CAREER POINT

JEE Main Exam 2018

Paper & Solution

| 8th April 2018 |

Test Booklet Code

A

ALL THE GRAPHS/DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE.

Part A – PHYSICS

Q.1 The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5 % and 1%, the maximum error in determining the density is -

(1) 2.5 %

(2) 3.5 %

(3) 4.5 %

(4) 6 %

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Unit Dimension & Error, Based on Theory (6. 3)*]

Ans. **[3]**

Sol. $\rho = \frac{m}{V}$

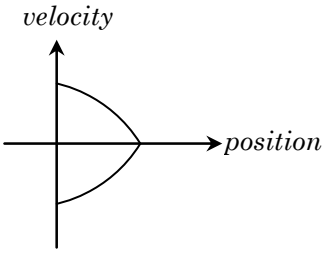
$\rho = \frac{m}{a^3}$

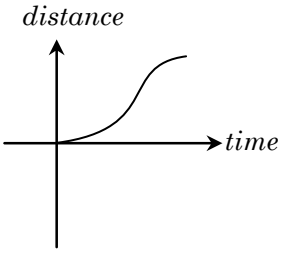
$\frac{d\rho}{\rho} = \frac{dm}{m} + \frac{3da}{a}$

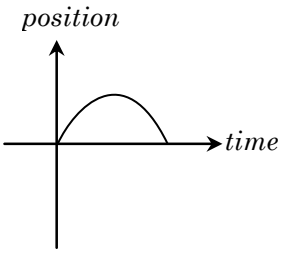
$\% = 1.5 + 3 \times 1$

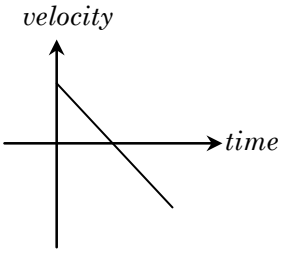
$= 4.5\ \%$

Q.2 All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up -

(1)


(2)


(3)


(4)


Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Motion in One dimension, Based on theory Pg. (12)*]

Ans. **[2]**

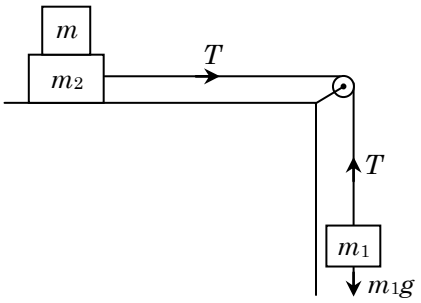
Sol. $s = at - bt^2$ shows (3)

$v = a - 2bt$ shows (4)

(1) is also correct because velocity at middle is zero.

(2) $\rightarrow$ incorrect.

Q.3 Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is -



(1) 18.3 kg

(2) 27.3 kg

(3) 43.3 kg

(4) 10.3 kg

Students may find similar question in CP exercise sheet :
[JEE Advance, Chapter : Newtons laws of motion, Level # 1, Q. No. 57]

Ans. **[2]**

Sol. $m_1 g = \mu(m_2 + m)$

$5 = .15\ (10 + m)$

$\frac{500}{15} = 10 + m$

$m = \frac{350}{15} = 23.3$

so answer will be (2).

Q.4 A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$.

Its total energy is -

(1) $-\frac{k}{4a^2}$

(2) $\frac{k}{2a^2}$

(3) zero

(4) $-\frac{3}{2}\ \frac{k}{a^2}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Work, Power & Energy, Similar in Exercise # 5 (B), Q. No. 3]

Ans. **[3]**

Sol. $F = -\frac{dU}{dr} = -\frac{k}{2}(-2)\ r^{-2-1}$

$$= \frac{k}{r^3}$$

$$\frac{k}{r^3} = \frac{mv^2}{r}$$

$$mv^2 = \frac{k}{r^2}$$

$$KE = K = \frac{k}{2r^2}$$

$$TE = \frac{-k}{2r^2} + \frac{k}{2r^2} = 0$$

Q.5 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % grater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is -

(1) $\frac{v_0}{4}$

(2) $\sqrt{2}\, v_0$

(3) $\frac{v_0}{2}$

(4) $\frac{v_0}{\sqrt{2}}$

Students may find similar question in CP exercise sheet :
[*JEE Main, Chapter : Conservation laws*]

Ans. **[2]**

Sol. $mv_0 = mv_1 + mv_2$ ....(1)

(momentum conservation)

$KE_f = KE_i + 0.5\, KE_i$

$KE_f = 1.5\, KE_i$

$$\frac{1}{2} m(v_1^2 + v_2^2) = \frac{1}{2} mv_0^2 \times 1.5 \quad \text{....(2)}$$

From (1) $v_1 + v_2 = v_0$

From (2) $v_1^2 + v_2^2 = 1.5\, v_2^2$

$$v_1 - v_2 = \sqrt{(v_1 + v_2)^2 - 4v_1v_2}$$

$(v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$

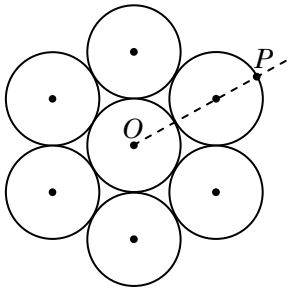
$v_0^2 = 1.5\, v_0^2 + 2v_1v_2$

$2v_1v_2 = -0.5\, v_0^2$

$$v_1 - v_2 = \sqrt{v_0^2 - 2(-0.5v_0^2)}$$

$= \sqrt{2}\, v_0$

Q.6 Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is -



(1) $\frac{19}{2}MR^2$

(2) $\frac{55}{2}MR^2$

(3) $\frac{73}{2}MR^2$

(4) $\frac{181}{2}MR^2$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Rotational Motion, Exercise # 1, Q. No. 90*]

Ans. **[4]**

Sol. $I_{about\ O} = \frac{MR^2}{2} + \left[\frac{MR^2}{2} + M4R^2\right] \times 6$

$= \frac{MR^2}{2} + \frac{9MR^2}{2} \times 6$

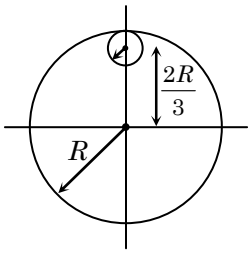
$= \frac{55}{2}MR^2$

$I_{about\ P} = \frac{55MR^2}{2} + 7M(9R^2)$

$= \left(\frac{55 + 63 \times 2}{2}\right)MR^2$

$= \frac{181MR^2}{2}$

Q.7 From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is -



(1) 4 MR^2

(2) $\frac{40}{9}$ MR^2

(3) 10 MR^2

(4) $\frac{37}{9}$ MR^2

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Rotational Motion, Exercise # 5(A), Q. No. 19]

Ans. **[1]**

Sol. $I_{at\ centre} = I_{big\ disc} - I_{small\ disc}$

$$= \frac{9MR^2}{2} - \left(M\left(\frac{R}{3}\right)^2 \frac{1}{2} + M\left(\frac{2R}{3}\right)^2 \right]$$

$$= \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{M \times 4R^2}{9} \right]$$

$$= \frac{9MR^2}{2} - \frac{MR^2}{2}$$

$$= 4\ MR^2$$

Q.8 A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then -

(1) $T \propto R^{3/2}$ for any n

(2) $T \propto R^{\frac{n}{2}+1}$

(3) $T \propto R^{(n+1)/2}$

(4) $T \propto R^{n/2}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Gravitation, Exercise # 4, Q. No. 10]

Ans. **[3]**

Sol. $F \propto \frac{1}{R^n}$

$F = \frac{C}{R^n}$

$\frac{mv^2}{R} = \frac{C}{R^n}$

$v^2 = \frac{C}{m} R^{1-n}$

$v = \sqrt{\frac{C}{m}}\ R^{\frac{1-n}{2}}$

$T = \frac{2\pi R}{v}$

$T = \frac{2\pi R}{\sqrt{\frac{C}{m}} R^{\frac{1-n}{2}}}$

$T \propto R^{1-\frac{(1-n)}{2}}$

$\propto R^{\frac{2-1+n}{2}}$

$\propto R^{\frac{n+1}{2}}$

Q.9 A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is -

(1) $\frac{Ka}{mg}$

(2) $\frac{Ka}{3mg}$

(3) $\frac{mg}{3Ka}$

(4) $\frac{mg}{Ka}$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Properties of Matter (Elasticity)*]

Ans. **[3]**

Sol. Increase in pressure = $\frac{mg}{Area} = \frac{mg}{a}$

$$B = -\frac{dP}{\frac{dV}{V}}$$

$$\frac{dV}{V} = -\frac{dP}{B}$$

$$\frac{dV}{V} = -\frac{mg}{Ka}$$

$$v = \frac{4}{3}\pi R^3$$

$$\frac{dV}{V} = \frac{3dR}{R}$$

$$3\frac{dR}{R} = -\frac{mg}{Ka}$$

$$\left|\frac{dR}{R}\right| = \frac{mg}{3Ka}$$

Q.10 Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (a) the final temperature of the gas and (b) change in its internal energy -

(1) (a) 189 K (b) 2.7 kJ

(2) (a) 195 K (b) – 2.7 kJ

(3) (a) 189 K (b) –2.7 kJ

(4) (a) 195 K (b) 2.7 kJ

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Thermodynamics, Exercise # 4, Q. No. 22]

Ans. **[3]**

Sol. $T_i = 273 + 27 = 300$

$PV^{\gamma} = C$

$TV^{\gamma-1} = C$

$300\ V^{\gamma-1} = T(2V)^{\gamma-1}$

$$T = \frac{300}{2^{\gamma-1}}$$

$$\gamma = 1 + \frac{2}{3}$$

$$T = \frac{300}{2^{2/3}} = \frac{300}{4^{1/3}}$$

$T = 189\ K$

$\Delta U = \frac{f}{2} nR\Delta T$

$= \frac{3}{2} \times 2 \times 8.314(189 - 300)$

$= -2.7$ kJ

Q.11 The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly :

(1) 2.35×10^3 N/m^2

(2) 4.70×10^3 N/m^2

(3) 2.35×10^2 N/m^2

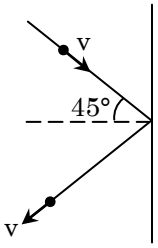
(4) 4.70×10^2 N/m^2

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : K.T.G, Exercise # 1, Q.No.21]

Ans. **[1]**

Sol.



Δp of one particle $= mv \cos 45° \times 2$

Force $= \dot{N}\Delta p$

$= 10^{23} \times \frac{mv}{\sqrt{2}} \times 2$

$= 10^{23} mv\sqrt{2}$

Pressure $= \frac{10^{23} \times 3.32 \times 10^{-27} \times 10^3 \sqrt{2}}{2 \times 10^{-4}}$

$= \frac{10^{23} \times 3.32 \times \sqrt{2}}{2}$

$= \frac{10^3 \times 4.7}{2}$

$= 2.35 \times 10^3$ N/m^2

Q.12 A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1})

(1) 6.4 N/m

(2) 7.1 N/m

(3) 2.2 N/m

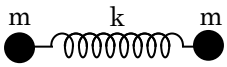
(4) 5.5 N/m

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : S.H.M, Formula Based [Class Notes]

Ans. **[2]**

Sol.



$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$\frac{1}{10^{12}} = 2 \times 3.14\sqrt{\frac{108}{6.02\times10^{23}k\times1000}}$$

$$\frac{1}{10^{24}} = \frac{4\times(3.14)^2\times108}{6.02\times10^{23}k\times1000}$$

$$k = \frac{4\times(3.14)^2\times108\times10}{6.02\times1000}$$

$$k = 7.1 \text{ N/m}$$

Q.13 A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modules is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ?

(1) 5 kHz

(2) 2.5 kHz

(3) 10 kHz

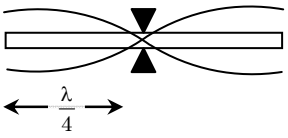
(4) 7.5 kHz

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Wave motion, Exercise # 2, Q. No.13]

Ans. **[1]**

Sol.



$\frac{\lambda}{4} = 0.3$

$\lambda = 1.2$ m

$v = n\lambda$

$n = \frac{v}{\lambda} = \frac{1}{1.2}\sqrt{\frac{Y}{d}}$

$= \frac{1}{1.2}\sqrt{\frac{9.27\times10^{10}}{2.7\times10^{23}}} = 4.8 \text{ kHz} \approx 5 \text{ kHz}$

Q.14 Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities + σ, – σ and + σ respectively. The potential of shell B is :

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

(2) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

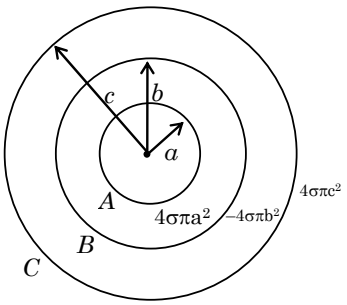
(3) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

(4) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Electrostatics, Exercise # 5 (B), Q. No.18]

Ans. **[2]**

Sol.



$V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$

$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma.4\pi a^2}{b} + \frac{1}{4\pi\varepsilon_0}\frac{(-\sigma)4\pi b^2}{b} + \frac{1}{4\pi\varepsilon_0}\frac{\sigma\times4\pi c^2}{c}$

$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$

Q.15 A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :

(1) 1.2 n C

(2) 0.3 n C

(3) 2.4 n C

(4) 0.9 n C

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Capacitance, Formula Based [Class Notes]

Ans. **[1]**

Sol. $q_{induced} = q\left[1 - \frac{1}{K}\right]$

Where q = KCV = KCV

$$q = \frac{5}{3} \times 90 \times 10^{-9} \times 20$$

$$q = 3000 \times 10^{-9}$$

$$q_{induced} = q\left[1 - \frac{1}{5} \times 3\right]$$

$$= \frac{q \times 2}{5}$$

$$= 3 \times 10^{-6} \times \frac{2}{5}$$

$$= 1.2 \text{ n C}$$

Q.16 In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\, t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively :

(1) 50, 10

(2) $\frac{1000}{\sqrt{2}}$, 10

(3) $\frac{50}{\sqrt{2}}$, 0

(4) 50, 0

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Alternating Current, Exercise # 1, Q. No. 81]

Ans. **[2]**

Sol. $e = 100 \sin 30\, t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

$$P = \frac{e_0 i_0 \cos\phi}{2}$$

$$= \frac{100 \times 20}{2} \times \cos\frac{\pi}{4}$$

$$P = \frac{1000}{\sqrt{2}} \text{ watt}$$

Wattless current

$$I = \frac{(I_0 \sin\phi)}{\sqrt{2}}$$

$$= \frac{20}{2} = 10 \text{ A}$$

Q.17 Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between :

(1) 11.6 V and 11.7 V

(2) 11.5 V and 11.6 V

(3) 11.4 V and 11.5 V

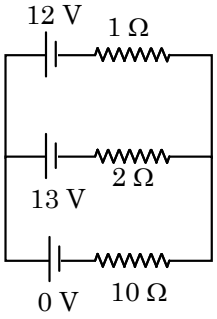
(4) 11.7 V and 11.8 V

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Current Electricity, Exercise # 4, Q. No.22*]

Ans. **[2]**

Sol.



$$V_{Load} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{10}}$$

$$= \frac{\frac{37}{2}}{\frac{10+5+1}{10}} = \frac{37}{2} \times \frac{10}{16}$$

$$\approx 11.56 \text{ volt}$$

Q.18 An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is :

(1) $r_e > r_p = r_\alpha$

(2) $r_e < r_p = r_\alpha$

(3) $r_e < r_p < r_\alpha$

(4) $r_e < r_\alpha < r_p$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Magnetic Effect of Current, Exercise # 5 (A),Q. No.4]

Ans. **[2]**

Sol. e^-, p^+, α

$$r = \frac{\sqrt{2mKE}}{qB}$$

$$r \propto \frac{\sqrt{m}}{q}$$

For electron $\frac{\sqrt{m}}{q} = \frac{\sqrt{m_e}}{1e}$

For proton $\frac{\sqrt{m}}{q} = \frac{\sqrt{m_p}}{1e}$

For α $\frac{\sqrt{m}}{q} = \frac{\sqrt{4u}}{2e}$

$$= \frac{2\sqrt{1u}}{2e}$$

$r_e < r_p = r_\alpha$

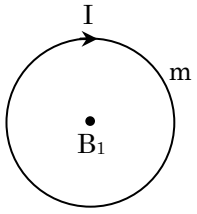
Q.19 The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is :

(1) 2 (2) $\sqrt{3}$ (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Magnetic Effect of Current]

Ans. **[3]**

Sol.



$\because$ $B = \frac{\mu_0 I(N)}{2r}$

$\therefore$ $M = NIA$

$= NI(\pi r^2)$

$r = \sqrt{\frac{M}{NI\pi}}$

$B = \frac{\mu_0 IN}{2\sqrt{\frac{M}{NI\pi}}}$

$\Rightarrow B \propto \frac{1}{\sqrt{M}}$

$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$

Q.20 For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :

(1) $\frac{\omega_0 L}{R}$

(2) $\frac{\omega_0 R}{L}$

(3) $\frac{R}{(\omega_0 C)}$

(4) $\frac{CR}{\omega_0}$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Alternating Current, Theory Booklet point no. 11, Page No. 15*]

Ans. **[1]**

Sol. $\therefore$ $\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \frac{X_L}{R}$

$= \frac{\omega_0 L}{R}$

Q.21 An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If $\in_{r_1}$ and $\in_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct ?

(1) $\frac{\in_{r_1}}{\in_{r_2}} = 4$

(2) $\frac{\in_{r_1}}{\in_{r_2}} = 2$

(3) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$

(4) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : EMW, Theory, Page No.76, Point No. 5.1*]

Ans. **[3]**

Sol. $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$

$\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$

In Air

$v_1 = \frac{2\pi\nu}{2\pi\nu/c} = c = \frac{1}{\sqrt{\mu_0 \in_{r_1}}}$ ..... (1)

In medium

$v_2 = \frac{kc}{2k} = \frac{c}{2} = \frac{1}{\sqrt{\mu_0 \in_{r_2}}}$ ..... (2)

by (1) & (2)

$\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$

Q.22 Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is

(1) 0^o

(2) 30^o

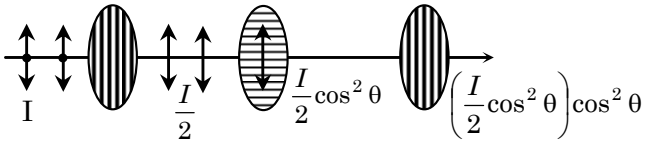
(3) 45^o

(4) 60^o

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Polarization, Ex.4, Q.No.8]

Ans. **[3]**

Sol.



$$\frac{I}{2}\cos^4\theta = \frac{I}{8}$$

$$\cos^4\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^o$$

Q.23 The angular width of the central maximum in a single slit diffraction pattern is 60^o. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ? (i.e. distance between the centres of each slit.)

(1) 25 μm

(2) 50 μm

(3) 75 μm

(4) 100 μm

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Wave Optics (Diffraction), Formula based in Class Notes]

Ans. **[1]**

Sol. For Ist minima

$\theta = 30$

$$\sin\theta = \frac{n\lambda}{a}$$

$$\frac{1}{2} = \frac{1\times\lambda}{1\mu m}$$

$\lambda = 0.5\ \mu m$

$D = 0.5\ m$

$\beta = 1 \times 10^{-2}\ m$

$d = ?$

$$\beta = \frac{D\lambda}{d}$$

$$10^{-2} = \frac{1\times 0.5\times 10^{-6}}{2d}$$

$$d = \frac{0.25\times 10^{-6}}{10^{-2}}$$

$d = 0.25 \times 10^{-4}\ m$

$d = 25 \times 10^{-6}\ m$

$d = 25\ \mu m$

Q.24 An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(2) $\Lambda_n \approx A + B\lambda_n$

(3) $\Lambda_n^2 \approx A + B\lambda_n^2$

(4) $\Lambda_n^2 \approx \lambda$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Atomic Structure]

Ans. **[1]**

Sol. $$E \Rightarrow -\frac{1}{n^2}(13.6\ eV)$$

$$E_n \Rightarrow -\frac{(137)^2 h^2 (13.6\ eV)}{m^2c^2\lambda_n^2}\ ,\quad \lambda = \frac{h}{mv} = \frac{hn\times 137}{mc}\ ,\quad n = \frac{mc\lambda}{137h}$$

$$E_n - E_1 \Rightarrow \frac{hc}{\lambda_n}$$

$$\frac{hc}{\lambda_n} = \frac{(137)^2 h^2 (13.6\ eV)}{m^2 c^2}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)$$

$$\frac{1}{\lambda_n} = \frac{(137)^2 h (13.6\ eV)}{m^2 c^3}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)$$

So, $\lambda_n = \frac{m^2 c^3}{(137)^2 h (13.6\ eV)}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)^{-1}$

using binomial approximation

$$\lambda_n = \frac{m^2 c^3}{(137)^2 h (13.6\ eV)}\left(\frac{1}{\lambda_g^2} + \frac{1}{\lambda_n^2}\right)$$

Let $\frac{m^2 c^3}{(137)^2 h (13.6\ eV)}$ be a constant Let it be B

then $\lambda_n = \left(\frac{B}{\lambda_g^2} + \frac{B}{\lambda_n^2}\right)$

λ_g^2 is also constant hence

$\frac{B}{\lambda_g^2}$ is another constant let called it A then

$$\lambda_n = \left(A + \frac{B}{\lambda_n^2}\right)$$

Q.25 If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is :

(1) 25 ν_L

(2) 16 ν_L

(3) $\nu_L/16$

(4) $\nu_L/25$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Atomic Structure, Theory Point No.7, Page No.12 &13]

Ans. **[4]**

Sol. Z = 1

$n \to \infty$ to n = 1

$$\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\nu_L = Rc\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$\boxed{\nu_L = Rc}$

$n \to \infty$ to n = 5

$$\nu_P = Rc\left(\frac{1}{5^2} - \frac{1}{\infty}\right)$$

$$\nu_P = \frac{Rc}{25}$$

$$\nu_P = \frac{\nu_L}{25}$$

Q.26 It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The value of p_d and p_c are respectively :

(1) (0.89, 0.28)

(2) (0.28, 0.89)

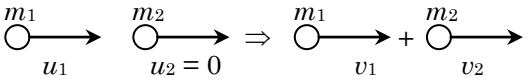
(3) (0, 0)

(4) (0, 1)

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Work-Power-Energy, Ex.4, Q.No.25]

Ans. **[1]**

Sol.



$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1$$

fractional loss of energy

$$= \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2}{\frac{1}{2}m_1u_1^2}$$

$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

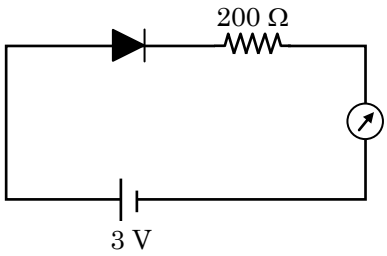
For neutron deutron collision : $m_1 = m$ and $m_2 = 2m$

$$p_d = 1 - \left(\frac{m - 2m}{m + 2m}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9} = 0.89$$

For neutron – carbon collision : $m_1 = m$ and $m_2 = 12m$

$$p_c = 1 - \left(\frac{m - 12m}{m + 12m}\right)^2 = 1 - \frac{121}{169} = 0.28$$

Q.27 The reading of the ammeter for a silicon diode in the given circuit is

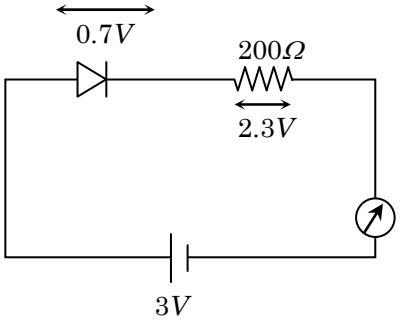


(1) 0
(2) 15 mA
(3) 11.5 mA
(4) 13.5 mA

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Semi Conductor & Electronics, Ex.5, Q.No.1]

Ans. **[3]**

Sol.



$i = \frac{2.3}{200} A = \frac{2300}{200} mA$

$i = 11.5\ mA$

Q.28 A telephonic communication service is working at carrier frequency of 10 GHz. Only 10 % of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ?

(1) 2×10^3
(2) 2×10^4
(3) 2×10^5
(4) 2×10^6

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Communication System, Ex.4, Q.No.8]

Ans. **[3]**

Sol. Working frequency range

$= \frac{10}{100} \times 10 \times 10^9 = 10^9$ Hz

Band width of each channel = 5×10^3 Hz

No. of channels = $\frac{10^9}{5 \times 10^3} = 2 \times 10^5$

Q.29 In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1 Ω

(2) 1.5 Ω

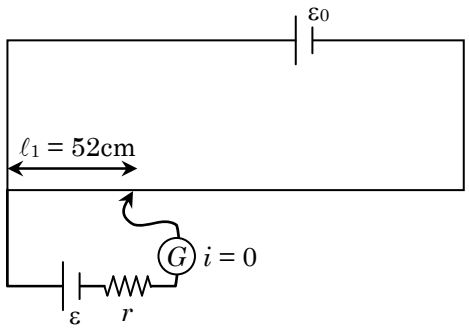
(3) 2 Ω

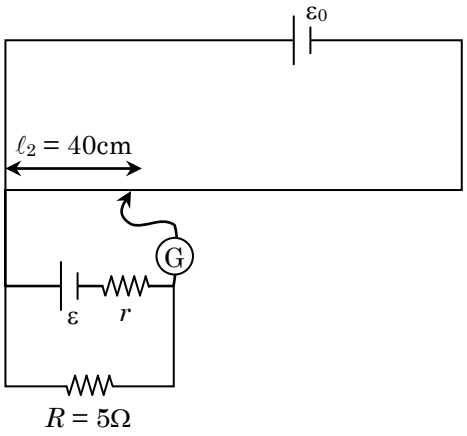
(4) 2.5 Ω

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Current Electricity, Ex.2, Q.No.52]

Ans. **[2]**

Sol.





$$r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$$

$$r = 5\left(\frac{52}{40} - 1\right)$$

$$r = \frac{52}{8} - 5 = 1.5\ \Omega$$

Q.30 On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ?

(1) 990 Ω

(2) 505 Ω

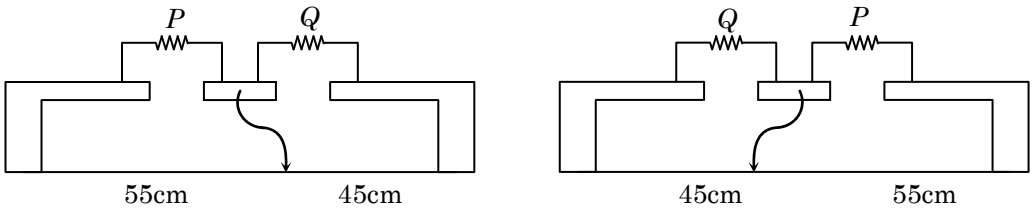
(3) 550 Ω

(4) 910 Ω

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Current Electricity, Ex.1, Q.No.75]

Ans. **[3]**

Sol. As on interchanging resistors balance point move towards left by 10 cm.



$$\frac{P}{Q} = \frac{55}{45}$$

$$\frac{P}{Q} = \frac{11}{9} \Rightarrow 9P = 11Q$$

$$P + Q = 1000$$

$$\frac{11Q}{9} + Q = 1000$$

$$\frac{20Q}{9} = 1000$$

$$Q = 50 \times 9 = 450\ \Omega$$

$$\therefore \quad P = 550\ \Omega$$

Part B – CHEMISTRY

Q.31 The ratio of mass percent of C and H of an organic compound ($C_xH_yO_z$) is 6 : 1. If one molecule of the above compound ($C_xH_yO_z$) contains half as much oxygen as required to burn one molecule of compound C_xH_y completely to CO_2 and H_2O. The empirical formula of compound $C_xH_yO_z$ is -

(1) $C_3H_6O_3$

(2) C_2H_4O

(3) $C_3H_4O_2$

(4) $C_2H_4O_3$

Students may find similar question in CP exercise sheet :
[*JEE Main, Chapter : Mole Concept, Ex. # 4, Page 24, Q. No. 19*]

Ans. **[4]**

Sol. $CxHy + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

1 mol $\left(x + \frac{y}{4}\right)$

amount of oxygen in the compound = $2 \times \frac{\left(x + \frac{y}{4}\right)}{z} = z$ or $\frac{x + y}{4} = z$

$x + \frac{y}{4} = z$

As C : H mass ratio 6 : 1

$\therefore$ mole ratio $\frac{x}{y} = \frac{\frac{6}{12}}{\frac{1}{1}} = \frac{1}{2}$

$\frac{x}{y} = \frac{1}{2}$

$y = 2x$

and $x : y : z$

putting y and z in terms of x

$x : 2x : \frac{3x}{2}$

$1 : 2 : \frac{3}{2}$

$2 : 4 : 3$

On putting the value of y in above equation

$\therefore \quad x + \frac{2x}{4} = z$

$\frac{3x}{2} = z$

Q.32 Which type of 'defect' has the presence of cations in the interstitials sites ?

(1) Schottky defect

(2) Vacancy defect

(3) Frenkel defect

(4) Metal deficiency defect

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Solid State, Theory notes, Page no. 14]

Ans. **[3]**

Sol. Frenkel defect result in arrangement of cation at interstitial site

Q.33 According to molecular orbital theory, which of the following will **not** be a viable molecule ?

(1) He_2^{2+}

(2) He_2^{+}

(3) H_2^{-}

(4) H_2^{2-}

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Bonding, Ex. # 4 Q. No. 34]

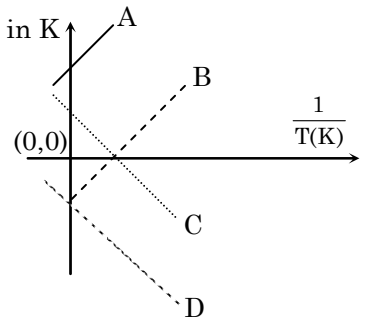
Ans. **[4]**

Sol. H_2^{2-} $4e^- = \sigma 1s^2,\ \sigma^* 1s^2$

$$\text{B.O.} = \frac{2-2}{2} = 0$$

$\therefore$ It does not exist as B.O. is zero

Q.34 Which of following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction ?



(1) A and B

(2) B and C

(3) C and D

(4) A and D

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Equilibrium, Ex # 4, Page 45, Q. No. 10]

Ans. **[1]**

Sol. $\ln K = -\frac{\Delta H}{RT} + \text{const.}$

Since ΔH = –ve for exothermic reaction Slope is +ve

i.e. (A) & (B)

Q.35 The combustion of benzene (ℓ) gives $CO_2(g)$ and $H_2O(\ell)$. Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be - ($R = 8.314\ JK^{-1}\ mol^{-1}$)

(1) 4152.6

(2) –452.46

(3) 3260

(4) –3267.6

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Energetic, Ex # 4, Page 35, Q. No. 30]

Ans. **[4]**

Sol. $C_6H_6(\ell) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(\ell)$

$\Delta ng = \frac{6-15}{2} = -\frac{3}{2}$

$\Delta H = \Delta E + \Delta ng\ RT$

$= -3263.9 - \frac{3}{2} \times \frac{8.314 \times 298}{1000}$

$= -3263.9 - 3.716358$

$= -3267.616$ kJ / mol

Q.36 For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point ?

(1) $[Co(H_2O)_6]Cl_3$

(2) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(4) $[Co(H_2O)_3Cl_3].3H_2O$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Solution and Colligative Properties, Ex # 2, Page 35 Q. No. 85]

Ans. **[1]**

Sol. ΔT_f (Freezing point depression) $\propto$ n(no. of particles)

$\therefore$ If no. of particles $\downarrow$, freezing point is more

Q.37 An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is -

(1) 5×10^{-8}

(2) 3×10^{-20}

(3) 6×10^{-21}

(4) 5×10^{-19}

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Equilibrium, Solved Example, Page 33 Q. No. 61]

Ans. **[2]**

Sol. $H_2S \rightleftharpoons 2H^+ + \overset{-2}{S}$

$K_{eq.} = 1 \times 10^{-7} \times 1.2 \times 10^{-13}$

$$= \frac{[H^+]^2[S^{-2}]}{[H_2S]}$$

$$1.2\times10^{-20} = \frac{(0.2)^2 \times S^{-2}}{0.1}$$

$$[S^{-2}] = \frac{0.1\times1.2\times10^{-20}}{0.04}$$

$$= 3\times10^{-20} \text{ M}$$

Q.38 An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+} ?

(1) 5×10^{-9} M

(2) 2×10^{-9} M

(3) 1.1×10^{-9} M

(4) 1.0×10^{-10} M

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Ionic Equilibrium, Ex #4, Page 50, Q. No. 9]

Ans. **[3]**

Sol. $K_{sp} = [Ba^{+2}]\,[SO_4^{-2}]$, In 500 ml volume SO_4^{-2} conc. is

$M_1V_1 = M_2V_2$

$1 \times 50 = M_2 \times 500$

$M_2 = 0.1$ M

$1 \times 10^{-10}\,[Ba^{+2}]\,[0.1]$

$[Ba^{+2}] = 1 \times 10^{-9}$ M

Initial conc. will be,

$M_1V_1 = M_2V_2$

$M_1 \times 450 = 1 \times 10^{-9} \times 500$

$M_1 = 1.11 \times 10^{-9}$ M

Q.39 At 518°C, the rate of decomposition of sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is -

(1) 2

(2) 3

(3) 1

(4) 0

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Chemical Kinetics, Page 15, Example No. 8*]

Ans. **[1]**

Sol.

$$\underset{363}{CH_3CHO} \longrightarrow CH_4 + CO$$

x (−)	x (+)	x (+)
363 − x	x	x

if $x = \frac{5}{100} \times 363 = 5 \times 3.63$

$= 18.15$

rate of decomposition $r = -\frac{d(CH_3CHO)}{dt} = K[A]^X$

$1.00 = K \cdot (363 - 18.15)^X$

similarly $0.5 = K\,(363 - 119.29)^X$

$$2 = \left(\frac{344.85}{243.21}\right)^X$$

$2 = (1.41)^X$

or $x = 2$

Q.40 How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane ? (Atomic weight of B = 10.8 u)

(1) 6.4 hours

(2) 0.8 hours

(3) 3.2 hours

(4) 1.6 hours

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Electro Chemistry, Ex # 4, Q. No.19]

Ans. **[3]**

Sol. $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

$\frac{27.66}{27.6}$ $\downarrow$

3 mole

= 1 mole = 3 × 22.4 (at N.T.P. volume in litre)

$$\frac{V}{V_e} = \frac{i \times t}{96500}$$

$$\frac{3 \times 22.4}{5.6} = \frac{100 \times t}{96500}$$

t = 11580 sec = 3.21 hours

Q.41 The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to :

(1) $[CaF_2]$

(2) $[3(CaF_2).Ca(OH)_2]$

(3) $[3Ca_3(PO_4)_2.CaF_2]$

(4) $[3\{Ca(OH)_2\}.CaF_2]$

Ans. **[3]**

Sol. $\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} \xrightarrow[\substack{1\text{ ppm}\\ \text{or}\\ 1\text{ mg dm}^{-3}}]{F^{\ominus}} \underset{\substack{\text{Fluorapatite}\\ \text{[HARD ENAMEL]}}}{[3Ca_3(PO_4)_2.CaF_2]}$

Q.42 Which of the following compounds contains(s) no covalent bond(s) ?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, B_2H_6, PH_3

(2) KCl, H_2SO_4

(3) KCl

(4) KCl, B_2H_6

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Bonding, Ex # 1, Q. No. 37]

Ans. **[3]**

Sol. KCl is an ionic compound it does not content covalent bond.

Q.43 Which of the following are Lewis acids ?

(1) PH_3 and BCl_3

(2) $AlCl_3$ and $SiCl_4$

(3) PH_3 and $SiCl_4$

(4) BCl_3 and $AlCl_3$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Boron Carbon Family Q. No. 7 Ex#2, page 14, Page 10 Theory notes]

Ans. **[2, 4]**

Sol. BCl_3 , $AlCl_3$ and $SiCl_4$ acts as Lewis acid.

Q.44 Total number of lone pair of electrons in I_3^- ion is

(1) 3

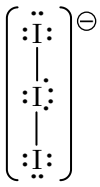
(2) 6

(3) 9

(4) 12

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Bonding Q. No. 11, Ex#5 (IIT 2005)]

Ans. **[3]**

Sol.  (9 pair of e^-)

Q.45 Which of the following salts is the most basic in aqueous solution ?

(1) $Al(CN)_3$

(2) CH_3COOK

(3) $FeCl_3$

(4) $Pb(CH_3COO)_2$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Ionic Equilibrium Q. No. 6, Ex # 5]

Ans. **[2]**

Sol. CH_3COOK

WASB salt

Upon hydrolysis KOH gives strongest basic solution

Q.46 Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively.

(1) $(H_2O + O_2)$ and H_2O

(2) $(H_2O + O_2)$ and $(H_2O + OH^-)$

(3) H_2O and $(H_2O + O_2)$

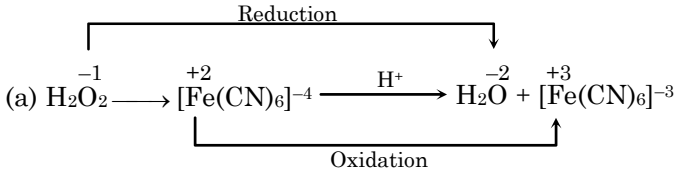
(4) H_2O and $(H_2O + OH^-)$

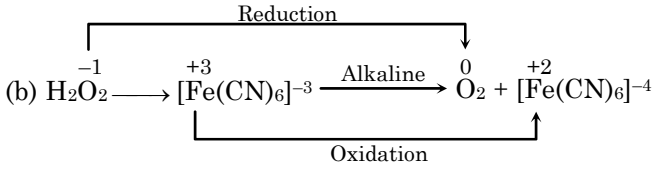
Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Hydrogen & Its compounds, Theory notes page 7]

Ans. **[3]**

Sol.





Q.47 The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are :

(1) +3, +4, and +6

(2) +3, +2, and +4

(3) +3, 0, and +6

(4) +3, 0, and +4

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Coordination Compounds Q. No. 45, Ex # 1]

Ans. **[3]**

Sol. $[Cr(H_2O)_6]Cl_3$

$x + 6(0) + 3(-1) = 0$

$x = +3$

$[Cr(C_6H_6)_2]$

$x = 2(0) = 0$

$x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2 + x + 2(-1) + 2(-2) + (-2) + (0) = 0$

$x = +6$

Q.48 The compound that **does not** produce nitrogen gas by the thermal decomposition is :

(1) $Ba(N_3)_2$

(2) $(NH_4)_2Cr_2O_7$

(3) NH_4NO_2

(4) $(NH_4)_2SO_4$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : p-Block Q. No. 2, Ex # 5, IIT 2011, 1991*]

Ans. **[4]**

Sol. (1) $BaN_3 \longrightarrow Ba + 3N_2$

(2) $(NH_4)_2Cr_2O_7 \longrightarrow N_2 + Cr_2O_3 + N_2O$ — All these gives N_2

(3) $NH_4NO_2 \longrightarrow N_2 + 2H_2O$

Q.49 When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

(1) Zn

(2) Ca

(3) Al

(4) Fe

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Boron family & Metallurgy Q. No. 5, Board Pattern Exercise*]

Ans. **[3]**

Sol. $M + NaOH \longrightarrow \underset{(x)}{Al(OH)_3\downarrow} \xrightarrow{NaOH} NaAlO_2$

$\downarrow$

Al_2O_3

$\therefore$ Element x = Al

Q.50 Consider the following reaction and statements :

$[CO(NH_3)_4Br_2]^+ + Br^- \rightarrow [CO(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.

(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.

(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.

(IV) Only one isomers is produced if the reactant complex ion is a *cis*-isomer.

(1) (I) and (II)

(2) (I) and (III)

(3) (III) and (IV)

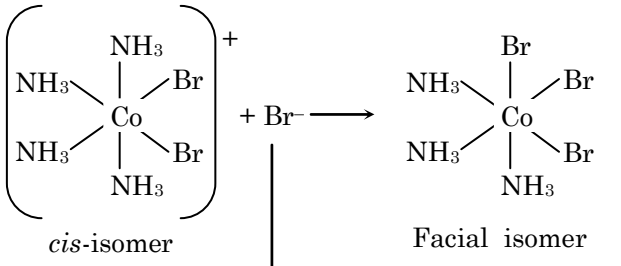
(4) (II) and (IV)

Students may find similar question in CP exercise sheet :

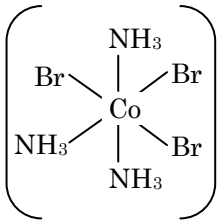
[*JEE Main, Chapter : Coordination Compound, Theory notes page 15*]

Ans. **[4]**

Sol. When *cis*-isomer reacts -

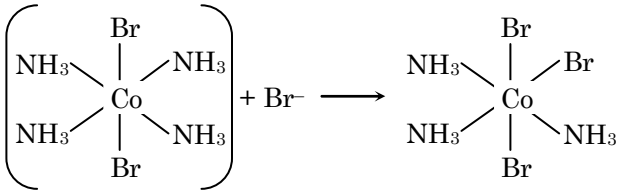


cis-isomer Facial isomer



Meridional isomer

When *trans*-isomer reacts -



trans-isomer Only one isomer

$\therefore$ *cis* isomer gives two isomer where as *trans* isomer gives one isomer

Q.51 Glucose on prolonged heating with HI gives :

(1) *n*-Hexane

(2) 1-Hexene

(3) Hexanoic acid

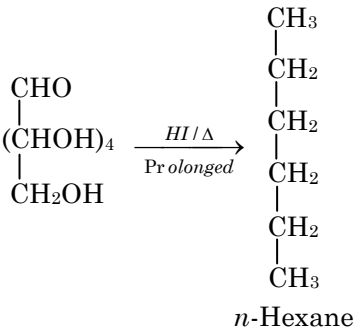
(4) 6-iodohexanal

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Biomolecules notes*]

Ans. **[1]**

Sol.



n-Hexane

This reaction gives the proof about straight chain structure of Glucose.

Q.52 The *trans*-alkenes are formed by the reduction of alkynes with :

(1) H_2 – Pd/C, $BaSO_4$

(2) $NaBH_4$

(3) Na/liq. NH_3

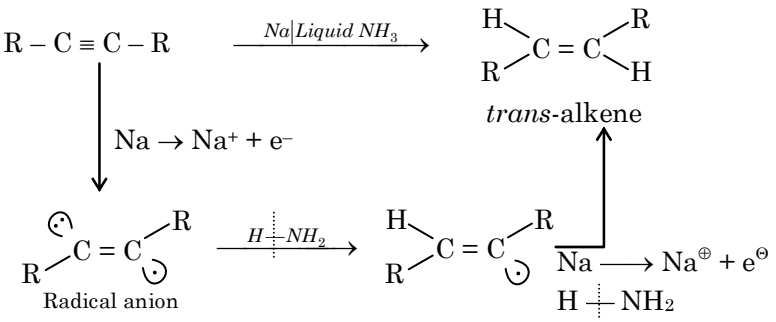
(4) Sn – HCl

Students may find similar question in CP exercise sheet :

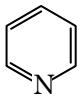
[*JEE Main, Chapter : Hydrocarbon, Q. No. 21 & 28 (solved) Ex # 4*]

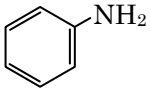
Ans. **[3]**

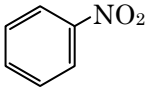
Sol.

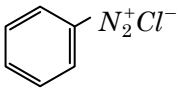


Q.53 Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(1) 

(2) 

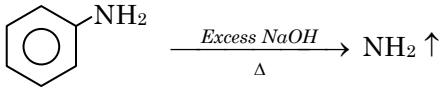
(3) 

(4) 

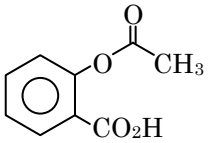
Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Purification & Characterisation of organic compounds Q. No. 10, Ex # 3]

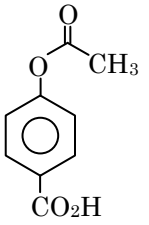
Ans. **[2]**

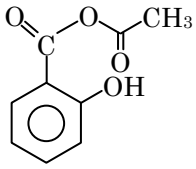
Sol. In Kjeldahl's method nitrogen is estimated in the form of NH_3. We can not use pyridine, Nitro and diazo compound for this

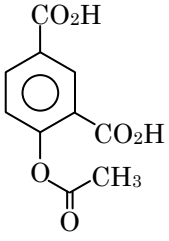


Q.54 Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

(1) 

(2) 

(3) 

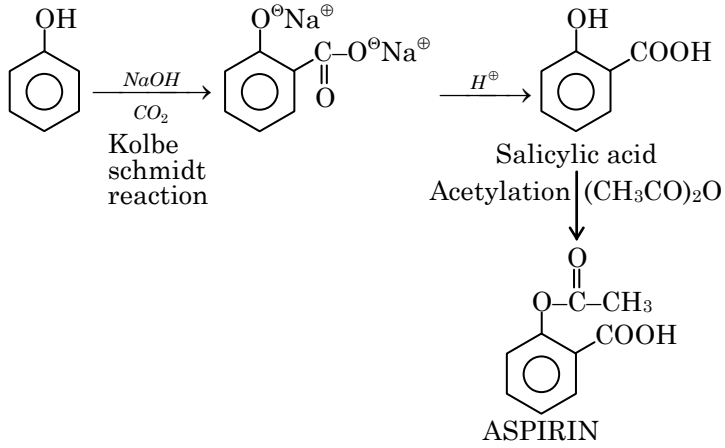
(4) 

Students may find similar question in CP exercise sheet :

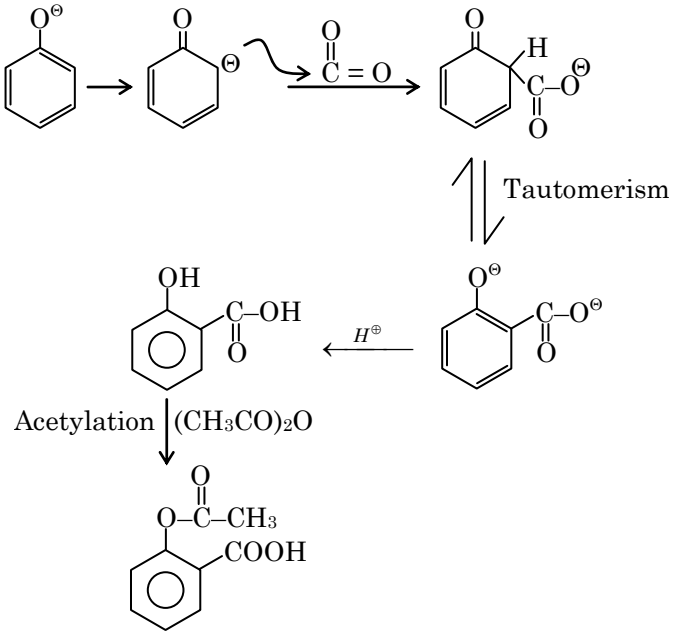
[*JEE Main, Chapter : Oxygen containing compounds-I, Q. No.12, Ex # 4*]

Ans. **[1]**

Sol.



Mechanism :

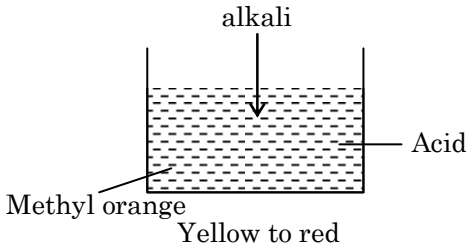


Q.55 An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination ?

	Base	Acid	End point
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to pinkish red
(4)	Strong	Strong	Pink to colourless

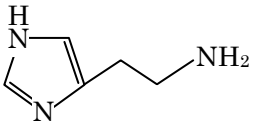
Ans. **[3]**

Sol.

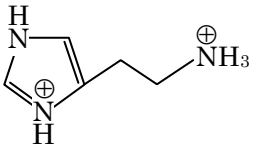


Q.56 The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

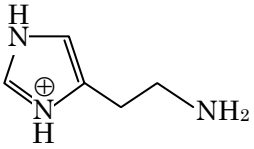
(1)



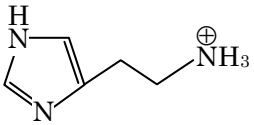
(2)



(3)

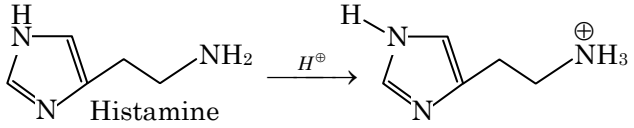


(4)



Ans. **[4]**

Sol.



Blood pH = 7.3

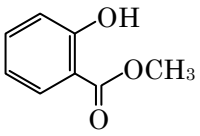
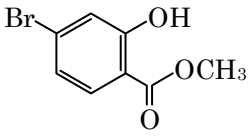
1st pK_a of N = 9.9

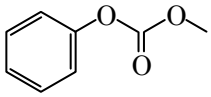
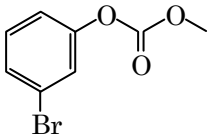
2nd pK_a of N = 5.88

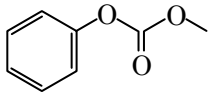
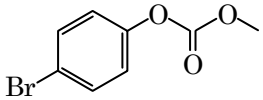
Histamine : It has three nitrogen

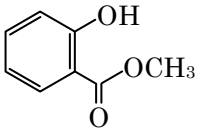
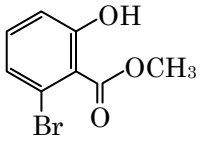
In normal physiological condition it exist as monoprotonated and aliphatic nitrogen is most basic so it will be protonated.

Q.57 Phenol reacts with methyl chloroformate in the presence of NaOH to form product A, A reacts with Br_2 to form product B. A and B are respectively :

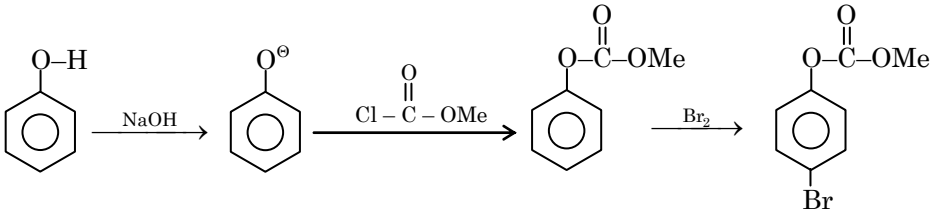
(1)  and 

(2)  and 

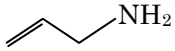
(3)  and 

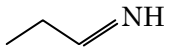
(4)  and 

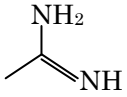
Ans. **[3]**

Sol. 

Q.58 The increasing order of basicity of the following compounds is :

(a) 

(b) 

(c) 

(d) 

(1) (a) < (b) < (c) < (d)

(2) (b) < (a) < (c) < (d)

(3) (b) < (a) < (d) < (c)

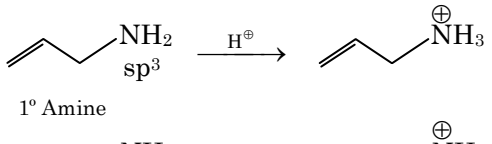
(4) (d) < (b) < (a) < (c)

Students may find similar question in CP exercise sheet :

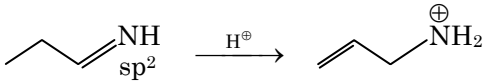
[JEE Advance, Chapter : GOC, Q. No.17, Ex # 5]

Ans. **[3]**

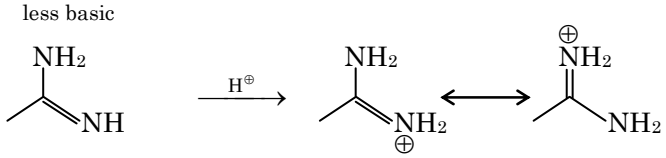
Sol.



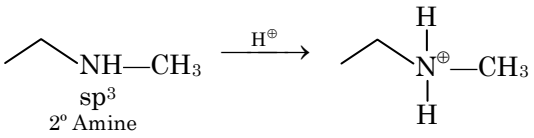
1° Amine



More % s character so less basic



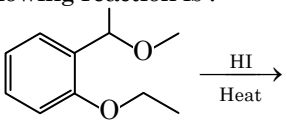
Most stabilized cation

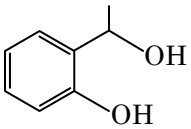


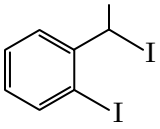
2° Amine

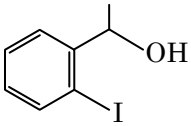
Basic strength C > d > a > b

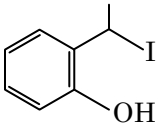
Q.59 The major product formed in the following reaction is :



(1) 

(2) 

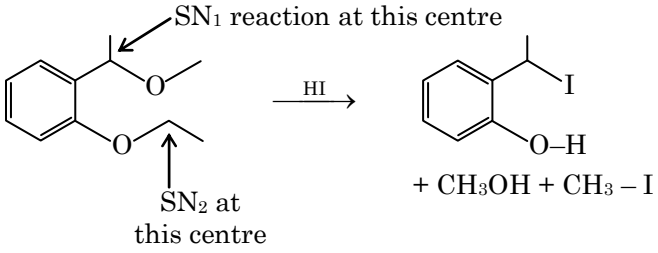
(3) 

(4) 

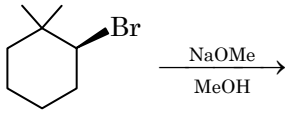
Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Oxygen containing compound-I, Q. No. 6, Ex-4]

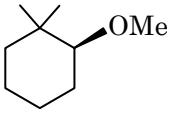
Ans. **[4]**

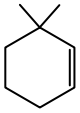
Sol.

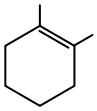


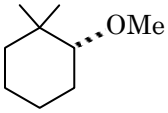
Q.60 The major product of the following reaction is :



(1) 

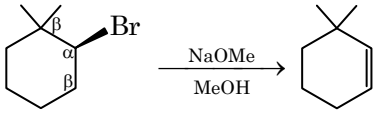
(2) 

(3) 

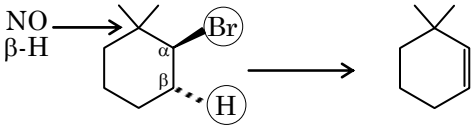
(4) 

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Hydrocarbon, Q. No. 11, Ex # 3]

Ans. **[2]**

Sol. 

Strong base sodium methoxide is used so E_2 elimination will occur and removed of Br and β-H occur simultaneously C anti elimination



Part C – MATHEMATICS

Q.61 Two sets A and B are as under:

$A = \{(a, b) \in \mathbf{R} \times \mathbf{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in \mathbf{R} \times \mathbf{R} : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$. Then

(1) $B \subset A$

(2) $A \subset B$

(3) $A \cap B = \phi$ (an empty set)

(4) neither $A \subset B$ nor $B \subset A$

Ans. **[2]**

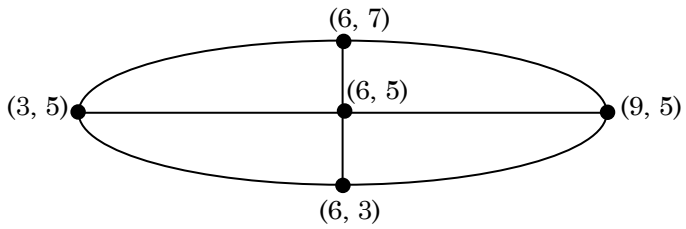
Sol. From set A

$\|a - 5\| < 1$	$\|b - 5\| < 1$
$-1 < a - 5 < 1$	$-1 < b - 5 < 1$
$4 < a < 6$	$4 < b < 6$

$A = \{4 < a < 6, 4 < b < 6\}$

From set B

$$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \le 1$$



$B = \{3 < a < 9, 3 < b < 7\}$

$A \subset B$

Q.62 Let $S = \{x \in \mathbf{R} : x \ge 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

(1) is an empty set

(2) contains exactly one element

(3) contains exactly two elements

(4) contains exactly four elements

Ans. **[3]**

Sol. $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$

Let $\sqrt{x} = t$

$2|t - 3| + t^2 - 6t + 6 = 0$

case (i)

$t < 3$ ...(1)

$-2t + 6 + t^2 - 6t + 6 = 0$

$t^2 - 8t + 12 = 0$

t = 2, 6

Since t < 3,

hence t = 2

$\sqrt{x} = 2$

$x = 4$

case (ii)

t ≥ 3

$2t - 6 + t^2 - 6t + 6 = 0$

$t^2 - 4t = 0$

t = 4, 0

Since t ≥ 3

hence t = 4

$\sqrt{x} = 4$

$x = 16$

Q.63 If α, β ∈ **C** are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(1) –1

(2) 0

(3) 1

(4) 2

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Quadratic equation, Ex. 2, Q. No. 20]

Ans. **[3]**

Sol. We have,

$x^2 - x + 1 = 0$

$\alpha, \beta = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$

$\alpha = \frac{1 + i\sqrt{3}}{2} = -\omega^2$

and $\beta = \frac{1 - i\sqrt{3}}{2} = -\omega$

$\alpha^{101} + \beta^{107} = (-\omega^2)^{101} + (-\omega)^{107}$

$= -\omega^{202} - \omega^{107}$

$= -\omega - \omega^2$

$= -(\omega + \omega^2)$

$= 1$

Q.64 If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(1) (–4, –5)

(2) (–4, 3)

(3) (–4, 5)

(4) (4, 5)

Ans. **[3]**

Sol. $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$

Put x = 0 both side, we get

$\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = (A + 0)(0 - A)^2$

$-4^3 = A^3$

$A = -4$

Put x = 1 both side, we get

$\begin{vmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = (A + B)(1 - A)^2$

$-3(9 - 4) - 2(-6 - 4) + 2(4 + 6) = (-4 + B)(1 + 4)^2$

$-15 + 20 + 20 = (-4 + B) \cdot 5^2$

$25 = (-4 + B) \cdot 25$

$-4 + B = 1$

$B = 5$

Q.65 If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z) then $\frac{xz}{y^2}$ is equal to

(1) –10

(2) 10

(3) –30

(4) 30

Ans. **[2]**

Sol. Linear equation has non zero solution

Hence $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0$

$-3k + 8 + 9k - 4k + 36 - 6k = 0$

$-4k = -44$

$k = 11$

Hence equations are

$x + 11y + 3z = 0$ ...(1)

$3x + 11y - 2z = 0$ ...(2)

$2x + 4y - 3z = 0$ ...(3)

add (1) and (3)

$3x + 15y = 0$

$x = -5y$

from (3)

$2x + 4y - 3z = 0$

$\because \; x = -5y$

$-10y + 4y - 3z = 0$

$-6y - 3z = 0$

$z = -2y$

consider $\frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

Q.66 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 1000

(2) less than 500

(3) at least 500 but less than 750

(4) at least 750 but less than 1000

Students may find similar question in CP exercise sheet :
[*JEE Main, Chapter : P & C, Ex. 4, Q. No. 14*]

Ans. **[1]**

Sol. ${}^6C_4 \times {}^3C_1 \times 4!$

$= 45 \times 24$

$= 1080$

Q.67 The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x>1)$ is

(1) –1

(2) 0

(3) 1

(4) 2

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Binomial Theorem, Ex.5 Q. No.1]

Ans. **[4]**

Sol. $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$= 2\left[{}^5C_0(x)^5+{}^5C_2(x)^3\left(\sqrt{x^3-1}\right)^2+{}^5C_4(x)^1\left(\sqrt{x^3-1}\right)^4\right]$

$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$

$= 2[x^5 + 10x^6 - 10x^3 + 5x(x^6 - 2x^3 + 1)]$

$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

$= 2[1 - 10 + 5 + 5]$

$= 2$

Q.68 Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \ldots\ldots + a_{17}^2 = 140\,m$, then m is equal to :

(1) 66

(2) 68

(3) 34

(4) 33

Ans. **[3]**

Sol. If $a_1 + a_5 + a_9 + a_{13} + \ldots\ldots\ldots + a_{49} = 416$

$6(a_1 + a_{49}) + a_{25} = 416$

$6(a_1 + a_1 + 48d) + (a_1 + 24d) = 416$

$12(a_1 + 24d) + (a_1 + 24d) = 416$

$13(a_1 + 24d) = 416$

$a_1 + 24d = 32$ ...(1)

and $a_9 + a_{43} = 66$

$a_1 + 8d + a_1 + 42d = 66$

$2a_1 + 50d = 66$

$(a_1 + 25d) = 33$ ...(2)

By (1) and (2)

$d = 1$

$a_1 = 8$

$(a_1^2 + a_2^2 + ... + a_{17}^2) = 140$ m

$a_1^2 + (a_1 + 1)^2 + (a_1 + 2)^2 + ... + (a_1 + 16)^2 = 140$ m

$17a_1^2 + 2a_1 (1 + 2 + 3 + ... + 16) + (1^2 + 2^2 + ... + 16^2) = 140$ m

$$17 \times 64 + 2 \times 8\left(\frac{16\times17}{2}\right) + \frac{16\times33\times17}{6} = 140 \text{ m}$$

$1088 + 2176 + 1496 = 140$ m

$4760 = 140$ m

$34 = m$

Q.69 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ If $B - 2A = 100\lambda$, then λ is equal to :

(1) 232

(2) 248

(3) 464

(4) 496

Ans. **[2]**

Sol. Sum of first 20 terms

$A = (1^2 + 3^2 + ... + 19^2) + 2(2^2 + 4^2 + ... + 20^2)$

$A = (1^2 + 2^2 + 3^2 + ... + 19^2 + 20^2) + (2^2 + 4^2 + ... + 20^2)$

$$A = \frac{20\times41\times21}{6} + 2^2(1^2 + 2^2 + ... + 10^2)$$

$$A = 2870 + 4 \times \frac{10\times11\times21}{6}$$

$A = 2870 + 1540$

$A = 4410$

Now, sum of first 40 terms

$B = (1^2 + 2^2 + ... + 40^2) + (2^2 + 4^2 + ... + 40^2)$

$$= \frac{40\times41\times81}{6} + 2^2(1^2 + 2^2 + ... + 20^2)$$

$$= \frac{40\times41\times81}{6} + 4\left(\frac{20\times21\times41}{6}\right)$$

$= 22140 + 11480$

$= 33620$

So, $B - 2A = 100\ \lambda$

$33620 - 8820 = 100\ \lambda$

$24800 = 100\ \lambda$

$\lambda = 248$

Q.70 For each t ∈ **R**, let [t] be the greatest integer less than or equal to t.

Then $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$

(1) is equal to 0.

(2) is equal to 15.

(3) is equal to 120.

(4) does not exist (in **R**)

Ans. **[3]**

Sol. $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$

$= \lim_{x\to 0^+} x\left\{\left(\frac{1}{x}+\frac{2}{x}+....+\frac{15}{x}\right)-\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+....\left\{\frac{15}{x}\right\}\right)\right\}$

where { } denotes fractional part of x

$\lim_{x\to 0^+} x\left(\frac{1}{x}+\frac{2}{x}+...+\frac{15}{x}-0\right)$

$= 1 + 2 + ... + 15$

$= \frac{15(15+1)}{2}$

$= 15 \times 8$

$= 120$

Q.71 Let S = {t ∈ **R** : $f(x) = |x-\pi|\cdot(e^{|x|}-1)\sin|x|$ is not differentiable at t}. Then the set S is equal to :

(1) ϕ (an empty set)

(2) {0}

(3) {π}

(4) {0, π}

Ans. **[1]**

Sol. S = {t ∈ R, $f(x) = |x-\pi|\cdot(e^{|x|}-1)\cdot\sin|x|$}

Doubtful points, $x = 0, \pi$

At $x = \pi$

$$f(x) = |x-\pi|(e^x-1)\sin x$$

$$= \begin{cases} (x-\pi)(e^x-1)\sin x & x>\pi \\ -(x-\pi)(e^x-1)\sin x & x<\pi \end{cases}$$

$f'(\pi + h) = (x-\pi)(e^x-1)\cos x + (x-\pi)\sin x \cdot e^x + (e^x-1)\sin x \cdot 1$

at $x = \pi \;\; = 0 + 0 + 0$

similarly $f'(\pi - h) = 0$

hence at $x = \pi$, differentiable

at $x = 0$

$$f(x)=\begin{cases}-(x-\pi)(e^{x}-1)\sin x & x>0\\ +(x-\pi)(e^{-x}-1)\sin x & x<0\end{cases}$$

$f'(0+h) = -\{(x-\pi)(e^x-1)\cos x + (e^x-1)\sin x\cdot 1 + (x-\pi)\sin x\cdot e^x\}$

$x=0 \quad = -\{(-\pi)\cdot 0 + 0 + 0\}$

similarly $f'(0-h)=0$

hence differentiable at x = 0 also differentiable every where

Q.72 If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) 6

(2) $\frac{7}{2}$

(3) 4

(4) $\frac{9}{2}$

Students may find same question in CP exercise sheet :
[JEE Main, Chapter : Tangent & Normal, Ex. 2, Q. No. 10]

Ans. **[4]**

Sol. $y^2 = 6x$ …(1)

and $9x^2 + by^2 = 16$ …(2)

slope of tangent of first curve

$$2y\cdot\frac{dy}{dx}=6$$

$$m_1=\frac{dy}{dx}=\frac{6}{2y} \qquad \ldots(3)$$

slope of tangent of second curve

$$18x+2by\frac{dy}{dx}=0$$

$$m_2=\frac{dy}{dx}=\frac{-18x}{2by}=\frac{-9x}{by} \qquad \ldots(4)$$

Since curve intersest at right angle

$\therefore \quad m_1 m_2 = -1$

$$\Rightarrow \quad \left(\frac{6}{2y}\right)\left(-\frac{9x}{by}\right)=-1$$

$\Rightarrow \quad -27x = -by^2$

$\Rightarrow \quad -27x = -b(6x)$ (from (1))

$$\Rightarrow \quad b=\frac{27}{6}=\frac{9}{2}$$

Q.73 Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbf{R} - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

(1) 3

(2) – 3

(3) $-2\sqrt{2}$

(4) $2\sqrt{2}$

Ans. **[4]**

Sol. $f(x) = x^2 + \frac{1}{x^2}$, $g(x) = x - \frac{1}{x}$

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}}$$

$$h(x) = x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}}$$

Differentiate both side

$$h'(x) = 1 + \frac{1}{x^2} - \frac{2}{\left(x - \frac{1}{x}\right)^2}\left(1 + \frac{1}{x^2}\right)$$

$$= \left(1 + \frac{1}{x^2}\right)\left[\frac{\left(x - \frac{1}{x}\right)^2 - 2}{\left(x - \frac{1}{x}\right)^2}\right]$$

$$= \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2}\left[\frac{(x^2 - 1)^2 - 2x^2}{x^2}\right]$$

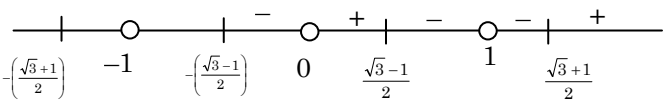
$$= \frac{\left(1 + x^2\right)}{\left(x - \frac{1}{x}\right)^2 x^4}[x^4 + 1 - 4x^2] = 0$$

$= (x^2 - 2)^2 - 3 = 0$

$= x^2 = 2 \pm \sqrt{3}$

$= x^2 = \frac{(\sqrt{3} \pm 1)^2}{2}$

$= x = \pm\left(\frac{\sqrt{3} \pm 1}{\sqrt{2}}\right)$



$$x = \frac{\sqrt{3}+1}{\sqrt{2}}$$

$$\frac{1}{x} = \frac{\sqrt{2}}{\sqrt{3}+1}\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$x - \frac{1}{x} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

Q.74 The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2}\,dx$ is equal to

(1) $\frac{1}{3(1+\tan^3 x)} + C$

(2) $\frac{-1}{3(1+\tan^3 x)} + C$

(3) $\frac{1}{1+\cot^3 x} + C$

(4) $\frac{-1}{1+\cot^3 x} + C$

(where C is a constant of integration)

Ans. **[2]**

Sol. $$\int \frac{\sin^2 x\ \cos^2 x\ dx}{(\sin^5 x + \cos^3 x\ \sin^2 x + \sin^3 x\ \cos^2 x + \cos^5 x)^2}$$

$$= \int \frac{\sin^2 x \cdot \cos^2 x\ dx}{\{(\sin^2 x(\sin^3 x + \cos^3 x) + \cos^2 x(\sin^3 x + \cos^3 x)\}^2}$$

$$= \int \frac{\sin^2 x \cdot \cos^2 x\ dx}{\{(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)\}^2}$$

$$= \int \frac{\sin^2 x\ \cos^2 x\ dx}{(\sin^3 x + \cos^3 x)^2}$$

divide by $\cos^3 x$ in nominator and denominator we get

$$= \int \frac{\sec^2 x \cdot \tan^2 x}{(\tan^3 x + 1)^2} dx$$

Let $1 + \tan^3 x = t$

$3 \tan^2 x \sec^2 x\, dx = dt$

$$= \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3}\frac{1}{t} + C$$

$$= -\frac{1}{3(1+\tan^3 x)} + C$$

Q.75 The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) $\frac{\pi}{8}$

(2) $\frac{\pi}{2}$

(3) 4π

(4) $\frac{\pi}{4}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Definite integration Q. No. 41]

Ans. **[4]**

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x\, dx}{1+2^x}$

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2(0-x)}{1+2^{0-x}} dx \qquad \left[Use \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right]$

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} + \frac{2^x \cdot \sin^2 x}{2^x+1} dx$

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}(1+2^x)dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$ [by using property $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, when f(x) is even]

$2I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$

$= \frac{\pi}{4}$

Q.76 Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is:

(1) $\frac{1}{2}(\sqrt{3}-1)$

(2) $\frac{1}{2}(\sqrt{3}+1)$

(3) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(4) $\frac{1}{2}(\sqrt{2}-1)$

Ans. **[1]**

Sol. $18x^2 - 9\pi x + \pi^2 = 0$

$18x^2 - 3\pi x - 6\pi x + \pi^2 = 0$

$3x(6x - \pi) - \pi(6x - \pi) = 0$

$(6x - \pi)(3x - \pi) = 0$

$x = \frac{\pi}{6}, \frac{\pi}{3}$

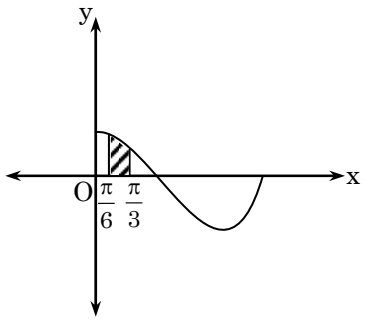
$\alpha < \beta$

$\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$

Now

$y = gof(x)$

$g(f(x)) = g(\sqrt{x}) = \cos x$



$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\, dx = (\sin x)_{\pi/6}^{\pi/3}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

Q.77 Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{4}{9\sqrt{3}}\pi^2$

(2) $\frac{-8}{9\sqrt{3}}\pi^2$

(3) $-\frac{8}{9}\pi^2$

(4) $-\frac{4}{9}\pi^2$

Ans. **[3]**

Sol. $\sin x \frac{dy}{dx} + y \cdot \cos x = 4x$

Divide by $\sin x$

$\Rightarrow \frac{dy}{dx} + y \cdot \cot x = \frac{4x}{\sin x}, x \neq 0$

Now integrating factor

I.F. $= e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

$\therefore$ solution

$y \cdot \sin x = \int \frac{4x}{\sin x} \cdot \sin x\, dx$

$y \cdot \sin x = \int 4x\, dx$

$y \cdot \sin x = 2x^2 + c$ ....(1)

at $x = \frac{\pi}{2}$, $y = 0$

$\therefore c = -\frac{\pi^2}{2}$

From (1)

$\Rightarrow y \cdot \sin x = 2x^2 - \frac{\pi^2}{2}$

at $x = \frac{\pi}{6}$

$y \cdot \frac{1}{2} = 2 \times \frac{\pi^2}{36} - \frac{\pi^2}{2}$

$y = \frac{\pi^2}{9} - \pi^2$

$= -\frac{8\pi^2}{9}$

Q.78 A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) $3x + 2y = 6$

(2) $2x + 3y = xy$

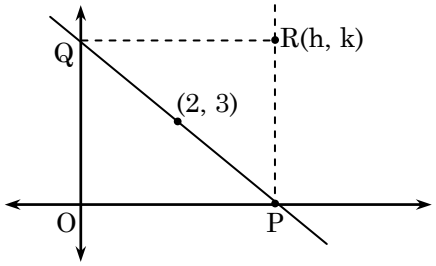
(3) $3x + 2y = xy$

(4) $3x + 2y = 6xy$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Straight line Q. No. 12]

Ans. **[3]**

Sol.



Let R be (h, k)

$\therefore$ equation of line PQ

$$\frac{x}{h}+\frac{y}{k}=1$$

It passes through (2, 3)

$\therefore \frac{2}{h}+\frac{3}{k}=1$

$\therefore$ locus $\frac{2}{x}+\frac{3}{y}=1$

$2y + 3x = xy$

Q.79 Let the orthocenter an centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $\sqrt{10}$

(2) $2\sqrt{10}$

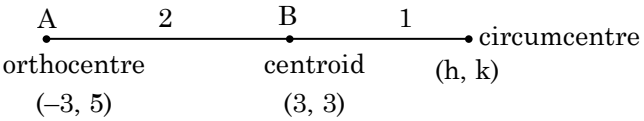
(3) $3\sqrt{\frac{5}{2}}$

(4) $\frac{3\sqrt{5}}{2}$

Ans. **[3]**

Sol. orthocentre A(–3 , 5)

Centroid B(3, 3)



By section formula

$$\frac{2h+(-3)}{3}=3 \qquad \frac{2k+5}{3}=3$$
$$2h-3=9 \qquad 2k+5=9$$
$$h=6 \qquad k=2$$

C(6, 2)

A(–3, 5)

diameter AC = $\sqrt{9^2+3^2}=3\sqrt{9+1}=3\sqrt{10}$

Radius = $\frac{3\sqrt{10}}{2}=3\sqrt{\frac{5}{2}}$

Q.80 If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is:

(1) 195

(2) 185

(3) 85

(4) 95

Ans. **[4]**

Sol. Curve $x^2 = y - 6$

$\Rightarrow 2x = \frac{dy}{dx} - 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,7)} = 2$

Equation of tangent at (1, 7)

$y - 7 = 2(x - 1)$

$\Rightarrow 2x - y + 5 = 0$ ....(1)

Line (1) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$

$\therefore \boxed{p = r}$

$$\left|\frac{2(-8)-(-6)+5}{\sqrt{4+1}}\right| = \sqrt{64+36-c}$$

$\Rightarrow \frac{25}{5} = 100 - c$

$\Rightarrow 5 = 100 - c$

$\Rightarrow c = 95$

Q.81 Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is:

(1) $\frac{1}{2}$

(2) 2

(3) 3

(4) $\frac{4}{3}$

Ans. **[2]**

Sol. $y^2 = 16x$

Slope of tangent $\frac{dy}{dx} = \frac{16}{2y} = \frac{8}{y}$

$\left(\frac{dy}{dx}\right)_{(16,16)} = \frac{1}{2}$

Equation of tangent

$y - 16 = \frac{1}{2}(x - 16)$

$\Rightarrow 2y - 32 = x - 16$

$\Rightarrow x - 2y + 16 = 0$ ....(1)

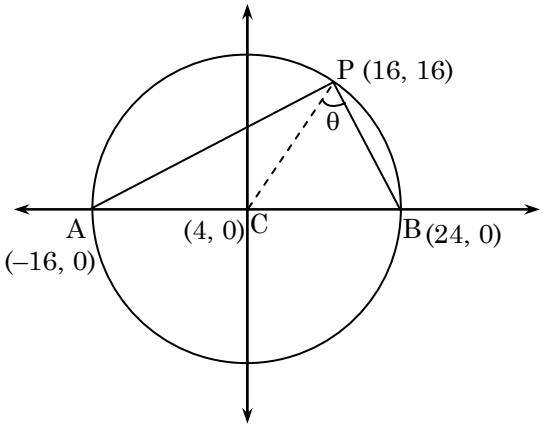
Equation of normal

$y - 16 = -2(x - 16)$

$2x + y - 48 = 0$ ....(2)

Tangent & normal intersect the axis of parabola

$\therefore$ A(–16, 0) B(24, 0)



(slope of AP) (slope of PB)

$\Rightarrow \left(\frac{16-0}{16+16}\right)\left(\frac{16-0}{16-24}\right) = -1$

$\therefore$ AB is diameter of circle i.e. centre C(4, 0)

$m_{PC} = \frac{16-0}{16-4} = \frac{4}{3}$

$$m_{PB} = \frac{16-0}{16-24} = -2$$

$$\tan\theta = \left|\frac{\frac{4}{3}-(-2)}{1+\left(\frac{4}{3}\right)(-2)}\right| = 2$$

Q.82 Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $45\sqrt{5}$

(2) $54\sqrt{3}$

(3) $60\sqrt{3}$

(4) $36\sqrt{5}$

Ans. **[1]**

Sol. Hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$

PQ is a chord of contact w.r.t T (0, 3)

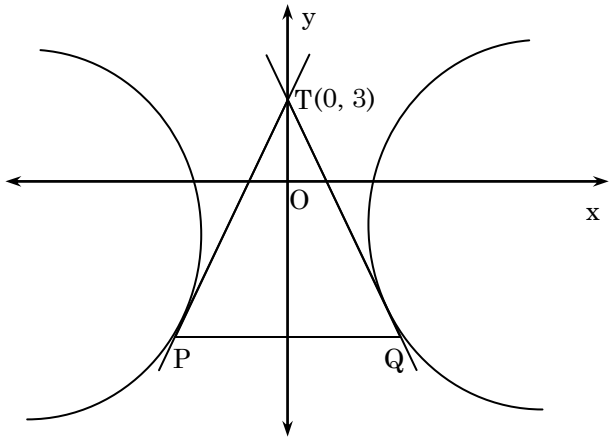
Eq. of PQ

$\boxed{T = 0}$

$\Rightarrow \frac{x(0)}{9} - \frac{y(3)}{36} = 1$

$y = -12$

$y = -12$ intersect the hyperbola at P & Q



$\therefore$ $P(-\sqrt{45}, -12)$ $\quad$ $Q(\sqrt{45}, -12)$

$$\text{Area of } \Delta PTQ = \frac{1}{2}\begin{vmatrix} -\sqrt{45} & -12 & 1 \\ 0 & 3 & 1 \\ \sqrt{45} & -12 & 1 \end{vmatrix} = 45\sqrt{5}$$

Q.83 If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is:

(1) $\frac{1}{4\sqrt{2}}$

(2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{\sqrt{2}}$

Ans. **[2]**

Sol. Eq. of line L_1 is

$$\frac{x+5}{1} = \frac{y-0}{1} = \frac{z-4}{0}$$

Eq. of line L_2 is

$$\frac{x-\frac{5}{7}}{3} = \frac{y-\frac{8}{7}}{-5} = \frac{z-0}{-7}$$

Eq. of plane contain line L_1 & L_2 given by

$-7(x + 5) + 7(y - 0) - 8(z - 4) = 0$

Where $(-7, 7, -8)$ are Dr's of normal of plane obtained by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix}$$

So eq. of plane is $-7x + 7y - 8z - 3 = 0$

Perpendicular distance from origin is $\frac{1}{3\sqrt{2}}$

Q.84 The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane, $x + y + z = 7$ is:

(1) $\frac{2}{\sqrt{3}}$

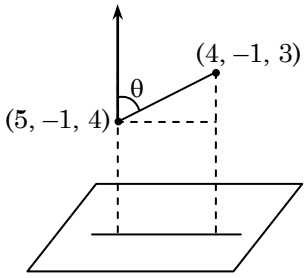
(2) $\frac{2}{3}$

(3) $\frac{1}{3}$

(4) $\sqrt{\frac{2}{3}}$

Ans. **[4]**

Sol.



$1, 0, 1 \rightarrow$ dr's of line

$\cos\theta = \dfrac{1+0+1}{\sqrt{2}\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

$AB = \sqrt{1+1} = \sqrt{2}$

$\text{Projection} = AB \sin\theta = \sqrt{2}\sqrt{1-\dfrac{2}{3}}$

$= \sqrt{2}\sqrt{\dfrac{1}{3}} = \sqrt{\dfrac{2}{3}}$

Q.85 Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to:

(1) 336

(2) 315

(3) 256

(4) 84

Ans. **[1]**

Sol. Let $\vec{u} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{u}$ is coplanar with vectors $\vec{a}$ & $\vec{b}$

$$\therefore \begin{vmatrix} a_1 & b_1 & c_1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow a_1(4) - b_1(2) + c_1(2) = 0$

$\Rightarrow 4a_1 - 2b_1 + 2c_1 = 0$ ....(1)

$\vec{u}$ is perpendicular to $\vec{a}$

$\therefore \vec{u} \cdot \vec{a} = 0$

$\Rightarrow 2a_1 + 3b_1 - c_1 = 0$ ....(2)

and $\vec{u} \cdot \vec{b} = 24$

$\Rightarrow b_1 + c_1 = 24$ . ...(3)

Solving (1) (2) & (3) we get

$a_1 = -4, b_1 = 8, c_1 = 16$

$\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$|\vec{u}|^2 = 16 + 64 + 256$

$|\vec{u}|^2 = 336$

Q.86 A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{3}{10}$

(2) $\frac{2}{5}$

(3) $\frac{1}{5}$

(4) $\frac{3}{4}$

Ans. **[2]**

Sol. If Red ball is drawn & finally Black ball is drawn $= \frac{4}{10} \times \frac{6}{12} = \frac{24}{120}$

If Black ball is drawn & finally Red ball is drawn $= \frac{6}{10} \times \frac{4}{12} = \frac{24}{120}$

$$P(A) = \frac{24}{120} + \frac{24}{120} = 2 \times \frac{24}{120} = \frac{2}{5}$$

Q.87 If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items x_1, x_2, ……, x_9 is :

(1) 9

(2) 4

(3) 2

(4) 3

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Statistics, Ex.4, Q. No. 6]

Ans. **[3]**

Sol. $V_{ar}(x) = \frac{1}{n}\sum d_i^2 - \left(\frac{1}{n}\Sigma d_i\right)^2$ where d_i = derivation ; $d_i = (x_i - A)$

$= \frac{1}{9}(45) - \left(\frac{1}{9} \times 9\right)^2$

$= 5 - 1$

$V_{ar}(x) = 4$

Stand. dev. (σ) = 2

Q.88 If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in [0, π] is $k\pi$, then k is equal to :

(1) $\frac{2}{3}$

(2) $\frac{13}{9}$

(3) $\frac{8}{9}$

(4) $\frac{20}{9}$

Ans. **[2]**

Sol. $8\cos x\left\{\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right\}=1$

$\Rightarrow 4\cos x\left\{2\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-1\right\}=1$

$\Rightarrow 4\cos x\left\{\cos 2x+\cos\frac{\pi}{3}-1\right\}=1$

$\Rightarrow 4\cos x\left\{\cos 2x-\frac{1}{2}\right\}=1$

$\Rightarrow 4\cos x\left\{2\cos^2 x-1-\frac{1}{2}\right\}=1$

$\Rightarrow 8\cos^3 x-6\cos x=1$

$\Rightarrow 2(4\cos^3 x-3\cos x)=1$

$\cos 3x=\frac{1}{2}$

$\cos 3x = \cos 60°, \cos 30°, \cos 420°$

$x = 20°, 100°, 140°$

$20° + 100° + 140° = k\pi.$

$\Rightarrow 260° = k(180°)$

$\Rightarrow k=\frac{260°}{180°}=\frac{13}{9}$

Q.89 PQR is a triangular park with PQ = PR = 200 *m*. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) 100

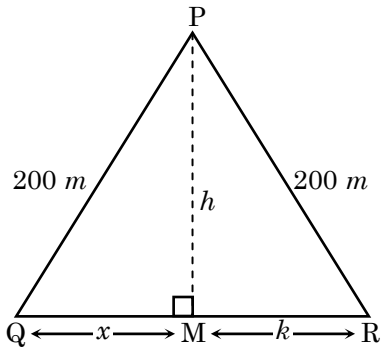
(2) 50

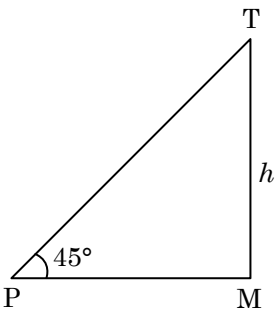
(3) $100\sqrt{3}$

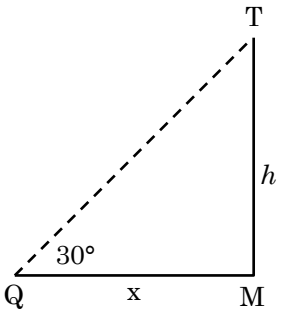
(4) $50\sqrt{2}$

Ans. **[1]**

Sol.







$\therefore$ PM = h $\quad$ $\tan 30° = \frac{h}{x}$

$$x = \sqrt{3}\,h$$

Now In ΔPOM

$(200)^2 = h^2 + 3h^2$

$4h^2 = 4 \times (100)^2$

$h = 100$ meter

Q.90 The Boolean expression $\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p$

(2) p

(3) q

(4) $\sim q$

Students may find same question in CP exercise sheet :
[JEE Main, Chapter : Mathematical Reasoning, Ex.3, Q. No. 9]

Ans. **[1]**

Sol. By Truth Table

p	q	$p \vee q$	$\sim (p \vee q)$	$\sim p$	$\sim p \wedge q$	$\sim (p \vee q) \vee (\sim p \wedge q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T