CAREER POINT

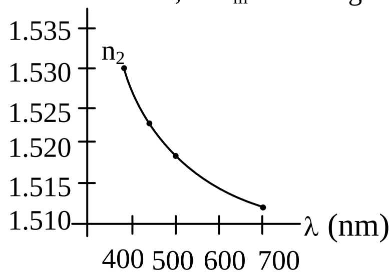
JEE Main Online Exam 2019

Questions & Solutions

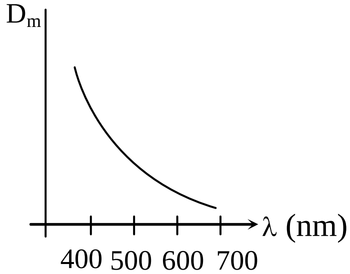
11th January 2019 | Shift - I

PHYSICS

Q.1 The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



(1)



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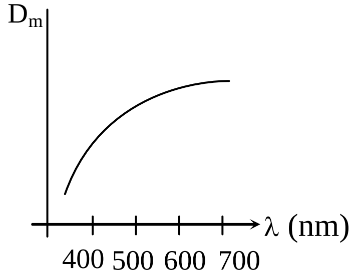

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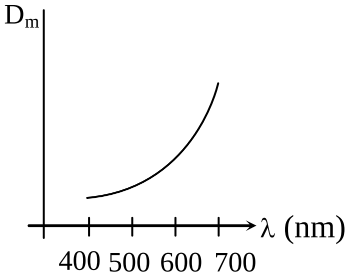

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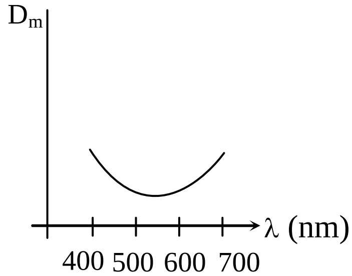


Ans. **[1]**

Sol. $D_m \Rightarrow (\mu - 1) A$

as λ increases μ decreases and hence deviation (D_m) also decreases

Q.2 A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$.

The ratio of kinetic to potential energy of this particle at t = 210 s will be:

(1) $\frac{1}{9}$ (2) 3 (3) 2 (4) 1

Ans. **[Bonus]**

Sol. P.E $\Rightarrow \frac{1}{2}$ $KA^2 \sin^2 \omega t$

K.E = $\frac{1}{2}mv^2 = \frac{1}{2}m(A\omega \cos\omega t)^2$

$\Rightarrow \frac{1}{2}$ $KA^2 \cos^2 \omega t$

$\frac{K.E}{P.E} = \cot^2 \omega t = \cot^2\left[\frac{\pi}{90} \times 210\right]$

$\Rightarrow \cot^2\left(\frac{7\pi}{3}\right)$

$\Rightarrow \frac{1}{3}$

Q.3 A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. considering only translational and rotational modes, the total internal energy of the system is :

(1) 12 RT (2) 20 RT (3) 4 RT (4) 15 RT

Ans. **[4]**

Sol. $U_{mix} \Rightarrow U_1 + U_2 = \frac{5}{2} \times 3RT + \frac{3}{2} \times 5RT$

$\Rightarrow$ 15 RT

Q.4 Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = 4.2J/g/°C Specific heat of Ice = 2.1J/g/°C Heat of fusion of water at 0°C= 334J/g)

(1) 100 g (2) 60 g (3) 50 g (4) 40 g

Ans. **[4]**

Sol. Let m mass ice added to water

$m \times (2.1) \times 20 + (m - 20) \times 334 = 50 \times 4.2 \times 40$

$42m + 334m - 6680 = 210 \times 40 = 8400$

m = 40.1

Q.5 In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to :

(1) 0.94 (2) 0.85 (3) 0.74 (4) 0.80

Ans. **[2]**

Sol. Let intensity of each waves be I

The intensity at the centre of bright fringe is 4I

$\Delta x = \frac{\lambda}{8}$

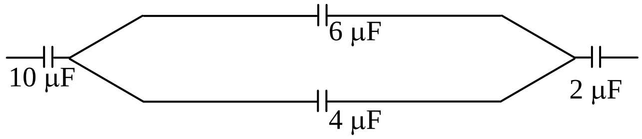
$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi = \frac{\frac{\lambda}{8}}{\lambda} \times 2\pi = \frac{\pi}{4}$

$I^1 \Rightarrow I + I + 2I \cos\left(\frac{\pi}{4}\right)$

$\Rightarrow 2I + \sqrt{2}I \Rightarrow 3.41\ I$

Ratio $\Rightarrow \frac{I'}{4I} = \frac{3.41}{4} = 0.85$

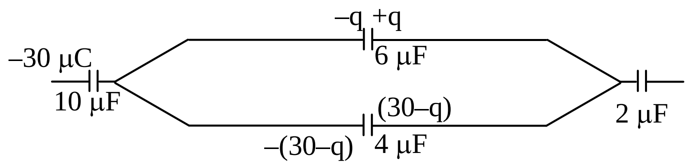
Q.6 In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30μC. The charge on the right plate of the 6 μF capacitor is :



(1) + 12 μC (2) + 18 μC (3) – 18 μC (4) – 12 μC

Ans. **[2]**

Sol.



Let charge on right plate of 6μF capacitor be q

$Q = CV ; V = \frac{Q}{C}$

$\frac{q}{6} = \frac{(30-q)}{4}$

$4q = 180 - 6q$

$10\ q = 180$

$q = 18$

NTA has given answer as option (3) but it should be option (2)

Q.7 A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is :

(1) $\frac{5}{3}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{2}{3}$

Ans. **[2]**

Sol. $TV^{\gamma-1}$ = constant (for adiabatic process)

$\gamma - 1 = x$

γ for diatomic = $\frac{7}{5}$s

so $\frac{7}{5} - 1 = x = \left(\frac{2}{5}\right)$

Q.8 A satellite is revolving in a circular orbit at a height h form the earth surface, such that h<<R where R is the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is :

(1) $\sqrt{gR}(\sqrt{2}-1)$ (2) $\sqrt{2gR}$ (3) $\sqrt{gR}$ (4) $\frac{\sqrt{gR}}{2}$

Ans. **[1]**

Sol. $\frac{mv^2}{R} = \frac{GmM}{R^2}$

$v = \sqrt{\frac{GM}{R}}$

$KE_i = \frac{1}{2}mv^2 = \frac{GmM}{2R}$

$P.E_i \Rightarrow -\frac{GMm}{2R}$

To escape $\frac{1}{2}mv'^2 + \left(\frac{-GMm}{R}\right) = 0$

$v' = \sqrt{\frac{2GM}{R}}$

Change in velocity $\Rightarrow v' - v$

$\Rightarrow \sqrt{\frac{2Gm}{R}} - \sqrt{\frac{GM}{R}}$

$\Rightarrow \sqrt{\frac{GM}{R}}\left(\sqrt{2}-1\right)$

$\Rightarrow \sqrt{gR}\left(\sqrt{2}-1\right)$

Q.9 In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron = 1.6×10^{-19} C Mass of the electron = 9.1×10^{-31} kg]

(1) 7.5×10^{-4} m (2) 7.5×10^{-3} m

(3) 7.5 m (4) 7.5×10^{-2} m

Ans. **[1]**

Sol. $r = \frac{mv}{qB} \Rightarrow \frac{\sqrt{2mK}}{qB}$

$\Rightarrow \frac{\sqrt{2mqV}}{qB}$

$\Rightarrow \frac{1}{B}\sqrt{\frac{2mqV}{q}}$

$\Rightarrow \frac{1}{100\times10^{-3}}\sqrt{\frac{2\times9.1\times10^{-31}\times500}{1.6\times10^{-19}}}$

$\Rightarrow 7.5 \times 10^{-4}$

Q.10 A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980Å. The radius of the atom in the excited state, in terms of Bohr radius a_0 will be : (hc = 12500 eVÅ)

(1) $4a_0$ (2) $9a_0$ (3) $25a_0$ (4) $16a_0$

Ans. [4]

Sol. $r = a_0 \frac{n^2}{z}$

$\Delta E = \frac{hc}{\lambda} = \frac{12500}{980} = 12.75$

$n_f = 4$

so $r = a_0 \frac{n^2}{z} \Rightarrow 16a_0$

Q.11 An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)] \sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given $\pi = 22/7$]

(1) 892.5 and 857.5 (2) 89.25 and 85.75 (3) 1785 and 1715 (4) 178.5 and 171.5

Ans. [2]

Sol. $V(t) = 10 [1 + 0.3 \cos (2.2 \times 10^4 t) \sin(5.5 \times 10^5 t)]$

$V(t) = 10 + 1.5[\sin(572 \times 10^3 t) + \sin(528 \times 10^3 t)]$

we get, $\omega_L + \omega_C = 572 \times 10^3 = 2\pi f_1$

$f_1 = \frac{572 \times 10^3}{2\pi} = 91$ KHz

$\omega_L - \omega_C = 528 \times 10^3 = 2\pi f_2$

$f_2 = \frac{572 \times 10^3}{2\pi} = 84$ KHz

Q.12 An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be :

(1) 1.16×10^{-3} m/s towards the lens (2) 2.26×10^{-3} m/s away from the lens

(3) 3.22×10^{-3} m/s towards the lens (4) 0.92×10^{-3} m/s away from the lens

Ans. [1]

Sol. $m = \frac{f}{f+u} = \frac{0.3}{0.3-20} = \frac{0.3}{-19.7}$

$v_I = (m)^2 v_O$

$v_I = \left(\frac{0.3}{-19.7}\right)^2 \times (-5) = 1.15 \times 10^{-3}$ m/s towards the lens

Q.13 If the de Broglie wavelength of an electron is equal to the 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to : (Speed of light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} J.s, Mass of electron = 9.1×10^{-31} kg)

(1) 1.7×10^6 m/s (2) 1.45×10^6 m/s (3) 1.1×10^6 m/s (4) 1.8×10^6 m/s

Ans. [2]

Sol. For photon $\nu = \frac{c}{\lambda}$

$$\lambda_p = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^{14}} = \frac{1}{2}\times 10^{-6}\ \text{m}$$

for electron

$$\lambda_e = 10^{-3}\times\lambda_p = \frac{10^{-9}}{2}\ \text{m}$$

$$\lambda = \frac{h}{p} \quad\Rightarrow\quad p = mv$$

$$v = \frac{h}{\lambda m} = \frac{6.63\times10^{-34}}{\frac{1}{2}\times10^{-9}\times9.1\times10^{-31}}$$

$v = 1.45\times10^6$ m/s

Q.14 A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(1) zero (2)10 m/s (3) $10\sqrt{2}$m/s (4) $10\sqrt{3}$m/s

Ans. **[2]**

Sol. $\Delta v = v_f - v_i$

$$\Rightarrow \sqrt{v^2+v^2+2v^2\cos(120)}$$

$$\Rightarrow \sqrt{v^2+v^2+2v^2\left(-\frac{1}{2}\right)}$$

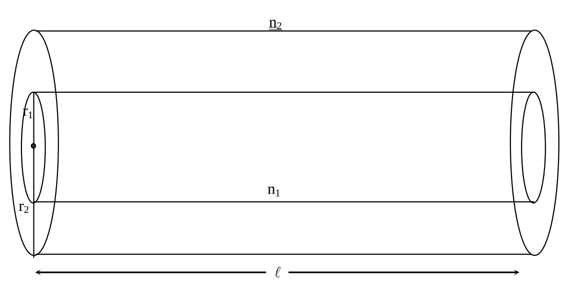
$\Rightarrow v = 10$ m/s

Q.15 There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self - inductance of the inner-coil is :

(1) $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}$ (3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$

Ans. **[2]**

Sol.



Self inductance 'L'

$\phi = Li$

$n_1\ell(\mu_0 n_1 i\, \pi r_1^2) = Li$

$L = \mu_0 n_1^2 \pi r_1^2 \ell$

Mutual inductance 'M'

$\phi_1 = Mi_2$

$n_1\ell(\mu_0 n_2 i_2\, \pi r_1^2) = Mi_2$

$M = \mu_0 n_1 n_2 \ell \pi r_1^2$

$$\frac{M}{L} = \frac{\mu_0 n_1 n_2 \ell \pi r_1^2}{\mu_0 n_1^2 \ell \pi r_1^2} = \frac{n_2}{n_1}$$

Q.16 The force of interaction between tow atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is :

(1) $M^2L^2T^{-2}$ (2) M^2LT^{-4} (3) MLT^{-2} (4) $M^0L^2T^{-4}$

Ans. **[2]**

Sol. $\frac{x^2}{\alpha KT}$ = dimensionless

So dimension of α will be

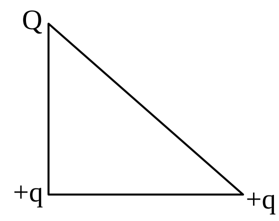
$\alpha = \frac{x^2}{KT}$

dimension of F = dimension of αβ

dimension of β = dimension of $\frac{F}{\alpha}$

$= \frac{FKT}{x^2} = \frac{MLT^{-2} \times ML^2T^{-2}}{L^2} = M^2LT^{-4}$

Q.17 Three charges Q, + q and + q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is :



(1) $\frac{-q}{1+\sqrt{2}}$ (2) + q (3) – 2q (4) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

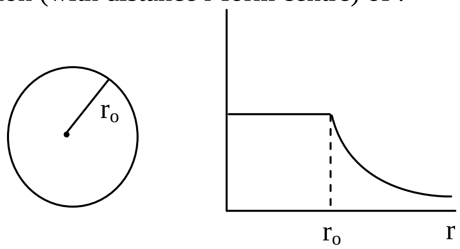
Ans. **[4]**

Sol. $\frac{1}{4\pi \in_0}\left(\frac{q^2}{a}+\frac{Q^2}{a}+\frac{Qq}{\sqrt{2}a}\right) = 0$

Qn Solving

$Q = \frac{-\sqrt{2}q}{(\sqrt{2}+1)}$

Q.18 The given graph shows variation (with distance r form centre) of :



(1) Electric field of a uniformly charged sphere
(2) Electric field of a uniformly charged spherical shell
(3) Potential of a uniformly charged sphere
(4) Potential of a uniformly charged spherical shell

Ans. **[4]**

Sol. Theoritical

Q.19 Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be :

(1) 240 W (2) 60 W (3) 30 W (4) 120 W

Ans. **[1]**

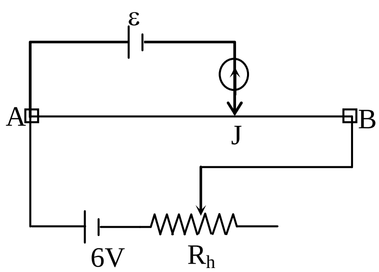
Sol. $\frac{V^2}{2R} = 60$ power in series

$\frac{V^2}{R} = 120$

Power in parallel

$\frac{V^2}{R_{eq}} = \frac{2V^2}{R} = 2\,(120) = 240$ W

Q.20 The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf ε = 0.5 V and rheostat resistance R_h = 2 Ω the null point is obtained at some point J. When the cell is replaced by another one of emf ε = ε_2 the same null point J is found for R_h = 6 Ω. The emf ε_2 is, :



(1) 0.3 V (2) 0.6 V (3) 0.5 V (4) 0.4 V

Ans. **[1]**

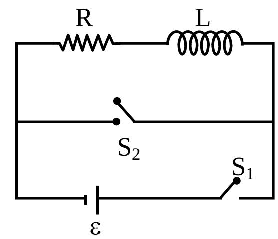
Sol. Let of length of null point J be x

$$E_1\left(\frac{6}{4+2}\right) \times \frac{4}{L}\times x = \frac{1}{2}$$

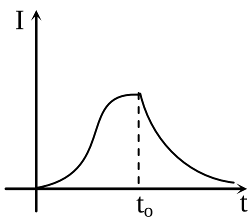
$$\frac{x}{L}=\frac{1}{8}$$

$$E_2 = \left(\frac{6}{4+6}\right) \times \frac{4}{L}\times x = \frac{6}{10}\times\frac{4}{1} \times \frac{1}{8} = 0.3V$$

Q.21 In the circuit shown,



the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time(t_0), the switch S_1 is opened and S_2 is closed. The behavior of the current I as a function of time 't' is given by :

(1)


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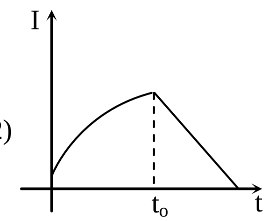

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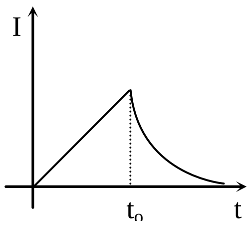
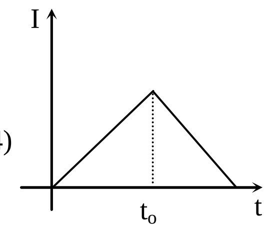


(4)


Ans. **[Bonus]**

Sol. $\varepsilon - L\frac{di}{dt} - iR = 0$

$$\varepsilon = L\frac{di}{dt} + iR$$

$$\varepsilon - iR = L\frac{di}{dt}$$

$$\int\frac{dt}{L} = \int\frac{di}{\varepsilon - iR}$$

Let $\varepsilon - iR = P$

$$\int\frac{dt}{L} = \frac{-1}{R}\int\frac{dP}{P}$$

$$\frac{-R}{L}t = \ell n\, P$$

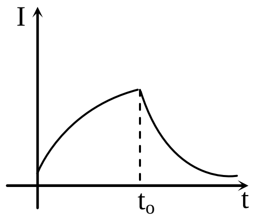
$$\frac{-R}{L}t = \ell n\left(\frac{\varepsilon - iR}{\varepsilon}\right)$$

$$e^{-\frac{R}{L}t} = 1 - \frac{iR}{\varepsilon}$$

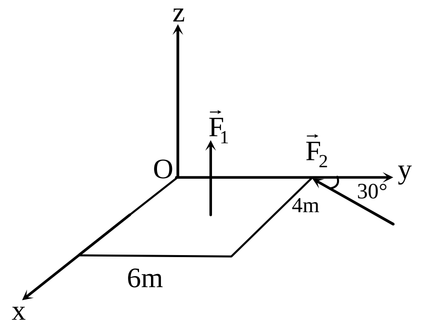
$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \text{ during growth}$$

$$\tau = \left(\frac{L}{R}\right)$$

$$i = i_{max}\, e^{-\frac{1}{\tau}} \text{ during decay}$$



Q.22 A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force $\vec{F}_1$ acts along z=axis at the point $\left(2\vec{i} + 3\vec{j}\right)$. The moment of these forces about point O will be :



(1) $(3\hat{i} - 2\hat{j} - 3\hat{k})\,F$ (2) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$ (3) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$ (4) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$

Ans. **[4]**

Sol. $\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$$\Rightarrow \left(2\vec{i} + 3\vec{j}\right) \times F\hat{k} + 6\hat{j} \times \left(-F\frac{\sqrt{3}}{2}\hat{j} - \frac{F}{2}\hat{i}\right)$$

$$\Rightarrow 2F(-\hat{j}) + (3F\hat{i}) + (3F\hat{k})$$

$$\Rightarrow (3\hat{i} - 2\hat{j} + 3\hat{k})F$$

Q.23 A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed ν and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:

(1) $\frac{3}{4}\rho\nu^2$ (2) $\frac{1}{4}\rho\nu^2$ (3) $\frac{1}{2}\rho\nu^2$ (4) $\rho\nu^2$

Ans. **[1]**

Sol. $F = \frac{dP}{dt}$

Force due to 25%, which loses all its momentum $\Rightarrow \frac{\rho AV}{4} \times V$

Force due to 25%, which comes back with same speed $\Rightarrow \frac{\rho AV}{4} \times 2V$

$\Rightarrow \frac{\rho AV^2}{2}$

Total pressure $\Rightarrow \frac{3}{4}\frac{\rho AV^2}{A} = \frac{3}{4}\rho V^2$

Q.24 A body of mass 1 kg falls freely from a height of 100 m, on a platform mass 3 kg which is mounted on a spring having spring constant k = 1.25×10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to :

(1) 8 cm (2) 4 cm (3) 40 cm (4) 80 cm

Ans. **[Bonus]**

Sol. initial compression

mg = kx

$30 = 1.25 \times 10^6 \left(\frac{x}{100}\right)$

$3 \Rightarrow 1250\ x$

$x = \frac{3}{1250}$ negligible

$P_i = P_f$

$1 \times \sqrt{2 \times 10 \times 100} = 4v$

$20\sqrt{5} = 4v$

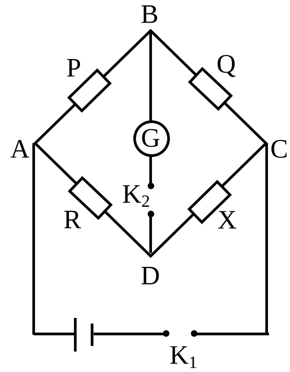
$v = 5\sqrt{5}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$4 \times 5 \times 5 \times 5 = 1.25 \times 10^6 \left(\frac{x}{100}\right)^2$

x = 2cm

Q.25 In a Wheatstone bridge(see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405 Ω. The value of X is close to :



(1) 402.5 ohm (2) 401.5 ohm (3) 403.5 ohm (4) 404.5 ohm

Ans. **[1]**

Sol. $\frac{P}{Q} = \frac{400}{x}$ I[st] condition

$\frac{Q}{P} = \frac{405}{x}$ II[nd] condition

$\frac{x}{400} = \frac{405}{x}$

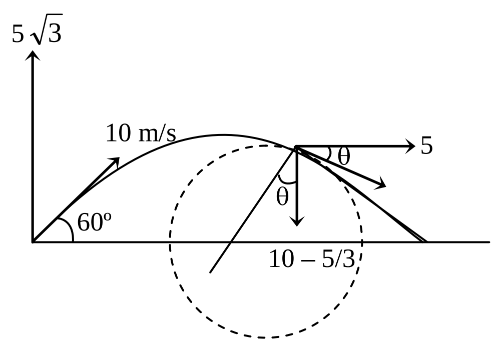
$x^2 = 400 \times 405$

$x = 20\sqrt{405} = 402.5$

Q.26 A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is :

(1) 2.8 m (2) 5.1 m (3) 2.5 m (4) 10.3m

Ans. **[1]**



Sol.

at t = 1

$V = |5\sqrt{3} - 10\times 1| = (10 - 5\sqrt{3})$

$\tan\theta = \frac{10 - 5\sqrt{3}}{5}$

$\tan\theta = (2 - \sqrt{3})$

$a_c = g\cos\theta = \frac{V^2}{R}$

$R = \frac{V^2}{g\cos\theta} = \frac{25 + (10 - 5\sqrt{3})^2}{10 \times 1} \times \sqrt{8 - 4\sqrt{3}}$

$\Rightarrow \frac{25 + 100 + 75 - 100\sqrt{3}}{10 \times \frac{1}{\sqrt{8 - 4\sqrt{3}}}}$

$\Rightarrow \frac{200 - 100\sqrt{3}}{10 \times \frac{1}{\sqrt{8 - 4\sqrt{3}}}}$

$\Rightarrow (20 - 10\sqrt{3})\ \sqrt{8 - 4\sqrt{3}}$

$\Rightarrow 2.679$

Q.27 Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin(450 t – 9x) where distance and time are measured in SI units. The tension in the string is :

(1) 10 N (2) 7.5 N (3) 5 N (4) 12.5 N

Ans.[4]

Sol. $v = \sqrt{\frac{T}{\mu}}$

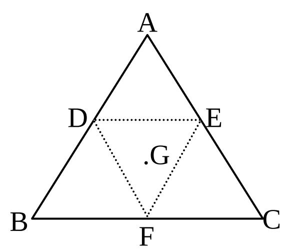
$T = v^2\mu$

$v = \frac{\omega}{k} = \left(\frac{450}{9}\right)^2 \times \left(\frac{5 \times 10^{-3}}{1}\right)$

$\Rightarrow 2500 \times 5 \times 10^{-3}$

$\Rightarrow 12.5$ N

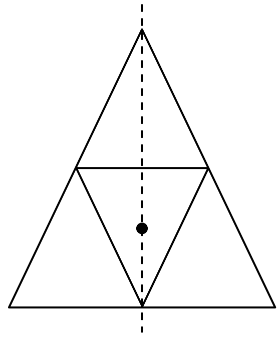
Q.28 An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. then :



(1) $I = \frac{I_0}{4}$ (2) $I = \frac{15}{16}I_0$ (3) $I = \frac{9}{16}I_0$ (4) $I = \frac{3}{4}I_0$

Ans. **[2]**

Sol.



$I = Km\ell^2$, where k any constant

$I_{complete} = I_{remaining} + I_{removed}$

$I = I_{remaining} + K\left(\frac{M}{4}\right)\left(\frac{\ell}{2}\right)^2$

$\Rightarrow I_{remain} + \frac{KM\ell^2}{16}$

$I = I_{remaining} + \frac{I}{16}$

$I - \frac{I}{16} = I_{remain}$

$\frac{15I}{16} = I_{remaining}$

Q.29 An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by:

(1) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (2) $\left(\sqrt{n}, \sqrt{n}\right)$ (3) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (4) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

Ans. **[3]**

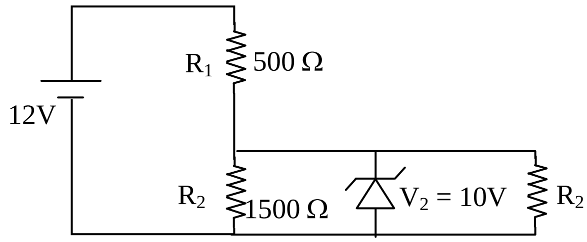
Sol. $\left(\frac{E_i}{B_i}\right) = c$

$\left(\frac{E_f}{B_f}\right) = v$

$\left(\frac{E_i}{E_f}\right) \times \frac{1}{\left(\frac{B_i}{B_f}\right)} = \frac{c}{v} = n = \frac{\sqrt{n}}{\frac{1}{\sqrt{n}}}$ $\left(\frac{\varepsilon_i}{\varepsilon_f} = \sqrt{n}, \frac{B_i}{B_f} = \frac{1}{\sqrt{n}}\right)$

$\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

Q.30 In the given circuit the current through Zener Diode is close to:



(1) 0.0 mA (2) 6.7 mA (3) 4.0 mA (4) 6.0 mA

Ans. **[1]**

Sol. When potential drop across 1500 Ω is 10 V, then electric current flowing through it will be –

$$I_2 = \frac{10}{1500} = \frac{20}{3} \text{ mA} = 6.61 \text{ mA}$$

Now, 2V will be the potential difference across 500 Ω

∴ Electric current flowing through it

$$I_1 = \frac{2}{500} = 4 \text{ mA}$$

$I_2 > I_1$ will not be possible

∴ potential drop across zener diode will be less than 10 V. Hence no current will flow through it.

JEE Main Online Exam 2019

Questions & Solutions

11th January 2019 | Shift - I

CHEMISTRY

Q.1 The amphoteric hydroxide is :

(1) $Ca(OH)_2$ (2) $Be(OH)_2$

(3) $Mg(OH)_2$ (4) $Sr(OH)_2$

Ans. **[2]**

Sol. $Be(OH)_2$ is amphoteric

Q.2 If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives straight line with a gradient (–y) unit.

The energy required to activate the reactant is :

(1) y unit (2) y/R unit

(3) yR unit (4) –y unit

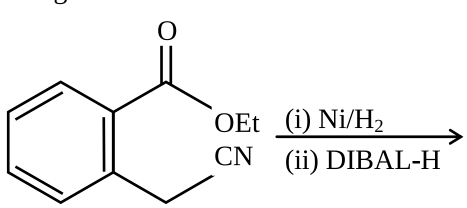
Ans. **[1]**

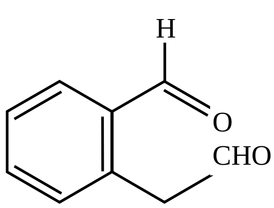
Sol. $\ell nk = \ell n\,A - \frac{E_a}{RT}$

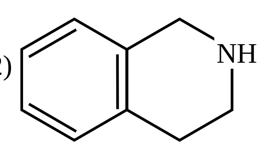
$m = -E_a = -y$

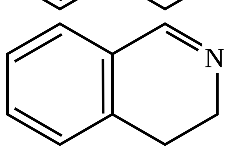
$y = E_a$

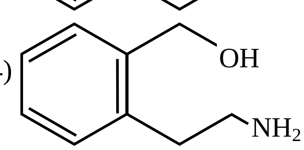
Q.3 The major product of the following reaction is :



(1) 

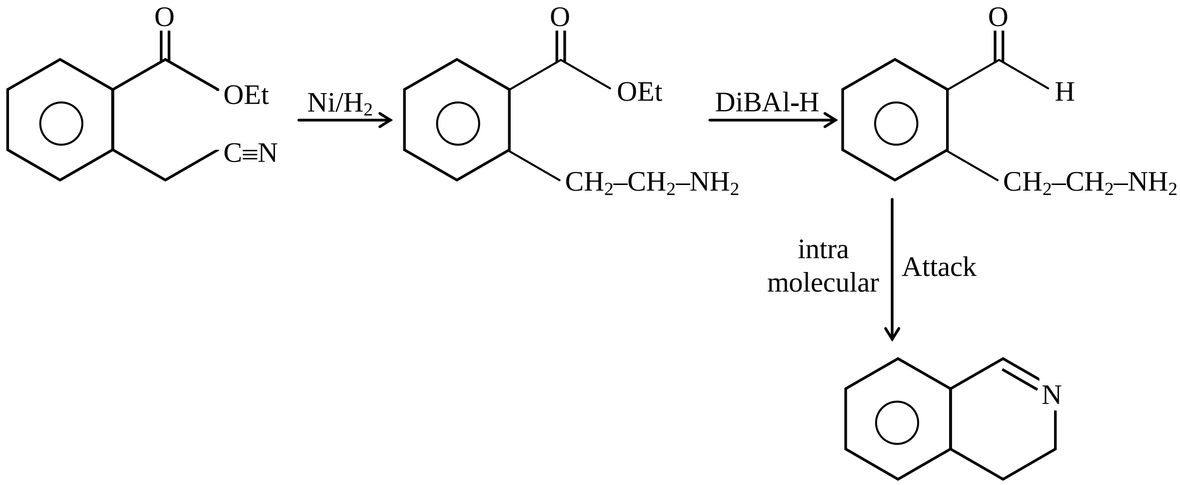
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



Q.4 The concentration of dissolved oxygen (DO) in cold water can go upto :

(1) 14 ppm (2) 16 ppm (3) 8 ppm (4) 10 ppm

Ans. **[4]**

Sol. The concentration of dissolved O_2 in cold water can go up to 10 ppm

Q.5 Match the metals (column I) with the coordination compound(s)/enzyme(s) (column II) :

(Column I) Metals		**(Column II) Coordination compounds(s) enzyme(s)**
(A) Co	(i)	Wilkinson catalyst
(B) Zn	(ii)	Chlorophyll
(C) Rh	(iii)	Vitamin B_{12}
(D) Mg	(iv)	Carbonic anhydrase

(1) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii) (2) (A)-(iv); (B)-(iii); (C)-(i); (D)-(ii)

(3) (A)-(i); (B)-(ii); (C)-(iii); (D)-(iv) (4) (A)-(ii); (B)-(i); (C)-(iv); (D)-(iii)

Ans. **[1]**

Sol.

(A) Co	Cynocobalmine vitamin B_{12}
(B) Zn	Carbonic anhydrase
(C) Rh	Wilkinson catalyst $[Rh\,Cl\,(PPh_3)_3]$
(D) Mg	Chlorophyll

Q.6 The chloride that CANNOT get hydrolysed is :

(1) $SnCl_4$ (2) $SiCl_4$ (3) $PbCl_4$ (4) CCl_4

Ans. **[4]**

Sol. CCl_4 do not possess d orbital so it do not get hydrolysed

Q.7 The correct statements among (a) to (d) regarding H_2 as a fuel are :

(a) It produces less pollutants than petrol.

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy.

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$.

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 KJ, respectively.

(1) (a) and (c) only (2) (b) and (d) only (3) (a), (b) and (c) only (4) (b), (c) and (d) only

Ans. **[3]**

Sol. Fact.

Q.8 Peroxyacetyl nitrate (PAN), an eye irritant is produced by :

(1) classical smog (2) acid rain (3) photochemical smog (4) organic waste

Ans. **[3]**

Sol. Peroxy Acetyl Nitrate (PAN), an eye irritant is produced by photochemical smog

Q.9 The element that usually does NOT show variable oxidation states is :

(1) V (2) Cu (3) Sc (4) Ti

Ans. **[3]**

Sol. Scandium [Ar $3d^1 4s^2$] shows only +3 oxidation state so variable oxidation state is not shown by scandium

Sc $\Rightarrow$ +3

V $\Rightarrow$ +2 to +5

Ti $\Rightarrow$ +2 to +4

Cu $\Rightarrow$ +1, +2

Q.10 For the cell Zn(s) $|Zn^{2+}$ (aq)$|$ $|M^{x+}$ (aq)$|$ M(s), different half cells and their standard electrode potentials are given below :

M^{X+} (aq)/M(s)	Au^{3+}(aq)/Au(s)	Ag^{+}(aq)/Ag(s)	Fe^{3+}(aq)/Fe^{2+} (aq)	Fe^{2+}(aq)/Fe(s)
$E^0_{M^{x+}/M}$/(V)	1.40	0.80	0.77	–0.44

If $E^0_{Zn^{2+}/Zn} = -0.76$ V, which cathode will give maximum value of E^0_{cell} per electron transferred?

(1) Ag^+/Ag (2) Fe^{3+}/Fe^{2+} (3) Au^{3+}/Au (4) Fe^{2+}/Fe

Ans. **[1]**

Sol. Zn(s) $|Zn^{+2}$ (aq)$|$ $|Ag^{+}$(aq)$|$ Ag(s)

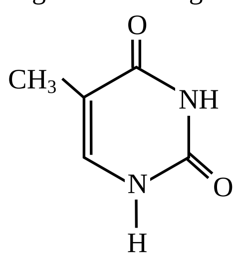
E^0_{cell} = SRP(Cathode) – SRP(anode)

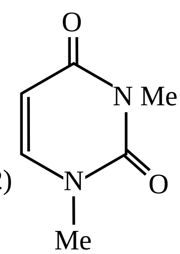
= 0.8 – (–0.76)

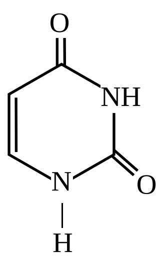
= 1.56 V

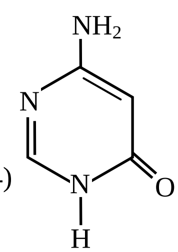
$\therefore$ Maximum value of E^0_{cell} per electron transferred will be of Ag^+/Ag electrode.

Q.11 Among the following compounds, which one is found in RNA ?

(1)


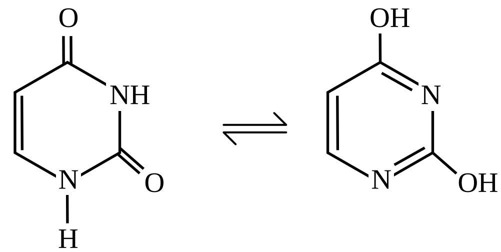
(2)


(3)


(4)


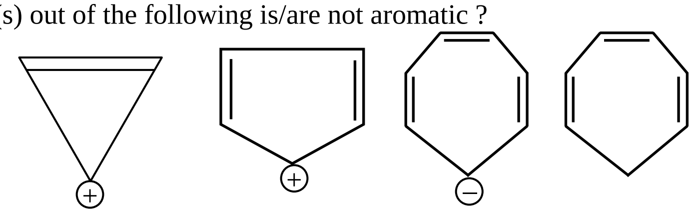
Ans. [3]

Sol. Uracil base is present in RNA



Uracil (u) (Keto form) Uracil (u) (enol form)

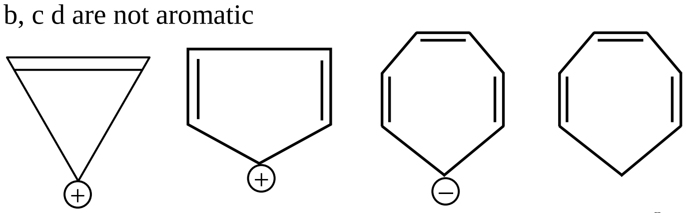
Q.12 Which compound (s) out of the following is/are not aromatic ?



(1) (A) and (C) (2) (C) and (D) (3) (B) (4) (B), (C) and (D)

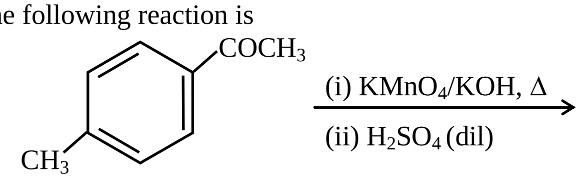
Ans. [4]

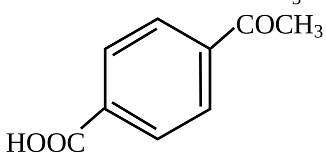
Sol. b, c d are not aromatic

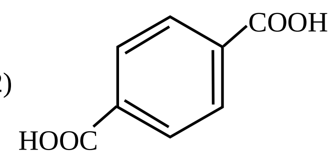


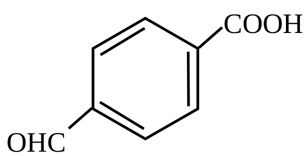
$2\pi\ e^-$	$4\pi\ e^-$	$8\pi\ e^-$	SP^3
Aromatic	Antiaromatic	Not aromatic	Non aromatic

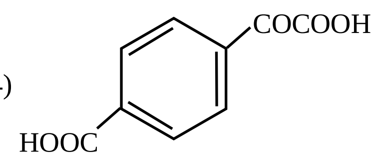
Q.13 The major product of the following reaction is



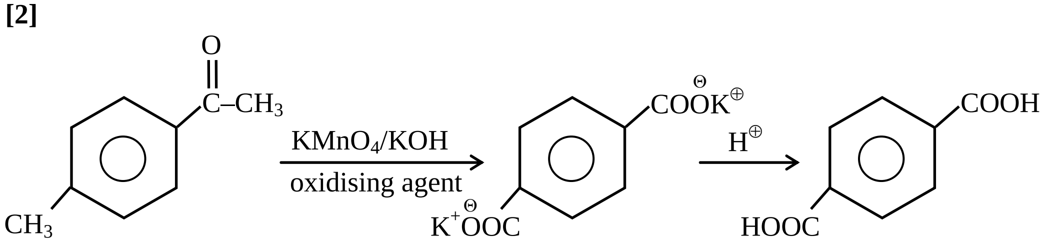
(1) 

(2) 

(3) 

(4) 

Ans. [2]

Sol. 

Q.14 A 10 mg effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet ?

[Molar mass of $NaHCO_3$ = 84 g mol^{-1}]

(1) 33.6 (2) 0.84 (3) 8.4 (4) 16.8

Ans. **[3]**

Sol. $2NaHCO_3 + H_2C_2O_4 \rightarrow Na_2CO_3 + H_2O + CO_2$

.25 ml

$$n_{CO_2} = \frac{.25}{25} \times 10^{-3} = 10^{-5} \text{ mole}$$

$$n_{NaHCO_3} = 1 \times 10^{-5} \text{ mole}$$

$$W_{NaHCO_3} = 1 \times 84 \times 10^{-5} \text{ gm}$$

$$= 84 \times 10^{-5} \text{ gm}$$

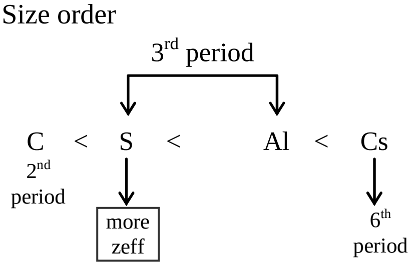
$$\% \, NaHCO_3 = \frac{84 \times 10^{-5}}{10 \times 10^{-3}} \times 100 = 8.4 \text{ gm}$$

Q.15 The correct order of the atomic radii of C, Cs, Al, and S is :

(1) S < C < Al < Cs (2) S < C < Cs < Al (3) C < S < Al < Cs (4) C < S < Cs < Al

Ans. **[3]**

Sol. Size order



Q.16 Match the ores (column A) with the metals (column B) :

(Column A) ores	**(Column B) Metals**
(I) Siderite	(a) Zinc
(II) Kaolinite	(b) Copper
(III) Malachite	(c) Iron
(IV) Calamine	(d) Aluminium

(1) (I)-(c); (II)-(d); (III)-(b); (IV)-(a) (2) (I)-(a); (II)-(b); (III)-(c); (IV)-(d)

(3) (I)-(b); (II)-(c); (III)-(d); (IV)-(a) (4) (I)-(c); (II)-(d); (III)-(a); (IV)-(b)

Ans. **[1]**

Sol. Siderite $\Rightarrow FeCO_3$

kaolinite $\Rightarrow Al_2(OH)_4 \, Si_2O_5$

Malachite $\Rightarrow CuO_3. \, Cu(OH)_2$

Calamine $\Rightarrow ZnCO_3$

Q.17 An example of solid sol is :

(1) Butter (2) Hair cream (3) Gem stones (4) Paint

Ans. **[3]**

Sol. Butter $\Rightarrow$ GEL

Hair cream $\Rightarrow$ Emulsion

Gem stone $\Rightarrow$ solid sol

Paints $\Rightarrow$ Sol

Q.18 NaH is an example of :

(1) metallic hydride (2) saline hydride (3) electron-rich hydride (4) molecular hydride

Ans. **[2]**

Sol. NaH is saline hydride

Q.19 Consider the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_p. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium)

(1) $\dfrac{3^{3/2}K^{1/2}P^2}{4}$ (2) $\dfrac{K_P^{1/2}P^2}{4}$ (3) $\dfrac{3^{3/2}K_P^{1/2}P^2}{16}$ (4) $\dfrac{K_P^{1/2}P^2}{16}$

Ans. **[3]**

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$K_p = \frac{P_{NH_3}^2}{P_{N_2} \times P_{H_2}^3}$$

$$K_p = \frac{P_{NH_3}^2}{\left(\frac{P}{4}\right)\left(\frac{3P}{4}\right)^3}$$

$$P_{NH_3}^2 = K_p\left(\frac{P}{4}\right)\left(\frac{3P}{4}\right)^3$$

$$P_{NH_3} = \frac{K_P^{1/2}P^2 3^{3/2}}{16}$$

Q.20 The correct match between item (I) and item (II) is :

Item-I	**Item-II**
(A) Norethindrone	(P) Anti-biotic
(B) Ofloxacin	(Q) Anti-fertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

(1) (A)$\rightarrow$(R); (B)$\rightarrow$(P); (C)$\rightarrow$(R) (2) (A)$\rightarrow$(R); (B)$\rightarrow$(P); (C)$\rightarrow$(S)

(3) (A)$\rightarrow$(Q); (B)$\rightarrow$(P); (C)$\rightarrow$(R) (4) (A)$\rightarrow$(Q); (B)$\rightarrow$(R); (C)$\rightarrow$(S)

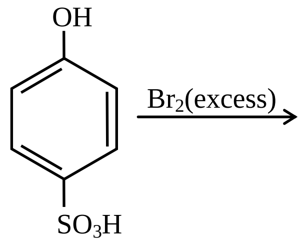
Ans. **[3]**

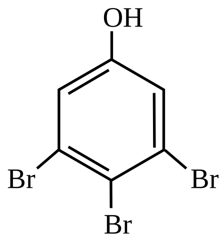
Sol. (A) Norethindrone $\rightarrow$ (Q) antifertility

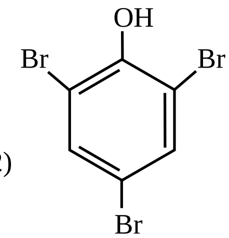
(B) Ofloxacin $\rightarrow$ (P) antibiotic

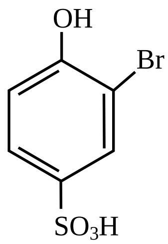
(C) Equanil $\rightarrow$ (R) Hyper tension

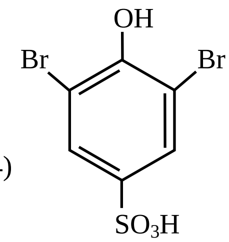
Q.21 The major product of the following reaction is :



(1) 

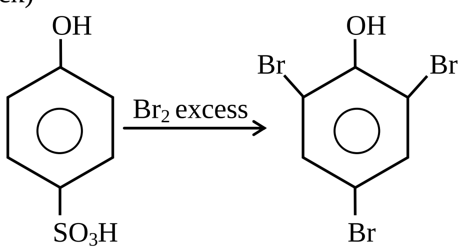
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. Due to activating nature of phenol it get tribrominated (even – COOH and-SO_3H can be replaced by IPSO attack)



Q.22 A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid?

[Avogadro constant $\cong 6 \times 10^{23}$ mol^{-1}, $\pi \cong 3$]

(1) 0.0305 kg mol^{-1} (2) 0.4320 kg mol^{-1} (3) 0.0216 kg mol^{-1} (4) 0.0432 kg mol^{-1}

Ans. **[1]**

Sol. $d = \dfrac{N \times M}{N_A \times a^3}$

$$9 \times 10^3 = \frac{4 \times M}{6 \times 10^{23} \times (200\sqrt{2} \times 10^{-12})^3}$$

M = 0.0305 kg/mol

Q.23 For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as

$\Delta_r G°$ (in kJ mol^{-1}) = $120 - \frac{3}{8}T$.

The major component of the reaction mixture at T is :

(1) Y if T = 300 K (2) Y if T = 280 K (3) X if T = 350 K (4) X if T = 315 K

Ans. **[4]**

Sol. $\Delta G° = 120 - \frac{3}{8}T$

for non spontaneous reaction $\Delta G° > 0$

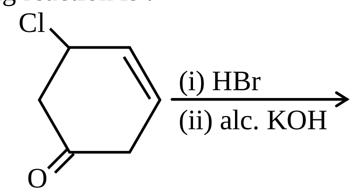
$120 - \frac{3}{8}T > 0$

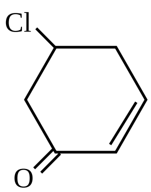
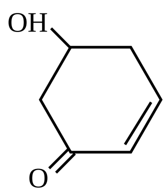
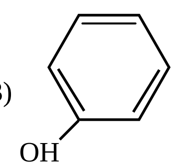
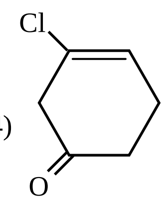
$120 > \frac{3}{8}T$

$T < \frac{120 \times 8}{3}$

$T < 320\ K \rightarrow$ x is major product

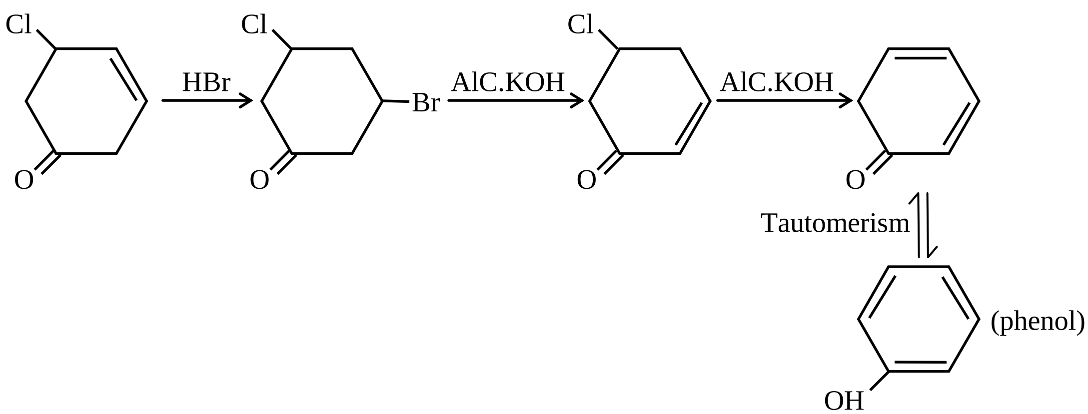
Q.24 The major product of the following reaction is :



(1)  (2)  (3)  (4) 

Ans. **[3]**

Sol.



Q.25 Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose ?

$[R_H = 1 \times 10^5\ cm^{-1}, h = 6.6 \times 10^{-34}\ Js, c = 3 \times 10^8\ ms^{-1}]$

(1) Balmer, $\infty \rightarrow 2$ (2) Paschen, $5 \rightarrow 3$ (3) Paschen, $\infty \rightarrow 3$ (4) Lyman, $\infty \rightarrow 1$

Ans. **[3]**

Sol. $\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= 10^5 \times (1)^2 \left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$

$\lambda = 900$ nm

Q.26 The correct match between items I and II is :

Item-I (Mixture)	**Item-II (Seperation method)**
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

(1) A→(R); (B)→(P); (C)→(S)
(2) A→(Q); (B)→(R); (C)→(S)
(3) A→(Q); (B)→(R); (C)→(P)
(4) A→(S); (B)→(R); (C)→(P)

Ans. **[2]**

Sol. H_2O – Sugar → (Q) Recrystallisation
H_2O – Aniline → (R) Steam Distillation
H_2O – Toluene → (S) Differential Extraction

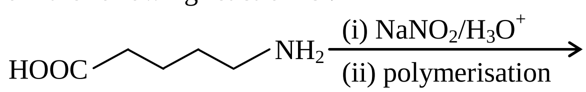
Q.27 The freezing point of a diluted milk sample is found to be –0.2°C, while it should have been –0.5°C for pure milk. How much water has been added to pure milk to make the diluted sample?

(1) 1 cup of water to 2 cups of pure milk
(2) 2 cups of water to 3 cups of pure milk
(3) 3 cups of water to 2 cups of pure milk
(4) 1 cup of water to 3 cups of pure milk

Ans. **[3]**

Sol. $\Delta T_f = K_f\, m$
$0.5 = K_f \times m_1$
$0.2 = K_f \times m_2$
$\frac{5}{2} = \frac{m_1}{m_2} = \frac{W_{A_2}}{W_{A1}}$
Therefore 3 cups of water is added in 2 cups of pure milk

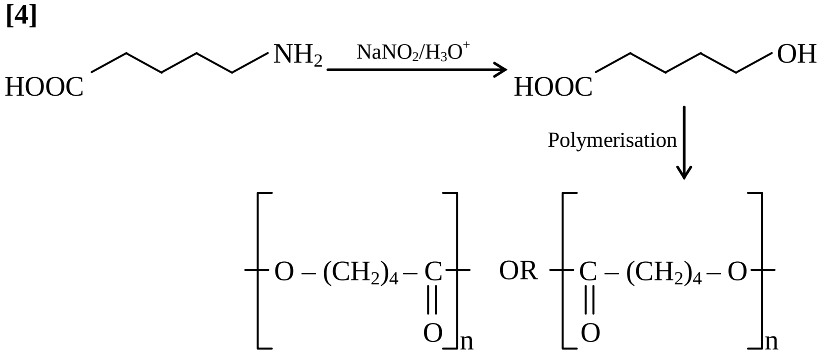
Q.28 The polymer obtained from the following reaction is :



(1) $\left[\overset{O}{\overset{||}{C}} - (CH_2)_4 - \overset{H}{\overset{|}{N}} \right]_n$
(2) $\left[HN\overset{O}{\overset{||}{C}} - (CH_2)_4 - \overset{O}{\overset{||}{C}} - \overset{H}{\overset{|}{N}} \right]_n$
(3) $\left[O\overset{O}{\overset{||}{C}} - (CH_2)_4 O \right]_n$
(4) $\left[O - (CH_2)_4 - \overset{O}{\overset{||}{C}} \right]_n$

Ans. **[4]**

Sol.



Q.29 An organic compound is estimated through Duma's method and was found to evolve 6 moles of CO_2. 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is :

(1) C_6H_8N (2) $C_6H_8N_2$ (3) $C_{12}H_8N$ (4) $C_{12}H_8N_2$

Ans. **[2]**

Sol. Organic compound $\xrightarrow[\text{method}]{\text{DUMA}}$ 6 mole CO_2 (6C) + 4 mole of H_2O (8H) + 1 mole of N_2 (2N)

($C_6H_8N_2$)

Q.30 Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is :

(1) $2C_p \text{ In}\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$ (2) $2C_p \text{ In}\left[\frac{(T_1+T_2)}{2T_1T_2}\right]$

(3) $C_p \text{ In}\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (4) $2C_p \text{ In}\left[\frac{(T_1+T_2)}{4T_1T_2}\right]$

Ans. **[3]**

Sol. $\Delta S_T = C_p\int_{T_1}^{Tf} dT + C_P\int_{T_2}^{Tt} dT$

$= C_P\, \ell n\, \frac{T_f^2}{T_1T_2}$ $\left\{T_f = \frac{T_1+T_2}{2}\right\}$

$= C_P\, \ell n\, \frac{(T_1+T_2)^2}{4T_1T_2}$

JEE Main Online Exam 2019

Questions & Solutions

11th January 2019 | Shift - I

MATHEMATICS

Q.1 The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ (where [x] denotes the greatest integer less than or equal to x) is

(1) 0 (2) 4 (3) 4 – sin 4 (4) sin 4

Ans. **[1]**

Sol. $I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$

$$I = \int_{0}^{2} \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} \right) dx$$

$$\left[\frac{x}{\pi}\right] + \left[\frac{-x}{\pi}\right] = -1 \quad \text{as} \quad x \neq n\pi$$

$$I = \int_{0}^{2} \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}} \right) dx = 0$$

Q.2 In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{3}$ (3) $\frac{c}{\sqrt{3}}$ (4) $\frac{3}{2}y$

Ans. **[3]**

Sol. $a + b = x$ and $ab = y$

$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$$

$$\cos C = -\frac{1}{2}$$

$\angle C = \frac{2\pi}{3}$

$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$

Q.3 Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and g(x) = |f(x)| + f(|x|). Then, in the interval (–2, 2), g is :

(1) non continuous

(2) differentiable at all points

(3) not differentiable at two points

(4) not differentiable at one point

Ans. **[4]**

Sol. $|f(x)| = \begin{cases} 1 & -2 \le x < 0 \\ 1 - x^2 & 0 \le x < 1 \\ x^2 - 1 & 1 \le x \le 2 \end{cases}$

$(f|x|) = x^2 - 1 \quad x \in [-2, 2]$

$g(x) = \begin{cases} x^2 & x \in [-2,\ 0) \\ 0 & x \in [0, 1) \\ 2(x^2 - 1) & x \in [1,2] \end{cases}$

not differentiable at x = 1

Q.4 The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$.

Then the common ratio of this series is :

(1) $\frac{4}{9}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{2}{9}$

Ans. **[3]**

Sol. $\frac{a}{1-r} = 3$

$\frac{a^3}{1-r^3} = \frac{27}{19}$

$\frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$6r^2 - 13r + 6 = 0$

r = 2/3 as |r| < 1

Q.5 The value of r for which ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + ... + {}^{20}C_0\,{}^{20}C_r$ is maximum, is

(1) 20 (2) 15 (3) 10 (4) 11

Ans. **[1]**

Sol. Given : sum = coefficient of x^r in expansions of $(1 + x)^{20}\,(1 + x)^{20}$ which is equal to ${}^{40}C_r$. it is maximum when

r = 20

Q.6 Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is

(1) $x + y + 1 = 0$ (2) $4x + 2y + 1 = 0$ (3) $x - 2y + 4 = 0$ (4) $x + 2y + 4 = 0$

Ans. **[4]**

Sol. Equation of tangent to parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$. Also tangent to hyperbola $xy = 2$ is

$$x\left(mx + \frac{1}{m}\right) = 2$$

$$x^2m + \frac{x}{m} - 2 = 0$$

$$D = 0\ ;\ m = \frac{1}{2}$$

equation of tangent is $2y + x + 4 = 0$

Q.7 If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non zero real numbers, has more than one solution, then :

(1) $b - c - a = 0$ (2) $a + b + c = 0$ (3) $b - c + a = 0$ (4) $b + c - a = 0$

Ans. **[1]**

Sol. $p_1 : 2x + 2y + 3z = a$

$p_2 : 3x - y + 5z = b$

$p_3 : x - 3y + 2z = c$

we find $p_1 + p_3 = p_2$

$a + c = b$

Q.8 If $\int \frac{\sqrt{1-x^2}}{x^4}\,dx = A(x)\left(\sqrt{1-x^2}\right)^m + c$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

(1) $\frac{1}{27x^6}$ (2) $\frac{-1}{27x^9}$ (3) $\frac{1}{9x^4}$ (4) $\frac{1}{3x^3}$

Ans. **[2]**

Sol. $$\int \frac{\sqrt{1-x^2}}{x^4}\,dx$$

$$\int \frac{x\sqrt{\frac{1}{x^2} - 1}}{x^4}\,dx$$

$$\frac{1}{x^2} - 1 = t$$

$$\left(\frac{-2}{x^3}\right)dx = dt$$

$$\int \sqrt{t}\left(-\frac{1}{2}\right).dt$$

$$= -\frac{2}{2}\frac{(t)^{3/2}}{3} + c$$

$$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2} + c$$

$$= -\frac{1}{3}\left(\frac{1-x^2}{x^2}\right)^{3/2} + c$$

$$= -\frac{1}{3x^3}\sqrt{1-x^2}(1-x^2) + c$$

$$= -\frac{1}{3x^3}(\sqrt{1-x^2})^3 + c$$

$$A(x) = \frac{-1}{3x^3}, \quad m = 3$$

$$(A(x))^m = \left(\frac{-1}{3x^3}\right)^3 = \frac{-1}{27x^9}$$

Q.9 The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

(1) 0 (2) 8 (3) 6 (4) 4

Ans. **[1]**

Sol. n = 8

$$T_{\text{middle term}} = \left(\frac{n}{2}+1\right)^{th}$$

$$= (5^{th})$$

$$^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$$

$$^8C_4 \times \frac{x^{12}}{3^4} \times \frac{3^4}{x^4} = 5670$$

$$x = \pm\sqrt{3}$$

sum = 0

Q.10 Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is :

(1) $-10\hat{i} - 5\hat{j}$ (2) $-10\hat{i} + 5\hat{j}$ (3) $-14\hat{i} + 5\hat{j}$ (4) $-14\hat{i} - 5\hat{j}$

Ans. **[2]**

Sol. $\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2-1 \end{vmatrix} = 0$

$[(\lambda^3 - \lambda) - 16] -2(\lambda^2 - 1 - 8) + 4(4 - 2\lambda) = 0$

$(\lambda^3 - \lambda - 16) - 2\lambda^2 + 18 + 16 - 8\lambda = 0$

$\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\lambda = 2, 3, -3$

$\lambda = 2$ is only option (4) $\lambda = \pm 3$

$\vec{a}$ is $\parallel$ $\vec{c}$

$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$

Q.11 The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(1) $(-2, 2, 2)$ (2) $(2, 2, 0)$ (3) $(2, 0, -2)$ (4) $(0, -2, 2)$

Ans. **[3]**

Sol. normal of required plane $(2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k}) = -8\hat{i} + 8\hat{j} + 8\hat{k}$

required plane whose dr of normal is $(-1, 1, 1)$ is

$-(x - 3) + (y + 2) + (z - 1) = 0$

$-x + y + z + 4 = 0$

satisfied by $(2, 0, -2)$

Q.12 The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is :

(1) -222 (2) -122 (3) 122 (4) 222

Ans. **[3]**

Sol. **S :** $x^2 - 11x + 30 \le 0$

$(x - 6)(x - 5) \le 0$

$5 \le x \le 6$

$f'(x) = 3.3x^2 - 18.2x + 27$

$= 9x^2 - 36x + 27$

$= 9(x^2 - 4x + 3)$

for maximum & minimum $f'(x) = 0$ at $x = 1$, $x = 3$

But in $[5, 6]$ $f'(x) > 0$

So $f(x) \uparrow$ function

maximum at $x = 6$

$3(6)^3 - 18(6)^2 + 27(6) - 40 = 122$

Q.13 If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology ?

(1) $p \wedge r$ (2) $(p \vee r) \to (p \wedge r)$ (3) $p \vee r$ (4) $(p \wedge r) \to (p \vee r)$

Ans. **[4]**

Sol. If q is false $p \wedge q \leftrightarrow r$ is true

p	q	r	$p \wedge q$	$p \wedge q \leftrightarrow r$
T	F	T	F	F
T	F	F	F	T
F	F	T	F	F
F	F	F	F	T

check option $(p \wedge r) \rightarrow (p \vee r)$

If $p \wedge q \leftrightarrow r$ is true

then

p	q	r	$p \wedge r$	$p \vee r$	$(p \wedge r) \rightarrow (p \vee r)$
T	F	F	F	T	T
F	F	F	F	F	T

Q.14 The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}$ – d each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}$ + d each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals :

(1) $\frac{2}{3}$ (2) $\frac{\sqrt{5}}{2}$ (3) $\sqrt{2}$ (4) 2

Ans. **[3]**

Sol. Variance is independent of origin we shift data by $\frac{1}{2}$

$$\frac{10d^2 + 10 \times (0)^2 + 10(d)^2}{30} - (0)^2 = \frac{4}{3}$$

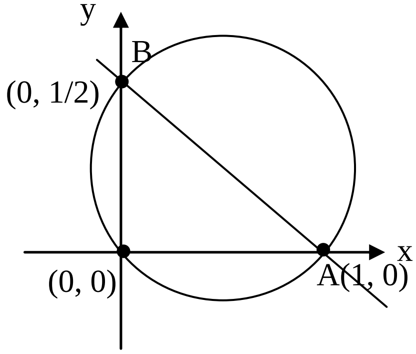
$$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$$

Q.15 The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is

(1) $4\sqrt{5}$ (2) $\frac{\sqrt{5}}{2}$ (3) $2\sqrt{5}$ (4) $\frac{\sqrt{5}}{4}$

Ans. **[2]**

Sol.



equation of circle

$(x-1)(x) + (y-0)\left(y-\frac{1}{2}\right) = 0$

$x^2 - x + y^2 - \frac{y}{2} = 0$

equation of tangent at (0, 0)

$x(0) - \frac{1}{2}(x+0) + y(0) - \frac{1}{4}(y+0) = 0$

$-\frac{x}{2} - \frac{y}{4} = 0$

$\frac{x}{2} + \frac{y}{4} = 0$

$2x + y = 0$

Sum of $\perp$ distance

$= \frac{|2|}{\sqrt{5}} + \frac{\left|\frac{1}{2}\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}} = \frac{4+1}{2\sqrt{5}} = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$

Q.16 If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is :

(1) –81 (2) –300 (3) 100 (4) 144

Ans. **[2]**

Sol. Roots α, α^3

$\alpha + \alpha^3 = \frac{-k}{81}$

$\alpha \cdot \alpha^3 = \frac{256}{81}$

$\alpha^4 = \frac{256}{81} = \frac{4^4}{3^4}$

$\alpha = \frac{4}{3}$

$\frac{4}{3} + \frac{64}{27} = \frac{-k}{81}$

$\frac{36+64}{27} = \frac{-k}{81}$

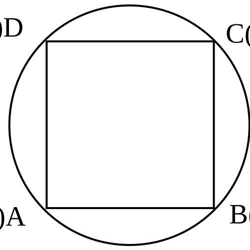
$\Rightarrow k = -300$

Q.17 A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is :

(1) $\sqrt{137}$ (2) 6 (3) $\sqrt{41}$ (4) 13

Ans. **[3]**

Sol. $R = \sqrt{9+16+103} = 8\sqrt{2}$



OA = 13

OB = $\sqrt{265}$

OC = $\sqrt{137}$

OD = $\sqrt{41}$

Q.18 If y(x) is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$, x > 0, where $y(1) = \frac{1}{2}e^{-2}$, then :

(1) $y(\log_e 2) = \log_e 4$ (2) y(x) is decreasing in (0, 1)

(3) $y(\log_e 2) = \frac{\log_e 2}{4}$ (4) y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

Ans. **[4]**

Sol. $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$, (x > 0)

I.F = $e^{\int \frac{2x+1}{x}dx} = e^{\int\left(2+\frac{1}{x}\right)dx}$

$e^{2x + \ln x} = e^{2x} . e^{\ln x} = xe^{2x}$

Sol. given by

$y.(xe^{2x}) = \int e^{-2x} x.e^{2x} \, dx + c$

$yxe^{2x} = \frac{x^2}{2} + c$

at x = 1

$\frac{1}{2}e^{-2} \times 1 \times e^2 = \frac{1}{2} + c$

c = 0

$yxe^{2x} = \frac{x^2}{2}$

$y = \frac{xe^{-2x}}{2}$

$\frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x + 1)$

f(x) is ↓ in $\left(\frac{1}{2}, 1\right)$

Q.19 Let [x] denote the greatest integer less than or equal to x. Then $\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$

(1) equals $\pi + 1$ (2) equals 0 (3) does not exist (4) equals π

Ans. **[3]**

Sol. $\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$

RHL :

$\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+(x)^2}{x^2}$

$= \lim_{x\to 0^+} 1+\frac{\tan(\pi\sin^2 x)}{x^2\pi\sin^2 x}\times\pi\sin^2 x$

$= 1 + \lim_{x\to 0^+}\pi\frac{\sin^2 x}{x^2}$

$= 1 + \pi$

LHL :

$\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(x-\sin(x))^2}{x^2}$

$= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(x-\sin x)^2}{x^2}$

$= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(x^2+\sin^2 x-2x\sin x)}{x^2}$

$= 1 + \lim_{x\to 0^-}\left(\frac{\tan\pi(\sin^2 x)}{x^2}+\frac{\sin^2 x}{x^2}-\frac{2\sin x}{x}\right)$

$= 1 + (\pi + 1 - 2) = \pi$

RHL $\neq$ LHL

limit dose not exist

Q.20 The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are :

(1) 2, –1, 1 (2) $2\sqrt{3}$, 1, –1 (3) $\sqrt{2}$, 1, –1 (4) 2, $\sqrt{2}$, – $\sqrt{2}$

Ans. **[3]**

Sol. Plane passing through the point (0, –1, 0) & (0, 0, 1)

$a(x - 0) + b(y + 1) + c(z - 0) = 0$

where (a, b, c) are d.r. of normal to plane also pass through (0, 0, 1)

$a(0) + b(1) + c(1) = 0$

$b + c = 0$ ...(1)

also making an $\pi/4$ angle with $y - z + 5 = 0$

$\cos\left(\frac{\pi}{4}\right)=\frac{|a.(0)+b(1)+c(-1)|}{\sqrt{a^2+b^2+c^2}\sqrt{1+1}}$

$a^2 + b^2 + c^2 = (b - c)^2$

put $b = - c$

$a^2 + c^2 + c^2 = (-2c)^2$

$a^2 + 2c^2 = 4c^2$

$a^2 = 2c^2$

$a = \pm\sqrt{2}\,c$

$\pm\sqrt{2}\,c,\ -c,\ c$

$(\pm\sqrt{2}, -1, 1)$
$(\sqrt{2}, -1, 1)$ or $(-\sqrt{2}, -1, 1)$
$(\sqrt{2}, -1, 1)$ or $(\sqrt{2}, 1, -1)$
Both can be possible

Q.21 Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{3}}$

Ans. **[1]**

Sol. $AA^T = I_3$

$$\begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}\begin{pmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2 \\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2 \\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Compare
$p^2 + q^2 + r^2 = 1$
$4q^2 + r^2 = 1$
$2q^2 = r^2$
$r^2 = 2p^2$
$p^2 = q^2 + r^2$
$p^2 = \frac{r^2}{2} + r^2$
$p^2 = \frac{3r^2}{2}$
$p^2 + \frac{3r^2}{2} = 1$
$p^2 + p^2 = 1$
$2p^2 = 1$
$p^2 = \frac{1}{2}$
$|p| = \frac{1}{\sqrt{2}}$

Q.22 If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve :

(1) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (2) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (3) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (4) $\frac{x^2}{4}+\frac{y^2}{2}=1$

Ans. **[2]**

Sol. equation of tangent $\frac{x}{a\sec\theta}+\frac{y}{b\operatorname{cosec}\theta}=1$

$a=\sqrt{2}$, $b = 1$

$$\frac{x}{\sqrt{2}\sec\theta} + \frac{y}{\operatorname{cosec}\theta} = 1$$

mid point (h, k)

$$h = \frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$$

similarly $\sin\theta = \frac{1}{2k}$

square and add

$$\frac{1}{2x^2}+\frac{1}{4y^2}=1$$

Q.23 If $x \log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then $\frac{dy}{dx}$ at x = e is equal to :

(1) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (4) $\frac{e}{\sqrt{4+e^2}}$

Ans. **[3]**

Sol. $x \log_e(\log_e x) - x^2 + y^2 = 4\ (y > 0)$

$$\log_e(\log_e x) + x.\left[\frac{1}{\log_e x}\times\frac{1}{x}\right] - 2x + 2y\frac{dy}{dx} = 0$$

at x = e

$$e.\log_e(\log_e e) - e^2 + y^2 = 4$$

$$y^2 = 4 + e^2$$

$$y = \sqrt{4+e^2}$$

at x = e

$$0 + \left[\frac{1}{\log_e e}\right] - 2e + 2\sqrt{4+e^2}\frac{dy}{dx} = 0$$

$$1 - 2e = -2\sqrt{4+e^2}\frac{dy}{dx}$$

$$2e - 1 = 2\sqrt{4+e^2}\frac{dy}{dx}$$

$$\frac{2e-1}{2\sqrt{4+e^2}} = \frac{dy}{dx}$$

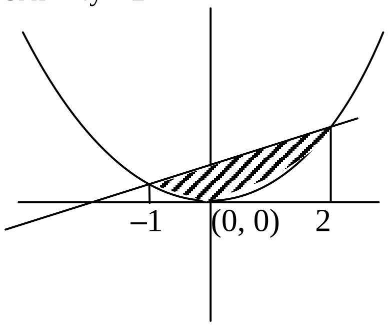
Q.24 The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is :

(1) $\frac{3}{4}$ (2) $\frac{5}{4}$ (3) $\frac{7}{8}$ (4) $\frac{9}{8}$

Ans. **[4]**

Sol. $x^2 = 4y$

& $x = 4y - 2$



$x + 2 = 4y$ ...(1)

$y = \frac{x+2}{4}$

$(4y - 2)^2 = 4y$

$16y^2 + 4 - 16y = 4y$

$16y^2 - 20y + 4 = 0$

$4y^2 - 5y + 1 = 0$

$4y^2 - 4y - y + 1 = 0$

$4y(y - 1) - 1(y - 1) = 0$

$y = 1$ or $y = \frac{1}{4}$

Put in (1)

$x = 2$ or $x = -1$

calculate = $\frac{9}{8}$

Q.25 Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is

(1) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (2) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (3) $(-1, 1) - \{0\}$ (4) $R - [-1, 1]$

Ans. **[1]**

Sol. $f(x) = \frac{x}{1+x^2}$; $x \in R$

Range

$y = \frac{x}{1+x^2}$

$y + yx^2 = x$

$yx^2 - x + y = 0$

$D \geq 0$

$1 - 4y^2 \geq 0$

$4y^2 - 1 \leq 0$

$y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Q.26 Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(1) 5^3 (2) $2(5^2)$ (3) $4(5^2)$ (4) 5^4

Ans. **[4]**

Sol. $\frac{ar^2}{a} = 25$

$\frac{a_9}{a_5} = \frac{a.r^8}{a.r^4} = r^4 = (25)^2 = 5^4$

Q.27 Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

(1) $\frac{1}{4}$ (2) $\frac{5}{12}$ (3) $\frac{-1}{12}$ (4) $\frac{1}{12}$

Ans. **[4]**

Sol. $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) - f_6(x)$

$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$= \frac{1}{4}(1 - 2\sin^2 x \cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x \cos^2 x)$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Q.28 Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $(i = \sqrt{-1})$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85 (3) –91 (4) 91

Ans. **[4]**

Sol. $\left(-2-\frac{i}{3}\right)^3 = \frac{x+iy}{27}$

$-1\left(2+\frac{i}{3}\right)^3 = \frac{x+iy}{27}$

$-\left[8+\frac{i^3}{27}+3.2.\frac{i}{3}\left(2+\frac{i}{3}\right)\right] = \frac{x+iy}{27}$

$-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] = \frac{x+iy}{27}$

$-\frac{22}{3}-\frac{107i}{27} = \frac{x+iy}{27}$

$\frac{x}{27} = -\frac{22}{3}$

$x = -198$

$-\frac{107}{27} = \frac{y}{27}$

$y = -107$

$y - x = 91$

Q.29 Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$ (2) $\frac{1}{2}$ (3) $\frac{7}{10}$ (4) $\frac{3}{5}$

Ans. **[1]**

Sol. Sum is even = Both are even + Both are odd

$= {}^5C_2 + {}^6C_2$

= 10 + 15

= 25

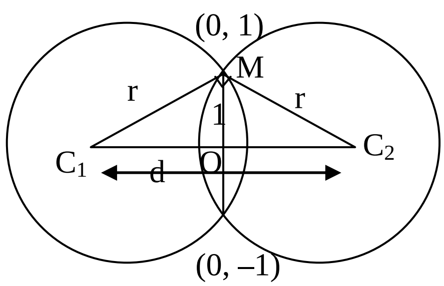
conditional probability that both are even $= \frac{10}{25} = \frac{2}{5}$

Q.30 Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3) 2 (4) 1

Ans. **[3]**

Sol. $r^2 + r^2 = d^2$



In ΔC_1MO

$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$r = \sqrt{2}$

distance between centre

$d^2 = 2(2)$

$d^2 = 4$

d = 2