

# Circle

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## Exercise 21A

**Q. 1. Find the equation of a circle with**

**Centre (2, 4) and radius 5**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

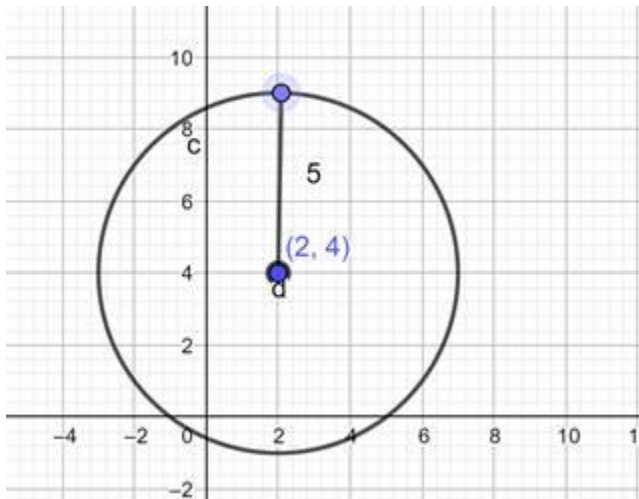
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in the general form:

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 5^2$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$$



Ans; equation of a circle with Centre (2, 4) and radius 5 is:

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$$

**Q. 2. Find the equation of a circle with**

**Centre ( - 3, - 2) and radius 6**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

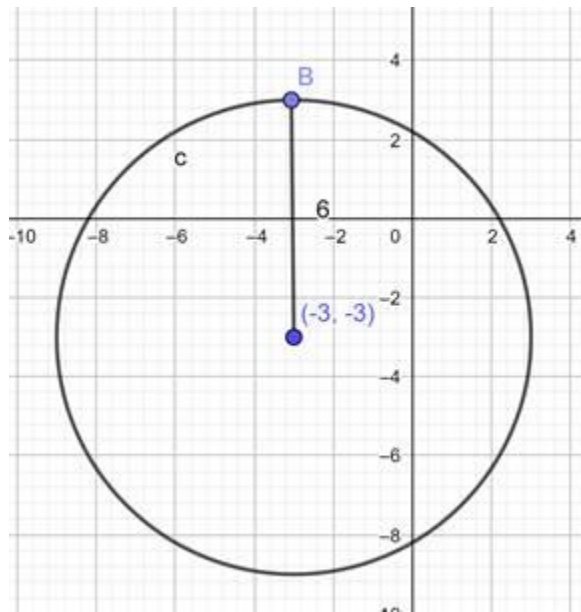
Substituting the centre and radius of the circle in the general form:

$$\Rightarrow (x - (-3))^2 + (y - (-2))^2 = 6^2$$

$$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$$

Ans; equation of a circle with Centre ( - 3, - 2) and radius 6 is:

$$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$$



**Q. 3. Find the equation of a circle with**

**Centre (a, a) and radius  $\sqrt{2}$**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in the general form:

$$\Rightarrow (x - a)^2 + (y - a)^2 = (\sqrt{2})^2$$

$$\Rightarrow (x - a)^2 + (y - a)^2 = 2$$

Ans; equation of a circle with Centre (a, a) and radius  $\sqrt{2}$

is:

$$(x - a)^2 + (y - a)^2 = 2$$

#### **Q. 4. Find the equation of a circle with**

**Centre (a cos  $\alpha$ , a sin  $\alpha$ ) and radius a**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in the general form:

$$(x - (a \cos \alpha))^2 + (y - (a \sin \alpha))^2 = a^2$$

$$\Rightarrow (x - a \cos \alpha)^2 + (y - a \sin \alpha)^2 = a^2$$

$$\Rightarrow x^2 - 2x a \cos \alpha + a^2 \cos^2 \alpha + y^2 - 2y a \sin \alpha + a^2 \sin^2 \alpha = a^2$$

$$\Rightarrow x^2 + y^2 + a^2 (\cos^2 \alpha + \sin^2 \alpha) - 2a(x \cos \alpha + y \sin \alpha) = a^2$$

$$\Rightarrow x^2 + y^2 + a^2 - 2a(x \cos \alpha + y \sin \alpha) = a^2 \dots ((\cos^2 \alpha + \sin^2 \alpha) = 1)$$

$$\Rightarrow x^2 + y^2 - 2a(x \cos \alpha + y \sin \alpha) = 0$$

Ans: equation of a circle with Centre (a cos  $\alpha$ , a sin  $\alpha$ ) and radius a is:

$$x^2 + y^2 - 2a(x\cos \alpha + y\sin \alpha) = 0$$

**Q. 5. Find the equation of a circle with**

**Centre ( - a, - b) and radius  $\sqrt{a^2 - b^2}$**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in the general form:

$$\Rightarrow (x - (-a))^2 + (y - (-b))^2 = (\sqrt{a^2 - b^2})^2$$

$$\Rightarrow (x + a)^2 + (y + b)^2 = a^2 - b^2$$

$$\Rightarrow x^2 + 2xa + a^2 + y^2 + 2ya + b^2 = a^2 - b^2$$

$$\Rightarrow x^2 + 2xa + y^2 + 2ya = a^2 - 2b^2$$

$$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$$

$$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$$

Ans; equation of a circle with Centre ( - a, - b) and radius

is:

$$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$$

**Q. 6. Find the equation of a circle with**

**Centre at the origin and radius 4**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

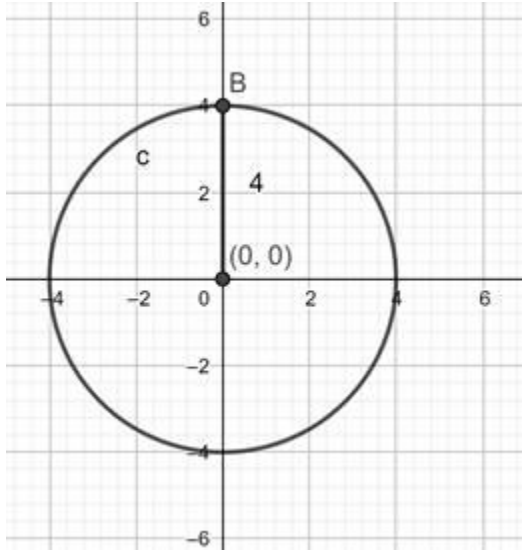
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in the general form:

$$\Rightarrow (x - 0)^2 + (y - 0)^2 = 4^2$$

$$\Rightarrow x^2 + y^2 = 16$$



Ans; equation of a circle with . Centre at the origin and radius 4 is:

$$x^2 + y^2 = 16$$

**Q. 7 A. Find the centre and radius of each of the following circles :**

$$(x - 3)^2 + (y - 1)^2 = 9$$

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$$h = 3, k = 1, r^2 = 9$$

$\Rightarrow$  centre = (3, 1) and radius = 3 units.

Ans: centre = (3, 1) and radius = 3 units.

**Q. 7 B. Find the centre and radius of each of the following circles :**

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{1}{16}$$

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$$h = 1/2, k = -1/3, r^2 = 1/16$$

⇒ centre = (1/2, -1/3) and radius = 1/4 units.

Ans: centre = (1/2, -1/3) and radius = 1/4 units.

**Q. 7 C. Find the centre and radius of each of the following circles :**

$$(x + 5)^2 + (y - 3)^2 = 20$$

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$$h = -5, k = 3, r^2 = 20$$

⇒ centre = (-5, 3) and radius =  $\sqrt{20} = 2\sqrt{5}$  units.

Ans: centre = (-5, 3) and radius =  $2\sqrt{5}$  units.

**Q. 7 D. Find the centre and radius of each of the following circles :**

$$x^2 + (y - 1)^2 = 2$$

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where,  $(h, k)$  is the centre of the circle.

$r$  is the radius of the circle.

Comparing the given equation of circle with general form we get:

$$h = 0, k = 1, r^2 = 2$$

$\Rightarrow$  centre =  $(0, 1)$  and radius =  $\sqrt{2}$ units.

Ans: centre =  $(0, 1)$  and radius =  $\sqrt{2}$ units.

**Q. 8. Find the equation of the circle whose centre is  $(2, -5)$  and which passes through the point  $(3, 2)$ .**

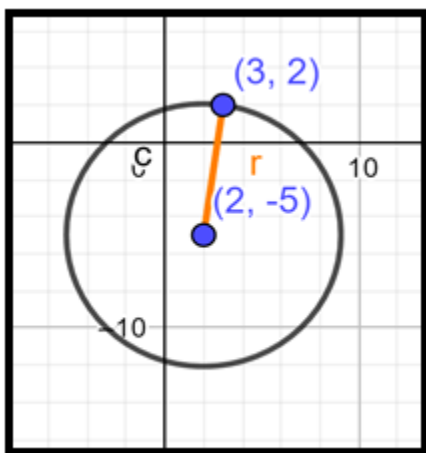
**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where,  $(h, k)$  is the centre of the circle.

$r$  is the radius of the circle.

In this question we know that  $(h, k) = (2, -5)$ , so for determining the equation of the circle we need to determine the radius of the circle.



Since the circle passes through  $(3, 2)$ , that pair of values for  $x$  and  $y$  must satisfy the equation and we have:

$$\Rightarrow (3 - 2)^2 + (2 - (-5))^2 = r^2$$

$$\Rightarrow 1^2 + 7^2 = r^2$$

$$\Rightarrow r^2 = 49 + 1 = 50$$

$$\therefore r^2 = 50$$

$\Rightarrow$  Equation of circle is:

$$(x - 2)^2 + (y - (-5))^2 = 50$$

$$\Rightarrow (x - 2)^2 + (y + 5)^2 = 50$$

$$\text{Ans: } (x - 2)^2 + (y + 5)^2 = 50$$

**Q. 9. Find the equation of the circle of radius 5 cm, whose centre lies on the y - axis and which passes through the point (3, 2).**

**Answer :** The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Since, centre lies on Y - axis,  $\therefore$  it's X - coordinate = 0, i.e. h = 0

Hence, (0, k) is the centre of the circle.

Substituting the given values in general form of the equation of a circle we get,

$$\Rightarrow (3 - 0)^2 + (2 - k)^2 = 5^2$$

$$\Rightarrow (3)^2 + (2 - k)^2 = 25$$

$$\Rightarrow 9 + (2 - k)^2 = 25$$

$$\Rightarrow (2 - k)^2 = 25 - 9 = 16$$

Taking square root on both sides we get,

$$\Rightarrow 2 - k = \pm 4$$

$$\Rightarrow 2 - k = 4 \text{ \& } 2 - k = -4$$



$$\Rightarrow k = 2 - 4 \text{ \& } k = 2 + 4$$

$$\Rightarrow k = -2 \text{ \& } k = 6$$

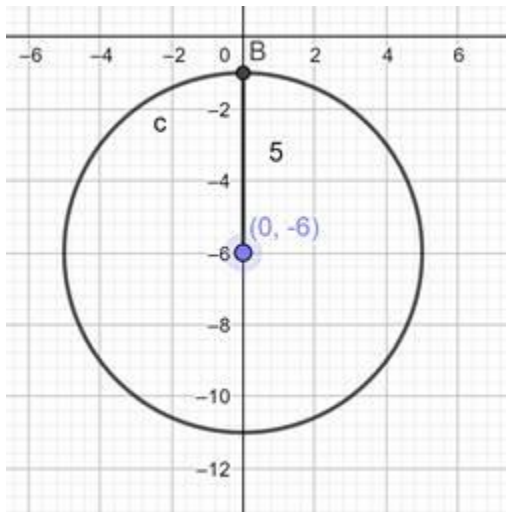
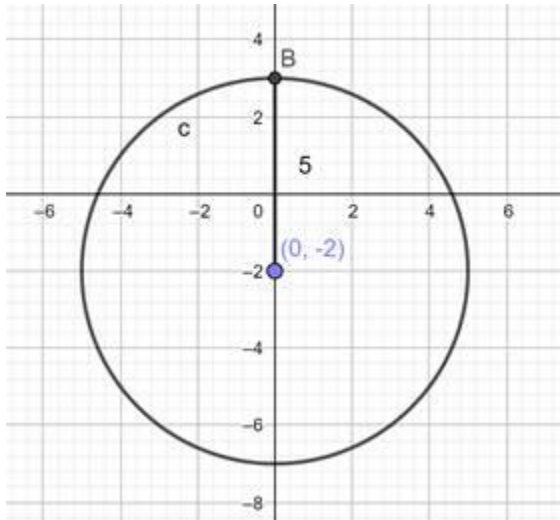
$\therefore$  Equation of circle when  $k = -2$  is:  $x^2 + (y + 2)^2 = 25$

Equation of circle when  $k = 6$  is:  $x^2 + (y - 6)^2 = 25$

Ans: Equation of circle when  $k = -2$  is:

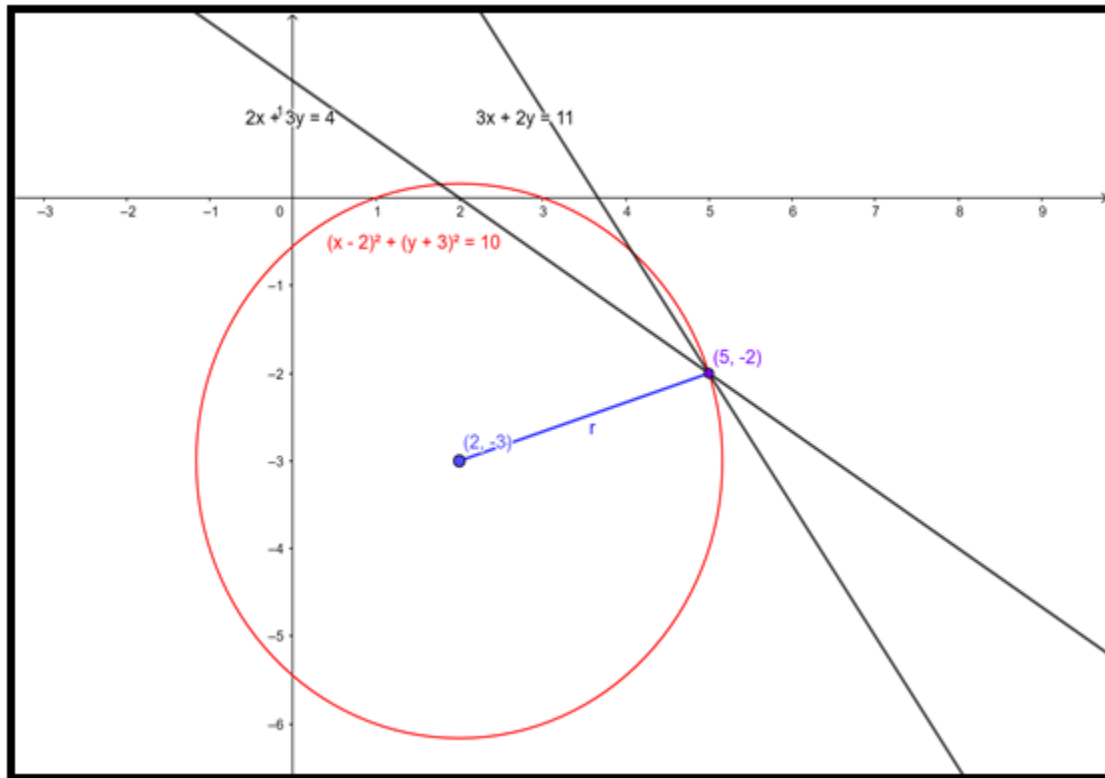
$$x^2 + (y + 2)^2 = 25$$

Equation of circle when  $k = 6$  is:  $x^2 + (y - 6)^2 = 25$



**Q. 10. Find the equation of the circle whose centre is  $(2, -3)$  and which passes through the intersection of the lines  $3x + 2y = 11$  and  $2x + 3y = 4$ .**

**Answer :**



The intersection of the lines:  $3x + 2y = 11$  and  $2x + 3y = 4$

Is  $(5, -2)$

∴ This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where,  $(h, k)$  is the centre of the circle.

$r$  is the radius of the circle.

In this question we know that  $(h, k) = (2, -3)$ , so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through  $(5, -2)$ , that pair of values for  $x$  and  $y$  must satisfy the equation and we have:

$$\Rightarrow (5 - 2)^2 + (-2 - (-3))^2 = r^2$$

$$\Rightarrow 3^2 + 1^2 = r^2$$

$$\Rightarrow r^2 = 9 + 1 = 10$$

$$\therefore r^2 = 10$$

$\Rightarrow$  Equation of circle is:

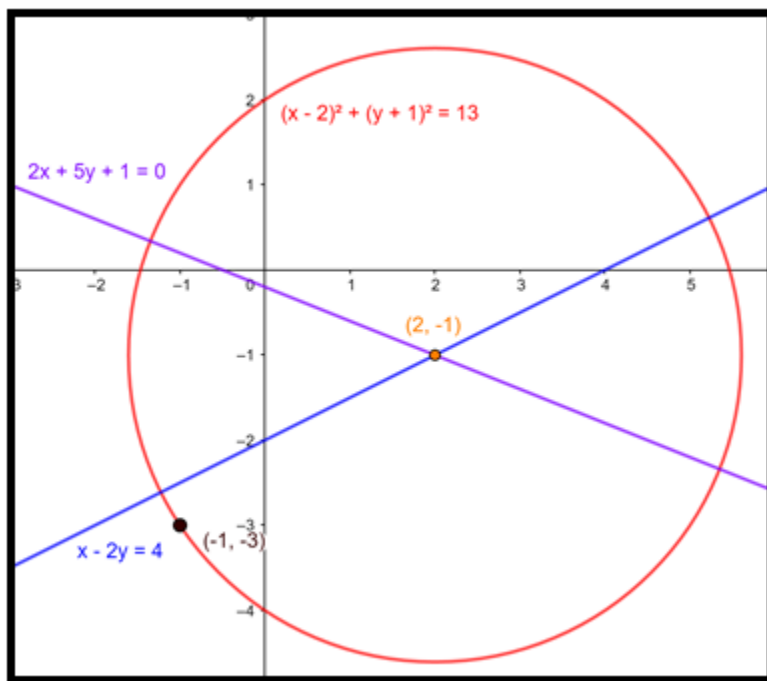
$$(x - 2)^2 + (y - (-3))^2 = 10$$

$$\Rightarrow (x - 2)^2 + (y + 3)^2 = 10$$

$$\text{Ans: } (x - 2)^2 + (y + 5)^2 = 10$$

**Q. 11. Find the equation of the circle passing through the point  $(-1, -3)$  and having its centre at the point of intersection of the lines  $x - 2y = 4$  and  $2x + 5y + 1 = 0$ .**

**Answer :**



The intersection of the lines:  $x - 2y = 4$  and  $2x + 5y + 1 = 0$ .

is  $(2, -1)$

$\therefore$  This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 1), so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through (- 1, - 3), that pair of values for x and y must satisfy the equation and we have:

$$\Rightarrow (- 1 - 2)^2 + (- 3 - (- 1))^2 = r^2$$

$$\Rightarrow (- 3)^2 + (- 2)^2 = r^2$$

$$\Rightarrow r^2 = 9 + 4 = 13$$

$$\therefore r^2 = 13$$

$\Rightarrow$  Equation of circle is:

$$(x - 2)^2 + (y - (- 1))^2 = 13$$

$$\Rightarrow (x - 2)^2 + (y + 1)^2 = 13$$

$$\text{Ans: } (x - 2)^2 + (y + 1)^2 = 13$$

**Q. 12. If two diameters of a circle lie along the lines  $x - y = 9$  and  $x - 2y = 7$ , and the area of the circle is 38.5 sq cm, find the equation of the circle.**

**Answer :** The point of intersection of two diameters is the centre of the circle.

$\therefore$  point of intersection of two diameters  $x - y = 9$  and  $x - 2y = 7$  is (11, 2).

$\therefore$  centre = (11, 2)

Area of a circle =  $\pi r^2$

$$38.5 = \pi r^2$$

$$\Rightarrow r^2 = \frac{38.5}{\pi}$$

$$\Rightarrow r^2 = 12.25 \text{ sq.cm}$$

the equation of the circle is:

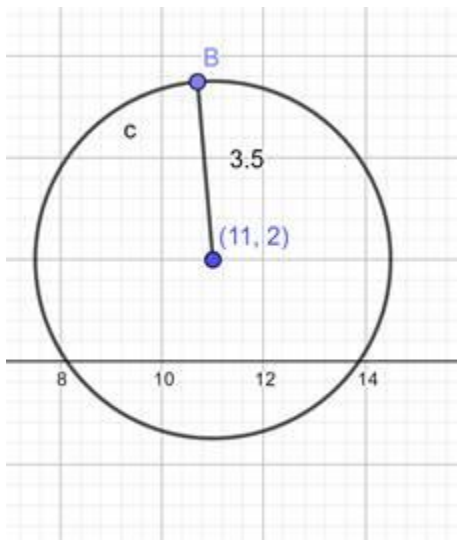
$$(x - h)^2 + (y - k)^2 = r^2$$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

$$\Rightarrow (x - 11)^2 + (y - 2)^2 = 12.25$$

Ans:  $(x - 11)^2 + (y - 2)^2 = 12.25$



**Q. 13 A. Find the equation of the circle, the coordinates of the end points of one of whose diameters are**

**A(3, 2) and B(2, 5)**

**Answer :** The equation of a circle passing through the coordinates of the end points of diameters is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substituting, values:  $(x_1, y_1) = (3, 2)$  &  $(x_2, y_2) = (2, 5)$

We get:

$$(x - 3)(x - 2) + (y - 2)(y - 5) = 0$$

$$\Rightarrow x^2 - 2x - 3x + 6 + y^2 - 5y - 2y + 10 = 0$$

$$\Rightarrow x^2 + y^2 - 5x - 7y + 16 = 0$$

$$\text{Ans: } x^2 + y^2 - 5x - 7y + 16 = 0$$

**Q. 13 B. Find the equation of the circle, the coordinates of the end points of one of whose diameters are**

**A(5, - 3) and B(2, - 4)**

**Answer :** The equation of a circle passing through the coordinates of the end points of diameters is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substituting, values:  $(x_1, y_1) = (5, - 3)$  &  $(x_2, y_2) = (2, - 4)$

We get:

$$(x - 5)(x - 2) + (y + 3)(y + 4) = 0$$

$$\Rightarrow x^2 - 2x - 5x + 10 + y^2 + 3y + 4y + 12 = 0$$

$$\Rightarrow x^2 + y^2 - 7x + 7y + 22 = 0$$

$$\text{Ans: } x^2 + y^2 - 7x + 7y + 22 = 0$$

**Q. 13 C. Find the equation of the circle, the coordinates of the end points of one of whose diameters are**

**A(- 2, - 3) and B(- 3, 5)**

**Answer :** The equation of a circle passing through the coordinates of the end points of diameters is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substituting, values:  $(x_1, y_1) = (- 2, - 3)$  &  $(x_2, y_2) = (- 3, 5)$

We get:

$$(x + 2)(x + 3) + (y + 3)(y - 5) = 0$$

$$\Rightarrow x^2 + 3x + 2x + 6 + y^2 - 5y + 3y - 15 = 0$$

$$\Rightarrow x^2 + y^2 + 5x - 2y - 9 = 0$$

$$\text{Ans: } x^2 + y^2 + 5x - 2y - 9 = 0$$

**Q. 13 D. Find the equation of the circle, the coordinates of the end points of one of whose diameters are**

**A(p, q) and B(r, s)**

**Answer :** The equation of a circle passing through the coordinates of the end points of diameters is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substituting, values:  $(x_1, y_1) = (p, q)$  &  $(x_2, y_2) = (r, s)$

We get:

$$(x - p)(x - r) + (y - q)(y - s) = 0$$

$$\Rightarrow x^2 - rx - px + pr + y^2 - sy - qy + qs = 0$$

$$\Rightarrow x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$$

$$\text{Ans: } x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$$

**Q. 14. The sides of a rectangle are given by the equations  $x = - 2$ ,  $x = 4$ ,  $y = - 2$  and  $y = 5$ . Find the equation of the circle drawn on the diagonal of this rectangle as its diameter.**

**Answer :** The intersection points in clockwise fashion are:  $(- 2, 5)$ ,  $(4, 5)$ ,  $(4, - 2)$ ,  $(- 2, - 2)$ .

The equation of a circle passing through the coordinates of the end points of diameters is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substituting, values:  $(x_1, y_1) = (- 2, 5)$  &  $(x_2, y_2) = (4, - 2)$

We get:

$$(x + 2)(x - 4) + (y - 5)(y + 2) = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 + y^2 + 2y - 5y - 10 = 0$$

$$\Rightarrow x^2 + y^2 - 2x - 3y - 18 = 0$$



Ans:  $x^2 + y^2 - 2x - 3y - 18 = 0$

### Exercise 21B

**Q. 1. Show that the equation  $x^2 + y^2 - 4x + 6y - 5 = 0$  represents a circle. Find its centre and radius.**

**Answer :** The general equation of a conic is as follows

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ where } a, b, c, f, g, h \text{ are constants}$$

For a circle,  $a = b$  and  $h = 0$ .

The equation becomes:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots (i)$$

Given,  $x^2 + y^2 - 4x + 6y - 5 = 0$



Comparing with (i) we see that the equation represents a circle with  $2g = -4 \Rightarrow g = -2$ ,  $2f = 6 \Rightarrow f = 3$  and  $c = -5$ .

$$\text{Centre } (-g, -f) = \{-(-2), -3\}$$

$$= (2, -3).$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-2)^2 + 3^2 - (-5)}$$

$$= \sqrt{4 + 9 + 5} = \sqrt{18} = 3\sqrt{2}.$$

**Q. 2. Show that the equation  $x^2 + y^2 + x - y = 0$  represents a circle. Find its centre and radius.**

**Answer :** The general equation of a conic is as follows

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ where } a, b, c, f, g, h \text{ are constants}$$

For a circle,  $a = b$  and  $h = 0$ .

The equation becomes:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots (i)$$

$$\text{Given, } x^2 + y^2 + x - y = 0$$

Comparing with (i) we see that the equation represents a circle with  $2g = 1$

$$\Rightarrow g = \frac{1}{2}, 2f = -1 \Rightarrow f = -\frac{1}{2} \text{ and } c = 0.$$

$$\text{Centre } (-g, -f) = \left\{-\frac{1}{2}, -\left(-\frac{1}{2}\right)\right\}$$

$$= \left(-\frac{1}{2}, \frac{1}{2}\right).$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\frac{1^2}{2} + \left(-\frac{1^2}{2}\right) - 0}$$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$$

### Q. 3

Show that the equation  $3x^2 + 3y^2 + 6x - 4y - 1 = 0$  represents a circle. Find its centre and radius.

**Answer :** The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  where a, b, c, f, g, h are constants

For a circle,  $a = b$  and  $h = 0$ .

The equation becomes:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots (i)$$

$$\text{Given, } 3x^2 + 3y^2 + 6x - 4y - 1 = 0 \Rightarrow x^2 + y^2 + 2x - \frac{4}{3}y - \frac{1}{3} = 0$$

Comparing with (i) we see that the equation represents a circle with  $2g = 2 \Rightarrow g = 1$ ,  $2f = -\frac{4}{3} \Rightarrow f = -\frac{2}{3}$  and  $c = -\frac{1}{3}$ .

$$\text{Centre } (-g, -f) = \left\{ -1, -\left(-\frac{2}{3}\right) \right\}$$

$$= \left( -1, \frac{2}{3} \right).$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{1^2 + \left(-\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)}$$

$$= \sqrt{1 + \frac{4}{9} + \frac{1}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}.$$

**Q. 4. Show that the equation  $x^2 + y^2 + 2x + 10y + 26 = 0$  represents a point circle. Also, find its centre.**

**Answer :** The general equation of a circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots (i) \text{ where } c, g, f \text{ are constants.}$$

$$\text{Given, } x^2 + y^2 + 2x + 10y + 26 = 0$$

Comparing with (i) we see that the equation represents a circle with  $2g = 2 \Rightarrow g = 1$ ,  $2f = 10 \Rightarrow f = 5$  and  $c = 26$ .

$$\text{Centre } (-g, -f) = (-1, -5).$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{1^2 + 5^2 - 26}$$

$$= \sqrt{26 - 26} = 0.$$

Thus it is a point circle with radius 0.

**Q. 5. Show that the equation  $x^2 + y^2 - 3x + 3y + 10 = 0$  does not represent a circle.**

**Answer :** Radius =

$$\sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2 - 10}$$

$$= \sqrt{\frac{9}{2} - 10} = \sqrt{-\frac{11}{2}},$$

which implies that the radius is negative. (not possible)

Therefore,  $x^2 + y^2 - 3x + 3y + 10 = 0$  does not represent a circle.

**Q. 6. Find the equation of the circle passing through the points**

**(i) (0, 0), (5, 0) and (3, 3)**

**(ii) (1, 2), (3, -4) and (5, -6)**

**(iii) (20, 3), (19, 8) and (2, -9)**

**Also, find the centre and radius in each case.**

**Answer : (i)** The required circle equation

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 0^2 + 0^2 & 0 & 0 & 1 \\ 5^2 + 0^2 & 5 & 0 & 1 \\ 3^2 + 3^2 & 3 & 3 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2) \begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 3 & 3 & 1 \end{vmatrix} - x \begin{vmatrix} 0 & 0 & 1 \\ 25 & 0 & 1 \\ 18 & 3 & 1 \end{vmatrix}$$

$$+ y \begin{vmatrix} 0 & 0 & 1 \\ 25 & 5 & 1 \\ 18 & 3 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 25 & 5 & 0 \\ 18 & 3 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 15(x^2 + y^2) - 75x - 15y = 0$$

$$\Rightarrow x^2 + y^2 - 5x - y = 0 \text{ is the equation with centre} = (2.5, 0.5)$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-2.5)^2 + (-0.5)^2 - 0} = 2.549$$

**(ii)** The required circle equation

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 1^2 + 2^2 & 1 & 2 & 1 \\ 3^2 + (-4)^2 & 3 & -4 & 1 \\ 5^2 + (-6)^2 & 5 & -6 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2) \begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ 5 & -6 & 1 \end{vmatrix} - x \begin{vmatrix} 5 & 2 & 1 \\ 25 & -4 & 1 \\ 61 & -6 & 1 \end{vmatrix} + y \begin{vmatrix} 5 & 1 & 1 \\ 25 & 3 & 1 \\ 61 & 5 & 1 \end{vmatrix} - \begin{vmatrix} 5 & 1 & 2 \\ 25 & 3 & -4 \\ 61 & 5 & -6 \end{vmatrix} = 0$$

$$\Rightarrow 8(x^2 + y^2) - 176x - 32y - 200 = 0$$

$\Rightarrow x^2 + y^2 - 22x - 4y - 25 = 0$  is the equation with centre = (11, 2)

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-11)^2 + (-2)^2 - 25} = 10$$

(iii) The required circle equation

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 20^2 + 3^2 & 20 & 3 & 1 \\ 19^2 + 8^2 & 19 & 8 & 1 \\ 2^2 + (-9)^2 & 2 & -9 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2) \begin{vmatrix} 20 & 3 & 1 \\ 19 & 8 & 1 \\ 2 & -9 & 1 \end{vmatrix} - x \begin{vmatrix} 409 & 3 & 1 \\ 425 & 8 & 1 \\ 85 & -9 & 1 \end{vmatrix} + y \begin{vmatrix} 409 & 20 & 1 \\ 425 & 19 & 1 \\ 85 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 409 & 20 & 3 \\ 425 & 19 & 8 \\ 85 & 2 & -9 \end{vmatrix} = 0$$

$$\Rightarrow 102(x^2 + y^2) - 1428x - 612y - 11322 = 0$$

$\Rightarrow x^2 + y^2 - 14x - 6y - 111 = 0$  is the equation with centre = (7, 3)

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-7)^2 + (-3)^2 - (-111)} = 13$$

**Q. 7. Find the equation of the circle which is circumscribed about the triangle whose vertices are A(-2, 3), B(5, 2) and C(6, -1). Find the centre and radius of this circle.**

**Answer :** The general equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$

...(i), where (h, k) is the centre and r is the radius.

Putting A(-2, 3), B(5, 2) and C(6, -1) in (i) we get

$$h^2 + k^2 + 4h - 6k + 13 = r^2 \dots(\text{ii})$$

$$h^2 + k^2 - 10h - 4k + 29 = r^2 \dots(\text{iii})\text{and}$$

$$h^2 + k^2 - 12h + 2k + 37 = r^2 \dots(\text{iv})$$

subtracting (ii) from (iii) and also from (iv),

$$-14h + 2k + 16 = 0 \Rightarrow -7h + k + 8 = 0$$

$$-16h + 8k + 24 = 0 \Rightarrow -2h + k + 3 = 0$$

Subtracting,

$$5h - 5 = 0 \Rightarrow h = 1$$

$$k = -1$$

Centre = (1, -1)

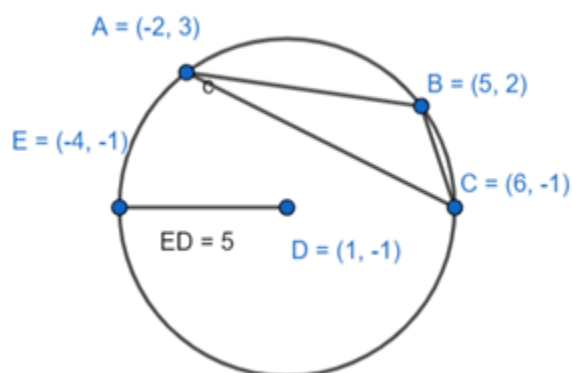
Putting these values in (ii) we get, radius

$$= \sqrt{1 + 1 + 4 + 6 + 13} = \sqrt{25} = 5$$

Equation of the circle is

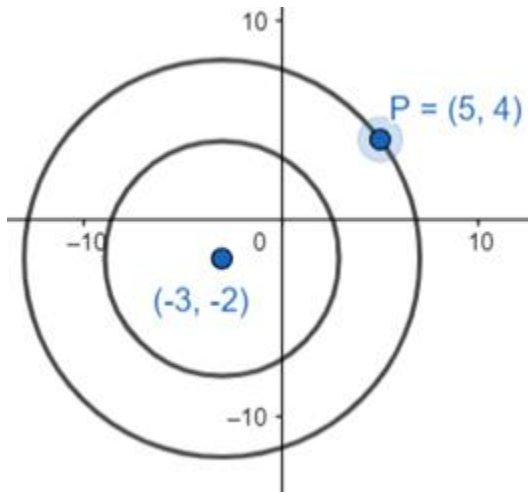
$$(x - 1)^2 + \{y - (-1)\}^2 = 5^2$$

$$(x - 1)^2 + (y + 1)^2 = 25.$$



**Q. 8. Find the equation of the circle concentric with the circle  $x^2 + y^2 + 4x + 6y + 11 = 0$  and passing through the point P(5, 4).**

**Answer :** 2 or more circles are said to be concentric if they have the same centre and different radii.



Given,  $x^2 + y^2 + 4x + 6y + 11 = 0$

The concentric circle will have the equation

$$x^2 + y^2 + 4x + 6y + c' = 0$$

As it passes through P(5, 4), putting this in the equation

$$5^2 + 4^2 + 4 \times 5 + 6 \times 4 + c' = 0$$

$$\Rightarrow 25 + 16 + 20 + 24 + c' = 0$$

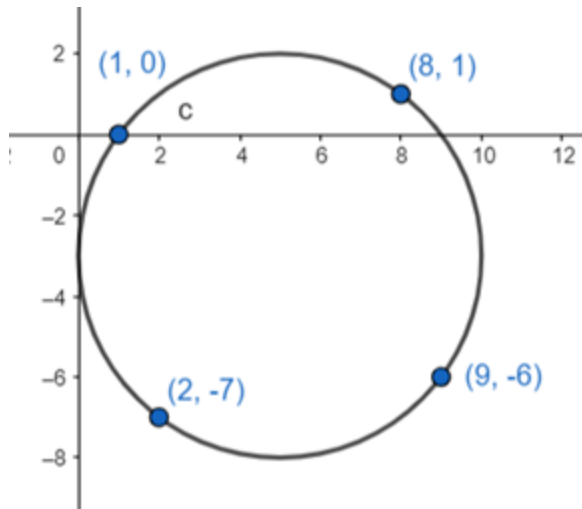
$$\Rightarrow c' = -85$$

The required equation is

$$x^2 + y^2 + 4x + 6y - 85 = 0$$

**Q. 9. Show that the points A(1, 0), B(2, -7), c(8, 1) and D(9, -6) all lie on the same circle. Find the equation of this circle, its centre and radius.**

**Answer :**



The general equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$

...(i), where  $(h, k)$  is the centre and  $r$  is the radius.

Putting  $(1, 0)$  in (i)

$$(1 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 + 1 - 2h = r^2 \text{ ..(ii)}$$

Putting  $(2, -7)$  in (i)

$$(2 - h)^2 + (-7 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 + 53 - 4h + 14k = r^2$$

$$\Rightarrow (h^2 + k^2 + 1 - 2h) + 52 - 2h + 14k = r^2$$

$$h - 7k - 26 = 0 \text{ ..(iii) [from (ii)]}$$

Similarly putting  $(8, 1)$

$$7h + k - 32 = 0 \text{ ..(iv)}$$

Solving (iii)&(iv)

$$h = 5 \text{ and } k = -3$$

centre  $(5, -3)$



Radius = 25

To check if (9, - 6) lies on the circle,  $(9 - 5)^2 + (- 6 + 3)^2 = 5^2$

Hence, proved.

**Q. 10. Find the equation of the circle which passes through the points (1, 3) and (2, - 1), and has its centre on the line  $2x + y - 4 = 0$ .**

**Answer :** The equation of a circle:  $x^2 + y^2 + 2gx + 2fy + c = 0 \dots(i)$

Putting (1, 3) & (2, - 1) in (i)

$$2g + 6f + c = - 10 \dots(ii)$$

$$4g - 2f + c = - 5 \dots(iii)$$

Since the centre lies on the given straight line,  $(-g, -f)$  must satisfy the equation as

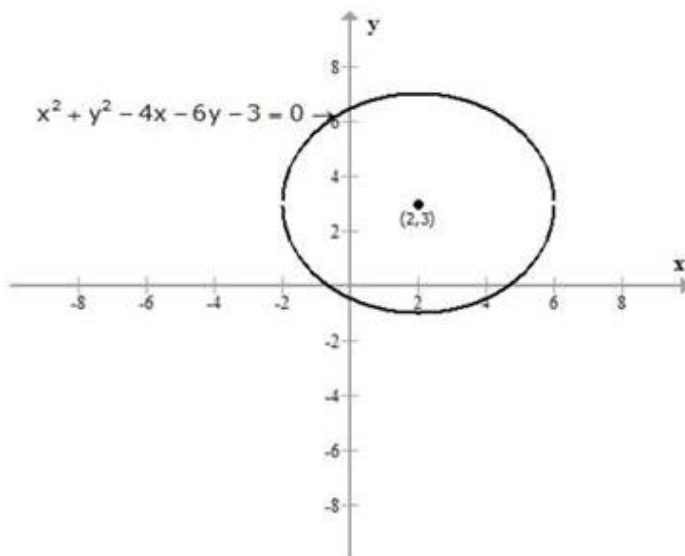
$$-2g - f - 4 = 0 \dots(iv)$$

Solving,  $f = - 1$ ,  $g = - 1.5$ ,  $c = - 1$

The equation is  $x^2 + y^2 - 3x - 2y - 1 = 0$

**Q. 11. Find the equation of the circle concentric with the circle  $x^2 + y^2 - 4x - 6y - 3 = 0$  and which touches the y-axis.**

**Answer :** The given image of the circle is:



We know that the general equation of the circle is given by:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Also,

Radius  $r =$

$$\sqrt{g^2 + f^2 - c}$$

Now,

$$r = \sqrt{(2)^2 + (3)^2 - (-3)}$$

$$r = \sqrt{4 + 9 + 3}$$

$r = 4$  units.

We need to find the equation of the circle which is concentric to the given circle and touches y-axis.

The centre of the circle remains the same.

Now, y-axis will be tangent to the circle.

Point of contact will be (0, 3)

Therefore, radius = 2

Now,

Equation of the circle:

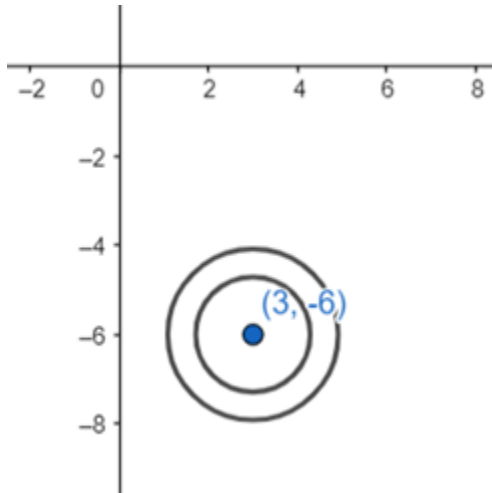
$$(x - 2)^2 + (y - 3)^2 = (2)^2$$

$$x^2 + 4 - 4x + y^2 + 9 - 6y = 4$$

$$x^2 + y^2 - 4x - 6y + 9 = 0$$

**Q. 12. Find the equation of the circle concentric with the circle  $x^2 + y^2 - 6x + 12y + 15 = 0$  and of double its area.**

**Answer :** 2 or more circles are said to be concentric if they have the same centre and different radii.



Given,  $x^2 + y^2 - 6x + 12y + 15 = 0$

Radius  $r =$

$$\sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + 6^2 - 15} = \sqrt{30}$$

The concentric circle will have the equation

$$x^2 + y^2 - 6x + 12y + c' = 0$$

Also given area of circle = 2x area of the given circle.

$$\Rightarrow r'^2 = 2 \times r^2 = 2 \times 30 = 60$$

We can get  $c' = 45 - 60 = -15$

The required equation is  $x^2 + y^2 - 6x + 12y - 15 = 0$ .

**Q. 13. Prove that the centres of the three circles  $x^2 + y^2 - 4x - 6y - 12 = 0$ ,  $x^2 + y^2 + 2x + 4y - 5 = 0$  and  $x^2 + y^2 - 10x - 16y + 7 = 0$  are collinear.**

**Answer :** Given,

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

centre  $(-g_1, -f_1) = (2, 3)$

$$x^2 + y^2 + 2x + 4y - 5 = 0$$

centre  $(-g_2, -f_2) = (-1, -2)$

$$x^2 + y^2 - 10x - 16y + 7 = 0$$

centre  $(-g_3, -f_3) = (5, 8)$

to prove that the centres are collinear,

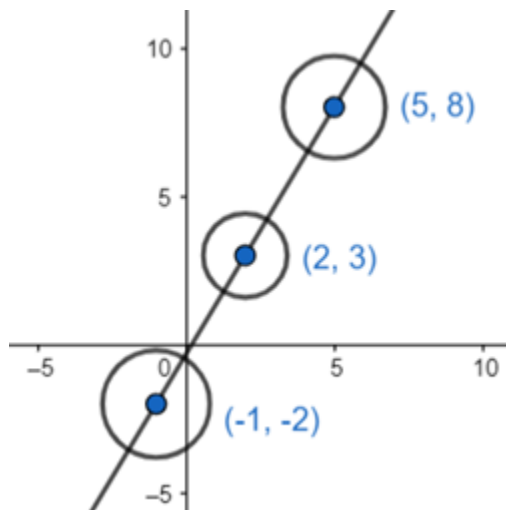
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Where  $x_1, y_1$  are the coordinates of the 1st centre and so on.

$$\Rightarrow \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$$= 2(-2 - 8) - 3(-1 - 5) + 1(-8 + 10)$$

$$= -20 + 18 + 2 = 0$$



The centres are collinear.

**Q. 14. Find the equation of the circle which passes through the points A(1, 1) and B(2, 2) and whose radius is 1. Show that there are two such circles.**

**Answer :** The general equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$

...(i), where (h, k) is the centre and r is the radius.

Putting A(1, 1) in (i)

$$(1 - h)^2 + (1 - k)^2 = 1^2$$

$$\Rightarrow h^2 + k^2 + 2 - 2h - 2k = 1$$

$$\Rightarrow h^2 + k^2 - 2h - 2k = -1 \text{..(ii)}$$

Putting B(2, 2) in (i)

$$(2 - h)^2 + (2 - k)^2 = 1^2$$

$$\Rightarrow h^2 + k^2 + 8 - 4h - 4k = 1$$

$$\Rightarrow h^2 + k^2 - 4h - 4k = -7$$

$$\Rightarrow (h^2 + k^2 - 2h - 2k) - 2h - 2k = -7$$

$$\Rightarrow -1 - 2h - 2k = -7 \text{ [from (ii)]}$$

$$\Rightarrow -2h - 2k = -6$$

$$\Rightarrow h + k = 3 \Rightarrow h = 3 - k$$

Putting it in (ii)

$$\Rightarrow (3 - k)^2 + k^2 - 2(3 - k) - 2k = - 1$$

$$\Rightarrow 9 + 2k^2 - 6k - 6 + 2k - 2k = - 1$$

$$\Rightarrow 2k^2 + 4 - 6k = 0$$

$$\Rightarrow k^2 - 3k + 2 = 0$$

$$\Rightarrow k = 2 \text{ or } k = 1$$

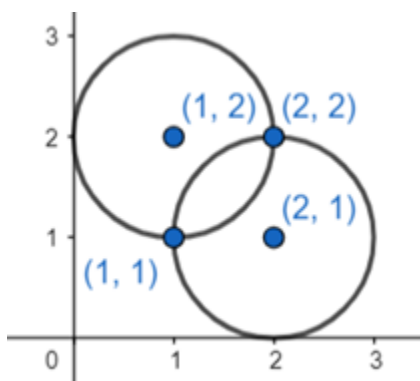
When  $k = 2$ ,  $h = 3 - 2 = 1$

Equation of 1 circle

$$(x - 1)^2 + (y - 2)^2 = 1$$

When  $k = 1$ ,  $h = 3 - 1 = 2$

$$(x - 2)^2 + (y - 1)^2 = 1$$

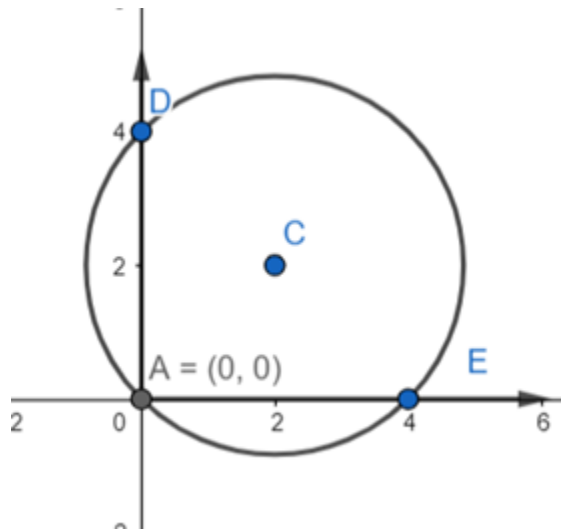


**Q. 15. Find the equation of a circle passing through the origin and intercepting lengths  $a$  and  $b$  on the axes.**

**Answer :** From the figure

AD = b units and AE = a units.

D(0, b), E(a, 0) and A(0, 0) lies on the circle. C is the centre.



The general equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$

... (i), where (h, k) is the centre and r is the radius.

Putting A(0, 0) in (i)

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 = r^2 \dots (ii)$$

Similarly putting D(0, b) in (i)

$$(0 - h)^2 + (b - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 + b^2 - 2kb = r^2$$

$$\Rightarrow r^2 + b^2 - 2kb = r^2$$

$$\Rightarrow b^2 - 2kb = 0$$

$$\Rightarrow (b - 2k)b = 0$$

$$\text{Either } b = 0 \text{ or } k = \frac{b}{2}$$

Similarly putting  $E(a, 0)$  in (i)

$$(a - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 + a^2 - 2ha = r^2$$

$$\Rightarrow r^2 + a^2 - 2ha = r^2$$

$$\Rightarrow a^2 - 2ha = 0$$

$$\Rightarrow (a - 2h)a = 0$$



Either  $a = 0$  or  $h = \frac{a}{2}$

Centre =  $C\left(\frac{a}{2}, \frac{b}{2}\right)$

$$r^2 = h^2 + k^2$$

$$\Rightarrow r^2 = \frac{a^2 + b^2}{4}$$

Putting the value of  $r^2$ ,  $h$  and  $k$  in equation (i)

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

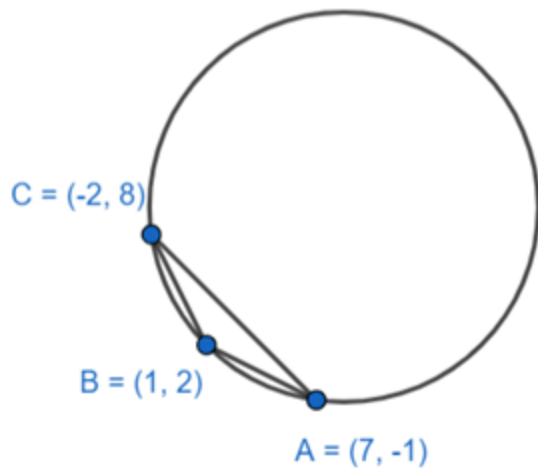
$$\Rightarrow x^2 + y^2 + \frac{a^2}{4} + \frac{b^2}{4} - xa - yb = \frac{a^2 + b^2}{4}$$

$$\Rightarrow x^2 + y^2 - xa - yb = 0$$

which is the required equation.

Q. 16. Find the equation of the circle circumscribing the triangle formed by the lines  $x + y = 6$ ,  $2x + y = 4$  and  $x + 2y = 5$ .

Answer : Solving the equations we get the coordinates of the triangle:



The required circle equation

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ (-2)^2 + 8^2 & -2 & 8 & 1 \\ 1^2 + 2^2 & 1 & 2 & 1 \\ 7^2 + (-1)^2 & 7 & -1 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2) \begin{vmatrix} -2 & 8 & 1 \\ 1 & 2 & 1 \\ 7 & -1 & 1 \end{vmatrix} - x \begin{vmatrix} 68 & 8 & 1 \\ 5 & 2 & 1 \\ 50 & -1 & 1 \end{vmatrix} + y \begin{vmatrix} 68 & -2 & 1 \\ 5 & 1 & 1 \\ 50 & 7 & 1 \end{vmatrix} -$$

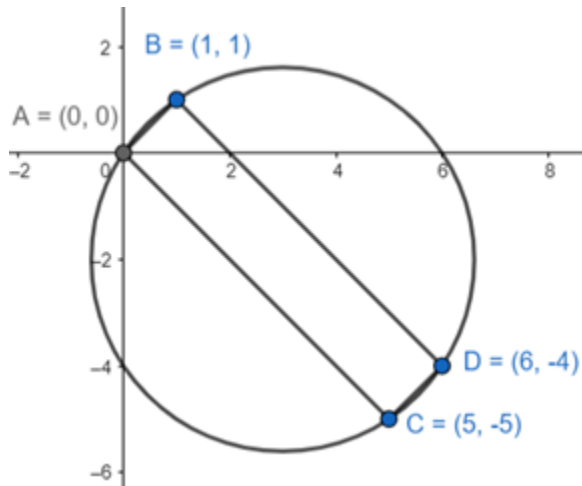
$$\begin{vmatrix} 68 & -2 & 8 \\ 5 & 1 & 2 \\ 50 & 7 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 27(x^2 + y^2) - 459x - 513y + 1350 = 0$$

$$\Rightarrow x^2 + y^2 - 17x - 19 + 50 = 0$$

**Q. 17. Show that the quadrilateral formed by the straight lines  $x - y = 0$ ,  $3x + 2y = 5$ ,  $x - y = 10$  and  $2x + 3y = 0$  is cyclic and hence find the equation of the circle.**

**Answer :** Solving the equations we get the coordinates of the quadrilateral.



$$\text{Slope of } AB = \frac{1-0}{1-0} = 1$$

$$\text{Slope of } CD = 1$$

$$AB \parallel CD$$

$$\text{Slope of } BD = AC = -1$$

$$AC \parallel BD$$

So they form a rectangle with all sides =  $90^\circ$

The quadrilateral is cyclic as sum of opposite angles =  $180^\circ$ .

Now, AD = diameter of the circle equation of the circle with extremities A(0, 0) & D(6, -4) is

$$(x - 0)(x - 6) + (y - 0)(y + 4) = 0$$

$$x^2 + y^2 - 6x + 4y = 0$$

**Q. 18.** If  $(-1, 3)$  and  $(\alpha, \beta)$  are the extremities of the diameter of the circle  $x^2 + y^2 - 6x + 5y - 7 = 0$ , find the coordinates  $(\alpha, \beta)$ .

**Answer :** Given  $x^2 + y^2 - 6x + 5y - 7 = 0$

$$\text{Centre} \left( 3, -\frac{5}{2} \right)$$

As  $(-1, 3)$  &  $(\alpha, \beta)$  are the 2 extremities of the diameter, using mid - point formula we can write

$$\frac{\alpha - 1}{2} = 3$$

$$\Rightarrow \alpha = 7$$

$$\text{and } \frac{\beta + 3}{2} = -\frac{5}{2}$$

$$\Rightarrow \beta = -8$$

$$(\alpha, \beta) = (7, -8)$$