CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th April 2019 | Shift - I

Physics

Q.1 A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ?

(1) $\sim 10^{12}$ (2) $\sim 10^{10}$ (3) $\sim 10^{13}$ (4) $\sim 10^{11}$

Ans. **[1]**

Sol. collision frequency = $\frac{v_{av}}{\lambda}$

$$v_{av} = \sqrt{\frac{8}{3\pi}}\ v_{rms}$$

$$v_{av} = \sqrt{\frac{8}{3\pi}} \times 200$$

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 NP} \qquad \because P = \frac{RT}{V}$$

$$\lambda = \frac{V}{\sqrt{2}\pi d^2} = \frac{25 \times 10^{-3}}{1.4 \times \pi \times 9 \times 10^{-20}}$$

put values frequency $\simeq$ 0. 2 × 10^{10}/sec

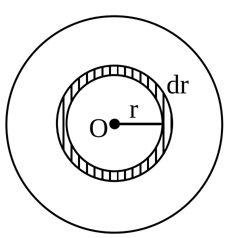
Answer by NTA is given as 10^{12} per sec.

Q.2 A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\frac{2MR^2}{3}$ (2) $\frac{MR^2}{6}$ (3) $\frac{MR^2}{3}$ (4) $\frac{MR^2}{2}$

Ans. **[1]**

Sol.



assume a ring of radius r and with dr

$dI = (dm)r^2$

$$I = \int_0^R dm\ r^2 = \int_0^R \sigma(2\pi r)dr\ r^2$$

$$= 2\pi k \int_0^R r^5 dr$$

$$= 2\pi k\left(\frac{R^6}{6}\right) = \frac{\pi k R^6}{3}$$

mass of disc $M = k\int_0^R 2\pi r^3\ dr$

$$M = k2\pi\left(\frac{R^4}{4}\right)$$

$$k = \frac{4M}{2\pi R^4} \text{ put the value}$$

$$I = \frac{\pi}{3}\left(\frac{4M}{2\pi R^4}\right)R^6$$

$$= \frac{2MR^2}{3}$$

Q.3 A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is :

(1) 10 Ω (2) 500 Ω (3) 100 Ω (4) 200 Ω

Ans. **[Drop by NTA]**

Sol. i_g = 0.1 mA , V = 5V, $R = 2 \times 10^6$

$V = i_g (G + R)$

$5 = 0.1 \times 10^{-3} (G + R)$

$G + R = 5 \times 10^4$

$G = 5 \times 10^4 - 2 \times 10^6$

= Negative

Not possible

Q.4 An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is .

(1) 10^2 (2) 10^4 (3) 60 (4) 6×10^2

Ans. **[1]**

Sol. $R_i = 100$, $R_o = 10^4$

Power gain ⇒ $60 = 10 \log\left(\frac{P_o}{P_i}\right)$

$$\frac{P_o}{P_i} = 10^6 = \beta^2 \frac{R_o}{R_i}$$

$\beta^2 = 10^6 \times \frac{R_i}{R_o}$

$= 10^6 \times \frac{100}{10^4}$

$\beta^2 = 10^4$

$\beta = 100$

Q.5 A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

(1) 1.0 m^2/Vs (2) 1.8 m^2/Vs (3) 1.5 m^2/Vs (4) 1.3 m^2/Vs

Ans. **[1]**

Sol. $\rho = \frac{m}{ne^2\tau} = \frac{1}{ne\mu}$

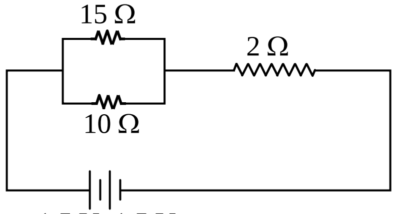
$i = neA\,v_d$

$n = \frac{i}{eAv_d}$

$\mu = \frac{Av_d}{i\rho} = \frac{1.1\times10^{-3}\times\pi\times25\times10^{-6}}{5\times1.7\times10^{-8}}$

$= \frac{17.27\times10^{-1}}{1.7} \approx 1.0$ m^2/Vs

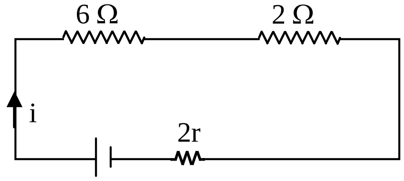
Q.6 In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2V. The internal resistance r, of each cell is :



(1) 1 Ω (2) 0.5 Ω (3) 1.5 Ω (4) 0 Ω

Ans. **[2]**

Sol.



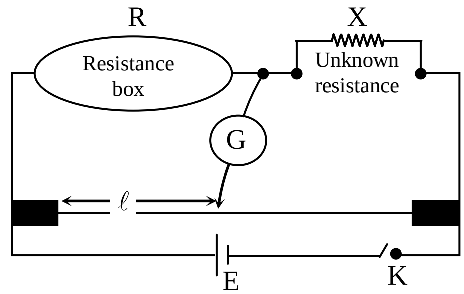
current $i = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3}(8 + 2r) = 3$

$2r = 1$

$r = 0.5\ \Omega$

Q.7 In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.



Sl. No.	R (Ω)	ℓ (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the reading is inconsistent ?

(1) 3 (2) 4 (3) 2 (4) 1

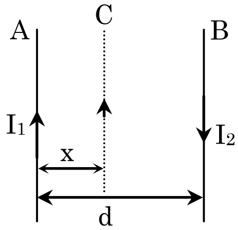
Ans. **[2]**

Sol. By using $\frac{X}{R} = \frac{100-\ell}{\ell}$

$X = R\left(\frac{100-\ell}{\ell}\right)$

The value of X for reading 4 is totally different

Q.8 Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are :



(1) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

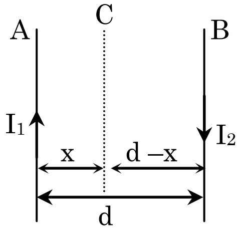
(2) $x = \pm \frac{I_1 d}{(I_1 - I_2)}$

(3) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \frac{I_2}{(I_1 - I_2)}d$

(4) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

Ans. **[2]**

Sol.



F_{net} on C = 0

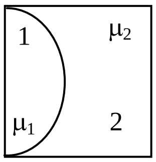
$$\frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi(d-x)} = 0$$

$$\frac{I_1}{x} = \frac{I_2}{x-d}$$

$$x = \frac{I_1 d}{I_1 - I_2}$$

Here, x > d, hence the wire C should be placed outside of A and B.

Q.9 One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\frac{R}{2-(\mu_1-\mu_2)}$ (2) $\frac{R}{2(\mu_1-\mu_2)}$ (3) $\frac{R}{\mu_1-\mu_2}$ (4) $\frac{2R}{\mu_1-\mu_2}$

Ans. **[3]**

Sol. $\frac{1}{r_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$

$$\frac{1}{r_2} = \frac{\mu_2 - 1}{-R}$$

Therefore $\frac{1}{r_{eq}} = \frac{R}{\mu_1 - \mu_2}$

Q.10 A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

(1) 0.25 ; 2 × 10^8 Hz
(2) 4 ; 1 × 10^8 Hz
(3) 4 ; 2 × 10^8 Hz
(4) 0.25 ; 1 × 10^8 Hz

Ans. **[1]**

Sol. $\omega_s = 2\pi f_s$, $f_s = 100 \times 10^6$, $E_s = 100$ V

$\omega_c = 2\pi f_c$, $f_c = 300 \times 10^9$ Hz, $E_c = 400$ V

modulation index m $= \frac{E_s}{E_c} = \frac{100}{400} = 0.25$

side band $\Delta f = f_{max} - f_{min}$

$= (f_s + f_c) - (f_c + f_s)$

$= 10^8[(3000+1)-(3000-1)]$

$= 10^8(2) = 2 \times 10^8$

Q.11 A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]

(1) 748 J (2) 350 J (3) 374 J (4) 700 J

Ans. **[1]**

Sol. $Q = nC_v dT$

= 748 Joule

Q.12 A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s)

(1) 12, 16 (2) 12, 18 (3) 16,14 (4) 8, 18

Ans. **[2]**

Sol. $n_A = \frac{n}{v}(v - v_A) = \frac{500}{300}[300 - V_A] = 480$

$300 - V_A = \frac{480 \times 3}{5} = 288$

$V_A = 300 - 288 = 12$ m/s

similarly $n_B = \frac{n}{v}(v + v_B)$

$V_B = 18$ m/s

Q.13 Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A.	Optical Fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(1) A-R, B-P, C-S, D-Q (2) A-S, B-Q, C-R, D-P

(3) A-Q, B-S, C-R, D-P (4) A-Q, B-S, C-P, D-R

Ans. **[4]**

Sol. According to frequency

$A \rightarrow Q$; $B \rightarrow S$; $C \rightarrow P$; $D \rightarrow R$

Q.14 The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0 \hat{i} \cos(kz) \cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by :

(1) $\vec{B} = \frac{E_0}{C}\hat{k}\ \sin(kz)\cos(\omega t)$ (2) $\vec{B} = \frac{E_0}{C}\hat{j}\ \sin(kz)\sin(\omega t)$

(3) $\vec{B} = \frac{E_0}{C}\hat{j}\ \sin(kz)\cos(\omega t)$ (4) $\vec{B} = \frac{E_0}{C}\hat{j}\ \cos(kz)\sin(\omega t)$

Ans. **[2]**

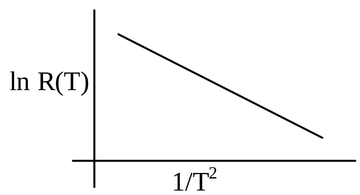
Sol. $\vec{E} = E_0 \cos kz\ \cos\omega t\ \hat{i}$

vibration of B = perpendicular to $\vec{E}$ and propagation of electro magnetic wave is in z direction. Hence the direction of vibration of $\vec{B}$ should be along $\hat{j}$.

$\frac{E_0}{B_0} = C$

$\vec{B} = \frac{E_0}{C}\sin\omega t \sin kz\ \hat{j}$

Q.15 In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line.



One may conclude that :

(1) $R(T) = R_0 e^{T^2/T_0^2}$ (2) $R(T) = \frac{R_0}{T^2}$ (3) $R(T) = R_0 e^{-T^2/T_0^2}$ (4) $R(T) = R_0 e^{-T_0^2/T^2}$

Ans. **[4]**

Sol. $y = -mx + c$

$\ln(R) = -\frac{m}{T^2} + c$

If $R = R_0 e^{\frac{-T_0^2}{T^2}}$

$\ln R = -\frac{T_0^2}{T^2} + \ln(R_0)$

satisfies the equation

Q.16 A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are ;

(1) 440 V and 20 A (2) 440 V and 5 A

(3) 220 V and 20 A (4) 220 V and 10 A

Ans. **[2]**

Sol. $n_p = 300, n_s = 150$

$$\frac{i_s}{i_p} = \frac{n_p}{n_s}$$

$$\Rightarrow i_p = i_s \frac{n_s}{n_p} = 5A$$

$$P_s = V_s i_s \Rightarrow V_s = \frac{2.2\times10^3}{10} = 220$$

$$V_P = V_s \frac{n_P}{n_s} = 220 \times \frac{300}{150}$$

$= 440V$

Q.17 In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be :

Given E (in eV) = $\frac{1237}{\lambda(\text{in nm})}$

(1) 3.0 eV (2) 1.5 eV (3) 4.5 eV (4) 15.1 eV

Ans. **[2]**

Sol. $\phi_0 = \frac{1237}{380}$, $E = \frac{1237}{260}$ of photon

K·E of electron $kE = E - \phi_0 = 1.5$ eV

Q.18 The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio (r_1/r_2), is then close to :

(1) 2/3 (2) 4/5 (3) 2/5 (4) 3/5

Ans. **[3]**

Sol. $h_{Hg} = h_{water}$

$$h = \frac{2T\cos\theta}{R\rho g}$$

$$R \propto \frac{T\cos\theta}{\rho}$$

$$\frac{R_{Hg}}{R_w} = \frac{2}{5} = 0.4$$

Q.19 A proton, an electron, and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(1) $r_e < r_p < r_{He}$ (2) $r_e > r_p = r_{He}$ (3) $r_e < r_p = r_{He}$ (4) $r_e > r_p > r_{He}$

Ans. **[3]**

Sol. $r = \frac{\sqrt{2mE}}{Bq}$ E = same

$$r \propto \frac{\sqrt{m}}{q}$$

proton $\frac{\sqrt{m_p}}{q_p} = \frac{\sqrt{m_\alpha}}{q_\alpha}$, He^{+2}

$\therefore$ $r_p = r_{He}$

For electron $\dfrac{\sqrt{m_e}}{q_e} < \dfrac{\sqrt{m_p}}{q_p}$ proton

$\therefore$ $r_e < r_p = r_{He}$

Q.20 A particle of mass m is moving along a trajectory given by

$x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

The torque, acting on the particle about the origin , at t = 0 is :

(1) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$ (2) Zero

(3) $+my_0 a\omega_1^2 \hat{k}$ (4) $m(-x_0 b + y_0 a)\omega_1^2 \hat{k}$

Ans. **[3]**

Sol. $x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$\vec{r} = x\hat{i} + y\hat{j}$

acceleration $\vec{a} = \dfrac{d^2\vec{r}}{dt^2}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (\vec{r} \times \vec{a})\, m$

$= (my_0 a\omega_1^2)\hat{k}$

Q.21 The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m)

(1) 6.4×10^6 m (2) 2.6×10^6 m (3) 1.6×10^6 m (4) 9.0×10^6 m

Ans. **[2]**

Sol. $g_h = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$, $g_h = \dfrac{g}{2}$

$\left(1+\dfrac{h}{R}\right)^2 = 2$

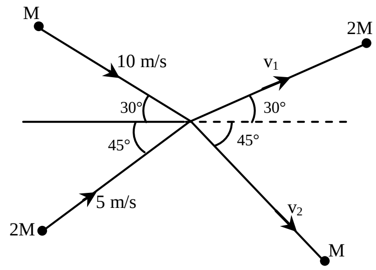
$1 + \dfrac{h}{R} = \sqrt{2}$

$h = R(\sqrt{2} - 1)$

$= (1.4 - 1) \times 64 \times 10^5$ m

$= 2.6 \times 10^6$ m

Q.22 Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly :



(1) 3.2 m/s and 12.6 m/s
(2) 3.2 m/s and 6.3 m/s
(3) 6.5 m/s and 6.3 m/s
(4) 6.5 m/s and 3.2 m/s

Ans. **[3]**

Sol. using momentum conservation in x direction

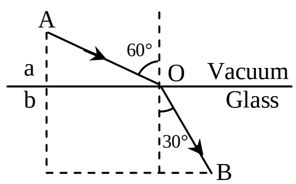
$m \times 10 \cos 30° + 2\,m \cos 45° = mv_2 \cos 45° + 2m\,v_1 \cos 30°$

in y direction

$2m\,(5) \sin 45° - m \times 10 \sin 30° = 2m\,v_1 \sin 30° - mv_2 \sin 45°$

solving equation $v_1 = 6.5$ m/s , $v_2 = 6.3$ m/s

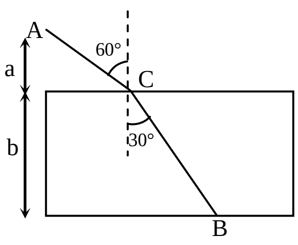
Q.23 A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :



(1) 2a + 2b
(2) $\frac{2\sqrt{3}}{a} + 2b$
(3) $2a + \frac{2b}{3}$
(4) $2a + \frac{2b}{\sqrt{3}}$

Ans. **[1]**

Sol.

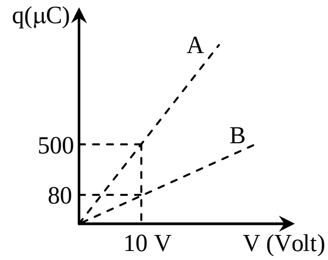


$AC = \frac{a}{\cos 60°}$, $BC = \frac{b}{\cos 30°}$, $\mu = \frac{\sin 60°}{\sin 30°}$

path = AC + μBC

= 2a + 2b

Q.24 Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitance are :



(1) 50 μF and 30 μF (2) 40 μF and 10 μF (3) 20 μF and 30 μF (4) 60 μF and 40 μF

Ans. **[2]**

Sol. In series $\frac{C_1C_2}{C_1+C_2} = \frac{q}{V} = \frac{80}{10} = 8 \times 10^{-6}$

In parallel $C_1 + C_2 = \frac{q}{V} = \frac{500}{10} = 50 \times 10^{-6}$

$C_1C_2 = 400 \times 10^{-6}$

$C_1 + C_2 = 50 \times 10^{-6}$

$C_1 = 10$ μF, $C_2 = 40$μF

Q.25 A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

(1) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(2) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

(3) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(4) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

Ans. **[1]**

Sol. $a = -(g + \rho v^2) = \frac{dv}{dt}$

$\int_{V_0}^{V}\frac{dv}{g+\rho v^2} = -\int_0^t dt$

By integrating

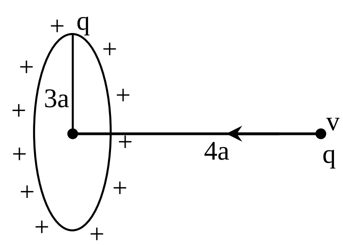
$t = \frac{1}{\sqrt{g\gamma}}\tan^{-1}\left[\sqrt{\frac{\gamma}{g}}V_0\right]$

Q.26 A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at z = 4a. The minimum value of v such that it crosses the origin is :

(1) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (3) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Ans. **[2]**

Sol.



$\Delta\frac{1}{2}mv^2 = \Delta PE$

$= \frac{2kq^2}{15a}$

$v = \sqrt{\frac{4}{15m}\frac{kq^2}{a}}$

Q.27 Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

(1) $\frac{I_1\omega_1^2}{6}$ (2) $\frac{3}{8}I_1\omega_1^2$ (3) $-\frac{I_1\omega_1^2}{12}$ (4) $-\frac{I_1\omega_1^2}{24}$

Ans. **[4]**

Sol. conservation of angular momentum

$L_i = L_F$

and kinetic energy = $\frac{1}{2}I\omega^2$

put the value KE = $-\frac{I_1\omega_1^2}{24}$

Q.28 n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(1) $\frac{nR}{C_V + nR}$ (2) $\frac{nR}{C_V - nR}$ (3) $\frac{4nR}{C_V + nR}$ (4) $\frac{4nR}{C_V - nR}$

Ans. **[1]**

Sol. In isobaric process w = nRdT

$w = Q - \Delta U$

$Q = nC_pdT$

$$\frac{w}{Q} = \frac{nRdT}{nC_p dT} = \frac{R}{C_p} = \frac{R}{C_V + R}$$

$$= \frac{nR}{nC_V + nR}$$

$$= \frac{nR}{C_V + nR}$$

Q.29 Two radioactive materials A and B have decay constants 10 λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time

(1) $\frac{1}{10\lambda}$ (2) $\frac{11}{10\lambda}$ (3) $\frac{1}{9\lambda}$ (4) $\frac{1}{11\lambda}$

Ans. **[3]**

Sol. $\lambda_1 = 10\lambda$, $\lambda_2 = \lambda$

$$\frac{N_1}{N_2} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = e^{-9\lambda t} = e^{-1}$$

$$t = \frac{1}{9\lambda}$$

Q.30 The displacement of a damped harmonic oscillator is given by

$x(t) = e^{-0.1t} \cos (10\pi t + \varphi)$. Here t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(1) 4 s (2) 13 s (3) 7 s (4) 27 s

Ans. **[3]**

Sol. $\frac{A_0}{2} = A_0 e^{-0.1t}$

$t = 10 \ln (2)$

$= 7$ sec

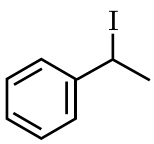
JEE Main Online Exam 2019

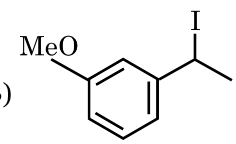
Questions & Solutions

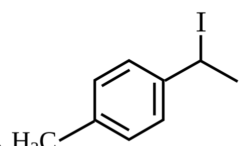
10th April 2019 | Shift - I

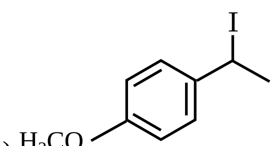
Chemistry

Q.1 Increasing rate of S_N1 reaction in the following compounds is :

(A) 

(B) 

(C) 

(D) 

(1) (B) < (A) < (D) < (C) (2) (A) < (B) < (C) < (D)

(3) (A) < (B) < (D) < (C) (4) (B) < (A) < (C) < (D)

Ans. **[4]**

Sol. Rate of S_N1– reaction $\propto$ stability of $C^{\oplus}$ – I.m.

Q.2 Amylopectin is compound of :

(1) α-D-glucose, $C_1 - C_4$ and $C_1 - C_6$ linkages

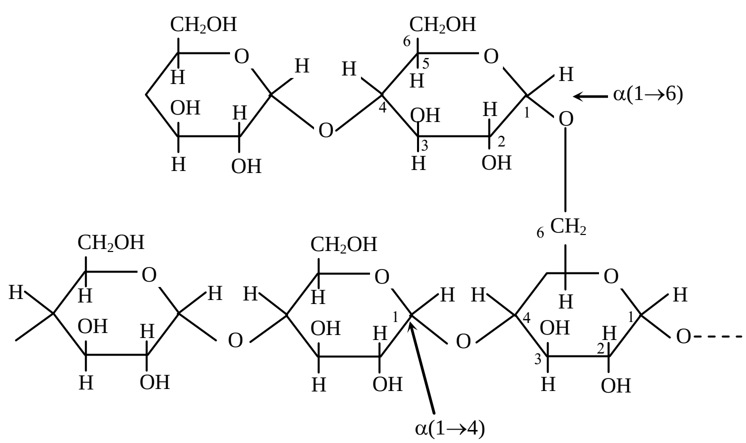
(2) β-D-glucose, $C_1 - C_4$ and $C_1 - C_6$ linkages

(3) β-D-glucose, $C_1 - C_4$ and $C_2 - C_6$ linkages

(4) α-D-glucose, $C_1 - C_4$ and $C_1 - C_6$ linkages

Ans. **[1]**

Sol.

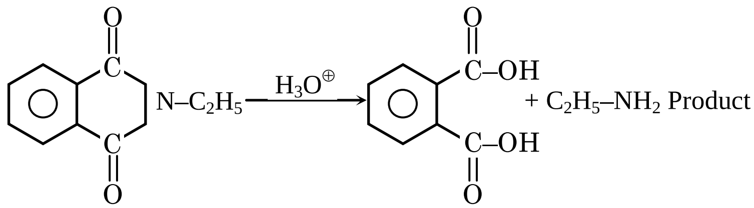


Q.3 Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphatalimide on treatment with :

(1) CaH_2 (2) H_2O (3) $NaBH_4$ (4) NH_2NH_2

Ans. **[4]**

Sol. It is the final step of Galbriel phatalimide synthesis reaction



Q.4 The isoelectronic set of ions is :

(1) F^-, Li^+, Na^+ and Mg^{2+}
(2) Li^+, Na^+, O^{2-} and F^-
(3) N^{3-}, O^{2-}, F^- and Na^+
(4) N^{3-}, Li^+, Mg^{2+} and O^{2-}

Ans. **[3]**

Sol. In this we have to choose isoelectronic set of ions
Isoelectronic species are those which have same no. of electron in total.
So option 3 : is correct.

Q.5 The regions of the atmosphere, where clouds form and where we live, respectively, are :

(1) Stratosphere and Stratosphere
(2) Stratosphere and Troposphere
(3) Troposphere and Stratosphere
(4) Troposphere and Troposphere

Ans. **[4]**

Sol. From Theory

Q.6 Consider the hydrated ions of Ti^{2+}, V^{2+}, Ti^{3+}, and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(1) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$
(2) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$
(3) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$
(4) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

Ans. **[2]**

Sol. As we know that

$\mu = \sqrt{n(n+2)}$

where n = no. of impaired electrons i.e. greater the no. of impaired electron more will be the spin-only magnetic moments.

$Ti^{2+} = 3d^2 \quad \therefore n = 2$

$Ti^{3+} = 3d^1 \quad \therefore n = 1$

$V^{2+} = 3d^3 \quad \therefore n = 3$

$Sc^{3+} = 3d^0 \quad \therefore n = 0$

The correct order of spin only magnetic moments is

$V^{2+} > Ti^{2+} > Ti^{3+} > Sc^{3}$

So option 2 is correct.

Q.7 Consider the statements S1 and S2

S1 : Conductivity always increases with decrease in the concentration of electrolyte.

S2 : Molar conductivity always increases with decrease in the concentration of electrolyte.

The correct option among the following is :

(1) Both S1 and S2 are wrong

(2) S1 is correct and S2 is wrong

(3) Both S1 and S2 are correct

(4) S1 is wrong and S2 is correct

Ans. **[4]**

Sol. We know that Here λ_m = molar conductivity

$\lambda_m = \frac{k}{c}$ k = conductivity

c = concentration

We the increase in the concentration conductivity always increase the molar conduction always increases with the decrease in the concentration

So option 4 is correct

Q.8 Consider the following table :

Gas	a/(k Pa dm^6 mol^{-1})	b/(dm^3 mol^{-1})
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are vander Waals constants. The correct statement about the gases is :

(1) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

(2) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

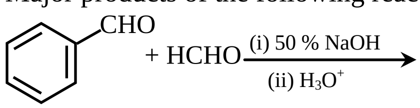
(3) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

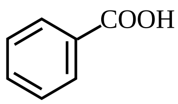
(4) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

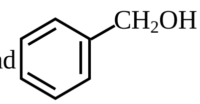
Ans. **[1]**

Sol. (

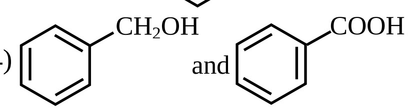
Q.9 Major products of the following reaction are :



(1) CH_3OH and 

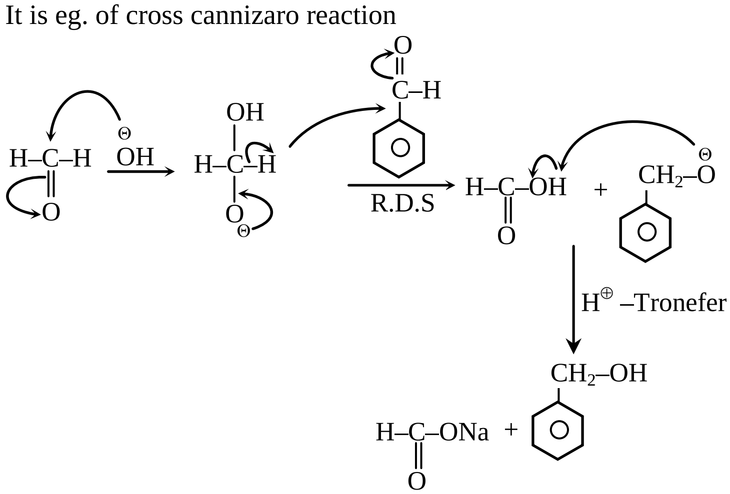
(2) HCOOH and 

(3) CH_3OH and HCO_2H

(4) 

Ans. **[2]**

Sol. It is eg. of cross cannizaro reaction

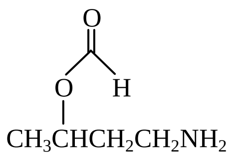


Q.10 The major product of the following reaction is :

$$CH_3CH(OH)CH_2CH_2NH_2 \xrightarrow[\text{triethylamine}]{\text{ethyl formate (1 equiv.)}}$$

(1) $CH_3\text{–}CH(OH)\text{–}CH = CH_2$

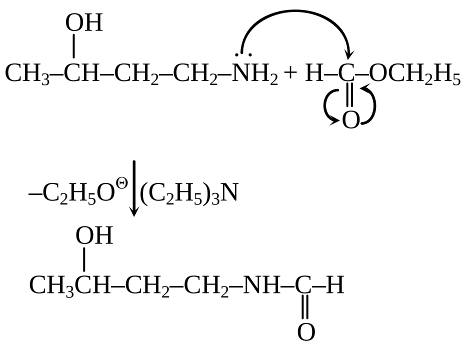
(2) $CH_3CH{=}CH\text{–}CH_2NH_2$

(3) 

(4) $CH_3CH(OH)CH_2CH_2NHCHO$

Ans. **[4]**

Sol. It is eg. of S_N2 Th– reaction



Q.11 The correct order of catenation is :

(1) C > Sn > Si ≈ Ge (2) Si > Sn > C > Ge (3) C > Si > Ge ≈ Sn (4) Ge > Sn > Si > C

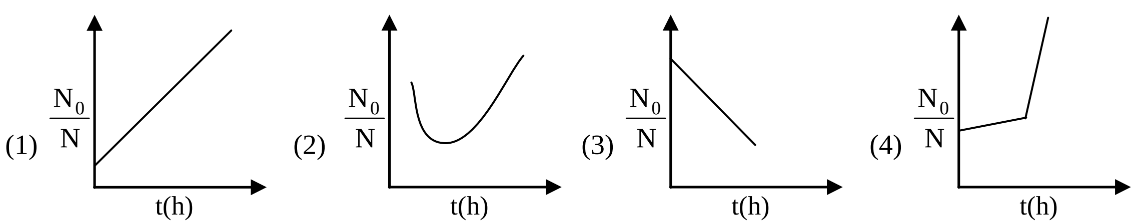
Ans. **[3]**

Sol. in this order of catenation is asked. catenation is a self-linking property here and for group 14 : self -linking is through covalent bonding.

C > Si > Ge ≈ Sn

in ethene is 2p-2p ouerlapping further 3p-3p, 4p – 4p and soon and the extent of overlapping is more in 2p-2p > 3p-3p >. 4p–4p ≈ 5p-5p.

Q.12 A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A does of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour ?

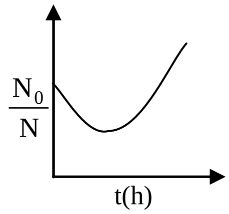


Ans. **[1]**

Sol. Initially

$N > N_0$

and N" is increasing through first -order kinetics. So $\frac{N_0}{N}$ in initial time decrease.



But after 1 hour the value of N decrease with a faster rate. So $\frac{N_0}{N}$ will increase.

Q.13 Consider the following statements

(a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3

(b) Ionic product of water is temperature dependent.

(c) A monobasic acid with $K_a = 10^{-5}$ has pH = 5. The degree of dissociation of this acid is 50 %.

(d) The Le Chatelier's principle is not applicable to common-ion effect.

The correct statements are :

(1) (a) and (b) (2) (a), (b) and (c) (3) (a), (b) and (d) (4) (b) and (c)

Ans. **[2]**

Sol. (a) $H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

Initially 40 m mole 40 m mole

Finally 20 m mole 0 20 m mole

$[H^+] = \frac{20 \times 10^{-3} \times 10^3 \times 2}{800}$

$[H^+] = \frac{1}{20}$

$-\log [H^+] = ...$ $......-\log \frac{1}{20}$

pH = log 20

pH = 1.3 approximately

(b) kw depends on temperature kw ↑ with temperature ↑

(c) pH = 5

$\therefore [H^+] = c\,\alpha = 10^{-5}$

$K_a = \frac{c\alpha^2}{(1-\alpha)}$

$k_a = \frac{[H^+]\alpha}{(1-\alpha)}$

$10^{-5} = \frac{10^{-5} \times \alpha}{1-\alpha}$

$\Rightarrow 1 = 2\alpha$

$\therefore \alpha = 0.5$

(d) Le Chatelier's principle is applicable to common-ion effect because commonion effect is itself depend on le chatelier's principle

So option-2 is correct

Q.14 The alloy used in the construction of aircrafts is :

(1) Mg-Zn (2) Mg-Al (3) Mg-Sn (4)Mg-Mn

Ans. **[2]**

Sol. It is completely memory based question for air-craft construction aluminum and its alloy is used because these are lighter

Q.15 A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increase with :

(1) Decrease in p and increase in T (2) Increase in p and decrease in T

(3) Decrease in p and decrease in T (4) Increase in p and increase in T

Ans. **[2]**

Sol. Increase in Pressure leads to the increase in adsorption capacity

And the physical adsorption is an exothermic process with the increase in temperature adsorption decrease

Q.16 A process will be spontaneous at all temperatures if :

(1) $\Delta H < 0$ and $\Delta S > 0$ (2) $\Delta H < 0$ and $\Delta S < 0$

(3) $\Delta H > 0$ and $\Delta S < 0$ (4) $\Delta H > 0$ and $\Delta S > 0$

Ans. **[1]**

Sol. At constant P and T and for the process to be spontaneous.

We should have $\Delta G = -ve$

and we know that

$\Delta G = \Delta H - T\Delta S$

If $\Delta H = -ve$ and $\Delta S = +ve$ then at all the temperature the process will be spontaneous

Q.17 At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion, and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :

(1) C_4H_7Cl (2) C_4H_6 (3) C_4H_8 (4) C_4H_{10}

Ans. **[2]**

Sol. $$C_x^0H_4(g) + \left(x + \frac{y}{4}\right) O_2 \longrightarrow x\,CO_2(g) + \frac{4}{2} H_2O\,(l)$$

10 mL 55 mL 40

Hence.

1 mL of hydrocarbon = x mL of CO_2 is produced

According to question

10 mL ... . = 10 x mL of CO_2

$\therefore$ 10 x = 40 mL

x = 4

$\left(\frac{x+y}{4}\right)$ mL of O_2 is required = x mL of CO_2

55mL mL .. .. .. $= \frac{x}{\left(x+\frac{y}{4}\right)} \times 55$ mL of CO_2

According to question.

$\Rightarrow \frac{x}{\left(x+\frac{y}{4}\right)} \times 55 = 40$

$\Rightarrow$ 55 x = 40 x + 10 y

$\Rightarrow$ 15 x = 10 y

15 × 4 = 10 y

$\Rightarrow \frac{60}{10} = y$

6 = y

Hence the compound is C_4H_6

Q.18 The principle of column chromatography is

(1) Gravitational force.

(2) Capillary action.

(3) Differential adsorption of the substances on the solid phase.

(4) Differential absorption of the substances on the solid phase.

Ans. **[3]**

Sol. The principle of column chromatography is differential adsorption of substance and hence option on 3 is correct.

Q.19 At room temperature, a dilute solution of urea is prepared by dissolving 0.60 of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mm Hg, lowering of vapour pressure will be. (molar mass of urea = 60 g mol^{-1})

(1) 0.031 mmHg (2) 0.017 mmHg (3) 0.028 mmHg (4) 0.027 mmHg

Ans. **[2]**

Sol. As we that relative lowering concept

i.e. $\frac{\Delta p}{p^0} = \frac{n}{N+n}$

or $\Delta p = p^0 \times \frac{n}{(N+n)}$

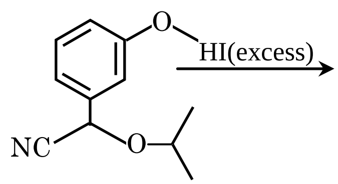
$\Delta p = \frac{35 \times \frac{0.6}{60}}{\left(\frac{360}{18} + \frac{0.6}{60}\right)}$

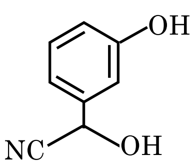
$$\Delta p = \frac{\frac{35\times 06}{600}}{\left(20+\frac{1}{100}\right)}$$

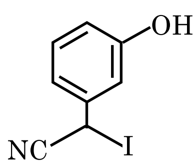
$$= \frac{35\times 100}{2001\times 100}$$

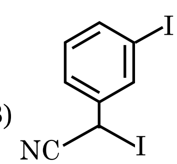
= 0.017 mm Hg

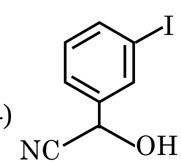
Q.20 The major product of the following reaction is :



(1) 

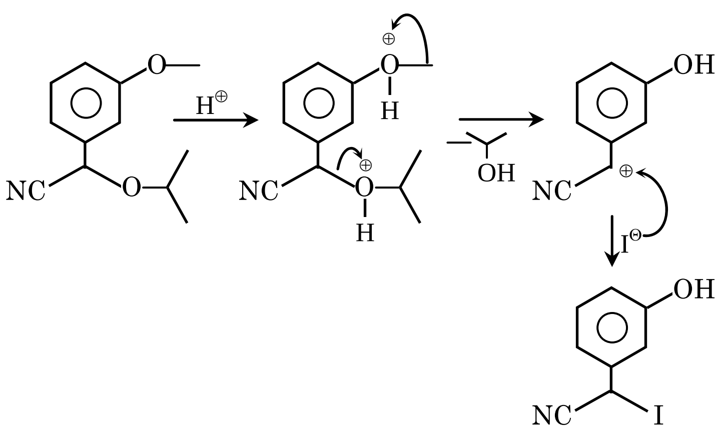
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



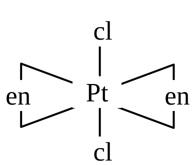
Q.21 The species that can have a trans-isomer is:

(en = ehane-1, 2-diamine, ox = oxalate)

(1) $[Cr(en)_2(ox)]^+$ (2) $[Pt(en)Cl_2]$ (3) $[Pt(en)_2Cl_2]^{2+}$ (4) $[Zn(en)C1_2]$

Ans. **[3]**

Sol. The trans-isomer of $[Pt(en)_2Cl_2]^{2+}$



Q.22 Three complexes,

$[CoCl(NH_3)_5]^{2+}$(I),

$[Co(NH_3)_5H_2O]^{3+}$ (II) and

$[Co(NH_3)_6]^{3+}$(III)

absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

(1) (III) > (I) > (II) (2) (III) > (II) > (I) (3) (I) > (II) > (III) (4) (II) > (I) > (III)

Ans. **[3]**

Sol. As we know that

strong lightnd α C.F.S.E. α $E_{absorbed} \propto \dfrac{1}{\lambda_{absorbed}}$

we have

$[CoCl(NH_3)_5]^{2+}$, $[Co(NH_3)_5H_2O]^{3+}$ and $[Co(NH_3)_6]^{3+}$

(I) (II) (III)

$\therefore$ III > II > I [as per the $E_{absorbed}$]

$\therefore$ $\lambda_{absorbed}$

I > II > III

Q.23 Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)	**Column II (Metals)**
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

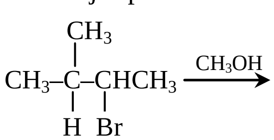
(1) (I)-(c) ; (II)-(a) ; (III)-(b) ; (IV)-(d) (2) (I)-(c) ; (II)-(d) ; (III)-(b) ; (IV)-(a)

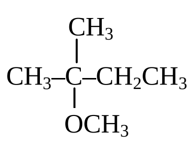
(3) (I)-(b) ; (II)-(d) ; (III)-(a) ; (IV)-(c) (4) (I)-(b) ; (II)-(c) ; (III)-(d) ; (IV)-(a)

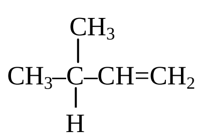
Ans. **[2]**

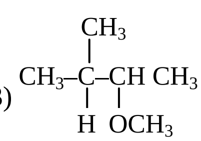
Sol. Here we known that from metallurgy for Ni monds process is done for Zr Van Arkel method, for Sn liquation is done and for "Ga" Zone Refining is done.

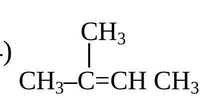
Q.24 The major product of the following reaction is :



(1) 

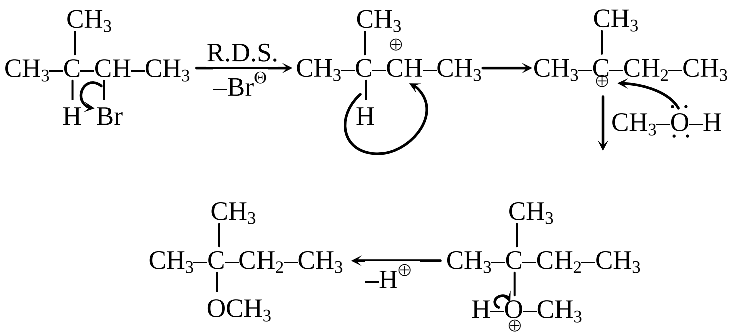
(2) 

(3) 

(4) 

Ans. **[1]**

Sol. It is SN'–Reaction



Q.25 During the change of O_2 to O_2^-, the incoming electron goes to the orbital :

(1) σ^*2p_z (2) $\pi 2p_y$ (3) π^*2p_x (4) $\pi\ 2p_x$

Ans. **[3]**

Sol. A/c to MOT

For O_2 and O_2^- we follow this

$O_2 \rightarrow \sigma/S^2, \sigma/1S^2, \sigma^*2S^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2$

$\pi^*2p_x^1 = \pi^*2p_y^1 \quad \sigma^*2p_z$

$O_2^- \rightarrow \sigma/S^2, \sigma^*/S^2 \quad \sigma 2S^2 \quad \sigma^*2S^2 \quad \sigma 2S_z^2 \quad \pi 2p_z^2 \quad \pi 2p_x^2 = \pi 2p_y^2$

$\pi^*2p_x^2 = \pi^*2p_y^1 \quad \sigma^*2p_z$

Q.26 The synonym for water gas when used in the production of methanol is :

(1) fuel gas (2) laughing gas (3) syn gas (4) natural gas

Ans. **[3]**

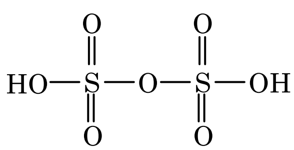
Sol. The synonym for water gas is syn gas.

Q.27 The oxoacid of sulphur that does not contain bond between sulphur atoms is :

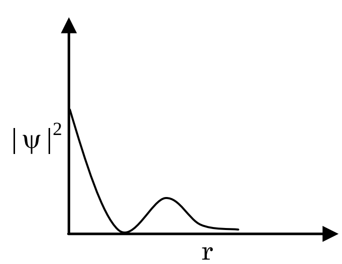
(1) $H_2S_2O_7$ (2) $H_2S_2O_3$ (3) $H_2S_4O_6$ (4) $H_2S_2O_4$

Ans. **[1]**

Sol.



Q.28 The graph between $|\psi|^2$ and r (radial distance) is shown below. This represents :



(1) 3s orbital (2) 2s orbital (3) 2p orbital (4) 1s orbital

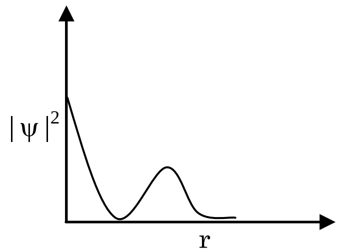
Ans. **[2]**

Sol. We know that for s-orbital graph starts from top and no. of radial mode = $n - \ell - 1$

$\therefore$ for 2 s orbital it will $= 2 - 0 - 1$

$= 1$

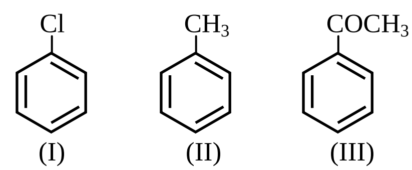
$\therefore$ the graph will be



is of 2 s

Hence option 2 is correct

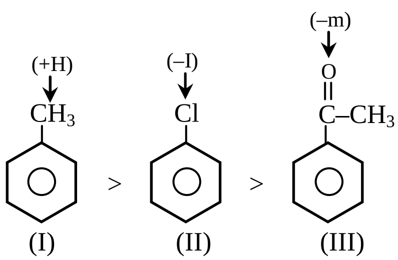
Q.29 The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :



(1) III < II < I (2) III < I < II (3) II < I < III (4) I < III < II

Ans. **[2]**

Sol. Reactivity for electrophilic α $e^{\oplus}$–density aromatic substitution reaction in aromatic ring



Q.30 Which of the following is a condensation polymer ?

(1) Neoprene (2) Buna-S (3) Nylon 6, 6 (4) Teflon

Ans. **[3]**

Sol. $H_2N–(CH_2)_6 NH_2 + HOOC–(CH_2)_4–COOH$

Hexamethylene diomine   Adipic acid

$\Delta \downarrow -n.\ H_2O$

$-\!\!\left[NH–(CH_2)_6NH–\overset{\displaystyle O}{\overset{\|}{C}}–(CH_2)_4–\overset{\displaystyle O}{\overset{\|}{C}} \right]\!\!-_n$

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

10th April 2019 | Shift - I

Mathematics

Q.1 The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a :

(1) rhombus of area $8\sqrt{2}$ sq. units

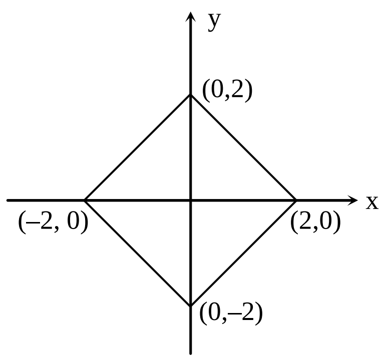
(2) square of side length $2\sqrt{2}$ units

(3) square of area 16 sq. units

(4) rhombus of side length 2 units

Ans. **[2]**

Sol. shown figure is square with side length $2\sqrt{2}$.



Q.2 All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation

(1) $\sin x = |\sin y|$

(2) $\sin x = 2 \sin y$

(3) $2 \sin x = \sin y$

(4) $2|\sin x| = 3 \sin y$

Ans. **[1]**

Sol. $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot 4^{-\sin^2 y} \le 1$

$\Rightarrow \quad 2^{\sqrt{(\sin x - 1)^2 + 4}} \le 4^{\sin^2 y}$

$$\underbrace{\sqrt{\underbrace{(\sin x - 1)^2}_{\ge 0} + 4}}_{\ge 2} \le \underbrace{2\sin^2 y}_{\le 2}$$

this is possible only if sin x = 1 & |sin y| = 1

Q.3 If α and β are the roots of the quadratic equation, $x^2 + x \sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then

$\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)\cdot(\alpha-\beta)^{24}}$ is equal to :

(1) $\frac{2^{12}}{(\sin\theta-8)^{6}}$ (2) $\frac{2^{6}}{(\sin\theta+8)^{12}}$ (3) $\frac{2^{12}}{(\sin\theta+8)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

Ans. **[3]**

Sol. $x^2 + x\sin\theta - 2\sin\theta = 0$ $\langle$ α, β

$\alpha + \beta = -\sin\theta$

$\alpha\beta = -2\sin\theta$

Now, $\frac{\alpha^{12}+\beta^{12}}{\left(\frac{1}{\alpha^{12}}+\frac{1}{\beta^{12}}\right)\cdot(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$$= \frac{(\alpha\beta)^{12}}{\left((\alpha+\beta)^2-4\alpha\beta\right)^{12}}$$

$$= \left(\frac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right)^{12}$$

$$= \left(\frac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right)^{12}$$

$$= \frac{2^{12}}{(\sin\theta+8)^{12}}$$

Q.4 If a > 0 and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{1}{5}+\frac{3}{5}i$ (2) $-\frac{1}{5}-\frac{3}{5}i$ (3) $\frac{1}{5}-\frac{3}{5}i$ (4) $-\frac{3}{5}-\frac{1}{5}i$

Ans. **[2]**

Sol. $z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

$|z| = \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

$\therefore\ \bar{z} = \frac{-2i(3-i)}{10}$

$\Rightarrow \frac{-1-3i}{5}$

Q.5 If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{1}{2} - e$ (2) $e - 2$ (3) $2 + \frac{1}{e}$ (4) $\frac{1}{e} - 2$

Ans. **[2]**

Sol. $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

$\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$

Let $\tan x = t \Rightarrow \sec^2 x = \frac{dt}{dx}$

$\therefore \frac{dy}{dt} = (t - y)$

$\frac{dy}{dt} + y = t$ (Linear differential equation)

After solving we get

$y e^t = e^t(t - 1) + c$

$\Rightarrow$ $y = (\tan x - 1) + ce^{-\tan x}$

$y(0) = 0 \Rightarrow c = 1$

$y = \tan x - 1 + e^{-\tan x}$

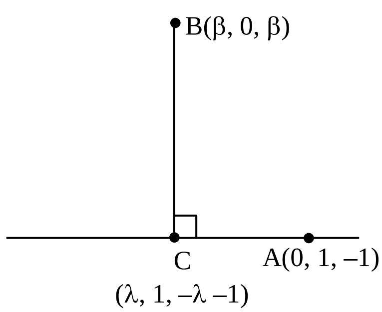
So, $y\left(-\frac{\pi}{4}\right) = e - 2$

Q.6 If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) 2 (2) 1 (3) –2 (4) –1

Ans. **[4]**

Sol.



$\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = \lambda$

A point on this line is A(0, 1, –1)

$\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$

we get $\lambda = -\frac{1}{2}$

$\therefore C \equiv \left(-\frac{1}{2}, 1, -\frac{1}{2}\right)$

$\left|\overrightarrow{BC}\right| = \sqrt{\frac{2}{3}}$

$\sqrt{\left(\beta+\frac{1}{2}\right)^2 + (1)^2 + \left(\beta+\frac{1}{2}\right)^2} = \sqrt{\frac{2}{3}}$

$\therefore\ \beta = 0, -1$

$\beta = -1\ (\beta \neq 0)$

Q.7 If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$ (2) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$ (4) $\Delta_1 - \Delta_2 = -2x^3$

Ans. **[2]**

Sol. $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$\Rightarrow -x^3$

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$

$\Rightarrow -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

Q.8 Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{10}$ (2) $\frac{1}{17}$ (3) $\frac{1}{11}$ (4) $\frac{1}{12}$

Ans. **[3]**

Sol. P (Boy) = P(girl) = $\frac{1}{2}$

Required probability = $\frac{\text{all four girls}}{\text{At least two girls}}$

$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4}$

$= \frac{1}{11}$

Q.9 Which one of the following Boolean expressions is a tautology ?

(1) $(p \vee q) \wedge (\sim p \vee \sim q)$ (2) $(p \vee q) \vee (p \vee \sim q)$

(3) $(p \wedge q) \vee (p \wedge \sim q)$ (4) $(p \vee q) \wedge (p \vee \sim q)$

Ans. **[2]**

Sol. from options

$(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim (p \wedge q) \rightarrow$ Not a tautology

$(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \rightarrow$ tautology

$(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \rightarrow$ Not a tautology

$(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \rightarrow$ Not a tautology

Q.10 Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{ x \in R : f(x) \in A\}$. If $S = [0,4]$, then which one of the following statements is not true ?

(1) $g(f(S)) \neq S$ (2) $f(g(S)) = S$ (3) $f(g(S)) \neq f(S)$ (4) $g(f(S)) = g(S)$

Ans. **[4]**

Sol. $g(S) = [-2, 2]$

$f(g(S)) = [0,4] = S$

$f(S) = [0, 16] \Rightarrow f(g(S)) \neq f(S)$

$g(f(S)) = [-4, 4] \neq g(S)$

therefore , $g(f(S)) \neq S$

Q.11 If $\lim_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is :

(1) $\frac{3}{2}$ (2) $\frac{8}{3}$ (3) $\frac{4}{3}$ (4) $\frac{3}{8}$

Ans. **[2]**

Sol. $\lim_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim_{x\to 1} (x + 1)(x^2 + 1)$ .....(i)

$\lim_{x\to k} \frac{x^3 - k^3}{x^2 - k^2} = \frac{k^2 + k^2 + k^2}{2k}$ ....(ii)

(i) = (ii)

$\Rightarrow k = \frac{8}{3}$

Q.12 If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 27 = 0$ (2) $4e^4 - 24e^2 + 35 = 0$ (3) $4e^4 - 12e^2 - 27 = 0$ (4) $4e^4 + 8e^2 - 35 = 0$

Ans. **[2]**

Sol. Let hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & passes through $(4, -2\sqrt{3})$ therefore

$\frac{16}{a^2} - \frac{12}{b^2} = 1$ .....(i) $\because b^2 = a^2(e^2 - 1)$

$x = \frac{4\sqrt{5}}{5} = \frac{a}{e}$

$\Rightarrow a^2 = \frac{16}{5}e^2$ ....(ii)

one solving (i) & (ii)

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

Q.13 If $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$ is continuous at x = 0, then the ordered pair (p, q) is equal to :

(1) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (2) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (3) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{5}{2}, \frac{1}{2}\right)$

Ans. **[3]**

Sol. RHL = $\lim_{x \to 0^+} \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$

$= \lim_{x \to 0^+} \dfrac{\sqrt{1 + x} - 1}{x} = \dfrac{1}{2}$

LHL = $\lim_{x \to 0^-} \dfrac{\sin(p+1)x + \sin x}{x}$

= (p + 1) + 1

= p + 2

for function to be continuous

LHL = RHL = f(0)

$\Rightarrow (p, q) = \left(-\frac{3}{2}, \frac{1}{2}\right)$

Q.14 If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a,b) is equal to :

(1) (28, 861) (2) (28, 315) (3) (–21, 714) (4) (–54, 315)

Ans. **[2]**

Sol. coefficient of x^2 = ${}^{15}C_2 \times 9 - 3a\ ({}^{15}C_1) + b = 0$

$\Rightarrow {}^{15}C_2 \times 9 - 45a + b = 0$ ...(i)

coefficient of $x^3 = -27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$

$\Rightarrow -273 + 21a - b = 0$ ...(ii)

(i) + (ii)

$-24a + 672 = 0 \Rightarrow a = 28$

$b = 315$

Q.15 If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for :

(1) exactly two values of K (2) no value of K.

(3) exactly one value of K (4) infinitely many values of K

Ans. **[2]**

Sol. equation of common chord

$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$ ...(i)

and given line $4x + 5y - k = 0$ ...(ii) on comparing (i) & (ii) we get

$k = \frac{1}{10} = \frac{k + 1/2}{-k}$

$\Rightarrow$ No. real value of k exist

Q.16 The sum $\frac{3\times1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + ...$ upto 10^{th} term is :

(1) 660 (2) 680 (3) 600 (4) 620

Ans. **[1]**

Sol. $T_n = \frac{(3+(n-1)\times2)(1^3+2^3+......+n^3)}{(1^2+2^2+........+n^2)}$

$= \frac{3}{2}\ n(n + 1)$

$S_n = \sum T_n$

$= \sum \frac{3}{2} n(n+1)$

on solving $S_n = \frac{n(n+1)(n+2)}{2} \Rightarrow S_{10} = 660$

Q.17 Let f : R $\rightarrow$ R be differentiable at c $\in$ R and f(c) = 0. If g(x) = $|f(x)|$, then at x = c, g is :

(1) differentiable if f ' (c) = 0

(2) differentiable if f ' (c) $\neq$ 0

(3) not differentiable

(4) not differentiable if f ' (c) = 0

Ans. **[1]**

Sol. $g'(c) = \lim_{h\to0} \frac{|f(c+h)| - |f(c)|}{h}$

$= \lim_{h\to0} \frac{|f(c+h)|}{h}$ $(\because\ f(c) = 0)$

$= \lim_{h\to0} \left|\frac{f(c+h)-f(c)}{h}\right| \cdot \frac{|h|}{h}$

$= \lim_{h\to0} |f'(c)| \frac{|h|}{h} = 0$ if $f'(c) = 0$

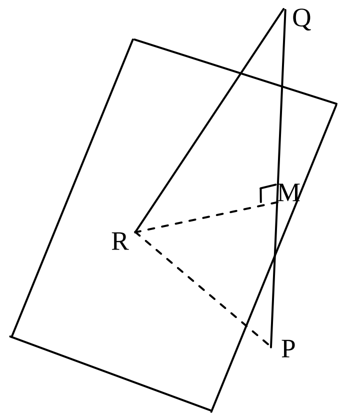
i.e. g(x) is differentiable at x = c if f '(c) = 0

Q.18 If Q(0, –1 –3) is the image of the point P in the plane 3x – y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $2\sqrt{13}$ (3) $\frac{\sqrt{91}}{2}$ (4) $\frac{\sqrt{91}}{4}$

Ans. **[3]**

Sol.



$$MQ = \frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}}$$

$$= \sqrt{\frac{13}{2}}$$

$PM = \sqrt{26}$

$RQ = \sqrt{9+1} = \sqrt{10}$

$$RM = \sqrt{10-\frac{13}{2}} = \sqrt{\frac{7}{2}}$$

$$Ar(\Delta PQR) = \frac{1}{2} \times \sqrt{26} \times \sqrt{\frac{7}{2}} = \sqrt{\frac{91}{2}}$$

Q.19 If $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$ where C is a constant of integration then :

(1) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(2) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(4) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

Ans. **[2]**

Sol. $$\int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{\left((x-1)^2+9\right)^2}$$

Let $x - 1 = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$$\therefore \frac{1}{27}\int \cos^2\theta\, d\theta = \frac{1}{54}\int (1+\cos 2\theta)d\theta = \frac{1}{54}\left(\theta + \frac{\sin 2\theta}{2}\right)$$

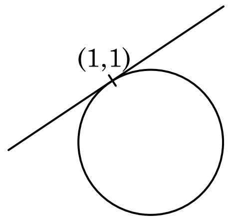
$$= \frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right) + C$$

Q.20 The line x = y touches a circle at the point (1,1). If the circle also passes through the point (1, – 3), then its radius is :

(1) 3 (2) 2 (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Ans. **[3]**

Sol.



Equation of circle is given as

$S + \lambda L = 0$

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

passes through(1, – 3)

$16 + \lambda \times 4 = 0 \quad \Rightarrow \lambda = -4$

$\therefore (x-1)^2 + (y-1)^2 - 4(x-y) = 0$

$r = 2\sqrt{2}$

Q.21 The value of $\int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function is :

(1) 2π (2) π (3) -2π (4) $-\pi$

Ans. **[4]**

Sol. $I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$

$2I = \int_0^{2\pi} \left([\sin 2x(1+\cos 3x)] + [-\sin 2x - \sin 2x\cos 3x]\right)dx$

$2I = \int_0^{2\pi} -dx$

$2I = 2\int_0^{\pi} -dx$

$I = \int_0^{\pi} -dx \Rightarrow -\pi$

Q.22 Let A (3,0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{15}}$ (2) $\frac{1}{6\sqrt{10}}$ (3) $\frac{1}{\sqrt{30}}$ (4) $\frac{1}{2\sqrt{15}}$

Ans. **[1]**

Sol. G will be centroid of ΔABC

$G \equiv (2, 4, 2)$

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$\cos(\angle GOA) = \frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}|\,|\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$

Q.23 If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 10 (2) 9 (3) 12 (4) 7

Ans. **[1]**

Sol. $x + 3y + \lambda z - \mu = a(x + y + z - 5) + b(x + 2y + 2z - 6)$

comparing coefficients we get

$a + b = 1$ and $a + 2b = 3$

$(a, b) \equiv (-1, 2)$

So, $x + 3y + \lambda z - \mu = x + 3y + 3z - \lambda$

$\Rightarrow \mu = 7, \lambda = 3$

Q.24 The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated is :

(1) 36 (2) 60 (3) 72 (4) 48

Ans. **[2]**

Sol. Let the six digit number be abcdef for this number to be divisible by 11, $|(a + c + e) - (b + d + f)|$ must be multiple of 11

$\therefore$ possibility is $a + c + e = b + d + f = 12$

Case : 1 $\{a, c, e\} = \{7, 5, 0\}$

& $\{b, d, f\} = \{9, 2, 1\}$

So, number of numbers $= 2 \times 2! \times 3! = 24$

Case : 2 $\{a, c, e\} = \{9, 2, 1\}$

& $\{b, d, f\} = \{7, 5, 0\}$

So, number of numbers $= 3! \times 3! = 36$

total $\Rightarrow 24 + 36$

$= 60$

Q.25 If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x + 1)^2$	$2x - 5$	$x^2 - 3x$	x

then the mean of the marks is

(1) 3.0 (2) 2.8 (3) 2.5 (4) 3.2

Ans. **[2]**

Sol. Mean $\bar{x} = \dfrac{\sum x_i f_i}{\sum f_i}$

$\because \sum f_i = (x + 1)^2 + (2x - 5) + (x^2 - 3x) + x = 20$

$\Rightarrow x = 3, -4$(rejected)

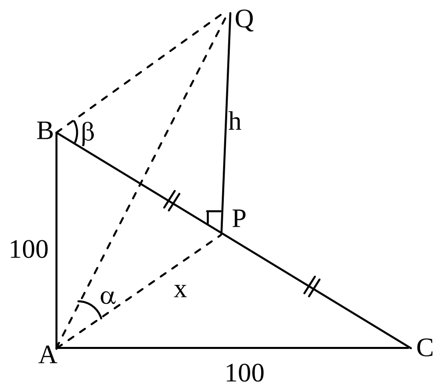
$\therefore \bar{x} = \dfrac{\sum x_i f_i}{\sum f_i} = 2.8$

Q.26 ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(1) $\frac{100}{3\sqrt{3}}$ (2) 25 (3) 20 (4) $10\sqrt{5}$

Ans. **[3]**

Sol.



$\text{cosec}\,\beta = 2\sqrt{2}$

$\cot\alpha = 3\sqrt{2}$

$\frac{x}{h} = 3\sqrt{2}$ ...(i)

So $\frac{h}{\sqrt{10^4 - x^2}} = \frac{1}{\sqrt{7}}$ .....(ii)

from (i) & (ii)

h = 20

Q.27 If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98 (3) 76 (4) 64

Ans. **[3]**

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$

$a_1 + a_{16} = 38$

So, $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 \Rightarrow 76$

Q.28 Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is :

(1) $[0, \infty)$ (2) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (3) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (4) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

Ans. **[4]**

Sol. h(x) = f(g(x))

h'(x) = f '(g(x)) g'(x)   and $f'(x) = e^x - 1$

$h'(x) = (e^{g(x)} - 1)\, g'(x)$

$h'(x) = (e^{x^2 - x} - 1)(2x - 1) \geq 0$

Case :1 $e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow x \in \left[0, \frac{1}{2}\right]$ .....(i)

Case : 2 $e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$ ....(ii)

from (i) & (ii)

$x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

Q.29 $\lim_{n\to\infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + ... + \frac{(2n)^{1/3}}{n^{4/3}} \right)$ is equal to :

(1) $\frac{4}{3}(2)^{3/4}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$ (3) $\frac{4}{3}(2)^{4/3}$ (4) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

Ans. **[2]**

Sol. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3}$

$= \int_0^1 (1+x)^{1/3}\, dx = \frac{3}{4}\,(2^{4/3} - 1)$s

Q.30 If the line x – 2y = 12 is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

(1) $8\sqrt{3}$ (2) $12\sqrt{2}$ (3) 5   (4) 9

Ans. **[4]**

Sol. Tangent at (3, – 9/2)

$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$

comparing with x – 2y = 12

$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$

$\Rightarrow a = 6$ & $b = 3\sqrt{3}$

length of latus rectum = $\frac{2b^2}{a} = 9$