezing point of a 4% aqueous solution of X is equal to freezing point of 12% aqueous solution of Y. If molecular weight of X is A, then molecular weight of Y is -

(1) 4A (2) 2A (3) 3A (4) A

Ans. **[3]**

Sol. $$K_f \frac{4/M_x}{W_{solvent}} = K_f \frac{12/M_y}{W_{solvent}}$$

$$\frac{4}{A} = \frac{12}{M_y}$$

$M_Y = 3A$

Q.14 A metal on combustion in excess air forms X. X upon hydrolysis with water yields H_2O_2 and O_2 along with another product. The metal is -

(1) Rb (2) Mg (3) Na (4) Li

Ans. **[1]**

Sol. $$Rb + \underset{(excess)}{O_2} \longrightarrow RbO_2$$

$$2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2$$

Q.15 The hardness of a water sample (in terms of equivalents of $CaCO_3$) containing 10^{-3} M $CaSO_4$ is (Molar mass of $CaSO_4$ = 136 g mol^{-1})

(1) 90 ppm (2) 100 ppm (3) 50 ppm (4) 10 ppm

Ans. **[2]**

Sol. ppm of $CaCO_3 = (10^{-3} \times 10^3) \times 100$

= 100 ppm

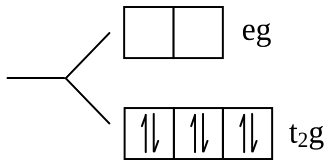
Q.16 The metal d-orbitals that are directly facing the ligands in $K_3[Co(CN)_6]$ are -

(1) $d_{x^2-y^2}$ and d_{z^2} (2) d_{xy}, d_{xz} and d_{yz} (3) d_{xz}, d_{yz} and d_{z^2} (4) d_{xy} and $d_{x^2-y^2}$

Ans. **[1]**

Sol. $K_3[Co(CN)_6]$

$Co^{+3} \longrightarrow [Ar]\ 3d^6$



d^2sp^3 uses $dx^2 - y^2$ and dz^2 orbital

Q.17 Poly β–hydroxybutyrate-co-β-hydroxyvalerate (PHBV) is a copolymer of
(1) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid
(2) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid
(3) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid
(4) 3-hydroxybutanoic acid and 4-hydroxypentanoic acid

Ans. **[3]**

Sol. PHBV is copolymer of 3 hydroxybutanoic acid + 3 hydroxypentanoic acid

Q.18 Decomposition of X exhibits a rate constant of 0.05 μg/year. How many year are required for the decomposition of 5μg of X into 2.5 μg?
(1) 50 (2) 20 (3) 25 (4) 40

Ans. **[1]**

Sol. Rate constant (K) = 0.05 μg/year

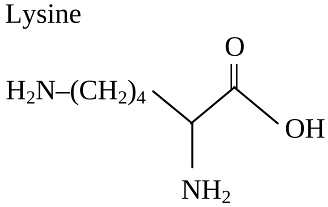
means zero order reaction

$$t_{\frac{1}{2}} = \frac{a_0}{2k} = \frac{5\mu g}{2 \times 0.05 \mu g / year} = 50 \text{ years}$$

Q.19 Among the following compounds most basic amino acid is -
(1) Lysine (2) Asparagine (3) Histidine (4) Serine

Ans. **[1]**

Sol. Lysine



Q.20 The standard electrode potential E^{Θ} and its temperature coefficient $\left(\frac{dE^{\Theta}}{dT}\right)$ for a cell are 2V and -5×10^{-4} VK^{-1} at 300 K respectively. The cell reaction is $Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$
The standard reaction enthalpy (ΔrH^{Θ}) at 300 K in kJ mol^{-1} is, [Use R = 8 JK^{-1} mol^{-1} and F = 96,000 $Cmol^{-1}$]
(1) –412.8 (2) –384.0 (3) 192.0 (4) 206.4

Ans. **[1]**

Sol. $$\Delta H = nfT\left(\frac{\Delta E}{\Delta T}\right)_P - nfE$$

$= 2 \times 96000 \times 300\ (-5 \times 10^{-4}) - 2 \times 96000 \times 2$
$= -412.8$ KJ/mol

Q.21 The molecule that has minimum /no role in the formation of photochemical smog, is -

(1) $CH_2 = O$ (2) O_3 (3) N_2 (4) NO

Ans. **[3]**

Sol. No role of N_2 (Nitrogen) in the formation of photochemical fog

Q.22 Water samples with BOD values of 4 ppm and 18 ppm, respectively are -

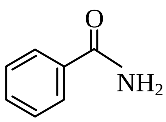
(1) Clean and Highly polluted (2) Highly polluted and Clean

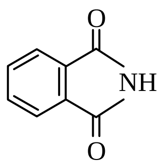
(3) Highly polluted and Highly polluted (4) Clean and Clean

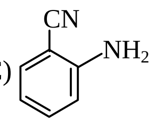
Ans. **[1]**

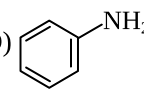
Sol. Clean water would have BOD value less than 5 ppm where as highly polluted water could have a BOD value of 17 ppm or more

Q.23 The increasing order of reactivity of the following compounds towards reaction with alkyl halides directly is-

(A) 

(B) 

(C) 

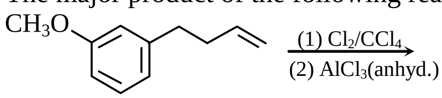
(D) 

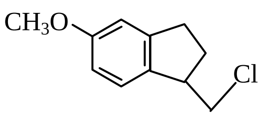
(1) B < A < C < D (2) A < C < D < B (3) A < B < C < D (4) B < A < D < C

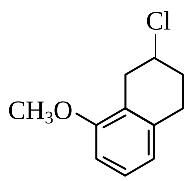
Ans. **[1]**

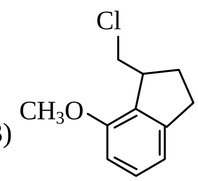
Sol. Nucleophillicity order B < A < C < D

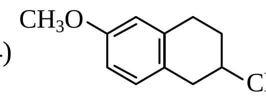
Q.24 The major product of the following reaction is



(1) 

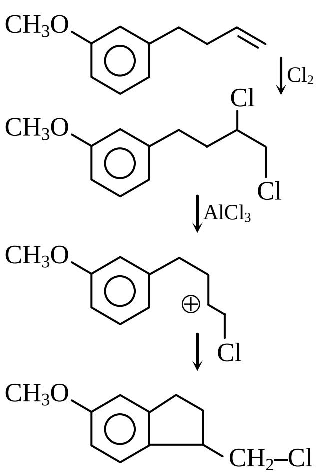
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.25 In a chemical reaction, $A + 2B \underset{}{\overset{K}{\rightleftharpoons}} 2C + D$, the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant (K) for the aforesaid chemical reaction is -

(1) 16 (2) 1 (3) 1/4 (4) 4

Ans. **[4]**

Sol.

	A +	2B	$\rightleftharpoons$	2C	+	D
t = 0	a	1.5a				
t = t	a – x	1.5a – 2x		2x		x

a – x = 1.5 a – 2x

x = 0.5 a

$$\therefore\ K = \frac{(2\times 0.5a)^2 \times 0.5a}{0.5a \times (0.5a)^2}$$

K = 4

Q.26 $Mn_2(CO)_{10}$ is an organometallic compound due to the presence of -

(1) C–O bond (2) Mn – Mn bond (3) Mn – O bond (4) Mn – C bond

Ans. **[4]**

Sol. Compounds having at least one bond between carbon and metal are known as organometallic compound.

Q.27 The volume of gas A is twice than that of gas B. The compressibility factor of gas A is thrice than that of gas B at same temperature. The pressures of the gases for equal number of moles are -

(1) $2P_A = 3P_B$ (2) $3P_A = 2P_B$ (3) $P_A = 3P_B$ (4) $P_A = 2P_B$

Ans. **[1]**

Sol. $V_A = 2V_B$

$Z_A = 3Z_B$

$$Z_A = \frac{P_A V_A}{nRT} \qquad \text{..... (i)}$$

$$Z_B = \frac{P_B V_B}{nRT} \qquad \text{..... (ii)}$$

$$\frac{Z_A}{Z_B} = \frac{P_A \times V_B \times 2}{P_B \times V_B}$$

$$\frac{P_A}{P_B} = \frac{3}{2}$$

$2P_A = 3P_B$

Q.28 Two solids dissociate as follows –

$A(s) \rightleftharpoons B(g) + C(g)\ ;\ k_{p_1} = x\ atm^2$

$D(s) \rightleftharpoons C(g) + E(g)\ ;\ k_{p_2} = y\ atm^2$

The total pressure when both the solids dissociated simultaneously is -

(1) (x + y) atm (2) $(\sqrt{x+y})$ atm (3) $2(\sqrt{x+y})$ atm (4) $x^2 + y^2$ atm

Ans. **[3]**

Sol. $A(s) \rightleftharpoons B(g) + C(g)$; $k_{p_1} = x\ atm^2$

$P_1 \quad P_1 + P_2$

$D(s) \rightleftharpoons C(g) + E(g)$; $k_{p_2} = y\ atm^2$

$P_1 + P_2 \quad P_2$

$P_1(P_1 + P_2) = x$

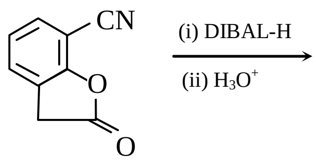
$P_2(P_1 + P_2) = y$

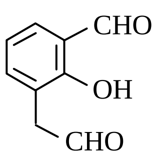
$(P_1 + P_2)^2 = x + y$

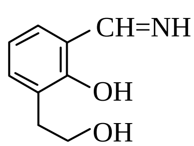
$\therefore P_1 + P_2 = \sqrt{x + y}$

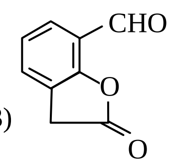
$\therefore$ Total pressure $= 2\,(P_1 + P_2) = 2\sqrt{x+y}$ atm

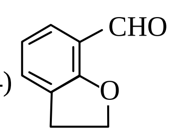
Q.29 The major product of the following reaction is –



(1) 

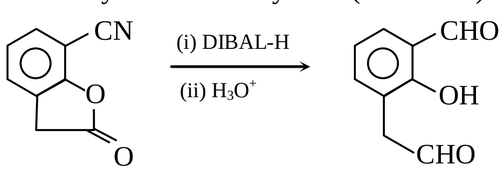
(2) 

(3) 

(4) 

Ans. [1]

Sol. Diisobutyl aluminium hydride (DIBAL-H) Reduces cyanide & ester into aldehyde group



Q.30 In the Hall-Heroult process, aluminum is formed at the cathode. The cathode is made out of -

(1) Copper (2) Platinum (3) Carbon (4) Pure aluminium

Ans. [3]

Sol. In Hall-Heroult process, the cathode is made up of carbon

JEE Main Online Exam 2019

Questions & Solution

12th January 2019 | Shift - I

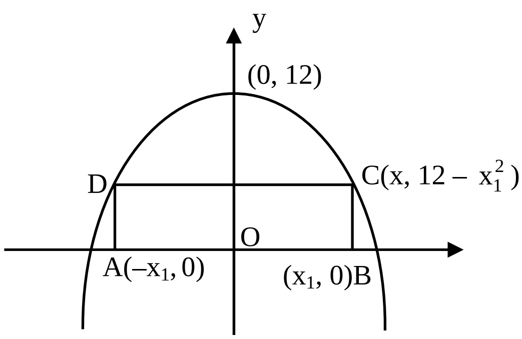
MATHEMATICS

Q.1 The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is :

(1) 36 (2) $20\sqrt{2}$ (3) $18\sqrt{3}$ (4) 32

Ans. **[4]**

Sol.



$AB = 2x_1$

$BC = 12 - x_1^2$

Area $A = 2x_1 (12 - x_1^2)$

Area $A = 24x_1 - 2x_1^3$

for max & min $\frac{dA}{dx} = 0 \Rightarrow x_1 = 2$

$(\text{Area})_{max} = 2(2)(12 - 4) = 32$

Q.2 In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{200}{6^5}$ (2) $\frac{225}{6^5}$ (3) $\frac{150}{6^5}$ (4) $\frac{175}{6^5}$

Ans. **[4]**

Sol. $= \frac{1\times5\times5+5\times1\times5+5\times5\times5}{6^5} = \frac{125+50}{6^5} = \frac{175}{6^5}$

OR

$= \frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1 5^2}{6^3}\right) = \frac{175}{6^5}$

Q.3 Let S = {1, 2, 3, … , 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :

(1) $2^{50} - 1$ (2) $2^{50}(2^{50} - 1)$ (3) $2^{100} - 1$ (4) $2^{50} + 1$

Ans. **[2]**

Sol. Total = 2^{100} (Lucluling nett)
Sunset warn = 2^{50} (Inclines natt)
Near of of nunin
Required = $2^{100} - 2^{50}$
= $2^{50}(2^{50} - 1)$

Q.4 If $\frac{z-\alpha}{z+\alpha}$ ($\alpha \in R$) is a purely imaginary number and | z | = 2, then a value of α is :

(1) $\frac{1}{2}$ (2) $\sqrt{2}$ (3) 2 (4) 1

Ans. **[3]**

Sol. If $\frac{z-\alpha}{z+\alpha}$ is purely imaginary

than $\frac{z-\alpha}{z+\alpha} + \left(\overline{\frac{z-\alpha}{z+\alpha}}\right) = 0$

$\frac{z-\alpha}{z+\alpha} + \left(\frac{\bar{z}-\alpha}{\bar{z}+\alpha}\right) = 0$

$z\bar{z} + z\alpha - \bar{z}\alpha - \alpha^2 + z\bar{z} - \alpha z + \alpha\bar{z} - \alpha^2 = 0$

$2|z|^2 - 2\alpha^2 = 0$

$|z|^2 - \alpha^2$

$y = \alpha^2$

$\alpha = \pm 2$

According to option $\alpha = 2$

Q.5 Let f and g be continuous functions on [0, a] such that f(x) = f(a – x) and g(x) + g(a – x) = 4, then $\int_0^a f(x)\,g(x)\,dx$ is equal to :

(1) $4\int_0^a f(x)dx$ (2) $-3\int_0^a f(x)dx$ (3) $\int_0^a f(x)dx$ (4) $2\int_0^a f(x)dx$

Ans. **[4]**

Sol. $I = \int_0^a f(x)g(x)dx$ ---------- (1)

$I = \int_0^a f(a-x)g(a-x)dx$

$I = \int_0^a f(x)\{4 - g(x)\}dx$

$I = \int_0^a 4f(x)dx - \int_0^a f(x)g(x)dx$

$I = 4\int_0^a f(x)dx - I$

$\Rightarrow 2I = 4\int_0^a f(x)dx$

$I = 2\int_0^a f(x)dx$

Q.6 The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k},\ \hat{i}+\mu\hat{j}+\hat{k},\ \hat{i}+\hat{j}+\mu\hat{k}$ are co-planar, is :

(1) 2 (2) –1 (3) 0 (4) 1

Ans. **[2]**

Sol. $\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$

$\mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1 - \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

at $\mu = 1$, $1 - 3 + 2 = 0$

$$\begin{array}{r|l} & \mu^3 + \mu - 2 \\ (\mu + 1) & \mu^3 - 3\mu + 2 \\ & \mu^3 - \mu^2 \\ & \quad + \\ \hline & \mu^2 - 3\mu + 2 \\ & \mu^2 - \mu \\ & \quad + \\ \hline & -2\mu + 2 \\ & -2\mu + 2 \\ \hline & \times \end{array}$$

$(\mu - 1)(\mu^2 + \mu - 2) = 0$

$(\mu - 1)(\mu - 1)(\mu + 2) = 0$

$\mu = 1, 1 \quad \mu = -2$

Sum of distinct root = 1 + (–2) = –1

Q.7 A ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is :

(1) $1:2(6)^{1/3}$ (2) $1:4(16)^{1/3}$ (3) $2(36)^{1/3}:1$ (4) $4(36)^{1/3}:1$

Ans. **[4]**

Sol. $\dfrac{(T_5)_{start}}{(T_5)_{end}} = \dfrac{{}^{10}C_4(2^{1/3})^4\left(\dfrac{1}{2\cdot 3^{1/3}}\right)^4}{{}^{10}C_4(2^{1/3})^4\left(\dfrac{1}{2\cdot 3^{1/3}}\right)^6}$

$= (2^{1/3})^2 \times (2 \cdot 3^{1/3})^2$

$= 2^{2/3 + 2} \cdot 3^{2/3}$

$= 2^{8/3} \cdot 3^{2/3}$

$= (2^8 \cdot 3^2)^{1/3}$

$= (2^2 \cdot 3^2)^{1/3}$

$= (2^2 \cdot 3^2)^{1/3} \cdot 2^2$

$= (36)^{1/3} \cdot 2^2$

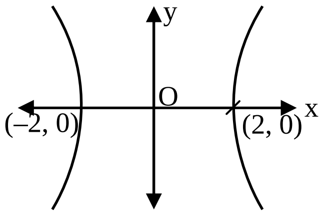
$= (36)^{1/3} \cdot 4 : 1$

Q.8 If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola ?

(1) $\left(6, 5\sqrt{2}\right)$ (2) $\left(2\sqrt{6}, 5\right)$ (3) $\left(-6, 2\sqrt{10}\right)$ (4) $\left(4, \sqrt{15}\right)$

Ans. **[1]**

Sol.



$2a = 4$

$a = 2$

$+ae = +3$

$ae = 3$

$2e = 3$

$e \cdot \frac{3}{2}$

$b^2 = a^2(e^2 - 1)$

$b^2 = 4\left(\frac{a}{4} - 1\right) = 4\left(\frac{5}{4}\right)$

Eq. of hyperbola

$\frac{x^2}{4} - \frac{y^2}{5} = 1$

$(6, 5\sqrt{2})$ does not satisfy the curve.

Q.9 If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :

(1) 31 (2) 50 (3) 51 (4) 30

Ans. **[1]**

Sol. By the give condition

$$\sum_{i=1}^{50} (x_i - 30) = 50$$

$$\sum_{i=1}^{50} x_i = 1500 + 50$$

$$(\bar{x}) = \frac{1550}{50} = 31$$

Q.10 Let $S_k = \frac{1+2+3+...+k}{k}$. If $S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to :

(1) 283 (2) 156 (3) 301 (4) 303

Ans. **[4]**

Sol. $S_k = \frac{k(k+1)}{2k} = \left(\frac{k+1}{2}\right)$

$s_1^2 + s_2^2 + ... + s_{10}^2 = \sum_{k=1}^{10} \frac{1}{4}(k+1)^2$

$= \frac{1}{4}\left\{2^2 + 3^2 + ... + (11)^2\right\}$

$= \frac{1}{4}[(1^2 + 2^2 + ... + 11^2) - 1^2] = \frac{505}{4} = \frac{5}{12}A$

$\Rightarrow A = 303$

Q.11 If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :

(1) (23, 31) (2) (2, 17) (3) [13, 23] (4) [12, 21]

Ans. **[4]**

Sol. Centre of circles are opposite side of line

$(3 + 4 - \lambda) . (27 + 4 - \lambda) < 0$

$(\lambda - 7).(\lambda - 31) < o$

$\lambda \in (7,31)$

distance form S_1

$\left|\frac{3+4-\lambda}{5}\right| \geq 1$

$\lambda \in (-\infty, 2] \cup [12, \infty)$

Distance from S_2

$\left|\frac{27+4-\lambda}{5}\right| \geq 2$

$\lambda \in (-\infty, 21] \cup [41, \infty)$

So, $\lambda \in [12, 21]$ s

Q.12 Let y = y(x) be the solution of the differential equation, $x\frac{dy}{dx} + y = x \log_e x, (x > 1)$. If $2y(2) = \log_e 4 - 1$, then y(e) is equal to :

(1) $-\frac{e}{2}$ (2) $-\frac{e^2}{2}$ (3) $\frac{e^2}{4}$ (4) $\frac{e}{4}$

Ans. **[4]**

Sol. $y - x = \int \ln x \cdot xdx + C$

$y - x = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx + C$

$= \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$xy = \ln x \cdot \frac{x^2}{2} - \frac{x^2}{4} + C$

at x = 2, $y = \frac{\ln 4 - 1}{2}$

$2\frac{(\ln 4 - 1)}{2} = \ln 2 \times \frac{4}{2} - \frac{1}{4} \times 4 + C$

ln 4 – 1 = 2 ln 2 – 1 + C

ln 4 – 1 = ln 4 – 1 + C

$xy = \ln x \cdot \frac{x^2}{2} - \frac{x^2}{4}$

$ey = \frac{e^2}{2} - \frac{e^2}{4}$

$y = \frac{e}{2} - \frac{e}{4}$

$y = \frac{e}{4}$

Q.13 Considering only the principal values of inverse functions, the set $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

(1) contains two elements (2) contains more than two elements

(3) is an empty set (4) is a singleton

Ans. **[4]**

Sol. $\tan\left(\frac{2x + 3x}{1 - 6x^2}\right) = \frac{\pi}{4}$

$\frac{5x}{1 - 6x^2} = 1$

$5x = 1 - 6x^2$ $= 6x^2 + 5x - 1 = 0$

$= 6x^2 + 6x - x - 1 = 0$

6x (x+1) – 1(x + 1) = 0

(x + 1) (6x – 1) = 0

$x = -1, \frac{1}{6}$

$x \geq 0$ hence $x = \frac{1}{6}$

Q.14 The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is :

(1) 36 (2) 28 (3) 32 (4) 24

Ans. **[2]**

Sol. $\frac{a}{r}$, a, ar

$\frac{a}{r} \times a \times ar = 512$

a = 8

$\frac{a}{r}+4$, a + 4, ar

$2(a+4)=\left(\frac{a}{r}+4\right)+ar$

$\frac{2(12)}{24}=\left(\frac{8}{r}+4+8r\right)$

$20=\frac{8}{r}+8r$

$5=\frac{2}{r}+2r$

$5=\frac{2+2r^2}{r}$

$5r = 2 + 2r^2$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r + 2 = 0$

$2r(r-2) - (r-2) = 0$

$r = 2 \qquad r=\frac{1}{2}$

So terms will be 4, 8, 16

Q.15 For x > 1, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to :

(1) $\frac{x\log_e 2x-\log_e 2}{x}$ (2) $\log_e 2x$

(3) $x \log_e 2x$ (4) $\frac{x\log_e 2x+\log_e 2}{x}$

Ans. **[1]**

Sol. X > 1

taking log both side

$2y \ln 2x = \ln 4 + 2x - 2y$

$y = \frac{x+\log 2}{1+\log 2x}$

$y^1=\frac{(1+\ln 2x)-(x+\ln 2)\frac{1}{x}}{(1+\ln 2x)^2}$

$y^1(1+\ln 2x)^2=\left(\frac{x\ln 2x-\ln 2}{x}\right)$

Q.16 The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is :

(1) $6\sqrt{11}$ (2) $\frac{11}{\sqrt{6}}$ (3) 11 (4) $11\sqrt{6}$

Ans. **[2]**

Sol. normal vector of plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$= \hat{i}(35-18) - \hat{j}(21-7) + \hat{k}(12-5)$

$= 7\hat{i} - 14\hat{j} + 7\hat{k}$

$= 7(\hat{i} - 2\hat{j} + \hat{k})$

Now, equation of plane

$7(x+2) - 14(y-2) + 7(z+5) = 0$

$X - 2y + z + 11 = 0$

$\frac{x-2y+z}{\sqrt{6}} = \frac{-11}{\sqrt{6}}$

Perpendicular distance from origin is $\frac{11}{\sqrt{6}}$

Q.17 If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :

(1) $\frac{35}{3}$ (2) -5 (3) $-\frac{35}{3}$ (4) 5

Ans. **[4]**

Sol. $\therefore\ m_1 m_2 = -1$

$\left(\frac{17-\beta}{-8}\right) \times \left(\frac{2}{3}\right) = -1$

$\beta = 5$

Q.18 The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to :

(1) $p \wedge q$ (2) $p \wedge (\sim q)$ (3) $p \vee (\sim q)$ (4) $(\sim p) \wedge (\sim q)$

Ans. **[4]**

Sol. Check by truth table

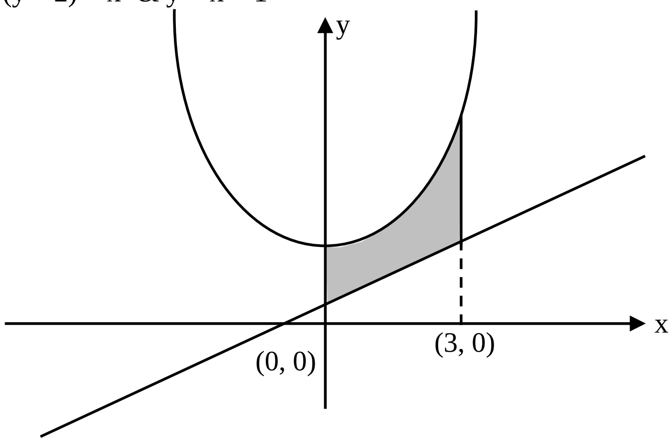
p	$\sim p$	q	$\sim q$	$(p \wedge q)$	$(p \vee \sim q)$	$(-p \wedge \sim q)$	$[(p \wedge q) \vee (p \vee \sim q)]$	$[(p \wedge q) \vee (p \vee \sim q)] \wedge [(\sim p \wedge \sim q)]$
T	F	T	F	T	T	F	T	F
T	F	F	T	F	T	F	T	F
F	T	T	F	F	F	F	F	F
F	T	F	T	F	T	T	T	T

Q.19 The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is

(1) $\frac{15}{4}$ (2) $\frac{15}{2}$ (3) $\frac{21}{2}$ (4) $\frac{17}{4}$

Ans. **[2]**

Sol. $(y - 2) = x^2$ & $y = x + 1$



$$\int_0^3 ((x^2 + 2) - (x + 1)\,dx$$

$$= \int_0^3 (x^2 - x + 1)\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x \Big|_0^3$$

$$= \frac{9}{2} + 3$$

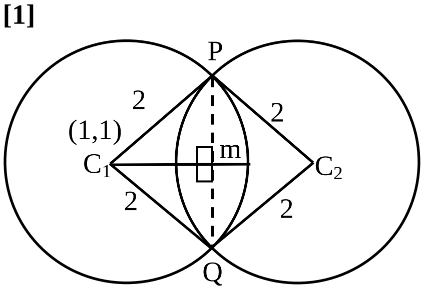
$$= \frac{15}{2}$$

Q.20 Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is:

(1) 4 (2) 6 (3) 9 (4) 8

Ans. **[1]**

Sol.



Equation of PQ

$(x - 1)^2 - (x - 3)^2 + (y - 1)^2 - (y - 3)^2 = 0$

$(x - 1 - x + 3)\ (x - 1 + x - 3) + (y - 1) - y + 3)\ (y - 1 - (r + 3)\ (y - 1 + y - 3)$

$(2x - 4) + (2)\ (2y - 4) = 0$

$2x - 4 + 2y - 4 = 0$

x + y – 4 = 0

$$PM = \frac{|1+1-4|}{\sqrt{2}} = \frac{(-2)}{\sqrt{2}} = \sqrt{2}$$

$$PM = \sqrt{4-2} = \sqrt{2}$$

$$PQ = 2\sqrt{2}$$

Area of = 4

PC_1QC_2

Q.21 $\lim_{x\to\pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is :

(1) $8\sqrt{2}$ (2) 4 (3) $4\sqrt{2}$ (4) 8

Ans. **[4]**

Sol. $$\lim_{x\to\frac{\pi}{4}} \frac{\frac{\cos^3 x}{\sin^3 x} - \frac{\sin x}{\cos x}}{\cos\left(x+\frac{\pi}{4}\right)} = \lim_{x\to\frac{\pi}{4}} \frac{\cos^4 x - \sin^4 x}{\sin^3 x \cos x \cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{(\cos^2 x - \sin^2 x)}{\sin^3 x \cos x \cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\sin^3 x . \cos x . \cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{h\to 0} \frac{\cos 2\left(\frac{\pi}{4}+h\right)}{\left(\frac{1}{\sqrt{2}}\right)^3 . \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{4}+h+\frac{\pi}{4}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+2h\right) \times 2\sqrt{2}.\sqrt{2}}{-\sin h}$$

$$\lim_{h\to 0} \frac{-\sin 2h}{-\sin h \times 2h} \times h \times 2$$

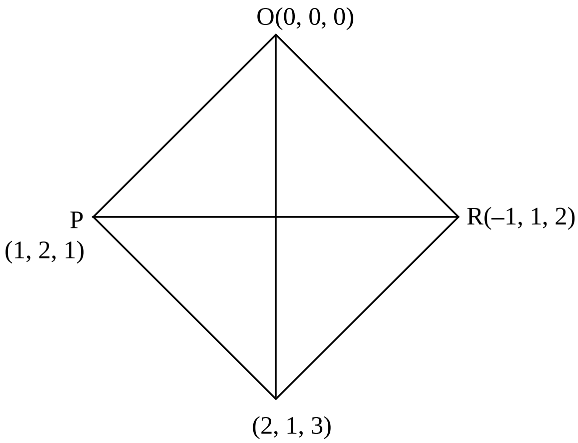
= 8

Q.22 A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1, 1, 2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

(1) $\cos^{-1}\left(\frac{17}{31}\right)$ (2) $\cos^{-1}\left(\frac{9}{35}\right)$ (3) $\cos^{-1}\left(\frac{19}{35}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

Ans. **[3]**

Sol.



$\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{i} - \hat{j} + 2\hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k})$

$= \hat{i} - 5\hat{j} - 3\hat{k}$

$\cos\theta = \dfrac{5+5+9}{(\sqrt{25+9+1})^2} = \dfrac{19}{35}$

Q.23 Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following ?

(1) $\left\{-\dfrac{\pi}{2}, -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{\pi}{2}\right\}$

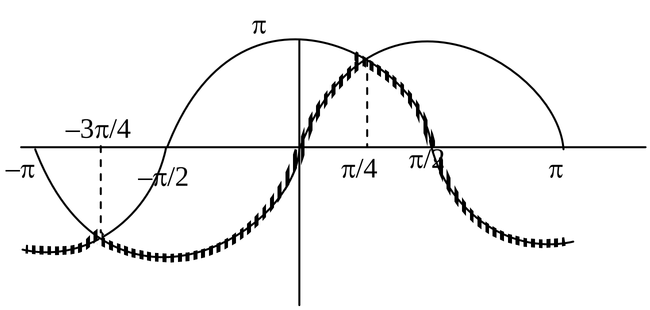
(2) $\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}\right\}$

(3) $\left\{-\dfrac{\pi}{4}, 0, \dfrac{\pi}{4}\right\}$

(4) $\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{\pi}{4}\right\}$

Ans. **[4]**

Sol.



non diff/ at $x = \pi/4$

on $x = -3\pi/4$

Q.24 The integral $\int \cos(\log_e x)\,dx$ is equal to : (where C is a constant of integration)

(1) $\dfrac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

(2) $x\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

(3) $\dfrac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

(4) $x\left[\cos(\log_e x) - \sin(\log_e x)\right] + C$

Ans. [3]

Sol. $I = x\cos(\ln x) + \int x.\sin(\ln x)\frac{dx}{x}$

$I = x\cos(\ln x) + x\sin(\ln x) - \int \cos \ln x\ dx$

$2I = x\cos(\ln x) + \sin(\ln x)$

$I = \frac{x}{2}(\cos(\ln x) + \sin(\ln x)) + c$

Q.25 The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{34}$ (2) $\sqrt{31}$ (3) $\sqrt{19}$ (4) $\frac{\sqrt{79}}{2}$

Ans. [3]

Sol. $f(\theta) = 3\cos\theta + 5\left[\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6}\right]$

$f(\theta) = 3\cos\theta + 5\left[\sin\theta.\frac{\sqrt{3}}{2} - \cos\theta\times\frac{1}{2}\right]$

$f(\theta) = \left(3 - \frac{5}{2}\right)\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$f(\theta) = \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

maximum = $\sqrt{\frac{1}{4} + \frac{25\times3}{4}} = \sqrt{\frac{76}{4}} = \sqrt{19}$

Q.26 An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha)x + \beta y + z = 2$

$\alpha x + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is :

(1) (–3, 1) (2) (1, –3) (3) (–4, 2) (4) (2, 4)

Ans. [4]

Sol. An ordered pair (Qa, β)

$(1 + \alpha)x + \beta y + z = 2$

$\alpha x + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has unique solution if $\Delta \neq 0$

$$\begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$\Rightarrow \alpha + \beta \neq -2$

option (4) is correct.

Q.27 Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\dfrac{q_{21} + q_{31}}{q_{32}}$ is equal to :

(1) 15 (2) 9 (3) 135 (4) 10

Ans. **[4]**

Sol. $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$

$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 3 & 1 \end{bmatrix}$

Similarly $P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$

$a_{21} = 15$, $a_{31} = 135$, $a_{32} = 15$

So, $\dfrac{15 + 135}{15} = 10$

Q.28 If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \dfrac{1}{\lambda} = 1$, is

(1) $-2 + \sqrt{2}$ (2) $4 - 3\sqrt{2}$ (3) $2 - \sqrt{3}$ (4) $4 - 2\sqrt{3}$

Ans. **[2]**

Sol. $3m^2x^2 + m(m - 4)x + 2 = 0$

$\lambda + \dfrac{1}{\lambda} = 1$ (λ is ratio of roots = α/β)

$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 1$

$\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\dfrac{m(m-4)}{3m^2}\right)^2 = \dfrac{3 \times 2}{3m^2}$

$\dfrac{(m-4)^2}{9m^2} = \dfrac{6}{3m}$

$(m - 4)^2 = 18$

$m = 4 \pm \sqrt{8}$

$= 4 \pm 3\sqrt{2}$

Q.29 Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2 = 4x$ and let x be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{625}{4}$ (2) $\frac{125}{4}$ (3) $\frac{75}{2}$ (4) $\frac{125}{2}$

Ans. **[2]**

Sol. $(\text{Area}) = \frac{1}{2}\begin{vmatrix} 4 & -4 & 1 \\ 9 & 6 & 1 \\ t^2 & 2t & 1 \end{vmatrix}$

$= \frac{1}{2}[4(6 - 2t) + 4(a - t^2) + 1(18t - 6t^2)]$

$A = \frac{1}{2}[24 - 8t + 36 - 4t^2 + 18t - 6t^2]$

$= \frac{1}{2}[-10t^2 + 10t + 60]$

$= [-5t^2 + 5t + 30]$

for maximum & minimum $\frac{dA}{dt} = -10t + 5 = 0$

$10t = 5$

$t = \frac{1}{2}$

$A = \left|\left(-5\times\frac{1}{4}+\frac{5}{2}+30\right)\right|$

$= \left|\left(-\frac{5}{4}+\frac{5}{2}+30\right)\right|$

$= \frac{5}{4} + 30 = \left(\frac{125}{4}\right)$

Q.30 Consider three boxes, each containing, 10 balls labelled 1, 2, … , 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 164 (2) 240 (3) 82 (4) 120

Ans. **[4]**

Sol. Total case

(1 + 2 + ...+ 8) + (1 + 2 +....+ 7) + (1 + 2 +... 6) + (1 + 2 + ... + 5) + (1 + 2 +...+ 4) + (1 + 2 + 3) + (1 + 2) + 1

= 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1

= 120