Three-Dimensional Geometry

Exercise 26A

Q. 1. If a point lies on the z-axis, then find its x-coordinate and y-coordinate.

Answer : X and y coordinates of a point are its distance from the origin along or parallel to the horizontal x-axis and y-axis. To measure the x and y coordinates, you must move either to the left of the origin or to its right. In case of a point on the z-axis, you do not move to the right or to the left of the origin. Hence x and y coordinates are 0 for a point on the z-axis.

Q. 2. If a point lies on yz-plane then what is its x-coordinate?

Answer : x-coordinate is the distance of a point from the origin parallel or along the x-axis. To measure the x coordinate, you must move either to the left of the origin or to its right. In case of a point lying on the yz-plane, you do not move to the right or to the left of the origin. Hence x coordinate is 0 for a point on the yz-plane.

Q. 3. In which plane does the point (4, -3, 0) lie?

Answer : Here the x, y, z coordinates of the point are 4, -3, 0. As the distance of point along the z-axis is 0, the plane in which the point lies is the xy-plane.

Q. 4. In which octant does each of the given points lie?

(i) (-4, -1, -6)
(ii) (2, 3, -4)
(iii) (-6, 5, -1)
(iv) (4, -3, -2)
(v) (-1, -6, 5)
(vi) (4, 6, 8)

Answer : The position of a point in a octant is signified by the signs of the x, y, z coordinates.

Here is a table showing signs of the x, y, z coordinates in all the octants.

Number	x sign	y sign	z sign
I	+	+	+
II	-	+	+
III	-	-	+
IV	+	-	+
V	+	+	-
VI	-	+	-
VII	-	-	-
VIII	+	-	-

According to the table

(i) (-4, -1, -6) lies in octant VII

(ii) (2, 3, -4) lies in octant V

(iii) (-6, 5, -1) lies in octant VI

(iv) (4, -3, -2) lies in octant VIII

(v) (-1, -6, 5) lies in octant III

(vi) (4, 6, 8) lies in octant I

Exercise 26B

Q. 1. **Find the distance between the points :**

(i) A(5, 1, 2) and B(4, 6, -1)
(ii) P(1, -1, 3) and Q(2, 3, -5)
(iii) R(1, -3, 4) and S(4, -2, -3)
(iv) C(9, -12, -8) and the origin

Answer :

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(i) A(5, 1, 2) and B(4, 6, -1)

Here, $(x_1,y_1,z_1) = (5, 1, 2)$

$(x_2,y_2,z_2)= (4, 6, -1)$

Therefore,

$$D = \sqrt{(4\ -5)^2+(6-1)^2+(-1-2)^2}$$

$$= \sqrt{(-1)^2+(5)^2+(-3)^2}$$

$$= \sqrt{1+25+9}$$

$=\sqrt{35}$

Distance between points A and B is

$\sqrt{35}$.

(ii) P(1, -1, 3) and Q(2, 3, -5)

Here, (x_1,y_1,z_1)= (1, -1, 3)

(x_2,y_2,z_2)= (2, 3, -5)

Therefore,

$$D = \sqrt{(2-1)^2 + (3-(-1))^2 + (-5-3)^2}$$

$$= \sqrt{(1)^2 + (4)^2 + (-8)^2}$$

$$= \sqrt{1+16+64}$$

$$= \sqrt{81} = 9$$

Distance between points P and Q are 9 units.

(iii) R(1, -3, 4) and S(4, -2, -3)

Here, (x_1,y_1,z_1)= (1, -3, 4)

(x_2,y_2,z_2)= (4, -2, -3)

Therefore,

$$D = \sqrt{(4-1)^2 + (-2-(-3))^2 + (-3-4)^2}$$

$$= \sqrt{(3)^2 + (1)^2 + (-7)^2}$$

$= \sqrt{9 + 1 + 49}$

$= \sqrt{59}$

Distance between points R and S is $\sqrt{59}$ units.

(iv) C(9, -12, -8) and the origin

Coordinates of origin are (0, 0, 0)

Here, (x_1,y_1,z_1)= (9, -12, -8)

(x_2,y_2,z_2)= (0, 0, 0)

Therefore,

$$D = \sqrt{(0-9)^2 + (0-(-12))^2 + (0-(-8))^2}$$

$$= \sqrt{(-9)^2 + (12)^2 + (8)^2}$$

$$= \sqrt{81 + 144 + 64}$$

$$= \sqrt{289} = 17$$

Distance between points C and origin is 17 units.

Q. 2. Show that the points A(1, -1, -5), b(3, 1,3) and C(9, 1, -3) are the vertices of an equilateral triangle.

Answer : To prove: Points A, B, C form equilateral triangle.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$$D=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Here,

(x_1,y_1,z_1)= (1, -1, -5)

(x_2,y_2,z_2)= (3, 1,3)

(x_3,y_3,z_3)= (9, 1, -3)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(3-1)^2+(1-(-1))^2+(3-(-5))^2}$

$= \sqrt{(2)^2+(2)^2+(8)^2}$

$= \sqrt{4+4+64}$

$= \sqrt{72} = 6\sqrt{2}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(9-3)^2+(1-1)^2+(-3-3)^2}$

$= \sqrt{(6)^2+(0)^2+(-6)^2}$

$= \sqrt{36+0+36}$

$= \sqrt{72} = 6\sqrt{2}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(9-1)^2 + (1-(-1))^2 + (-3-(-5))^2}$

$= \sqrt{(8)^2 + (2)^2 + (2)^2}$

$= \sqrt{64 + 4 + 4}$

$= \sqrt{72} = 6\sqrt{2}$

Hence, AB = BC = AC

Therefore, Points A, B, C make an equilateral triangle.

Q. 3. Show that the points A(4, 6, -5), B(0, 2, 3) and C(-4, -4, -1) from the vertices of an isosceles triangle.

Answer : To prove: Points A, B, C form isosceles triangle.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1,y_1,z_1)= (4, 6, -3)

(x_2,y_2,z_2)= (0, 2, 3)

(x_3,y_3,z_3)= (-4, -4, -1)

Length AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(0-4)^2 + (2-6)^2 + (3-(-3))^2}$

$= \sqrt{(-4)^2 + (-4)^2 + (6)^2}$

$= \sqrt{16+16+36}$

$= \sqrt{68} = 2\sqrt{17}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(-4-0)^2 + (-4-2)^2 + (-1-3)^2}$

$= \sqrt{(-4)^2 + (-6)^2 + (-4)^2}$

$= \sqrt{16+36+16}$

$= \sqrt{68} = 2\sqrt{17}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(-4-4)^2 + (-4-6)^2 + (-1-(-5))^2}$

$= \sqrt{(-8)^2 + (-10)^2 + (2)^2}$

$= \sqrt{64+100+4}$

$= \sqrt{168}$

Here, AB = BC

∴ vertices A, B, C forms an isosceles triangle.

Q. 4. Show that the points A(0, 1, 2), B(2, -1, 3) and C(1, -3, 1) are the vertices of an isosceles right-angled triangle.

Answer : To prove: Points A, B, C form isosceles triangle.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (0, 1, 2)

(x_2,y_2,z_2)= (2, -1, 3)

(x_3,y_3,z_3)= (1, -3, 1)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(2-0)^2+(-1-1)^2+(3-2)^2}$

= $\sqrt{(2)^2+(-2)^2+(1)^2}$

= $\sqrt{4+4+1}$

= $\sqrt{9}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

= $\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}$

= $\sqrt{(-1)^2+(-2)^2+(-2)^2}$

= $\sqrt{1+4+4}$

= $\sqrt{9}$

Length AC = $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

= $\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}$

= $\sqrt{(1)^2+(-4)^2+(-1)^2}$

= $\sqrt{1+16+1}$

= $\sqrt{18}$

Also, $AB^2 + BC^2 = 9 + 9 = 18 = AC^2$

Therefore, points A, B, C forms an isosceles right-angled triangle.

Q. 5. **Show that the points A(1, 1, 1), B(-2, 4, 1), C(1, -5, 5) and D(2, 2, 5) are the vertices of a square.**

Answer : To prove: Points A, B, C, D form square.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D= $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (1, 1, 1)

(x_2,y_2,z_2)= (-2, 4, 1)

(x_3,y_3,z_3)= (-1, 5, 5)

(x_4,y_4,z_4)= (2, 2, 5)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(-2-1)^2+(4-1)^2+(1-1)^2}$

= $\sqrt{(-3)^2+(3)^2+(0)^2}$

= $\sqrt{9+9+0}$

$= \sqrt{18}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(-1 + 2)^2 + (5 - 4)^2 + (5 - 1)^2}$

$= \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

Length CD = $\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

$= \sqrt{(2 + 1)^2 + (2 - 5)^2 + (5 - 5)^2}$

$= \sqrt{(3)^2 + (3)^2 + (0)^2}$

$= \sqrt{9 + 9 + 0}$

$= \sqrt{18}$

Length AD = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(2 - 1)^2 + (2 - 1)^2 + (5 - 1)^2}$

$= \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(-1-1)^2 + (5-1)^2 + (5-1)^2}$

$= \sqrt{(-2)^2 + (4)^2 + (4)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

Length BD = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(2+2)^2 + (2-4)^2 + (5-1)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{16 + 4 + 16}$

$= \sqrt{36}$

Here, AB = BC = CD = AD

Also, AC = BD

This means all the sides are the same and diagonals are also equal.

Hence vertices A, B, C, D form a square.

Q. 6. Show that the points A(1, 2, 3), B(-1, -2, -1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram. Show that ABCD is not a rectangle.

Answer : To prove: Points A, B, C, D form parallelogram.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1,y_1,z_1)= (1, 2, 3)

(x_2,y_2,z_2)= (-1, -2, -1)

(x_3,y_3,z_3)= (2, 3, 2)

(x_4,y_4,z_4)= (4, 7, 6)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}$

= $\sqrt{(-2)^2+(-4)^2+(-4)^2}$

= $\sqrt{4+16+16}$

= $\sqrt{36}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

= $\sqrt{(2+1)^2+(3+2)^2+(2+1)^2}$

= $\sqrt{(3)^2+(5)^2+(3)^2}$

= $\sqrt{9+25+9}$

= $\sqrt{43}$

Length CD = $\sqrt{(x_4-x_3)^2+(y_4-y_3)^2+(z_4-z_3)^2}$

= $\sqrt{(4-2)^2+(7-3)^2+(6-2)^2}$

= $\sqrt{(2)^2+(4)^2+(4)^2}$

= $\sqrt{4+16+16}$

$= \sqrt{36}$

Length AD = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$

$= \sqrt{(3)^2 + (5)^2 + (3)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(2-1)^2 + (3-2)^2 + (2-3)^2}$

$= \sqrt{(1)^2 + (1)^2 + (-1)^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

Length BD = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(4+1)^2 + (7+2)^2 + (6+1)^2}$

$= \sqrt{(5)^2 + (9)^2 + (7)^2}$

$= \sqrt{25 + 81 + 49}$

$= \sqrt{155}$

Here, AB = CD which are opposite sides of polygon.

BC = AD which are opposite sides of polygon.

Also the diagonals AC and BD are not equal in length.

Hence, the polygon is not a rectangle.

Q. 7. Show that the points P(2, 3, 5), Q(-4, 7, -7), R(-2, 1, -10) and S(4, -3, 2) are the vertices of a rectangle.

Answer : To prove: Points P, Q, R, S forms rectangle.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D= $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (2, 3, 5)

(x_2,y_2,z_2)= (-4, 7, -7)

(x_3,y_3,z_3)= (-2, 1, -10)

(x_4,y_4,z_4)= (4, -3, 2)

Length PQ = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(-4-2)^2+(7-3)^2+(-7-5)^2}$

= $\sqrt{(-6)^2+(4)^2+(-12)^2}$

= $\sqrt{36+16+144}$

= $\sqrt{196}$

Length QR = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

= $\sqrt{(-2+4)^2+(1-7)^2+(-10+7)^2}$

= $\sqrt{(2)^2 + (-6)^2 + (-3)^2}$

= $\sqrt{4 + 36 + 9}$

= $\sqrt{49}$

Length RS = $\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

= $\sqrt{(4 + 2)^2 + (-3 - 1)^2 + (2 + 10)^2}$

= $\sqrt{(6)^2 + (-4)^2 + (12)^2}$

= $\sqrt{36 + 16 + 144}$

= $\sqrt{196}$

Length PS = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

= $\sqrt{(4 - 2)^2 + (-3 - 3)^2 + (2 - 5)^2}$

= $\sqrt{(2)^2 + (-6)^2 + (-3)^2}$

= $\sqrt{4 + 36 + 9}$

= $\sqrt{49}$

Length PR = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

= $\sqrt{(-2 - 2)^2 + (1 - 3)^2 + (-10 - 5)^2}$

= $\sqrt{(-4)^2 + (-2)^2 + (-15)^2}$

= $\sqrt{16 + 4 + 225}$

$= \sqrt{245}$

Length QS = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(4 + 4)^2 + (-3 - 7)^2 + (2 + 7)^2}$

$= \sqrt{(8)^2 + (-10)^2 + (9)^2}$

$= \sqrt{64 + 100 + 81}$

$= \sqrt{245}$

Here, PQ = RS which are opposite sides of polygon.

QR = PS which are opposite sides of polygon.

Also the diagonals PR = QS.

Hence, the polygon is a rectangle.

Q. 8,. Show that the points P(1, 3, 4), Q(-1, 6, 10), R(-7, 4, 7) and S(-5, 1, 1) are the vertices of a rhombus.

Answer : To prove: Points P, Q, R, S forms rhombus.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1,y_1,z_1)= (1, 3, 4)

(x_2,y_2,z_2)= (-1, 6, 10)

(x_3,y_3,z_3)= (-7, 4, 7)

(x_4,y_4,z_4)= (-5, 1, 1)

Length PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$

$= \sqrt{(-2)^2+(3)^2+(6)^2}$

$= \sqrt{4+9+36}$

$= \sqrt{49}$

Length QR = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$

$= \sqrt{(-6)^2+(-2)^2+(-3)^2}$

$= \sqrt{36+34+9}$

$= \sqrt{49}$

Length RS = $\sqrt{(x_4-x_3)^2+(y_4-y_3)^2+(z_4-z_3)^2}$

$= \sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$

$= \sqrt{(2)^2+(-3)^2+(-6)^2}$

$= \sqrt{4+9+36}$

$= \sqrt{49}$

Length PS = $\sqrt{(x_4-x_1)^2+(y_4-y_1)^2+(z_4-z_1)^2}$

$= \sqrt{(-5-1)^2+(1-3)^2+(1-4)^2}$

$= \sqrt{(-6)^2+(-2)^2+(-3)^2}$

= $\sqrt{36 + 4 + 9}$

= $\sqrt{49}$

Length PR = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

= $\sqrt{(-7 - 1)^2 + (4 - 3)^2 + (7 - 4)^2}$

= $\sqrt{(-8)^2 + (1)^2 + (3)^2}$

= $\sqrt{64 + 1 + 9}$

= $\sqrt{74}$

Length QS = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

= $\sqrt{(-5 + 1)^2 + (1 - 6)^2 + (1 - 10)^2}$

= $\sqrt{(-4)^2 + (-5)^2 + (-9)^2}$

= $\sqrt{16 + 25 + 81}$

= $\sqrt{122}$

Here, PQ = RS =QR = PS .

Also the diagonals PR ≠ QS.

Hence, the polygon is a rhombus as all sides are equal and diagonals are not equal.

Q. 9. Show that D(-1, 4, -3) is the circumcentre of triangle ABC with vertices A(3, 2, -5), B(-3. 8, -5) and C(-3, 2, 1).

Answer : To prove: D is circumcenter of triangle ABC

Let us consider D as circumcenter of triangle ABC.

∴ AD = BC = CD.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D= $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (3, 2, -5)

(x_2,y_2,z_2)= (-3. 8, -5)

(x_3,y_3,z_3)= (-3, 2, 1)

(x_4,y_4,z_4) = (-1, 4, -3)

Length AD = $\sqrt{(x_4-x_1)^2+(y_4-y_1)^2+(z_4-z_1)^2}$

= $\sqrt{(-1-3)^2+(4-2)^2+(-3+5)^2}$

= $\sqrt{(-4)^2+(2)^2+(2)^2}$

= $\sqrt{16+4+4}$

= $\sqrt{24}$

Length BD = $\sqrt{(x_4-x_2)^2+(y_4-y_2)^2+(z_4-z_2)^2}$

= $\sqrt{(-1+3)^2+(4-8)^2+(-3+5)^2}$

= $\sqrt{(2)^2+(-4)^2+(2)^2}$

= $\sqrt{4+16+4}$

= $\sqrt{24}$

Length CD = $\sqrt{(x_4-x_3)^2+(y_4-y_3)^2+(z_4-z_3)^2}$

$= \sqrt{(-1+3)^2+(4-2)^2+(-3-1)^2}$

$= \sqrt{(2)^2+(2)^2+(-4)^2}$

$= \sqrt{4+4+16}$

$= \sqrt{24}$

Hence, the condition is consistent.

Hence, D is circumcenter of triangle ABC.

Q. 10 A. Show that the following points are collinear :

A(-2, 3, 5), B(1, 2, 3) and C(7, 0, -1)

Answer : To prove: the 3 points are collinear.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (-2, 3, 5)

(x_2,y_2,z_2)= (1, 2, 3)

(x_3,y_3,z_3)= (7, 0, -1)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

$= \sqrt{(3)^2+(-1)^2+(-2)^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{36 + 4 + 16}$

$= \sqrt{56} = 2\sqrt{14}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

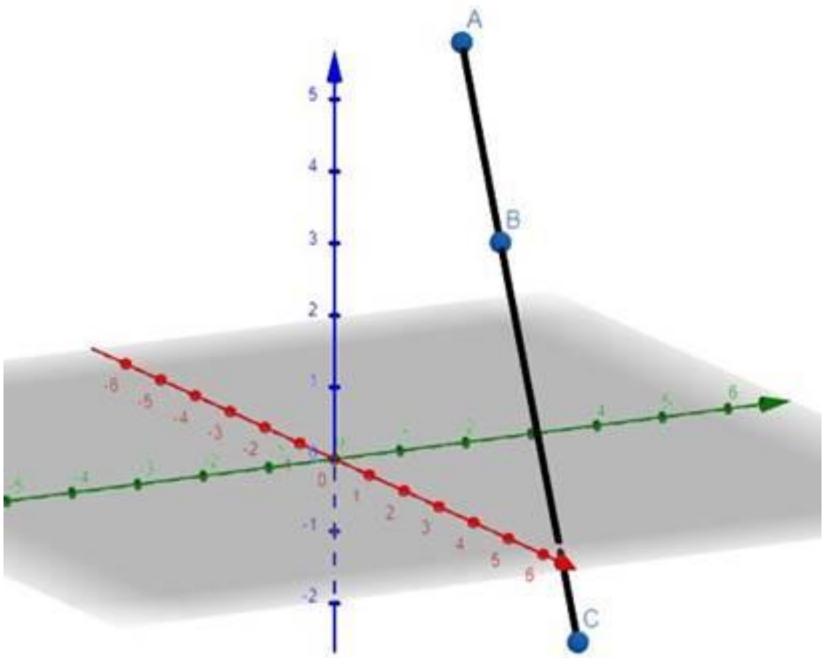
$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{81 + 9 + 36}$

$= \sqrt{126} = 3\sqrt{14}$

$AB + BC = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = AC$

Therefore A, B, C are collinear.



Q. 10 B. Show that the following points are collinear :

A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

Answer : To prove: the 3 points are collinear.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1,y_1,z_1)= (3, -5, 1)

(x_2,y_2,z_2)= (-1, 0, 8)

(x_3,y_3,z_3)= (7, -10, -6)

Length AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= $\sqrt{(-1 - 3)^2 + (0 + 5)^2 + (8 - 1)^2}$

$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90} = 3\sqrt{10}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(7 + 1)^2 + (-10 - 0)^2 + (-6 - 8)^2}$

$= \sqrt{(8)^2 + (-10)^2 + (-14)^2}$

$= \sqrt{64 + 100 + 196}$

$= \sqrt{360} = 6\sqrt{10}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(7 - 3)^2 + (-10 + 5)^2 + (-6 - 1)^2}$

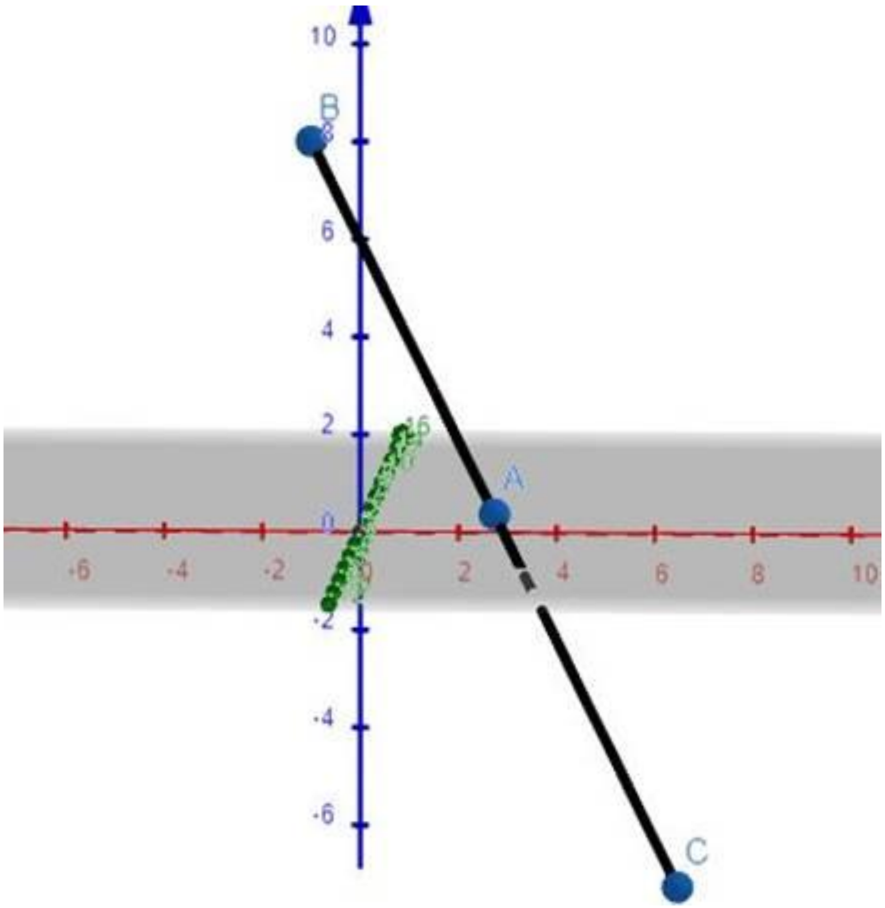
$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90} = 3\sqrt{10}$

BA + BC = $3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$ = BC

Therefore A, B, C are collinear.



Q. 10 C. Show that the following points are collinear :

P(3, -2, 4), Q(1, 1, 1) and R(-1, 4, 2)

Answer : To prove: the 3 points are collinear.

Formula: The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here, Vertices should be R(-1, 4, -2)

The solution according is

(x_1,y_1,z_1)= (3, -2, 4)

(x_2,y_2,z_2)= (1, 1, 1)

(x_3,y_3,z_3)= (-1, 4,- 2)

Length PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= $\sqrt{(1-3)^2 + (1+2)^2 + (1-4)^2}$

= $\sqrt{(-2)^2 + (3)^2 + (-3)^2}$

= $\sqrt{4+9+9}$

= $\sqrt{22}$

Length QR= $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

= $\sqrt{(-1-1)^2 + (4-1)^2 + (-2-1)^2}$

= $\sqrt{(-2)^2 + (3)^2 + (-3)^2}$

= $\sqrt{4+9+9}$

= $\sqrt{22}$

Length PR = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

= $\sqrt{(-1-3)^2 + (4+2)^2 + (-2-4)^2}$

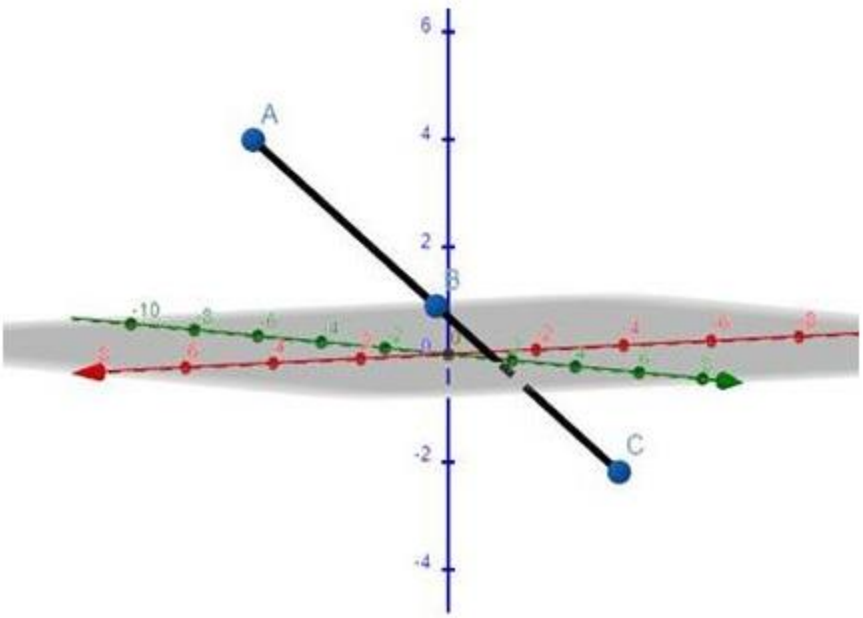
= $\sqrt{(-4)^2 + (6)^2 + (-6)^2}$

= $\sqrt{16+36+36}$

= $\sqrt{88}$ = $2\sqrt{22}$

PQ + QR= $\sqrt{22}$ + $\sqrt{22}$ =2 $\sqrt{22}$ = PR

Therefore P, Q, R are collinear.



Q. 11. Find the equation of the curve formed by the set of all points which are equidistant from the points A(-1, 2, 3) and B(3, 2, 1).

Answer : Consider, C(x,y,z) point equidistant from points A(-1, 2, 3) and B(3, 2, 1).

∴ AC = BC

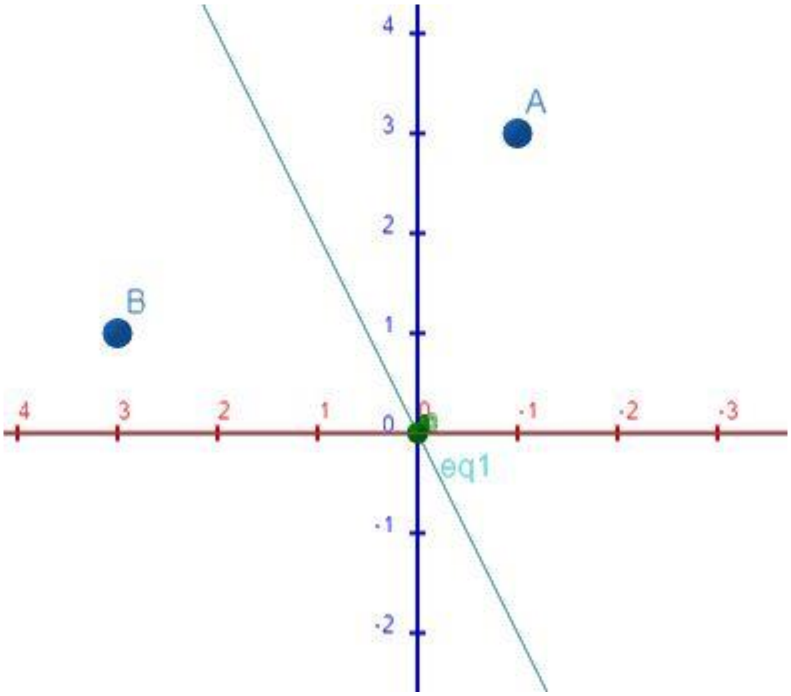
$$\sqrt{(x+1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-3)^2+(y-2)^2+(z-1)^2}$$

Squaring both sides,

$$(x+1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z-1)^2$$

$x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 2z + 1$

8x -4z = 0



Equation of curve is 8x-4z = 0

Q. 12. Find the point on the y-axis which is equidistant from the points A(3, 1, 2) and B(5, 5, 2).

Answer : Consider, C(0,y,0) point which lies on y axis and is equidistant from points A(3, 1, 2) and B(5, 5, 2).

∴ AC = BC

$$\sqrt{(0-3)^2+(y-1)^2+(0-2)^2}=\sqrt{(0-5)^2+(y-5)^2+(0-2)^2}$$

Squaring both sides,

$$(0-3)^2+(y-1)^2+(0-2)^2=(0-5)^2+(y-5)^2+(0-2)^2$$

9+ y^2 -2y +1 + 4 = 25 + y^2 -10y +25 + 4

8y = 40

Y = 5

The point C is (0,5,0).

Q. 13. Find the point on the z-axis which is equidistant from the points A(1, 5, 7) and B(5, 1, -4).

Answer : Consider, C(0,0,z) point which lies on z axis and is equidistant from points A(1, 5, 7) and B(5, 1, -4).

∴ AC = BC

$$\sqrt{(0-1)^2+(0-5)^2+(z-7)^2} = \sqrt{(0-5)^2+(0-1)^2+(z+4)^2}$$

Squaring both sides,

$$(0-1)^2+(0-5)^2+(z-7)^2 = (0-5)^2+(0-1)^2+(z+4)^2$$

1 +25+ z^2 -14z +49 = 25 + 1 + z^2 +8z +16

-22z = -33

Z = 1.5

The point C is (0,0,1.5).

Q. 14 Find the coordinates of the point which is equidistant from the points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

Answer : Consider, D(x,y,z) point equidistant from points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

∴ AD = 0D

$$\sqrt{(x-a)^2+(y-0)^2+(z-0)^2} = \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-a)^2+(y-0)^2+(z-0)^2 = (x-0)^2+(y-0)^2+(z-0)^2$$

x^2 +2ax + a^2 + y^2 + z^2 = x^2 + y^2 + z^2

a(2x-a) = 0

as a ≠ 0.

X = a/2

∴ BD = 0D

$$\sqrt{(x-a)^2+(y-0)^2+(z-0)^2} = \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-0)^2+(y-b)^2+(z-0)^2 = (x-0)^2+(y-0)^2+(z-0)^2$$

$x^2 + y^{2+}\ 2by + b^2 + z^2 = x^2 + y^2 + z^2$

b(2y-b) = 0

as b ≠ 0.

y= b/2

∴ CD = 0D

$$\sqrt{(x-0)^2+(y-0)^2+(z-c)^2} = \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-0)^2+(y-0)^2+(z-c)^2 = (x-0)^2+(y-0)^2+(z-0)^2$$

$x^2 + y^{2+}\ z^2 + 2cz + c^2 = x^2 + y^2 + z^2$

c(2z-c) = 0

as c ≠ 0.

z= c/2

Therefore, the pint D(a/2,b/2,c/2) is equidistant to points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

Q. 15. Find the point in yz-plane which is equidistant from the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

Answer : The general point on yz plane is D(0, y, z). Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

∴ AD = BD

$$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2}=\sqrt{(0-1)^2+(y+1)^2+(z-0)^2}$$

Squaring both sides,

$$(0-3)^2+(y-2)^2+(z+1)^2=(0-1)^2+(y+1)^2+(z-0)^2$$

9+ y^2 -4y + 4 + z^2 + 2z + 1 = 1+ y^2 +2y + 1+ z^2

-6y + 2z + 12 = 0 ….(1)

Also, AD = CD

$$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2}=\sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$$

Squaring both sides,

$$(0-3)^2+(y-2)^2+(z+1)^2=(0-2)^2+(y-1)^2+(z-2)^2$$

9+ y^2 -4y + 4 + z^2 + 2z + 1 = 4+ y^2 - 2y + 1+ z^2 – 4z + 4

-2y + 6z + 5 = 0 ….(2)

Simultaneously solving equation (1) and (2) we get

Y= 31/16, z = -3/16

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is (0, 31/16, -3/16).

Q. 16. Find the point in xy-plane which is equidistant from the points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

Answer : The general point on xy plane is D(x, y, 0). Consider this point is equidistant to the points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

∴ AD = BD

$$\sqrt{(x-2)^2+(y-0)^2+(0-3)^2}=\sqrt{(x-0)^2+(y-3)^2+(0-2)^2}$$

Squaring both sides,

$$(x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)^2$$

$X^2 - 4x + 4 + y^2 + 9 = X^2 + y^2 - 6y + 9 + 4$

-4x = -6y ….(1)

Also, AD = CD

$$\sqrt{(x-2)^2 + (y-0)^2 + (0-3)^2} = \sqrt{(x-0)^2 + (y-0)^2 + (0-1)^2}$$

Squaring both sides,

$$(x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$$

$X^2 - 4x + 4 + y^2 + 9 = X^2 + y^2 + 1$

-4x = -12 ….(2)

Simultaneously solving equation (1) and (2) we get

X = 3, y = 2.

The point which is equidistant to the points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) is (3, 2, 0).

Exercise 26C

Q. 1. Find the coordinates of the point which divides the join of A(3, 2, 5) and B(-4, 2, -2) in the ratio 4 : 3.

Answer : The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Point A(3, 2, 5) and B(-4, 2, -2), m and n are 4 and 3 respectively.

Using the above formula, we get,

$$= \left(\frac{4 \times -4 + 3 \times 3}{4+3}, \frac{4 \times 2 + 3 \times 2}{4+3}, \frac{4 \times -2 + 3 \times 5}{4+3}\right)$$

$$= (\frac{-7}{7}, \frac{14}{7}, \frac{7}{7})$$

(-1,2,1), is the point which divides the two points in ratio 4 : 3.

Q. 2. Let A(2, 1, -3) and B(5, -8, 3) be two given points. Find the coordinates of the point of trisection of the segment AB.

Answer : The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) in the ratio m: n are

$$(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n})$$

Point A(2, 1, -3) and B(5, -8, 3), m and n are 2 and 1 respectively.

Using the above formula, we get,

$$(\frac{2 \times 5 + 1 \times 2}{2 + 1}, \frac{2 \times -8 + 1 \times 1}{2 + 1}, \frac{2 \times 3 + 1 \times -3}{2 + 1})$$

$$(\frac{12}{3}, \frac{-15}{3}, \frac{3}{3})$$

(4,-5, 1), is the point of trisection of the segment AB.

Q. 3. Find the coordinates of the point that divides the join of A(-2, 4, 7) and B(3, -5, 8) extremally in the ratio 2 : 1.

Answer : The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) externally in the ratio m: n are

$$(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n})$$

Point A(-2, 4, 7) and B(3, -5, 8), m and n are 2 and 1 respectively.

Using the above formula, we get,

$$\left(\frac{2\times 3-\ 1\ \times -2}{2-1},\frac{2\times -5-\ 1\times 4}{2-1},\frac{2\times 8\ -\ 1\times 7}{2-1}\right)$$

= (8,-14,9), is the point that divides the two point A and B externally in the ratio 2:1.

Q. 4. Find the ratio in which the point R(5, 4, -6) divides the join of P(3, 2, -4) and Q(9, 8, -10).

Answer : Let the ratio be k:1 in which point R divides point P and point Q.

Using $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$, we get,

Here m and n are k and 1. The point which this formula gives is already given, i.e. R(5,4,-6) and the joining points are P(3, 2, -4) and Q(9, 8, -10).

$$(5,4,-6)\ =\ \frac{k\ \times 9+\ 1\ \times 3}{k+1},\frac{k\times 8\ +\ 1\times 2}{k+1},\frac{k\times -10\ +\ 1\times -4}{k+1})$$

Taking any point and finding the value of k, we get

$$5\ =\ \frac{k\ \times 9+\ 1\ \times 3}{k+1}$$

5k + 5 = 9k + 3

4k = 2

$$K\ =\ \frac{1}{2}$$

Therefore, the ratio be 1:2.

Q. 5. Find the ratio in which the point C(5, 9, -14) divides the join of A(2, -3, 4) and B(3, 1, -2).

Answer : Let the ratio be k:1 in which point R divides point P and point Q.

Using $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$, we get,

Here m and n are k and 1. The point which this formula gives is already given, i.e. R(5,9,-14) and the joining points are P(2, -3, 4) and Q(3, 1, -2).

$$(5, 9, -14) = \left(\frac{k \times 3 + 1 \times 2}{k+1}, \frac{k \times 1 + 1 \times -3}{k+1}, \frac{k \times -2 + 1 \times 4}{k+1}\right)$$

Taking any point and finding the value of k, we get

$$5 = \frac{k \times 3 + 1 \times 2}{k+1}$$

5k + 5 = 3k + 2

2k = -3

$$K = -\frac{3}{2}$$

Since, the ratio is -3:2. hence the division is external division.

The external division ratio is 3:2.

Q. 6. Find the ratio in which the line segment having the end points A(-1, -3, 4) and B(4, 2, -1) is divided by the xz-plane. Also, find the coordinates of the point of division.

Answer : Let the plane XZ divides the points A(-1, -3, 4) and B(4, 2, -1) in ratio k:1.

Hence, using section formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$, we get

$$= \left(\frac{k \times 4 + 1 \times -1}{k+1}, \frac{k \times 2 + 1 \times -3}{k+1}, \frac{k \times -1 + 1 \times 4}{k+1}\right)$$

On XZ plane, Y co- ordinate of every point be zero, therefore

$$\frac{k \times 2 + 1 \times -3}{k+1} = 0$$

2k – 3 = 0

$$K = \frac{3}{2}$$

The ratio is 3:2 in XZ plane which divides the line joined from points A and B.

Q. 7. Find the coordinates of the point where the line joining A(3, 4, 1) and B(5, 1, 6) crosses the xy-plane.

Answer : Let the plane XY divides the points A(3,4,1) and B(5, 1, 6) in ratio k:1.

Hence, using section formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$, we get

$$= \left(\frac{k \times 5 + 1 \times 3}{k+1}, \frac{k \times 1 + 1 \times 4}{k+1}, \frac{k \times 6 + 1 \times 1}{k+1}\right)$$

On XY plane, Z co- ordinate of every point be zero, therefore

$$\frac{k \times 6 + 1 \times 1}{k+1} = 0$$

6k + 1 = 0

$$K = -\frac{1}{6}$$

The ratio is 1:6 externally in XZ plane which divides the line joined from points A and B.

Q. 8. Find the ratio in which the plane x – 2y + 3z = 5 divides the join of A(3, -5, 4) and B(2, 3, -7). Find the coordinates of the point of intersection of the line and the plane.

Answer : Let the plane x – 2y + 3z = 5 divides the join of A(3, -5, 4) and B(2, 3, -7) in ratio k:1.

The point which will come by section formula will be in the plane. Putting that in the plane equation will give the point coordinates. The points are A(3, -5, 4) and B(2, 3, -7).

Using section formula,

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right),$$

we get

$$= \left(\frac{k \times 2 + 1 \times 3}{k+1}, \frac{k \times 3 + 1 \times -5}{k+1}, \frac{k \times -7 + 1 \times 4}{k+1}\right)$$

Putting this point in the plane equation, we get

$$\frac{2k+3}{k+1} - 2\left(\frac{3k-5}{k+1}\right) + 3\left(\frac{-7k+4}{k+1}\right) = 5$$

2k + 3 – 6k + 10 – 21k + 12 = 5k + 5

-25k + 25 = 5k + 5

-30k = -20

$$k = \frac{2}{3}$$

the ratio is 2:3. And the point of intersection of the plane and the line is

$\left(\frac{13}{5}, -\frac{9}{5}, -\frac{2}{5}\right)$.

Q. 9. The vertices of a triangle ABC are A(3, 2, 0), B(5, 3, 2) and C(-9, 6, -3). The bisector AD of ∠ A meets BC at D, find the fourth vertex D.

Answer : The given co-ordinates: A(3, 2, 0), B(5, 3, 2) and C(-9, 6, -3)

Now, $AB = \sqrt{(5-3)^2 + (3-2)^2 + (2-0)^2} = \sqrt{4+1+4} = 3$

Also, $AC = \sqrt{(-9-3)^2 + (6-2)^2 + (-3-0)^2} = \sqrt{144+16+9} = 13$

Now, we have, $\frac{AB}{AC} = \frac{3}{13}$

By the property of internal angle bisector,

$$\frac{AB}{AC} = \frac{BD}{CD}$$

Therefore, $\frac{BD}{CD} = \frac{3}{13}$