JEE Main Online Exam 2019

Questions & Solutions

10th April 2019 | Shift - II

Physics

Q.1 In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?

(1) $[M^{-3} L^{-2} T^8 A^4]$ (2) $[M^{-2} L^{-2} T^6 A^3]$ (3) $[M^{-1} L^{-2} T^4 A^2]$ (4) $[M^{-2} L^0 T^{-4} A^{-2}]$

Ans. **[1]**

Sol. $X = 5Y Z^2$

$X = \text{capacitance} = \frac{Q}{V}$

$= \frac{Q^2}{\omega} = \frac{Q^2}{F.L}$

$= \frac{A^2T^2}{M^1L^1T^{-2} \times L}$

$= A^2 M^{-1} L^{-2} T^4$

$= M^{-1} L^{-2} T^4 A^2$

$Z = B = \frac{F}{i\ell} = \frac{M^1L^1T^{-2}}{A^1L^1} = M^1L^0 T^{-2} A^{-1}$

$Z^2 = M^2 T^{-4} A^{-2}$

$Y = \frac{X}{Z^2} = \frac{M^{-1}L^{-2}T^4A^2}{M^2T^{-4}A^{-2}}$

$= M^{-3} L^{-2} T^8 A^4$

Q.2 In free space, a particle A of charge 1μC is held fixed at a point P. Another particle B of the same charge and mass 4μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is :

$\left[\text{Take} \frac{1}{4\pi \in_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}\right]$

(1) 1.0 m/s (2) 3.0×10^4 m/s (3) 2.0×10^3 m/s (4) 1.5×10^2 m/s

Ans. **[3]**

Sol. By conservation of energy

$\frac{1}{2}mv^2 = kq_1q_2\left[\frac{1}{d_1} - \frac{1}{d_2}\right]$

$= k \times 10^{-12}\left[1 - \frac{1}{9}\right] \times 10^3$

$= 9 \times 10^9 \times 10^{-9}\left[\frac{8}{9}\right] = 8$

$\frac{1}{2}$ mv^2 = 8

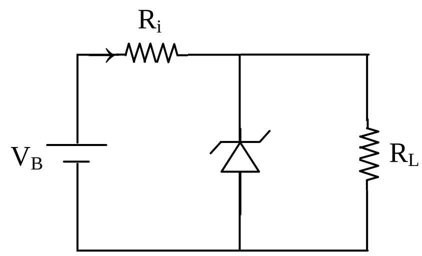
$v = \sqrt{\frac{16}{m}} = \sqrt{\frac{16}{4\times10^{-9}}}$

$= 2 \times 10^4 \times \sqrt{10}$

$\simeq 6 \times 10^4$ m/s

If mass is considered as m = 4×10^{-6} kg then $v = 2 \times 10^3$ m/s

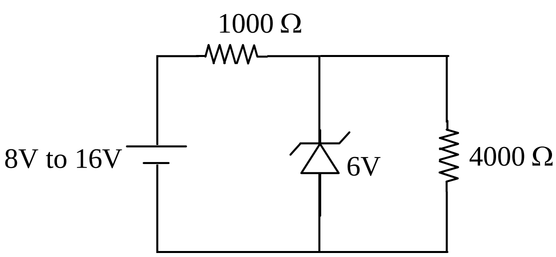
Q.3 The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, R_L = 4kΩ. The series resistance of the circuit is R_i = 1 kΩ. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?



(1) 0.5 mA; 8.5 mA (2) 1.5 mA; 8.5 mA (3) 1 mA; 8.5 mA (4) 0.5 mA; 6 mA

Ans. **[1]**

Sol.



When Battery = 8 V

total current = 2mA

I_z = 2 – 1.5 = 0.5 mA

Battery = 16 V

total current = 10 mA

I_z = 10 – 1.5 = 8.5 mA

∴ 0.5 mA, 8.5 mA

Q.4 A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2}N, the speed of the bullet after emerging from the other side of the wall is close to :

(1) 0.3 ms^{-1} (2) 0.1 ms^{-1} (3) 0.7 ms^{-1} (4) 0.4 ms^{-1}

Ans. **[3]**

Sol. M = 20 g u = 1m/s v = ?

$v^2 = u^2 + 2as$

$s = 20 \times 10^{-2}$ m

$a = -\dfrac{2.5\times10^{-2}}{2\times10^{-2}} = -1.25 \text{ m/s}^2$

$v^2 = 1 - 2 \times 1.25 \times 0.2$

$v^2 = 0.5 = \dfrac{1}{2}$

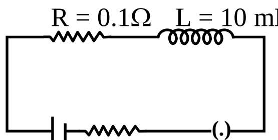
$v = \dfrac{1}{\sqrt{2}} = 0.7$ m/s

Q.5 A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is: [take ln 5 = 1.6]

(1) 0.324 s (2) 0.002 s (3) 0.103 s (4) 0.016 s

Ans. **[4]**

Sol.



$i = i_0\left[1 - e^{-\frac{Rt}{L}}\right]$

$0.8i_0 = i_0\left[1 - e^{-\frac{Rt}{L}}\right]$

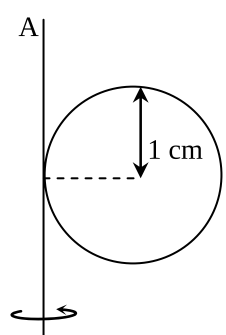
$e^{-\frac{Rt}{L}} = 0.2 = \dfrac{1}{5}$

$-\dfrac{Rt}{L} - \ln(5)$

$t = \dfrac{L}{R}\ln(5)$

= 0.016 sec

Q.6 A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to :



(1) 7.9×10^{-6} Nm (2) 4.0×10^{-6} Nm (3) 2.0×10^{-5} Nm (4) 1.6×10^{-5} Nm

Ans. **[3]**

Sol. $\alpha = \dfrac{d\omega}{dt} = 10\pi$

$\tau = I\alpha$

$= \dfrac{5}{4}(MR^2)\,\alpha$

$= 2 \times 10^{-5}$ N–m

Q.7 Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be :

(1) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ (2) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ (3) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$ (4) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$

Ans. **[4]**

Sol. Resistance of spherical shell is $R = \dfrac{\rho}{4\pi}\left[\dfrac{b-a}{ab}\right]$

Q.8 One mole of ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1-\dfrac{1}{2}\left(\dfrac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes form V_0 to $2V_0$

(1) $\dfrac{3}{4}\dfrac{P_0V_0}{R}$ (2) $\dfrac{1}{2}\dfrac{P_0V_0}{R}$ (3) $\dfrac{5}{4}\dfrac{P_0V_0}{R}$ (4) $\dfrac{1}{4}\dfrac{P_0V_0}{R}$

Ans. **[3]**

Sol. $P = P_0\left[1-2\left(\dfrac{V_0}{V}\right)^2\right]$

When $V_1 = V_0$ $\quad P_1 = \dfrac{P_0}{2}$

$V_2 = 2V_0$ $\quad P_2 = \dfrac{7}{8}P_0$

$\Delta T = T_2 - T_1 = (P_2V_2 - V_1P_1)\,\dfrac{1}{nR}$

$= \left(\dfrac{7}{4}-\dfrac{1}{2}\right)\dfrac{P_0V_0}{R}$

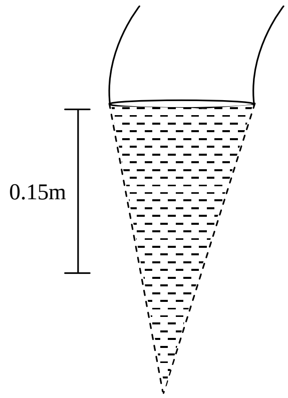
$= \dfrac{5}{4}\dfrac{P_0V_0}{R}$

Q.9 Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be : (Take g = 10 ms^{-2})

(1) 5×10^{-4} m^2 (2) 2×10^{-5} m^2 (3) 5×10^{-5} m^2 (4) 1×10^{-5} m^2

Ans. **[3]**

Sol.



$A_1V_2 = A_2V_2$

$10^{-4} \times 1 = A_2V_2$

$A_2V_2 = 10^{-4}$

Bernoulli's theorem

$\frac{1}{2}\rho\left(V_1^2 - V_2^2\right) = \rho gh$

$V_2^2 = V_1^2 + 2gh$

$V_2 = 2$

$\therefore A_2 = \frac{10^{-4}}{2} = 5 \times 10^{-5}\ m^2$

Q.10 A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. if this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be:

(1) $\frac{4m}{\pi}$ (2) $\frac{3m}{\pi}$ (3) $\frac{2m}{\pi}$ (4) $\frac{m}{\pi}$

Ans. **[1]**

Sol. $m = nIA = 1 \times I \times \pi a^2$

a = side of square

$\because 4a = 2\pi r$

$r = \frac{4a}{2\pi} = \frac{2a}{\pi}$

$\therefore m' = I\ \pi\left[\frac{2a}{\pi}\right]^2$. For circular loop

$m' = \frac{4m}{\pi}$

Q.11 A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}) :

(1) 600 m (2) 400 m (3) 300 m (4) 500 m

Ans. [3]

Sol. $P_1 = P_0 + \rho g d_1$

$P_2 = P_0 + \rho g d_2$

$\therefore \Delta P = \rho g [d_2 - d_1] = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

= 300 m

Q.12 Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be :

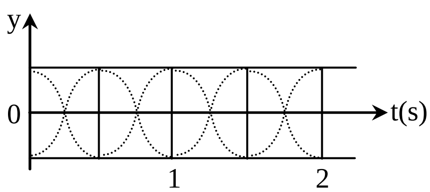
(1) $\frac{2}{\lambda}$ (2) $\frac{1}{4\lambda}$ (3) $\frac{1}{2\lambda}$ (4) $\frac{1}{\lambda}$

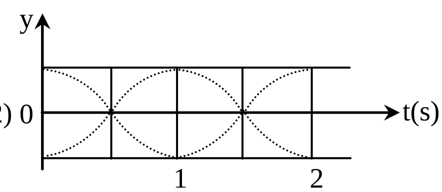
Ans. [3]

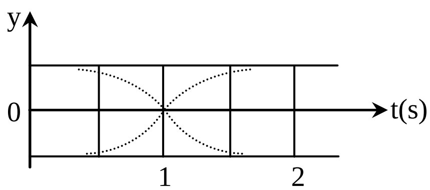
Sol. $\frac{N_1}{N_2} = \frac{e^{-5\lambda t}}{e^{-\lambda t}} = e^{-4\lambda t} = e^{-2}$

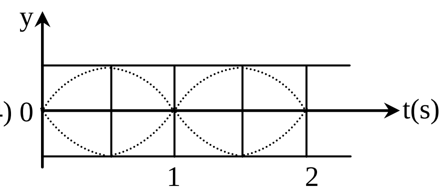
$\therefore t = \frac{1}{2\lambda}$

Q.13 The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is :

(1) 

(2) 

(3) 

(4) 

Ans. [1]

Sol. Beat frequency = 2 Beats/sec

Q.14 In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1) 25 : 9 (2) 4 : 1 (3) $(\sqrt{3} + 1)^4 : 16$ (4) 9 : 1

Ans. [4]

Sol. $I_1 = 4I_0$ $I_2 = I_0$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I_0$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I_0$

$\therefore \frac{I_{max}}{I_{min}} = \frac{9}{1}$

Q.15 A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given ; Mass of planet = 8×10^{22} kg, Radius of planet = 2×10^{6} m, Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2]

(1) 13 (2) 9 (3) 17 (4) 11

Ans. **[4]**

Sol. $F_g = \frac{mv^2}{2} = \frac{GMm}{r^2}$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times8\times10^{22}}{2.02\times10^{6}}}$$

$v = 1.62 \times 10^3$

$$T = \frac{2\pi r}{v}$$

$n(T) = 24 \times 60 \times 60$

$$n = \frac{24\times60\times60}{T}$$

Put the value of T

$n = 11$

Q.16 A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity of sound in air is 350 m/s)

(1) 750 Hz (2) 857 Hz (3) 807 Hz (4) 1143 Hz

Ans. **[1]**

Sol.

$$F' = \left(\frac{V-0}{V-50}\right) F_{source}$$

$$\therefore F_{source} = \frac{1000\times300}{350}$$

Now if source moves away from observer $f'' = \left(\frac{V}{V+50}\right) . F_{source}$

$$= \frac{300}{350}\times\frac{350}{400}\times1000$$

$= 750$ Hz

Q.17 In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ?

(Given : $H = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^{8}$ ms^{-1})

(1) 12.3 nm (2) 10.8 nm (3) 9.4 nm (4) 11.4 nm

Ans. **[2]**

Sol. $\Delta E = \frac{hc}{\lambda} = 13.6 \times 9 - 0.85 \times 9$

$\lambda = \frac{1237}{9 \times 12.75}$ nm

$\simeq 10.8$nm

Q.18 A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by:

(1) 65 (2) 140 (3) 185 (4) 285

Ans. **[2]**

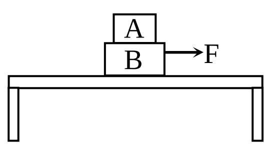
Sol. $I_1 = \frac{\left(\frac{7M}{8}\right)(2R^2)}{2} = \frac{7}{4}MR^2$

$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)R_1^2 = \frac{MR_1^2}{80}$

$R = 2R_1$

$\frac{I_1}{I_2} = \frac{7}{4} \times 80 = 140$

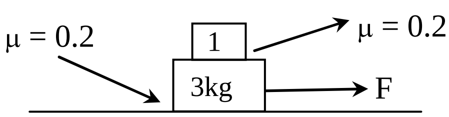
Q.19 Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. the coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is : [Take g = 10 m/s^2]



(1) 8 N (2) 40 N (3) 16 N (4) 12 N

Ans. **[3]**

Sol.



$a_{max} = \mu g = 2$

$F - 8 = 4 \times 2$

$F = 16$ N

Q.20 Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be :

(1) 6.3×10^{-4} Ns (2) 5.0×10^{-3} Ns (3) 1.4×10^{-6} Ns (4) 3.5×10^{-6} Ns

Ans. **[2]**

Sol. Pressure = $\frac{I}{C}$ where I = Intensity

$\therefore \frac{F}{A} = \frac{I}{C}$

$F = \frac{IA}{C} = \frac{\Delta P}{\Delta t}$

$\Delta P = \frac{I}{C} A \Delta t$

$= \frac{25\times10^4}{3\times10^8} \times 25 \times 10^{-4} \times 40 \times 60$

$= 5 \times 10^{-3}$ Ns

Q.21 The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is : [Take $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}]

(1) 3 μT (2) 18 μT (3) 9 μT (4) 1 μT

Ans. **[2]**

Sol. $B = \frac{3\mu_0 i}{\pi a}\sin\frac{\pi}{3}\tan\frac{\pi}{3}$

$= \frac{9\mu_0 i}{2\pi a}$

$= \frac{9\times4\pi\times10^{-7}\times10}{2\pi\times1}$

$= 18 \times 10^{-6}$ T

Q.22 The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is

(1) 36 $\hat{k}$ (2) – 48 $\hat{k}$ (3) – 34 ($\hat{k}$ – $\hat{i}$) (4) 48 ($\hat{i}$ +s $\hat{j}$)

Ans. **[2]**

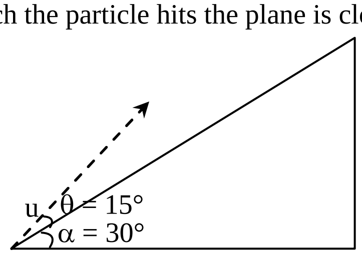
Sol. $\vec{L} = m\,v\,r\sin\theta\ \hat{n} = m(\hat{r}\times\vec{v})$

$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 12\hat{j}$

$\vec{r} \times \vec{v} = -24\ \hat{k}$

$\therefore \vec{L} = -48\hat{k}$

Q.23 A plane is inclined at an angle α = 30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1}, from the base of the plane, making an angle θ = 15° with respect to the plane as shown in the figure. the distance from the base, at which the particle hits the plane is close to : (Take g = 10 ms^{-2})



(1) 14 cm (2) 18 cm (3) 20 cm (4) 26 cm

Ans. [3]

Sol. $T = \dfrac{u\sin 15°}{g\cos 30}$

$S = t(2\cos 15) - \dfrac{g\sin 30 t^2}{2}$

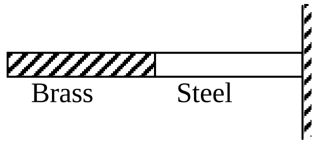
S = 20 cm

Q.24 In an experiment, brass and steel wires of length 1 m each with areas of cross section $1mm^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,
[Given, the Young's Modulus for steel and brass are, respectively, 120×10^9 N/m^2 and 60×10^9 N/m^2]

(1) 4.0×10^6 N/m^2 (2) 1.2×10^6 N/m^2 (3) 1.8×10^6 N/m^2 (4) 0.2×10^6 N/m^2

Ans. [Bonus]

Sol.



$k_1 = \dfrac{Y_1A_1}{\ell_1} = \dfrac{120\times10^9 A}{\ell}$

$k_2 = 60 \times 10^9 \dfrac{A}{\ell}$

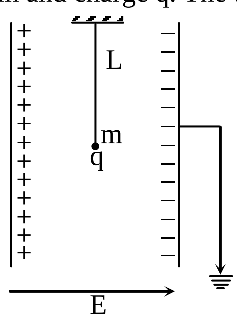
$k_{eq} = \dfrac{k_1k_2}{k_1+k_2}$

$F = k_{eq}(x)$

$\dfrac{F}{A} = \dfrac{k_{eq}}{A}(x) = 8\times10^6$

Solution gives this answer which is not matching with any option

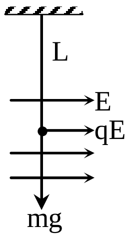
Q.25 A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by :



(1) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 - \dfrac{q^2E^2}{m^2}}}}$ (2) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$ (3) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ (4) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$

Ans. [4]

Sol.

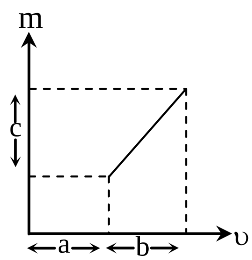


$$T = 2\pi\sqrt{\frac{L}{a_{eff}}}$$

$$a_{eff} = \frac{\sqrt{(mg)^2 + (qE)^2}}{m}$$

$$\therefore\ T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

Q.26 The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used?



(1) $\frac{b^2c}{a}$ (2) $\frac{b}{c}$ (3) $\frac{b^2}{ac}$ (4) $\frac{a}{c}$

Ans. **[2]**

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$1 - m = \frac{v}{f}$

$m = 1 - \frac{v}{f}$

$v = a,\ m = 1 - \frac{a}{f}$

at $v = a + b$, $m = 1 - \frac{a+b}{f}$

$m_2 - m_1 = c = \frac{b}{f}$

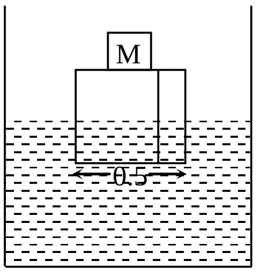
$\therefore\ f = \frac{b}{c}$

Q.27 A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m^3]

(1) 30.1 kg (2) 87.5 kg (3) 65.4 kg (4) 46.3 kg

Ans. **[2]**

Sol.



$M = \rho_w[0.5 \times 0.5 \times 0.35]$

$M = 87.5$ kg

Q.28 A 2 mW laser operates at wavelength of 500 nm. The number of photons that will be emitted per second is : [Given Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m/s]

(1) 5×10^{15} (2) 1.5×10^{16} (3) 1×10^{16} (4) 2×10^{16}

Ans. **[1]**

Sol. $p = \dfrac{nhc}{\lambda}$

$n = \dfrac{p\lambda}{hc} = \dfrac{2\times10^{-3}\times5\times10^{-7}}{2\times10^{-25}}$

$\approx 5 \times 10^{15}$

Q.29 When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. the heat required to produce the same change in temperature, at a constant pressure is :

(1) $\frac{7}{5}Q$ (2) $\frac{3}{2}Q$ (3) $\frac{2}{3}Q$ (4) $\frac{5}{3}Q$

Ans. **[1]**

Sol. $Q = mC_v\,dT$

$Q^1 = nC_p\,dT$

$\dfrac{Q^1}{Q} = \dfrac{C_p}{C_v} = \gamma = \dfrac{7}{5}$

Q.30 The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(1) 1.16 mm (2) 1.36 mm (3) 1.00 mm (4) 0.90 mm

Ans. **[1]**

Sol. Stress $= \dfrac{F}{A} = \dfrac{400\times4}{\pi d^2} = 379 \times 10^6$

$d = \sqrt{1.34} \times 10^{-3}$

$\approx 1.1 \times 10^{-3}$

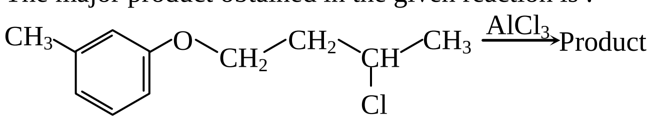
JEE Main Online Exam 2019

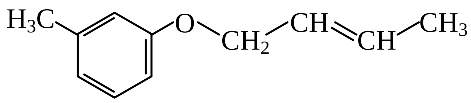
Questions & Solutions

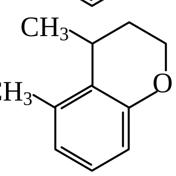
10th April 2019 | Shift - II

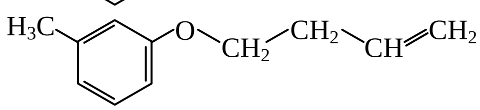
Chemistry

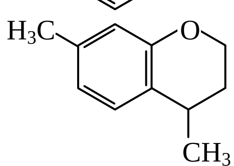
Q.1 The major product obtained in the given reaction is :



(1) 

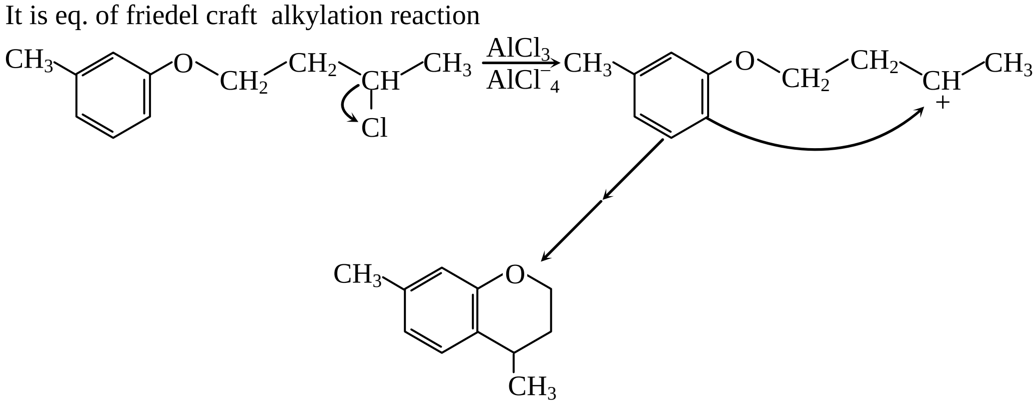
(2) 

(3) 

(4) 

Ans. **[4]**

Sol. It is eq. of friedel craft alkylation reaction



Q.2 Which of the following is not a correct method of the preparation of benzylamine from cyanobenzene ?

(1) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$

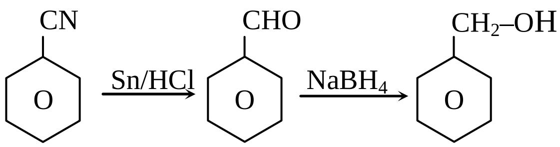
(2) H_2/Ni

(3) (i) $LiAlH_4$ (ii) H_3O^+

(4) (i) HCl/H_2O (ii) $NaBH_4$

Ans. **[4]**

Sol.



Q.3 The noble gas that does not occur in the atmosphere is :

(1) Ne (2) He (3) Kr (4) Ra

Ans. **[4]**

Sol. Radon is not naturally occurring in the atmosphere. Because radon is produced by the radioactive decay of radium – 226

Q.4 The correct statement is :

(1) zincite is a carbonate ore.

(2) Sodium cyanide cannot be used in the metallurgy of silver.

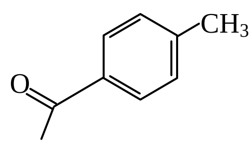
(3) aniline is a froth stabilizer.

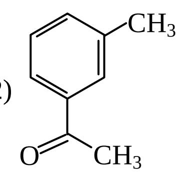
(4) zone refining process is used for the refining of titanium.

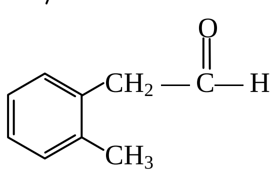
Ans. **[3]**

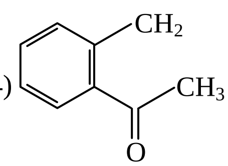
Sol. Aniline is a froth stabiliser

Q.5 Compound A($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH given acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is:

(1) 

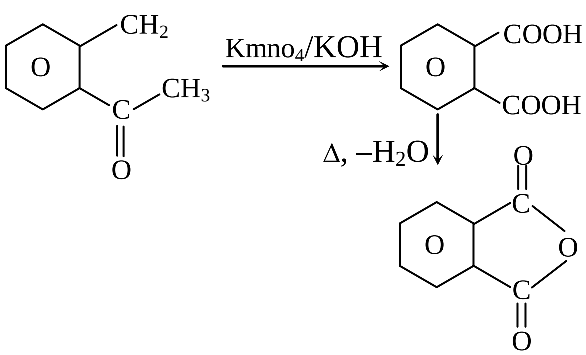
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



Phthalicarhydride

Q.6 The correct option among the following is :
(1) Addition of alum to water makes it unfit for drinking.
(2) Colloidal medicines are more effective because they have small surface area.
(3) Colloidal paraticles in lyophobic sols can be precipitated by electrophoresis.
(4) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.

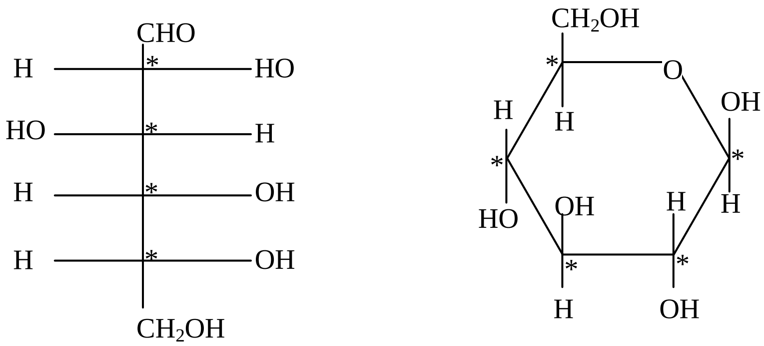
Ans. **[3]**

Sol. fact.

Q.7 Number of stereo centers present in linear and cyclic structures of glucose are respectively :
(1) 4 & 4 (2) 4 & 5 (3) 5 & 4 (4) 5 & 5

Ans. **[2]**

Sol.



Q.8 The highest possible oxidation states of uranium and plutonium, respectively are :
(1) 4 and 6 (2) 6 and 4 (3) 7 and 6 (4) 6 and 7

Ans. **[4]**

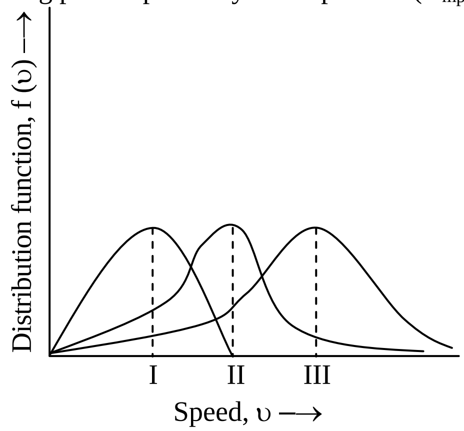
Sol. This highest oxidation state of uranium is +6 whereas of plutonium is + 7

Q.9 In chromatography, which of the following statements is incorrect for R_f ?
(1) R_f value depends on the type of chromatography.
(2) Higher R_f value means higher adsorption.
(3) R_f value is dependent on the mobile phase.
(4) The value of R_f can not be more than one.

Ans. **[2]**

Sol. Low polarity compounds are weekly absorbed and has greater R_f value

Q.10 Points I, II and III in the following plot respectively correspond to (V_{mp} : most probable velocity)



(1) V_{mp} of O_2 (400 K); V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K)
(2) V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K); V_{mp} of O_2 (400 K)
(3) V_{mp} of H_2 (300 K); V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K)
(4) V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K); V_{mp} of H_2 (300 K)

Ans. **[4]**

Sol. $V_{mp} = \sqrt{\frac{2RT}{m}}$

$V_{mp} \alpha \frac{1}{\sqrt{m}}$

O_2, N_2, H_2 ⟶

m decreases

So V_{mp} ⟶ increases

V_{mp} of $O_2 = \sqrt{\frac{2 \times R \times 400}{32}}$

V_{mp} of $N_2 = \sqrt{\frac{2 \times R \times 300}{28}}$

V_{mp} of $H_2 = \sqrt{\frac{2 \times R \times 300}{2}}$

V_{mp} of O_2 < V_{mp} of N_2 < V_{mp} of H_2
I II III

Q.11 The incorrect statement is :

(1) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

(2) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.

(3) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.

(4) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.

Ans. **[2]**

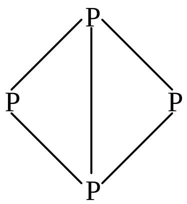
Sol. Because option-4 is incorrect because the hybridisation of $[Fe(H_2O)_6]^{2+}$ is sp^3d^2 whereas the hybridisation of $[Cr(H_2O)_6]^{2+}$ is d^2sp^3

Q.12 The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are :

(1) 12 and 3 (2) 20 and 3 (3) 20 and 4 (4) 12 and 4

Ans. **[4]**

Sol. Because there are almost 12 pentagons is C_{60} and 4 triangles in white phosphosphorus



Q.13 For the reaction of H_2 with I_2, the rate constant is 2.5×10^{-4} dm^3 mol^{-1} s^{-1} at 327°C and 1.0 dm^3 mol^{-1} at 527°C. The activation energy for the reaction, in kJ $mole^{-1}$ is : (R = 8.314 JK^{-1} mol^{-1})

(1) 59 (2) 166 (3) 72 (4) 150

Ans. **[2]**

Sol. $\log\frac{k_2}{k_1} = \frac{Eq}{2.303k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)$

$\log\frac{1}{2.5\times10^{-4}} = \frac{Eq}{2.3\times8.314}\left(\frac{1}{600}-\frac{1}{800}\right)$

$\log_{10} 4 \times 10^3 = \frac{Eq}{2.3\times8.3}\left(\frac{1}{2400}\right)$

$\frac{3.6\times2.3\times8.3\times2400}{1000}\text{kg/mol} = Eq$

164.93

Q.14 A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373 leads to an anhydrous white powder Z. X and Z, respectively, are:

(1) Baking soda and dead burnt plaster.
(2) Washing soda and soda ash
(3) Washing soda and dead burnt plaster.
(4) Baking soda and soda as.

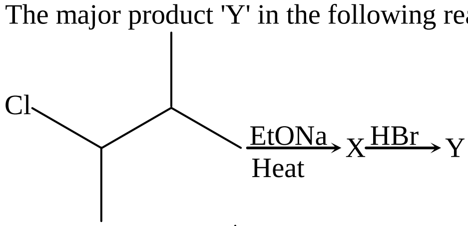
Ans. **[2]**

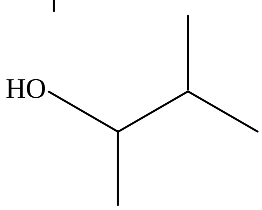
Sol.

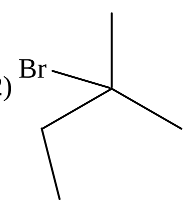
$$\underset{(X)}{Na_2CO_3.10H_2O} \xrightarrow[T<373K]{\Delta} \underset{(Y)}{Na_2CO_3.10H_2O} \xrightarrow[T>373K]{\Delta} \underset{(Z)}{Na_2CO_3}$$

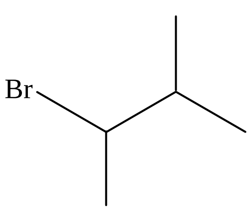
X = wading soda
Z = soda ash

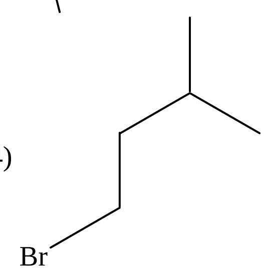
Q.15 The major product 'Y' in the following reactions is :



(1) 

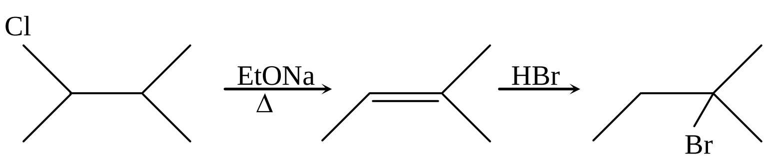
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Q.16 The correct match between Item-I and Item-II is :

	Item-I		Item-II
(a)	High density polythene	(I)	Peroxide catalyst
(b)	Polyacrylonitrile	(II)	Condensation at high temperature & pressure
(c)	Novolac	(III)	Ziegler-Natta Catalyst
(d)	Nylon 6	(IV)	Acid or base catalyst

(1) (a) → (IV), (b) → (II), (c) → (I), (d) → (III)
(2) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)
(3) (a) → (II), (b) → (IV), (c) → (I), (d) → (III)
(4) (a) → (III), (b) → (I), (c) → (II), (d) → (IV)

Ans. **[2]**

Sol. Nylon –6 is condensation polymer of capzalocatiom at higher temp and presser. Novalac is obtained by acid or base catalyored polymerization of phenol and termaldehyde high-density plythere is obtained by using Ziegler natta catalyst. Polyacrylonitrile is obtained by acrylonitrile using peroxide or catalyst

Q.17 The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are :

(1) Paschen and Pfund
(2) Balmer and Brackett
(3) Lyman and Paschen
(4) Brackett and Pfund

Ans. **[3]**

Sol. Ratio of shortest wavelength of two spectral series wavelength shortest means energy maximum

$$\frac{(\Delta E_{lyman})_{max}}{(\Delta E_{pascmen})_{max}} = \frac{13.6 \times 2^2\left(\frac{1}{2} - \frac{1}{\infty}\right)}{13.6 \times 2^2\left(\frac{1}{9} - \frac{1}{\infty}\right)}$$

$$\frac{(\Delta E_{lyman})_{max}}{(\Delta E_{pascmen})_{max}} = \frac{9}{1}$$

Q.18 The correct order of the first ionization enthalpies is :

(1) Ti < Mn < Zn < Ni
(2) Zn < Ni < Mn < Ti
(3) Mn < Ti < Zn < Ni
(4) Ti < Mn < Ni < Zn

Ans. **[4]**

Sol. From theory we locus that Zn > Ni > Mn > Ti

Q.19 For the reaxtion, $2SO_2(g) + O_2(g) = 2\ SO_3(g)$, $\Delta H = -57.2$ kJ mol^{-1} and $K_C = 1.7 \times 10^{16}$

Which of the following statement is incorrect ?

(1) The equilibrium will shift in forward direction as the pressure increase.
(2) The addition of inert gas at constant volume will be not affect the equilibrium constant.
(3) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
(4) The equilibrium constant decreases as the temperature increase.

Ans. [3]

Sol.

$$2SO_2 + O_2 \xrightleftharpoons[]{V_2O_5 \text{ (Catalyst)}} 2SO_{3(g)}\ T$$

Q.20 1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\frac{\Delta T_b(A)}{\Delta T_b(B)}$, is :

(1) 5 : 1 (2) 1 : 0.2 (3) 10 : 1 (4) 1 : 5

Ans. [4]

Sol. $\frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{(k_b \times m)_A}{(k_b \times m)_B}$

Molality is same for both the solution

$= \frac{1}{5} \times 1$

= 1 : 5

Q.21 The difference between ΔH and ΔU (ΔH – ΔU), when the combustion of one mole of heptane (i) is carried out at a temperature T, is equal to :

(1) 3 RT (2) 4 RT (3) – 3 RT (4) – 4 RT

Ans. [4]

Sol. $\underset{(\ell)}{C_7H_{16}}\ \underset{(g)}{HO_2} \longrightarrow \underset{(g)}{7CO_2} + \underset{(\ell)}{8H_2O}$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H - \Delta U = (7 - 11)\ RT = -4RT$

Q.22 The minimum amount of $O_2(g)$ consumed per gram of reactant is for the reaction : (Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

(1) $4\ Fe(s) + 3\ O_2(g) \rightarrow 2\ Fe_2O_3(s)$ (2) $P_4(s) + 5\ O_2(g) \rightarrow P_4O_{10}(s)$

(3) $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$ (4) $2Mg(s) + O_2(g) \rightarrow MgO(s)$

Ans. [1]

Sol. $4Fe_{(s)} + 3O_{2(g)} \longrightarrow 2Fe_2O_{3(g)}$

4 × 56 gm = 3 × 32 gm

1 gm = $\frac{3 \times 32}{4 \times 56}$ = 0.42gm

Which is least amount

Q.23 The pH of a 0.02 M NH_4Cl solution will be [given K_b $(NH_4OH) = 10^{-5}$ and log 2 = 0.301]

(1) 2.56 (2) 5.35 (3) 4.35 (4) 4.65

Ans. **[2]**

Sol. pH of 0.02 mNH_4Cl

$$pH = 7 - \frac{P}{2} - \frac{\log C}{2}$$

$$= 7 - \frac{5}{2} - \frac{\log^2 \times 10^{-2}}{2}$$

$$= 7 - 2.5 + \frac{1.7}{2}$$

= 4.50 + 0.85

5.35

Q.24 Which of these factors does not govern the stability of a conformation in acyclic compounds ?

(1) Angle strain
(2) Torsional strain
(3) Electrostatic forces of interaction
(4) Steric interactions

Ans. **[1]**

Q.25 The correct statements among (a) to (d) are :

(a) saline hydrides produce H_2 gas when reacted with H_2O.

(b) reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.

(c) PH_3 and CH_4 are electron - rich and electron - precise hydrides, respectively.

(d) HF and CH_4 are called as molecular

(1) (a), (b), (c) and (d)

(2) (a), (c) and (d) only

(3) (c) and (d) only

(4) (a), (b) and (c) only

Ans. **[1]**

Sol. Statement a, b, c and d are the correct statements as per the facts

Q.26 The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$ respectively, are :

(1) $-0.4\Delta_o$ and $-0.8\Delta_t$
(2) $-0.6\Delta_o$ and $-0.8\Delta_t$
(3) $-2.4\Delta_o$ and $-1.2\Delta_t$
(4) $-0.4\Delta_o$ and $-1.2\Delta_t$

Ans. **[1]**

Sol. $[FeCH_2O]^{2+}$

Fe^{2+} = [Ar] $3d^6$

H_2O is a weak field ligand so pairing do not take place

T_{2g}	e_g
2, 1, 1	11

$\therefore$ C.F.S.F = $-0.4 \times 4\Delta_0 + 0.6 \times 2\Delta_0 = -0\ 0.4\Delta_0$

In $[NiCl_4]^{2-}$

$Ni^{2+} \rightarrow$ [Ar] $3d^8$

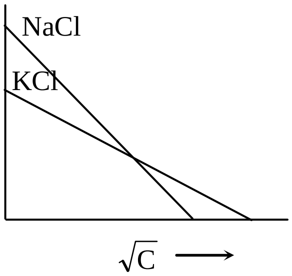
$Cl^- \rightarrow$ weak field ligand, Δ_0 paring do not takes place and have tetrahedral geometry

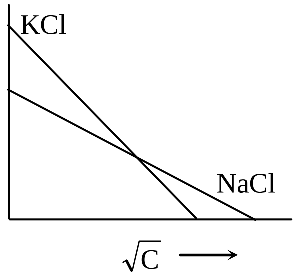
e_g	t_{2g}
2, 2	2, 11

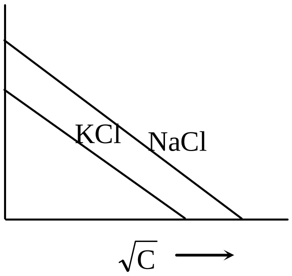
C.F.S.E = $-0.6 \times 4\ \Delta_t + 0.4 \times \Delta t$

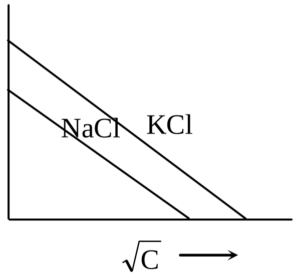
$= -2.4\ \Delta_t + 1.6\ \Delta_t = -0.8\Delta_t$

Q.27 Which one of the following graphs between molar conductivity (Λ_m) versus $\sqrt{C}$ is correct

(1) Λ_m 

(2) Λ_m 

(3) Λ_m 

(4) Λ_m 

Ans. **[4]**

Sol. At infinité olilution molar conductivity of KCl is greater than the molar conductivity of NaCl

Q.28 The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^{\ominus}$ (b) H_2O

(c) $CH_3SO_3^{\ominus}$ (d) $\overset{\ominus}{O}H$

(1) (b) < (c) < (a) < (d) (2) (b) < (c) < (d) < (a)

(3) (a) < (d) < (c) < (b) (4) (d) < (a) < (c) < (b)

Ans. **[1]**

Sol. Nucleophilicity order

$$H_2O < CH_3SO_3^{\ominus} < CH_3COO^{\ominus} < \overset{\ominus}{O}H$$

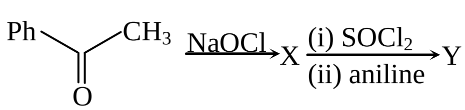
Q.29 Air pollution that occurs in sunlight is :

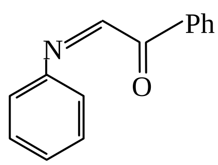
(1) oxidising smog (2) reducing smog

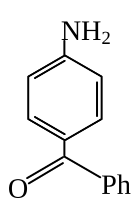
(3) fog (4) acid rain

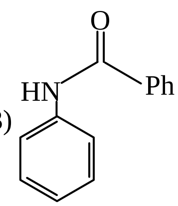
Ans. **[1]**

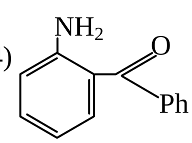
Sol. A/C to the fact that Oxidizing smog form when the primary pollutants are transformed through photochemical reaction into secondary pollutants the most important of which are oxidant gases, ozone and peoxyacetyl nitrate

Q.30 The major product 'Y' in the following reaction is : 

(1) 

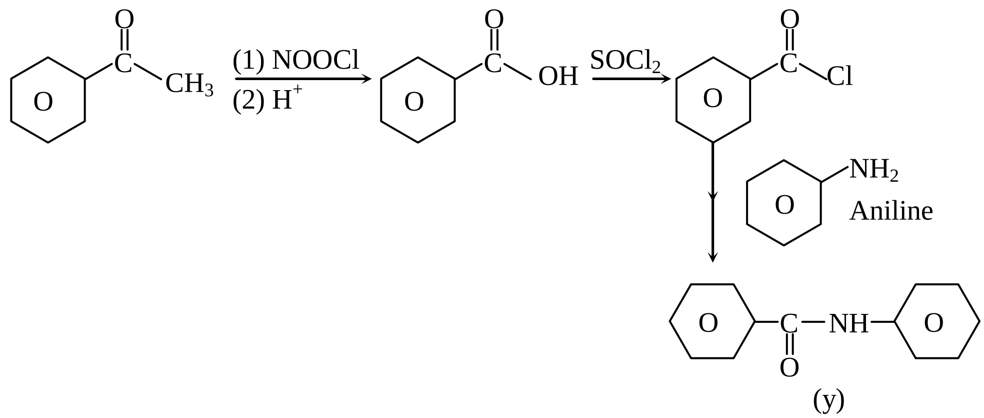
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th April 2019 | Shift - II

Mathematics

Q.1 If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then a + b is equal to :

(1) 1 (2) – 4 (3) – 7 (4) 5

Ans. **[3]**

Sol. $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$

$(1)^2 - a(1) + b = 0$

$1 - a + b = 0$

$a - b = 1$ ...(1)

Now

'L' hospital rule

$2x - a = 5$

$2 - a = 5$ ($\because$ x = 1)

$a = -3$ ....(2)

Put in (1)

$\therefore$ b = – 4

a + b = – 7

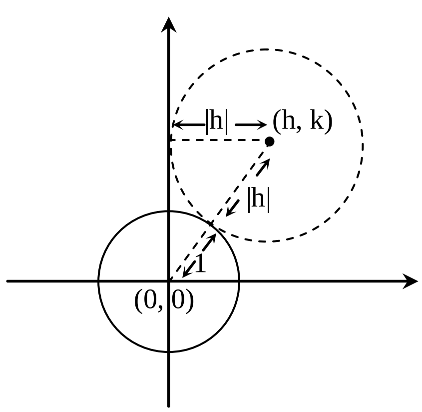
Q.2 The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is

(1) $x = \sqrt{1 + 2y}$, $y \ge 0$ (2) $y = \sqrt{1 + 2x}$, $x \ge 0$

(3) $y = \sqrt{1 + 4x}$, $x \ge 0$ (4) $x = \sqrt{1 + 4y}$, $y \ge 0$

Ans. **[2]**

Sol.



$\sqrt{h^2 + k^2} = 1 + |h|$

$h^2 + k^2 = 1 + h^2 + 2|h|$

$k^2 = 1 + 2|h|$

$y^2 = 1 + 2x$

Q.3 The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

(1) 2 (2) 1 (3) 3 (4) 4

Ans. **[2]**

Sol.

$2^x \geq 1$

$5 + 2^x - 1 = 2^x(2^x - 2)$

Let $2^x = t$

$5 + t - 1 = t(t - 2)$

$t = 4, -1$(rejected)

$2^x = 4$

$x = 2$

only 1 solution

$2^x < 1$

$5 + 1 - 2^x = 2^x(2^x - 2)$

$2^x = t$

$5 + 1 - t = t(t - 2)$

$0 = t^2 - t - 6$

$0 = (t - 3)(t - 2)$

$t = 3, -2$

$2^x = 3, 2^x = -2$

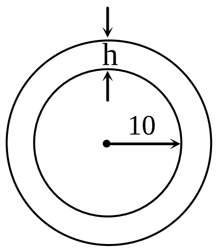
(rejected)

Q.4 A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{5}{6\pi}$ (2) $\frac{1}{9\pi}$ (3) $\frac{1}{36\pi}$ (4) $\frac{1}{18\pi}$

Ans. **[4]**

Sol.



$V = \frac{4}{3}\pi\left((10 + h)^3 - 10^3\right)$

$\frac{dV}{dt} = 4\pi(10 + h)^2 \frac{dh}{dt}$

$-50 = 4\pi(10 + 5)^2 \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{-1}{18\pi} \frac{cm}{min}$

Q.5 The sum $1 + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + ... + \frac{1^3 + 2^3 + 3^3 + ... + 15^3}{1 + 2 + 3 + ... + 15} - \frac{1}{2}(1 + 2 + 3 + ... + 15)$ is equal to :

(1) 620 (2) 1240 (3) 1860 (4) 660

Ans. **[1]**

Sol. Sum = $\sum_{n=1}^{15} \frac{1^3 + 2^3 + ... + n^3}{1 + 2 + ... + n} - \frac{1}{2}\frac{15\times16}{2}$

$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$

$= \frac{1}{2}\sum_{n=1}^{15} n^2 + \frac{1}{2}\sum_{n=1}^{15} n - 60$

$= \frac{1}{2}\times\frac{15\times16\times31}{6} + \frac{1}{2}\times\frac{15\times16}{2} - 60$

$= 620$

Q.6 If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 13 (2) 9 (3) 5 (4) 15

Ans. **[1]**

Sol. Distance formula

(i) $\frac{|\lambda - 6|}{\sqrt{16+4+16}} = \left|\frac{\lambda-6}{6}\right| = \frac{1}{3}$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

(ii) $\frac{|\mu - 3|}{\sqrt{4+4+1}} = \frac{2}{3}$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore (\mu + \lambda)_{max} = 13$

Q.7 Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

(1) $a\alpha^2 + b\alpha - a = -2\alpha^2$ (2) $a\alpha^2 + b\alpha + a = 0$

(3) $a\alpha^2 - b\alpha - a = 0$ (4) $a\alpha^2 + b\alpha - a = 1$

Ans. **[4]**

Sol. $f(g(x)) = -x \Rightarrow (f(g(x)))' = -1$

$f(g(\alpha)) = -\alpha = b \Rightarrow (f(g(\alpha)))' = -1 = a$

$\therefore b = -\alpha$

$a = -1$

Now check from options

Q.8 The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

(1) $2\sqrt{3}$ (2) $4\sqrt{3}$ (3) $\frac{4}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. [1]

Sol. $\because$ 2B = A + C

& A + B + C = π

3B = π

B = $\frac{\pi}{3}$

$\therefore$ A + C = $\frac{2\pi}{3}$ σ

$\frac{a}{b} = \frac{1}{\sqrt{3}}$

$\frac{2R\sin A}{2R\sin B} = \frac{1}{\sqrt{3}}$

sinA = $\frac{1}{2}$

$\therefore$ A = 30°

$\therefore$ a = 2, b = $2\sqrt{3}$, c = 4

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$

Q.9 Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 6 (2) 7 (3) 8 (4) 5

Ans. [2]

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\Rightarrow$ n = 7

Q.10 Let λ be a real number for which the system of linear equations x + y + x = 6, 4x + λy – λz = λ – 2, 3x + 2y – 4z = – 5 has infinitely many solutions. Then λ is a root of the quadratic equation:

(1) $\lambda^2 + \lambda - 6 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$ (3) $\lambda^2 - 3\lambda - 4 = 0$ (4) $\lambda^2 + 3\lambda - 4 = 0$

Ans. [2]

Sol. $\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

On solving we get $\lambda = 3$

Q.11 If z and w are two complex numbers such that |zw| = 1 and arg(z) – arg(w) = $\frac{\pi}{2}$, then:

(1) $z\overline{w} = \frac{1-i}{\sqrt{2}}$ (2) $\overline{z}w = i$ (3) $\overline{z}w = -i$ (4) $z\overline{w} = \frac{-1+i}{\sqrt{2}}$

Ans. **[3]**

Sol. $|zw| = 1$

$|z|\ |w| = 1$

Let $w = \frac{1}{r}e^{i\theta}$

then $z = re^{i\left(\theta+\frac{\pi}{2}\right)}$

$\bar{z}w = e^{-i\left(\theta+\frac{\pi}{2}\right)}.e^{i\theta} = e^{-i(\pi/2)} = -i$

& $z\bar{w} = e^{i\left(\theta+\frac{\pi}{2}\right)}.\ e^{-i\theta} = e^{i\pi/2} = i$

Q.12 The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is:

(1) 23 (2) 58 (3) 38 (4) 35

Ans. **[3]**

Sol. General term

$T_{r+1} = {}^nC_r\, x^{2n-2r}.\ x^{-3r}$

$\because 2n - 5r = 1 \Rightarrow 2n = 5r + 1$

$\therefore r = \frac{2n-1}{5}$

$\therefore$ Coeff. of $x = {}^nC_{\left(\frac{2n-1}{5}\right)} = {}^nC_{23}$

$\therefore \frac{2n-1}{5} = 23$ or $n - \left(\frac{2n-1}{5}\right) = 23$

$2n - 1 = 115$ $\quad$ $n = 38$

$n = 58$ $\quad$ $\therefore$ smallest $n = 38$

Q.13 The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to :

(1) – 4 (2) 0 (3) 1 (4) 6

Ans. **[2]**

Sol. Expand

$x(-3x \times (x+2) - 2x(x-3)) + (-6)\ (2(x+2) + 3\ (x-3)) + (-1)\ (4x + 3\ (-3x))$

$\Rightarrow -5x^3 + 30x - 30 + 5x = 0$

$x^3 - 7x + 6 = 0$

Sum of roots = 0

Q.14 If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(\frac{5}{3}, 0\right)$ (2) (5, 0) (3) (–5, 0) (4) $\left(-\frac{5}{3}, 0\right)$

Ans. **[3]**

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

a = 3

b = 4

$e^2 = 1 + \frac{b^2}{a^2}$

$e^2 = 1 + \frac{16}{9}$

$e = \frac{5}{3}$

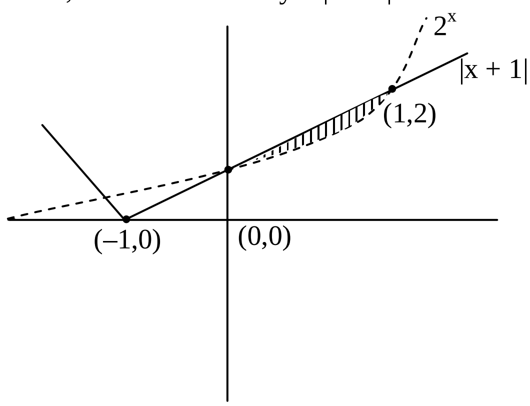
∴ focus is (–ae, 0) = (–5, 0)

Q.15 The area (in sq.units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{1}{2}$ (2) $\frac{3}{2} - \frac{1}{\log_e 2}$ (3) $\frac{3}{2}$ (4) $\log_e 2 + \frac{3}{2}$

Ans. **[2]**

Sol. $y = 2^x$; $y = |x + 1|$



Required Area = $\int_0^1 ((x+1) - 2^x)\,dx$

$= \left(\frac{x^2}{2} + x - \frac{2^x}{\log_e 2}\right)_0^1$

$= \frac{3}{2} - \frac{1}{\log_e 2}$

Q.16 If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then |c| is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{2}$

Ans. **[3]**

Sol. Tangent to the curve $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$

It is tangent to the circle $x^2 + y^2 = 1$

$\therefore \left|\dfrac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow m = \pm 1$

$\therefore$ tangent are $y = x + \sqrt{2}$ & $y = -x - \sqrt{2}$

Compare with $y = -ax + c$

$\Rightarrow a = \pm 1$ & $c = \pm\sqrt{2}$

Q.17 The integral $\displaystyle\int_{\pi/6}^{\pi/3} \sec^{2/3} x \,\text{cosec}^{4/3} x\, dx$ is equal to :

(1) $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$ (2) $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$ (3) $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$ (4) $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

Ans. **[4]**

Sol. $\displaystyle\int_{\pi/6}^{\pi/3} \sec^{2/3} x \,\text{cosec}^{4/3} x\, dx$

$= \displaystyle\int \frac{\sec^2 x}{\tan^{4/3} x} dx$

Let $\tan x = t$, $\sec^2 x\, dx = dt$

$= \displaystyle\int \frac{dt}{t^{4/3}}$

$I = -3\ (t^{-1/3})$

$= \left(-3(\tan x)^{\frac{-1}{3}}\right)_{\pi/6}^{\pi/3}$

$= 3\left(3^{\frac{1}{3}} - \dfrac{1}{3^{\frac{1}{6}}}\right)$

$= 3^{7/6} - 3^{5/6}$

Q.18 Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is :

(1) 180 (2) 170 (3) 190 (4) 210

Ans. **[2]**

Sol. $\dfrac{{}^{20}C_1 \times {}^{17}C_1}{2} = 170$

Q.19 Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \dfrac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P. is :

(1) a (2) $\dfrac{7}{3}a$ (3) $\dfrac{2}{3}a$ (4) 5a

Ans. **[1]**

Sol. a = a

b = ar

$c = ar^2$

3a, 7b, 15c $\longrightarrow$ A.P.

14b = 3a + 15c

$14(ar) = 3a + 15(ar^2)$

$15r^2 - 14r + 3 = 0$

$\Rightarrow r = \frac{1}{3}, \frac{3}{5}$ (rejected)

Common difference = 7b – 3a

= 7ar – 3a

$= \frac{7a}{3} - 3a$

$= -\frac{2}{3}a$

4^{th} term is $\Rightarrow 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

Q.20 If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

(1) – 1 (2) $-\frac{5}{2}$ (3) 1 (4) $-\frac{1}{2}$

Ans. **[2]**

Sol. Let $x^2 = t$

$\Rightarrow \frac{1}{2}\int t^2 e^{-t}\, dt$

$= \frac{1}{2}[-t^2e^{-t} + \int 2te^{-t}\, dt]$

$= \frac{-t^2e^{-t}}{2} - te^{-t} - e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + c$

$g(x) = -\frac{x^4}{2} - x^2 - 1$

$g(-1) = -\frac{1}{2} - 1 - 1$

$= -\frac{5}{2}$

Q.21 Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 1. Then :

(1) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(4) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Ans. **[2]**

Sol. $\frac{dy}{dx}$ + y (tan x) = 2x + x^2 tan x

I. F. = $e^{\int \tan x\, dx}$ = $e^{\ln \sec x}$ = sec x

y.sec x = $\int (2x + x^2 \tan x)\sec x\, dx$

y sec x = x^2sec x + λ

⇒ y = x^2 + λ cos x

y(0) = 0 + λ = 1 ⇒ λ = 1

y = x^2 + cos x

y' = 2x – sinx

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

Q.22 The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{68}{15}$ (4) $\frac{34}{15}$

Ans. **[3]**

Sol. $3x^2 + 5y^2 = 32$

6x + 10yy' = 0

$y' = \frac{-3x}{5y}$

$y'_{(2,\,2)} = -\frac{3}{5}$

Tangent $(y - 2) = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal $(y - 2) = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area = $\frac{1}{2}(QR) \times 2 = QR = \frac{68}{15}$

Q.23 If the tangent to the curve $y = \frac{x}{x^2 - 3}$, x ∈ ρ, $(x \neq \pm\sqrt{3})$, at a point (α, β) ≠ (0, 0) on it is parallel to the line 2x + 6y – 11 = 0, then :

(1) $|6\alpha + 2\beta| = 9$ (2) $|2\alpha + 6\beta| = 11$ (3) $|2\alpha + 6\beta| = 19$ (4) $|6\alpha + 2\beta| = 19$

Ans. **[4]**

Sol. $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given

$$\frac{-\alpha^2-3}{(\alpha^2-3)^2} = -\frac{1}{3}$$

$\Rightarrow \alpha = 0, \pm 3\ (\alpha \neq 0)$

Q.24 Lines are drawn parallel to the line 4x – 3y + 2 = 0, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines ?

(1) $\left(\frac{1}{4},-\frac{1}{3}\right)$ (2) $\left(-\frac{1}{4},\frac{2}{3}\right)$ (3) $\left(\frac{1}{4},\frac{1}{3}\right)$ (4) $\left(-\frac{1}{4},-\frac{2}{3}\right)$

Ans. **[2]**

Sol. Line parallel to 4x – 3y + 2 = 0

is given as $4x - 3y + \lambda = 0$

distance from origin is

$$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$$

$\lambda = \pm 3$

$\therefore$ required lines are 4x – 3y + 3 = 0 & 4x – 3y – 3 = 0

Now check options

Q.25 The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $\sim s \wedge \sim r$ (2) r (3) $s \wedge r$ (4) $s \vee r$

Ans. **[3]**

Sol. $\sim (\sim s \vee (\sim r \wedge s))$

$s \wedge (r \vee \sim s)$

$(s \wedge r) \vee (s \wedge \sim s)$

$(s \wedge r) \wedge (s)$

$(s \wedge r)$

Q.26 If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots\ldots, (x_{50} - 4)^2$ is :

(1) 400 (2) 480 (3) 380 (4) 525

Ans. **[1]**

Sol. $\text{Mean}(\mu) = \frac{\sum x_i}{50} = 16$

$\therefore \sum x_i = 16 \times 50$

$\text{S.D.}(\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$\Rightarrow \frac{\sum x_i^2}{50} = 256 \times 2$

Required mean = $\dfrac{\sum (x_i - 4)^2}{50}$

= $\dfrac{\sum x_i^2 + 16\times 50 - 8\sum x_i}{50}$

= 256 × 2 + 16 – 8 × 16

= 400

Q.27 Let a_1, a_2, a_3,.... be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1\, a_4\, a_5$, is :

(1) $\frac{3}{2}$ (2) $\frac{6}{5}$ (3) $\frac{8}{5}$ (4) $\frac{2}{3}$

Ans. **[3]**

Sol. first term = a

Common difference = d

$\therefore$ a + 5d = 2

$a_1 . a_4 . a_5 = a(a + 3d)(a + 4d)$

f(d) = (2 – 5d) (2 – 2d) (2 – d)

$f'(d) = 0 \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0$ at $d = \frac{8}{5}$

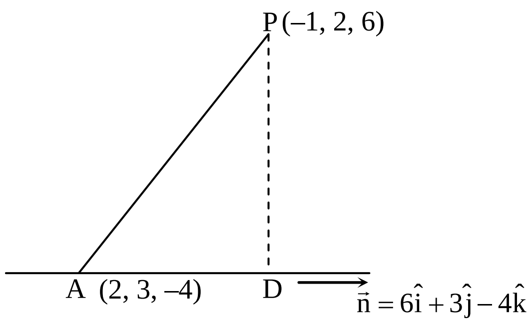
$\Rightarrow d = \frac{8}{5}$

Q.28 The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, – 4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is :

(1) 6 (2) 7 (3) $2\sqrt{13}$ (4) $4\sqrt{3}$

Ans. **[2]**

Sol.



AD = $\left|\dfrac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$

PD = $\sqrt{AP^2 - AD^2}$

= $\sqrt{110 - 61}$

= 7

Q.29 If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to :

(1) $4\sin^2\alpha$ (2) $2\sin^2\alpha$ (3) $4\sin^2\alpha - 2x^2y^2$ (4) $4\cos^2\alpha + 2x^2y^2$

Ans. **[1]**

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos\left(\cos^{-1}x - \cos^{-1}\left(\frac{y}{2}\right)\right) = \cos\alpha$

$x\frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\left(\cos\alpha - \frac{xy}{2}\right) = \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}$

Squaring both sides

$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

Q.30 A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are :

(1) (4, 0, – 1) (2) (2, 0, 1) (3) (1, 0, 2) (4) (– 1, 0, 4)

Ans. **[2]**

Sol. $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1} = \lambda$

Let a point P on the line is

$(2\lambda + 1, -\lambda - 1, +\lambda)$

Foot of $\perp^r$ Q is given by

$\frac{x-2\lambda-1}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = -\frac{(2\lambda-3)}{3}$

$\because$ Q lies on x + y + z = 3 & x – y + z = 3

$\Rightarrow$ x + z = 3 & y = 0

$\therefore y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda+3}{3} \Rightarrow \lambda = 0$

$\therefore$ Q is (2, 0, 1)