CAREER POINT

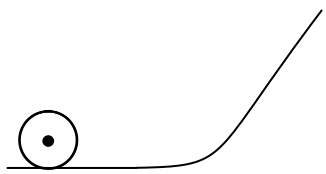
JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - II

Physics

Q.1 A Solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (See figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by



(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{14}{15}$ (3) $\frac{4}{5}$ (4) 1

Ans. **[2]**

Sol. $\frac{1}{2}mv^2(1+\eta) = mgh$

$h \propto 1 + \eta$

$$\frac{h_{sp}}{h_{cyl}} = \frac{1+\frac{2}{5}}{1+\frac{1}{2}} = \frac{7/5}{3/2} = \frac{14}{15}$$

Q.2 In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antena is 70m, then the minimum height of the transmitting antenaa should be – (Radius of the Earth = 6.4×10^6 m).

(1) 32 m (2) 51 m (3) 40 m (4) 20 m

Ans. **[1]**

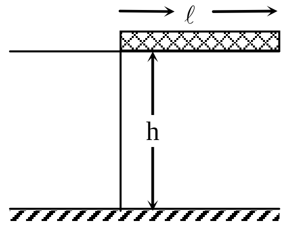
Sol. $D = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$50 \times 10^3 = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$h_T = \left[5\times10^4 - \sqrt{2\times6.4\times10^6\times70}\right]^2$

$h_T \simeq 32$ m

Q.3 A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to –



(1) 0.5 (2) 0.02 (3) 0.28 (4) 0.3

Ans. **[1]**

Sol. $\alpha = \frac{MgL/2}{ML^2/3} = \frac{3g}{2L}$

$\omega = \frac{3g}{2L}\tau = \frac{30(0.01)}{2(0.3)} = 0.5$

$t = \sqrt{\frac{2 \times 5}{10}} = 1\text{sec}$

$\Delta\theta = \omega t = (0.5)(1) = 0.5$ Rad

Q.4 Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to -

(1) 1.9 mm (2) 1.7 mm (3) 1.3 mm (4) 1.5 mm

Ans. **[2]**

Sol. $\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$

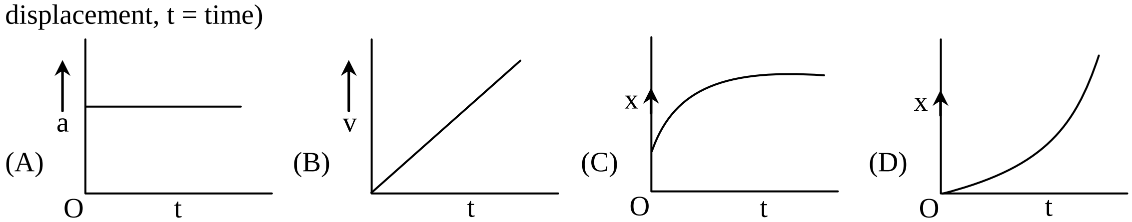
$F \propto \frac{AY}{\ell}$

$\frac{A_1Y_1}{\ell_1} = \frac{A_2Y_2}{\ell_2}$

$\frac{R^2(7)}{2} = \frac{2^2(4)}{1.5}$

R = 1.74

Q.5 A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

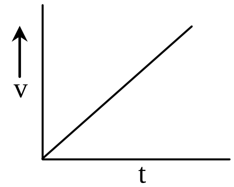


(1) (A) (2) (A), (B), (D) (3) (B), (C) (4) (A), (B), (C)

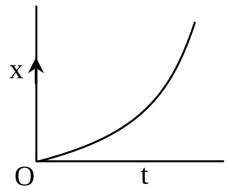
Ans. [2]

Sol. a = constant

$v \propto t$



$x \propto t^2$



Q.6 The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The value of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x =1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is

(1) –520 V (2) 180 V (3) –48 V (4) 320 V

Ans. [2]

Sol. $\Delta V = \frac{A}{2}[x^2]_1^{-5} + B[x]_1^{-5}$

= 10(24) – 60

= 180

Q.7 A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that of earth's volume is 64 times the volume of the moon.

(1) $\frac{E}{4}$ (2) $\frac{E}{32}$ (3) $\frac{E}{16}$ (4) $\frac{E}{64}$

Ans. [3]

Sol. $E \propto R^2$

$\frac{E'}{E} = \frac{R^2}{16R^2}$

$E' = \frac{E}{16}$

Q.8 A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to $\frac{1}{1000}$ of the original amplitude is close to -

(1) 50 s (2) 100 s (3) 10 s (4) 20 s

Ans. [4]

Sol. T = 2 sec

$(2)^{10} \simeq 1024$

So time t = 10 T

= 20 sec

Q.9 In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to -

(1) 0.7 % (2) 6.8 % (3) 0.2 % (4) 3.5 %

Ans. **[2]**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{4\pi^2 \ell}{T^2}$$

$$\left|\frac{\Delta g}{g}\right| = \left|\frac{\Delta \ell}{\ell}\right| + 2\left|\frac{\Delta T}{T}\right|$$

$$\% \left|\frac{\Delta g}{g}\right| = \frac{0.1}{55}\times 100 + 2\frac{1}{30}\times 100$$

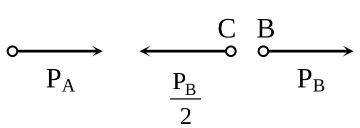
$$= 6.8\ \%$$

Q.10 A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-broglie wavelengths λ_B and λ_C of B and C are respectively -

(1) $\lambda_A, \frac{\lambda_A}{2}$ (2) $\lambda_A, 2\lambda_A$ (3) $2\lambda_A, \lambda_A$ (4) $\frac{\lambda_A}{2}, \lambda_A$

Ans. **[4]**

Sol.



$\lambda_B = \frac{\lambda_A}{2}$; $\lambda_C = 2\lambda_B = \lambda_A$

$P_A = P_B - \frac{P_B}{2} \Rightarrow$ $P_B = 2P_A$

Q.11 The magnetic field of an electromagnetic wave is given by –

$$\vec{B} = 1.6\times 10^{-6}\cos(2\times 10^7 z + 6\times 10^{15} t)(2\hat{i} + \hat{j})\frac{Wb}{m^2}$$

The associated electric field will be -

(1) $\vec{E} = 4.8\times 10^2 \cos(2\times 10^7 z + 6\times 10^{15} t)(-\hat{i} + 2\hat{j})\frac{V}{m}$

(2) $\vec{E} = 4.8\times 10^2 \cos(2\times 10^7 z - 6\times 10^{15} t)(2\hat{i} + \hat{j})\frac{V}{m}$

(3) $\vec{E} = 4.8\times 10^2 \cos(2\times 10^7 z + 6\times 10^{15} t)(\hat{i} - 2\hat{j})\frac{V}{m}$

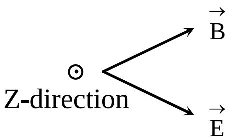
(4) $\vec{E} = 4.8\times 10^2 \cos(2\times 10^7 z - 6\times 10^{15} t)(-2\hat{j} + \hat{i})\frac{V}{m}$

Ans. **[3]**

Sol. $E_0 = CB_0$

$= 3 \times 10^8 \times 1.6 \times 10^{-6}$

$= 4.8 \times 10^2$



Q.12 A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is -

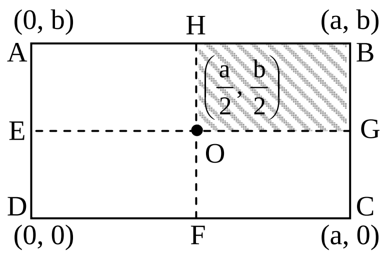
(1) 1V (2) 2V (3) 3V (4) 5V

Ans. **[1]**

Sol. $q_{cap} = \frac{4-2}{2} = 1\mu C$

$V = \frac{q}{C} = \frac{10^{-6}}{10^{-6}} = 1$ volt

Q.13 A uniform rectangular thin sheet ABCD of mas M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be -



(1) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$ (2) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (3) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$ (4) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

Ans. **[3]**

Sol. $X_C = \frac{2m\left(\frac{a}{4}\right) + m\left(\frac{3a}{4}\right)}{3m} = \frac{5a}{12}$

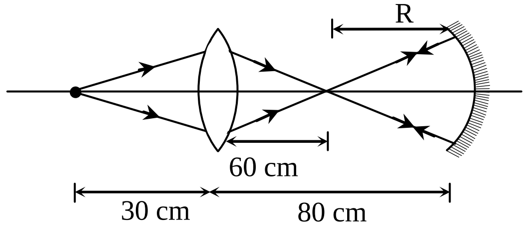
$Y_C = \frac{2m\left(\frac{b}{2}\right) + m\left(\frac{b}{4}\right)}{3m} = \frac{5b}{12}$

Q.14 A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be -

(1) 20 cm (2) 10 cm (3) 25 cm (4) 30 cm

Ans. **[2]**

Sol.



$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow v = 60cm$$

R = 2f = 20 cm

f = 10 cm

Max distance for virtual image = f = 10 cm

Q.15 A electric dipole is formed by two equal and opposite charge q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is -

(1) $\sqrt{\frac{qE}{2md}}$ (2) $\sqrt{\frac{2qE}{md}}$ (3) $2\sqrt{\frac{qE}{md}}$ (4) $\sqrt{\frac{qE}{md}}$

Ans. **[2]**

Sol. $T = 2\pi\sqrt{\frac{I}{pE}} = 2\pi\sqrt{\frac{md^2/2}{qdE}} = 2\pi\sqrt{\frac{md}{2qE}} \Rightarrow \omega = \sqrt{\frac{2qE}{md}}$

Q.16 A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when -

(1) R = 1000r (2) R = 0.001r (3) R = 2r (4) R = r

Ans. **[4]**

Sol. For maximum power transfer

$\boxed{R = r}$

Q.17 Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

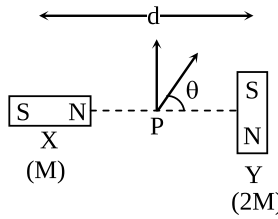
(1) 152.5×10^{-9} radian (2) 457.5×10^{-9} radian

(3) 610×10^{-9} radian (4) 305×10^{-9} radian

Ans. **[4]**

Sol. $\Delta\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = 305 \times 10^{-9}$ rad

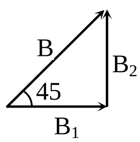
Q.18 Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle θ = 45 ° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)



(1) 0 (2) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$ (3) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$ (4) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3}\times qv$

Ans. **[1]**

Sol.



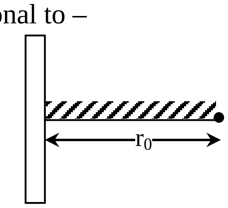
$B_1 = \frac{\mu_0}{4\pi}\frac{2(M)}{(d/2)^3}$

$B_2 = \frac{\mu_0}{4\pi}\frac{(2M)}{(d/2)^3}$

$B_1 = B_2$

$\vec{V}$ is along $\vec{B}$ thus $\vec{F}_{net} = \vec{0}$

Q.19 A positive point charge is released from rest at a distance r_0 from a position line chare with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional distance r from line charge, is proportional to –



(1) $v \propto e^{+r/r_0}$ (2) $v \propto \ell n\left(\frac{r}{r_0}\right)$ (3) $v \propto \left(\frac{r}{r_0}\right)$ (4) $v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

Ans. **[4]**

Sol. $E \propto \frac{1}{r}$

$\Delta V \propto \ell n\left(\frac{r}{r_0}\right)$

$\frac{1}{2}mv^2 \propto \ell n\left(\frac{r}{r_0}\right)$

$v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

Q.20 A circuit connected to an ac source of emf e = e_0 sin (100t) with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this ?

(1) RL circuit with R = 1 kΩ and L = 10 mH
(2) RL:circuit with R = 1 kΩ and L = 1 mH
(3) RC circuit with R = 1 kΩ and C = 1μF
(4) RC circuit with R = 1 kΩ and C = 10μF

Ans. **[4]**

Sol. $X_C = R$

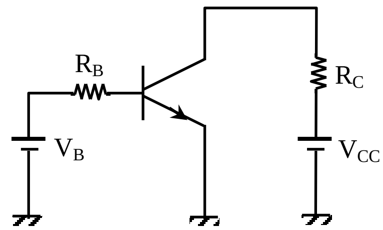
$\frac{1}{\omega C} = R$

$\frac{1}{100} = RC$

$R = 10^3\ \Omega$

$C = 10^{-5}$ F

Q.21 A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, R_C = 1 kΩ and V_{CC} = 10V.What is the minimum base current for V_{CE} to reach saturation ?



(1) 10 μA
(2) 100 μA
(3) 40 μA
(4) 7 μA

Ans. **[3]**

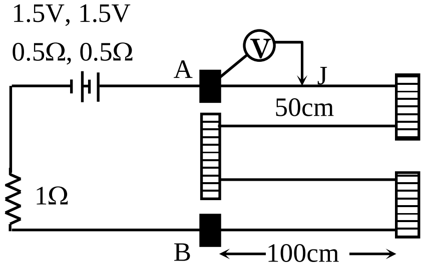
Sol. $V_{CC} - I_C R_C = 0$

$10 - I_C \times 10^3 = 0$

$I_C = 10^{-2}$

$I_B = \frac{I_C}{\beta} = \frac{10^{-2}}{250} = 40\mu A$

Q.22 In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be –



(1) 0.75 V
(2) 0.25 V
(3) 0.50 V
(4) 0.20 V

Ans. **[2]**

Sol. $i = \frac{3}{1+1+4} = 0.5A$

V = 0.5 × 0.5 = 0.25 volt

Q.23 The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to –
[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K ; Avogadro Number $N_A = 6.02 \times 10^{26}$/ kg;
Radius of Earth : 6.4×10^6 m; Gravitational acceleration on earth = 10 ms^{-2}]

(1) 650 K (2) 3×10^5 K (3) 800 K (4) 10^4 K

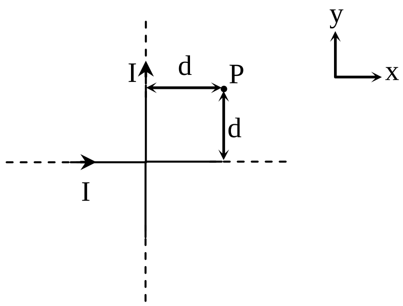
Ans. **[4]**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{2GM_E}{R_E}}$

$$\frac{3 \times 8.314 \times T}{2 \times 10^{-3}} = (11.2 \times 10^3)^2$$

$T = 10^4$ K

Q.24 Two very long, straight, and insulated wires are kept at 90° angle from each other in xy- plane as shown in the figure



These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be -

(1) Zero (2) $-\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$ (3) $\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$ (4) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

Ans. **[1]**

Sol. $\vec{B} = \frac{\mu_0 i}{2\pi d}\hat{k} - \frac{\mu_0 i}{2\pi d}\hat{k}$

$\vec{B} = \vec{0}$

Q.25 A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively . If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is -

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 + v_2$ (3) $v_4 - \frac{v_2}{2}$ (4) $v_4 - v_2$

Ans. **[4]**

Sol. $m_1v_1\hat{i} + m_2v_2\hat{i} = m_1(0.5v_1)\hat{i} + m_2v_4\hat{i}$

$m_1v_1\hat{i} + \frac{m_1v_2}{2}i = \frac{m_1v_1}{2}\hat{i} + \frac{m_1v_4}{2}\hat{i}$

$v_1 = v_4 - v_2$

Q.26 The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to -

(1) 0.1 (2) 2 (3) 5 (4)1

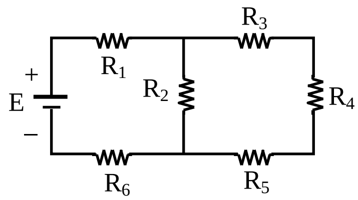
Ans. **[4]**

Sol. $\rho_{nucleus} \propto (A)^0$ [independent to mass number]

$\frac{\rho_{Ca}}{\rho_O} = \frac{1}{1}$

1 : 1

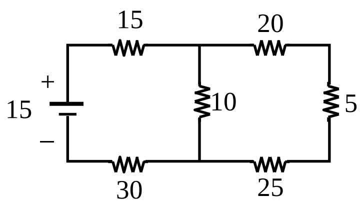
Q.27 In the figure shown, what is the current (in Ampere) drawn from the battery? You are given –
$R_1 = 15\Omega$, $R_2 = 10\Omega$, $R_3 = 20\Omega$, $R_4 = 5\Omega$, $R_5 = 25\Omega$, $R_6 = 30\Omega$, $E = 15V$



(1) 7/18 (2) 20/3 (3) 9/32 (4) 13/24

Ans. **[3]**

Sol.



$$i = \frac{15}{45 + \frac{25}{3}} = \frac{3}{9 + \frac{5}{3}} = \frac{9}{32}$$

Q.28 If surface tension (S), Moment of Inertia (I) and Plank's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be-

(1) $S^{3/2}I^{1/2}h^{0}$ (2) $S^{1/2}I^{1/2}h^{-1}$ (3) $S^{1/2}I^{3/2}h^{-1}$ (4) $S^{1/2}I^{1/2}h^{0}$

Ans. **[4]**

Sol. $p \propto \sqrt{\text{mass} \times \text{Energy}}$

$\propto \sqrt{mR^2S}$

$\propto \sqrt{IS}$

$[p] = \left[I^{\frac{1}{2}}\ S^{\frac{1}{2}} \right]$

Q.29 Let $|\overrightarrow{A_1}| = 3, |\overrightarrow{A_2}| = 5$ and $|\overrightarrow{A_1} + \overrightarrow{A_2}| = 5$. The value of $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \bullet (3\overrightarrow{A_1} - 2\overrightarrow{A_2})$ is -

(1) –112.5 (2) –106.5 (3) –118.5 (4) –99.5

Ans. **[3]**

Sol. $(2\vec{A}_1 + 3\vec{A}_2) \cdot (3\vec{A}_1 - 2\vec{A}_2)$

$= 6A_1^2 - 4\vec{A}_1 \cdot \vec{A}_2 + 9\vec{A}_1 \cdot \vec{A}_2 - 6A_2^2$

$= 6(A_1^2 - A_2^2) + 5A_1A_2 \cos\theta$

$= -96 + 5A_1A_2 \cos\theta$

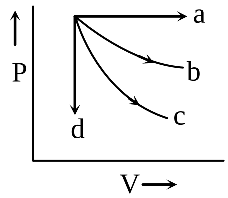
$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\theta$

$25 = 9 + 25 + 2A_1A_2 \cos\theta$

$A_1A_2 \cos\theta = -\frac{9}{2}$

$= -96 - \frac{5 \times 9}{2} = -118.5$

Q.30 The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by -



(1) d a b c (2) a d c b (3) a d b c (4) d a c b

Ans. **[1]**

Sol. a $\rightarrow$ isobaric

d $\rightarrow$ isochoric

b $\rightarrow$ isothermal

c $\rightarrow$ adiabatic

(d a b c)

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8^{th} April 2019 | Shift - II

Chemistry

Q.1 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane, 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaportates and a monolayer is formed. The distance from edge to centre of the watch glassf is 10 cm. What is the height of the monolayer ? [Density of fatty acid = 0.9 g cm^{-3}, π = 3]

(1) 10^{-2} m (2) 10^{-4} m (3) 10^{-8} m (4) 10^{-6} m

Ans. **[4]**

Sol. In 100ml gm of fatty acid = 0.27 gm

1 ml gm .... = $\frac{0.27}{100}$

10 ml gm .... = $\frac{0.27}{100}\times 10 = 0.027$

$d = \frac{m}{v}$

$d \times v = m$

$0.9\left(\frac{gm}{cm^3}\right) \times \text{area} \times \text{height} = 0.027$ gm

$0.9 \times (3) \times (10)^2 \times h = 0.027$

$\left[\begin{array}{l} \text{Area} = \pi r^2 \\ \quad = 3(10)^2 \end{array}\right]$

$h = 10^{-4}$ cm

$h = 10^{-6}$ m

Q.2 5 moles of and ideal gas at 100 K are allowed to undergo reversible compresion till its temperature becomes 200 K. If $C_V = 28 J K^{-1} mol^{-1}$, calculate ΔU and ΔpV

(1) ΔU = 14J; $\Delta(pV)$ = 0.8J (2) ΔU = 14kJ; $\Delta(pV)$ = 4kJ

(3) ΔU = 14kJ; $\Delta(pV)$ = 18J (4) ΔU = 2.8 kJ; $\Delta(pV)$ = 0.8kJ

Ans. **[2]**

Sol. $\Delta U = nC_V \Delta T$,

$= 5(28)(100)$ J

$= 14000$ J = 14 kj

$\Delta PV = P_2V_2 - P_1V_1$

$\Delta PV = nRT_2 - nRT_1$

$= nR(T_2 - T_1)$

$= 5(8)(100) = 4000$ J = 4 kJ

Q.3 The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are

(1) 2.84 and 5.92 (2) 0 amd 5.92 (3) 0 and 4.9 (4) 4.9 and 0

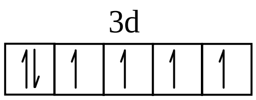
Ans. **[4]**

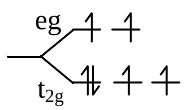
Sol. Compound is $[Fe(H_2O)_6]_2\ [Fe(CN)_6]$

Cation is $[Fe(H_2O)_6]^{+2}$

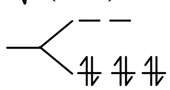
Anion is $[Fe(CN)_6]^{-4}$

Configuration of Fe^{+2} = (Ar) $3d^6$

= (Ar) 

For H_2O W.F. ligand 

4 umpaired e^-, $\therefore$ $\mu = \sqrt{4(4+2)} = 4.9$ = 4.9 B.M.

For CN^- & F. ligand 

No unpaired e^-

$\mu = 0$

Q.4 The compound that inhibits the growth of tumors is -

(1) cis-$[Pd(Cl)_2(NH_3)_2]$ (2) trans-$[Pd(Cl)_2(NH_3)_2]$

(3) cis-$[Pt(Cl)_2(NH_3)_2]$ (4) trans-$[Pt(Cl)_2(NH_3)_2]$

Ans. **[3]**

Sol. cis platin is used to inhibit growth of tumor

Q.5 Which of the following comlpounds will show the maximum 'enol' content ?

(1) CH_3COCH_3 (2) $CH_3COCH_2COOC_2H_5$

(3) $CH_3COCH_2COCH_3$ (4) $CH_3COCH_2CONH_2$

Ans. **[3]**

Sol. β-Dicarbonyl compound

β-Diketone

Extended conjugation and Intramolecular H-bonding in enolic form

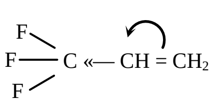
Q.6 Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product ?

(1) $CH_3O–CH=CH_2$ (2) $H_2N–CH=CH_2$

(3) $F_3C–CH=CH_2$ (4) $Cl–CH=CH_2$

Ans. **[3]**

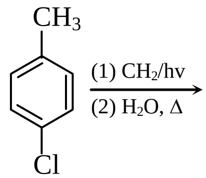
Sol.

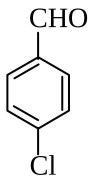


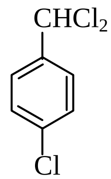
$– CF_3 \rightarrow$ – H effect

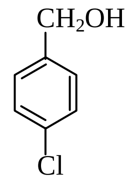
Most e^- withdrawing group

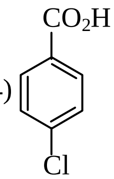
Q.7 The major product of the following reaction is –



(1) 

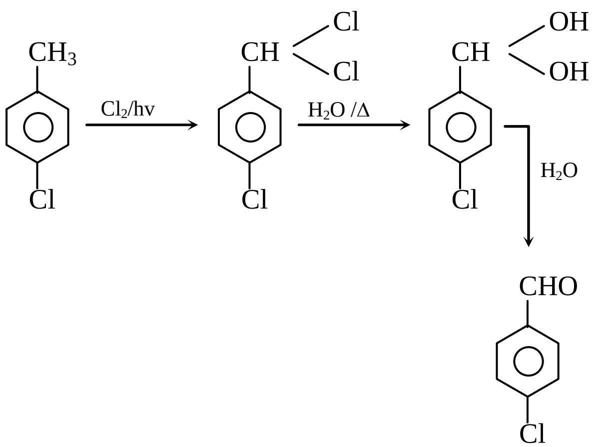
(2) 

(3) 

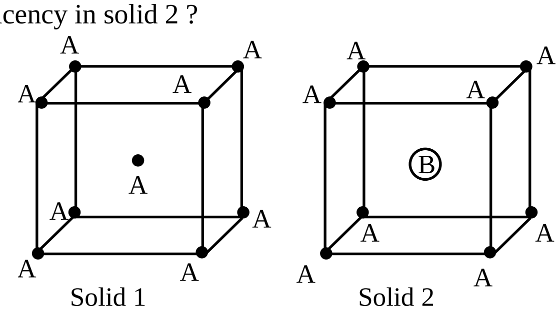
(4) 

Ans. **[1]**

Sol.



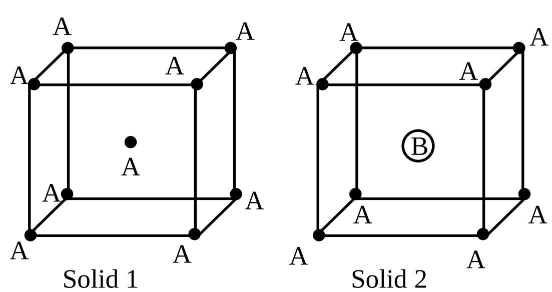
Q.8 Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficency in solid 2 ?



(1) 75% (2) 90% (3) 45% (4) 65%

Ans. **[2]**

Sol.



$r_B = 2r_A$ $\quad a_2 = 1.5a_1$

$4r_A = \sqrt{3}a_1 \quad , \quad a_1 = \dfrac{4r_A}{\sqrt{3}}$

$a_2 = 1.5\ a_1$

$= \dfrac{3}{2}\ \dfrac{4r_A}{\sqrt{3}}$

$a_2 = 2\sqrt{3}r_A$

$$PE_2 = \frac{\left(\frac{4}{3}\pi r_A^3 \times 1\right) + \left(\frac{4}{3}\pi r_B^3 \times 1\right)}{a_2^3}$$

$$= \frac{\frac{4}{3}\pi r_A^3 + \frac{4}{3}\pi (2r_A)^3}{(2\sqrt{3}r_A)^3}$$

$$= \frac{\frac{4}{3}\pi r_A^3 \times 9}{8 \times 3\sqrt{3}\, r_A^3} = \frac{\pi}{2\sqrt{3}} = 90.64\%$$

$= 90\%$

Q.9 If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function)

(1) $\frac{4}{9}\lambda$ (2) $\frac{2}{3}\lambda$ (3) $\frac{3}{4}\lambda$ (4) $\frac{1}{2}\lambda$

Ans. **[1]**

Sol. $E = \phi + KE$

$E = KE$

$$\frac{h_c}{\lambda} = \frac{1}{2}mv^2\left(\frac{m}{m}\right) = \frac{P^2}{2m}$$

$$p^2 \propto \frac{1}{\lambda}$$

$$\left(\frac{P_2}{P_1}\right)^2 = \frac{\lambda_1}{\lambda_2}$$

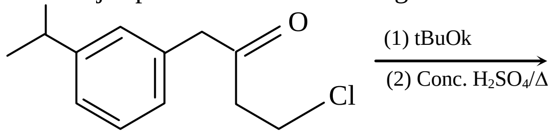
$$\left(\frac{1.5P_1}{P_1}\right)^2 = \frac{\lambda_1}{\lambda_2}$$

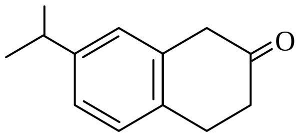
$$\left(\frac{3}{2}\right)^2 = \frac{\lambda_1}{\lambda_2}$$

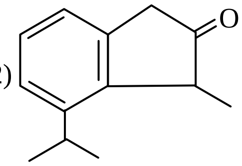
$$\frac{9}{4} = \frac{\lambda_1}{\lambda_2}$$

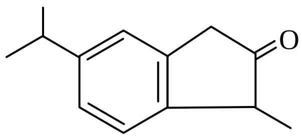
$$\lambda_2 = \frac{4}{9}\lambda_1$$

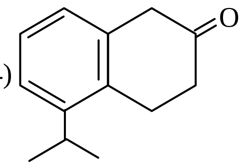
Q.10 The major product of the following reaction is –



(1) 

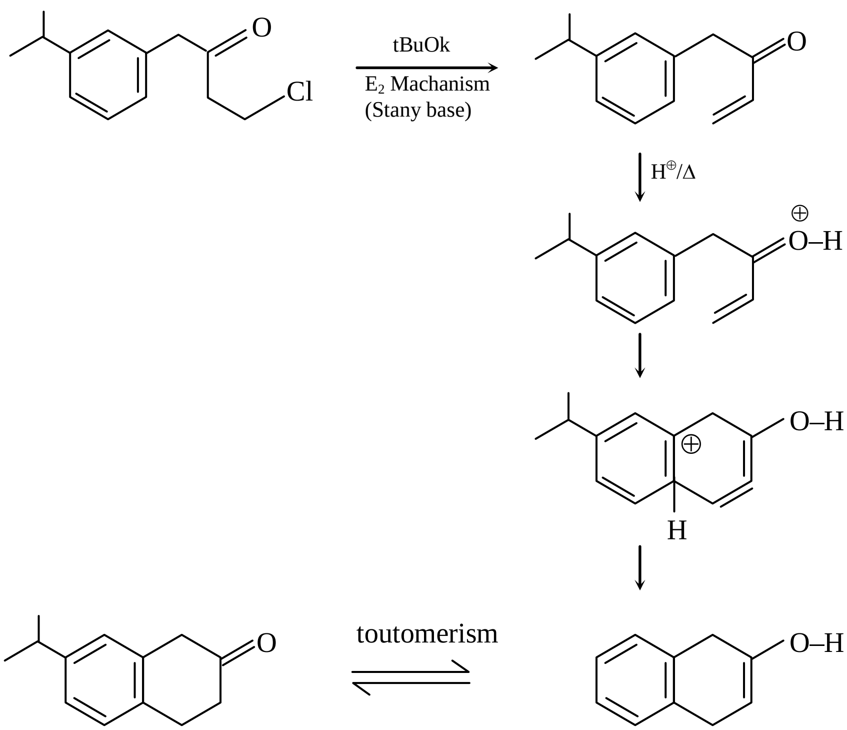
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.11 The Mond process is used for the-

(1) purification of Ni
(2) extraction of Zn
(3) extraction of Mo
(4) purification of Zr and Ti

Ans. **[1]**

Sol. Mond process is used for Ni

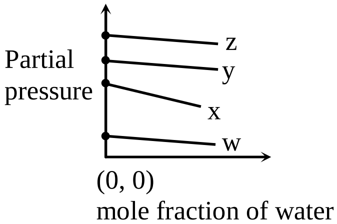
$$\underset{\text{impure}}{Ni} + 4CO \xrightarrow{330-350K} Ni(CO)_4$$

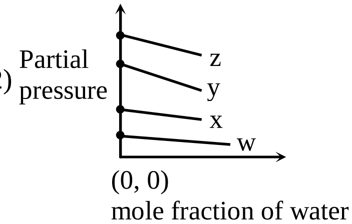
$$\underset{\text{impure}}{Ni} + 4CO \xrightarrow{330-350K} Ni(CO)_4$$

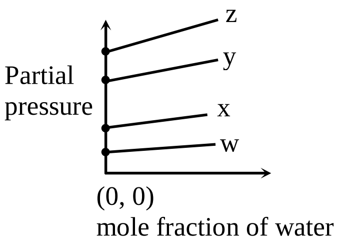
$\downarrow$ 450 – 470 K

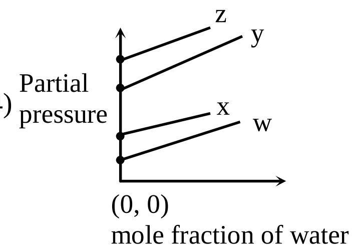
$\underset{\text{Pure}}{Ni} + 4CO\uparrow$

Q.12 For the solution of the gases w, x, y and z in water at 298 K, the Henry law constants (K_H) are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is -

(1)


(2)


(3)


(4)


Ans. **[2]**

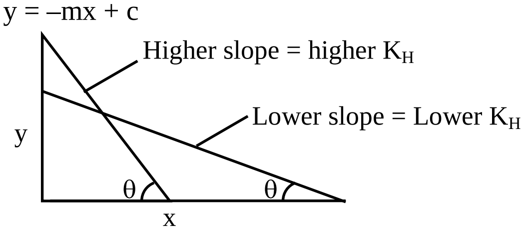
Sol. $k_H = \frac{p_g}{x_g} = \text{bar}$

$p_g = k_H x_g = k_H(1 - x_w)$

$= k_H - k_H x_w$

$= -k_H x_w + k_H$

$y = -mx + c$



$k_{H_z} > k_{H_y} > k_{H_x} > k_{H_w}$

Q.13 The IUPAC symbol for the element with atomic number 119 would be -

(1) uue (2) une (3) uun (4) unh

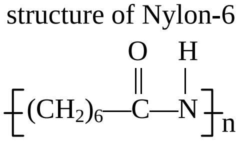
Ans. **[1]**

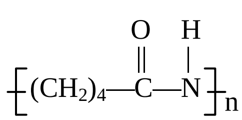
Sol. z = 119 is

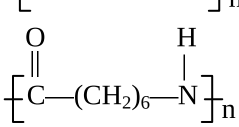
ununennium

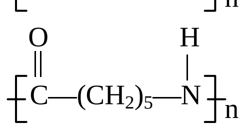
∴ uue

Q.14 The structure of Nylon-6 is ?

(1)


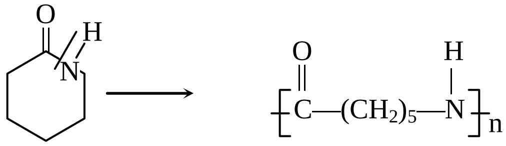
(2)


(3)


(4)


Ans. [4]

Sol.



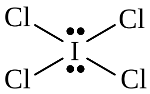
Caplactum Nylon-6

Q.15 The ion that has sp^3d^2 hybridazation for the central atom is -

(1) $[ICl_4]^-$ (2) $[ICl_2]^-$ (3) $[BrF_2]^-$ (4) $[IF_6]$–

Ans. [1]

Sol.



sp^3d^2

square planar geometry

Q.16 For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by -

(1) $(k_1 - k_2)$ [A] (2) k_1k_2[A] (3) $(k_1 + k_2)$ [A] (4) $\left(\frac{k_1}{k_2}\right)$[A]

Ans. [4]

Sol. $\frac{dB}{dt} = K_1[A] - K_2[B] = 0$

$K_1[A] = K_2[B]$

$[B] = \frac{K_1}{K_2}[A]$

$\frac{d_A}{dt} = K_1[A]$

$\frac{d_C}{dt} = K_2[B]$

Q.17 For the following reactions, equilibrium constants are given -

$S(s) + O_2(g) \rightleftharpoons SO_2(g); K_1 = 10^{52}$

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g); K_2 = 10^{129}$

The equilibrium constant for the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is

(1) 10^{154} (2) 10^{181} (3) 10^{25} (4) 10^{77}

Ans. [3]

Sol. $S + O_2 \rightleftharpoons SO_2 \quad ; K_1 = 10^{52}$

$2S + 3O_2 \rightleftharpoons 2SO_3 \quad ; K_2 = 10^{129}$

$2SO_2 + O_2 \rightleftharpoons 2SO_3 \quad ; K_3 = ?$

$$K_3 = K_1^{-2} \cdot K_2 = \frac{K_2}{K_1^2} = \frac{10^{129}}{10^{104}} = 10^{25}$$

Q.18 The statement that is incorrect about the intertitial compounds is -

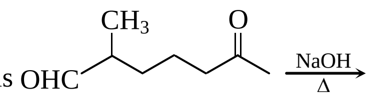
(1) they are very hard
(2) they have metallic conductivity
(3) they have high melting points
(4) they are chemically reactive

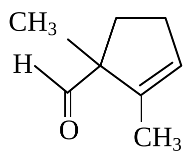
Ans. **[4]**

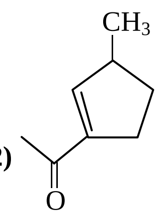
Sol. Intustitial compound are –

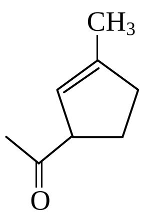
(i) hard
(ii) chemically inert
(iv) high m.p.

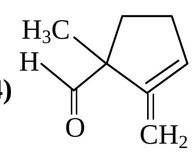
As interstitial compounds are chemically inert

Q.19 The major product obtained in the following reaction is 

(1) 

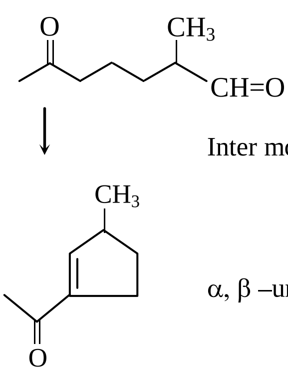
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.



Inter molecular aldol condensation

α, β –unsaturated carbonyl compound

Q.20 The percentage composition of carbon by mole in methane is -

(1) 75%
(2) 80%
(3) 20%
(4) 25%

Ans. **[3]**

Sol. % composition of C by mole in CH_4

$$\% \text{ C} = \frac{1}{5} \times 100$$

= 20%

Q.21 Among the following molecules /ions, C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2 which one is diamagnetic and has the shortest bond length

(1) N_2^{2-} (2) O_2 (3) C_2^{2-} (4) O_2^{2-}

Ans. **[3]**

Sol. O_2, N_2^{-2} = paramagnetic

C_2^{-2} and O_2^{-2} = diamagnetic

C_2^{-2} has B.O. = 3

$\therefore$ diamagnetic & shortest bond length, specie is C_2^{-2}.

Q.22 Polysubstition is a major drawback in -

(1) Reimer Tiemann reaction (2) Friedel Craft's acylation

(3) Friedel Craft's alkylation (4) Acetylation of aniline

Ans. **[3]**

Sol. Polysubstitution is a major drawback of Friedal –Craft alkylation

$-CH_3$ gp in highly activating group

due to +H effect of its

Q.23 Calculate the standard cell potential (in V) of the cell in which following reaction takes place –

Fe^{2+} (aq) + Ag^+ (aq) $\rightarrow$ Fe^{3+} (aq) + Ag(s)

Given that

$E^\circ_{Ag^+/Ag} = xV$

$E^\circ_{Fe^{2+}/Fe} = yV$

$E^\circ_{Fe^{3+}/Fe} = zV$

(1) x + 2y – 3z (2) x + y – z (3) x – y (4) x – z

Ans. **[1]**

Sol. $E^\circ_{Ag^+/Ag} = x$

$E^\circ_{Fe^{2+}/Fe} = z$

$E^\circ_{Fe^{3+}/Fe} = y$

$Fe^{2+} + Ag^+ \rightarrow Fe^{+3} + Ag$

$$E^\circ_{cell} = \underset{(R.P.)}{E^\circ_C} - \underset{(R.P.)}{E^\circ_A}$$

$$= E^\circ_{Ag^+/Ag} = E^\circ_{Fe^{+3}/Fe^{+2}}$$

= x – (3z – 2y)

= x + 2y – 3z

$$E_3 = \frac{\pm n_1, E_1 \pm n_2E_2}{n_3}$$

$$= \frac{3z - 2y}{1} = 3z - 2y$$

Q.24 The covalent alkaline earth metal halide (X = Cl, Br, I) is

(1) BeX_2 (2) SrX_2 (3) MgX_2 (4) CaX_2

Ans. **[1]**

Sol. Halides of Be are covalent

Q.25 The correct statement about ICl_5 and ICl_4^-

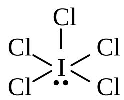
(1) both are isostructural

(2) ICl_5 is square pyramidal and ICl_4^- is square planner

(3) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral

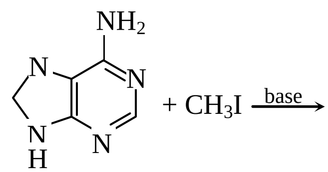
(4) ICl_5 is square pyramidal and ICl_4^- is tetrahedral

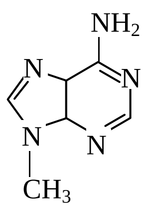
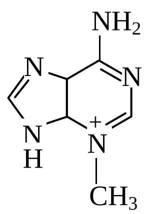
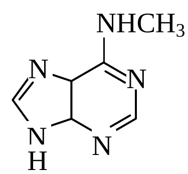
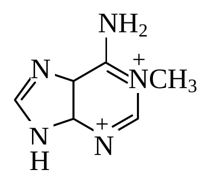
Ans. **[2]**

Sol. ICl_5 is  sp^3d^2 square pyramidal

ICl_4^- square planar sp^3d^2

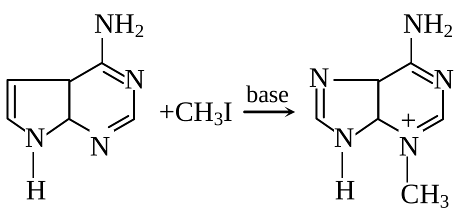
Q.26 The major product in the following reaction is ?



(1)  (2)  (3)  (4) 

Ans. **[1] Bonus**

Sol.



Official answer according to NTA → 1

Q.27 The maximum prescribed concntration of copper in dringking water is -

(1) 5 ppm (2) 0.5 ppm (3) 3 ppm (4) 0.05

Ans. **[3]**

Sol. The prescribed conc. of Cu in drinking water is 3ppm

Q.28 Fructose and glucose can be distinguished by -

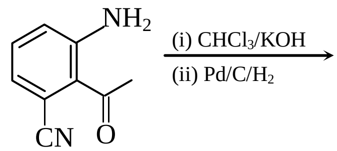
(1) Fehling's test (2) Seliwanoff's test

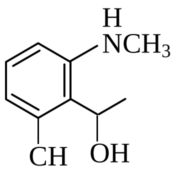
(3) Barfoed's test (4) Benedict's test

Ans. **[2]**

Sol. Glucose and fructose can be distinguised by seliwan eff's. It is used to distinguished aldose ketose group.

Q.29 The major product obtained in the following reaction is –



(1)


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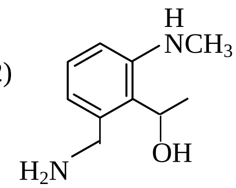

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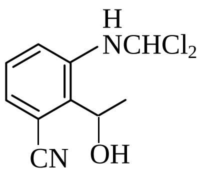

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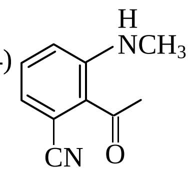
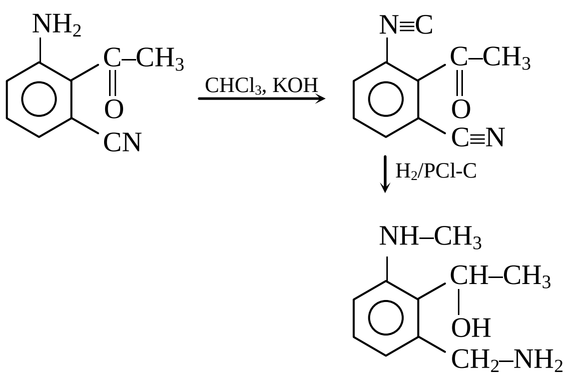


Ans. **[2]**

Sol.



Q.30 The strength of 11.2 volume solution of H_2O_2 is : [Given that molar mass of H = 1g mol^{-1} and O = 16g mol^{-1}]

(1) 1.7% (2) 34% (3) 3.4% (4) 13.6%

Ans. **[3]**

Sol. 11.2 vol. H_2O_2

m = ?

Vol. strength = 11.2 M

11.2 = 11.2 m

m = 1

1 mole H_2O_2 prompt in IL solution

34 gm in 1000 gm solution

$$\%\frac{w}{w} = \frac{(g_m)\text{solute}}{(g_m)\text{solution}} \times 100$$

$$= \frac{34}{1000} \times 100 = 3.4\%$$

CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

8^{th} April 2019 | Shift - II

(Memory Based)

MATHEMATICS

Q.1 If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct ?

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(3) d, e, f are in G.P.

(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Ans. **[2]**

Sol. $ax^2 + 2bx + c = 0$ $(b^2 = ac)$

$dx^2 + 2ex + f = 0$

$(af - cd)^2 = (2ae - 2bd)(2bf - 2ec)$

$a^2f^2 + c^2d^2 - 2a + cd = 4aebf - 4ae^2c - 4b^2df + 4bdec$

$a^2f^2 + c^2d^2 + 4b^2e^2 + 2afcd - 4aebf - 4bdec = 0$

$(af + cd - 2be)^2 = 0$

$af + cd = 2be$

$$\frac{af}{b^2} + \frac{cd}{b^2} = \frac{2be}{b^2}$$

$$\frac{af}{ac} + \frac{cd}{ac} = 2\left(\frac{e}{b}\right)$$

$$\frac{f}{c} + \frac{d}{a} = 2\left(\frac{e}{b}\right)$$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in AP.

Q.2 A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is -

(1) $\frac{10}{\sqrt{3}}$

(2) $\frac{100}{3}$

(3) $\frac{100}{\sqrt{3}}$

(4) $\frac{10}{3}$

Ans. **[1]**

Sol. Let unknown observation is x

$$\frac{45 + 54 + 41 + 57 + 43 + x}{6} = 48$$

$x = 48$

$$\sigma^2 = \left(\frac{1}{6}\left(45^2+54^2+41^2+51^2+43^2+48^2\right)\right)-(43)^2$$

$$\sigma^2 = \frac{1}{6}\times(14024)-(48)^2$$

$$\sigma^2 = \frac{14024-((48)^2\times 6)}{6}$$

$$\sigma^2 = \frac{14024-13824}{6}$$

$$\sigma^2 = \frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

Q.3 If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q (8, 0, 10), then the distance of R from the origin is -

(1) 6 (2) $\sqrt{53}$ (3) $2\sqrt{14}$ (4) $2\sqrt{21}$

Ans. **[3]**

Sol. Line PQ

$$\frac{x-2}{6} = \frac{y+3}{3} = \frac{3-4}{6} = \lambda$$

Let point R (6λ + 2, 3λ –3, 6λ + 4)

given that 6λ + 2 = 4

point R (4, –2, 6)

Distance between point R and origin = $\sqrt{16+4+36}$ = $\sqrt{56}$ = $2\sqrt{14}$

Q.4 Which one of the following statements is not a tautology ?

(1) $P \to (p \vee q)$ (2) $(p \wedge q) \to (\sim p) \vee q$

(3) $(p \wedge q) \to p$ (4) $(p \vee q) \to (p) \vee (\sim q))$

Ans. **[4]**

Sol.

p	q	$p \vee q$	$\vee q$	$p \vee \sim q$	$(p \vee q) \to (p \vee \sim q)$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	T

Q.5 Given that the slope of the tangent to a curve y = y(x) at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is -

(1) $x \log_e |y| = 2(x - 1)$ (2) $x \log_e |y| = 2(x - 1)$

(3) $x^2 \log_e |y| = -2(x - 1)$ (4) $x \log_e |y| = x - 1$

Ans. **[1]**

Sol. $\frac{dy}{dx}=\frac{2y}{x^2}$

$\frac{dy}{y}=\frac{2}{x^2}dx$

$\log_e|y|=-\frac{2}{x}+c$

process through (1, 1)

0 = –2 + c c = 2

$\log_e|y|=-\frac{2}{x}+2$

x $\log_e$ | y | = –2 + 2x

x $\log_e$ | y | = 2(x – 1)

Q.6 If the lengths of the sides of a traingle are in A. P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is **-**

(1) 5 : 6 : 7 (2) 4 : 5 : 6 (3) 3 : 4 : 5 (4) 5 : 9 : 13

Ans. **[2]**

Sol. Let length of sizes are

a = A-D (D > 0)

b = A

C = A + D

sinA, sinB, sinC are in AP

2 sin B = sin A + sin C

let A = 2θ

C = 2θ

B = π – (θ + 2θ)

B = π – 3θ

2 sin (π – 3θ) = sin θ + sin2θ

2 sin (3θ) = sin θ + 2 sin θ cosθ

2 (3 – 4 sin2θ) = 1 +2 cos θ

2 (4 $\cos^2$θ – 1) = 1 + 2 cosθ

8 $\cos^2$θ – 2 cos θ – 3 = 0

8 $\cos^2$θ – 6cosθ + 4 cosθ – 3 = 0

(2 cos θ + 1) (4 cosθ – 3) = 0

cosq = $\frac{-1}{2}$ cosθ = $\frac{3}{4}$

Not possible

$\cos\theta=\frac{(A+D)^2+A^2(A-D)^2}{2A(A+D)}=\frac{3}{4}$

$\frac{4AD+A^2}{2A(A+D)}=\frac{3}{4}$

$\frac{4AD+A^2}{2A(A+D)}=\frac{3}{4}\ \frac{4D+A^2}{2(A+D)}=\frac{3}{4}\ \Rightarrow A = 5D$

a = A – D = 4D

b = A = 5D

C = A + D = 6D

a : b : C : : 4 : 5 : 6

Q.7 Let f : [–1, 3] → R be defined as

$$f(x)=\begin{cases}|x|+[x] & , & -1\le x<1\\ x+|x| & , & 1\le x<2\\ x+[x] & , & 2\le x\le 3\end{cases}$$

Where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at -

(1) four or more points (2) only three points (3) only two points (4) only one point

Ans. **[2]**

Sol.
$$f(x)=\begin{cases}|x|+[x] & -1\le x<1\\ x+|x| & 1\le x<2\\ x+[x] & 2\le x\le 3\end{cases}$$

f×n is discontinuous

at x = 0, 1, 3

Q.8 The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0 is -

(1) $\vec{r}(\hat{i}-\hat{k})-2=0$ (2) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (3) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$ (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

Ans. **[3]**

Sol. equation of required plane

(x + y + z –1) + l (2x + 3y + 4z – 5) =0

x(1 + 2λ) + y (1 + 3λ) + 3 (1 + 4λ) – 5λ – = 0 … (i)

x – y + 3 = 0 … (ii)

Plane (1) & (2) are perpendicular to each other

(1) (1 + 2λ) + (–1) (1 + 3λ) + (1) (1 + 4λ) = 0

1 + 2λ –1 – 3λ + 1 + 4λ = 0

3λ = –1

$\lambda = -\frac{1}{3}$

put in equation (i)

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

x – z + 2 = 0

$\vec{r}\cdot(\hat{i}-\hat{k})=-2$

Q.9 The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to

(1) $2-\frac{21}{2^{20}}$ (2) $1-\frac{11}{2^{20}}$ (3) $2-\frac{3}{2^{17}}$ (4) $2-\frac{11}{2^{19}}$

Ans. **[4]**

Sol.
$$S=\sum_{k=1}^{20}\frac{k}{2k}$$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+.....+\frac{20}{2^{20}} \quad ....(i)$$

$$\frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^2}+.....+\frac{20}{2^{21}} \quad ....(ii)$$

equation (i) – (ii)

$$\frac{1}{2}S-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..........+\frac{1}{2^{20}}\right)-\frac{20}{2^{21}}$$

$$\frac{1}{2}S=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-\frac{20}{2^{21}}$$

$$\frac{1}{2}S=1-\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\frac{1}{2}S=1-\frac{1}{2^{20}}-\frac{10}{2^{20}}$$

$$\frac{1}{2}S=1-\frac{11}{2^{20}}$$

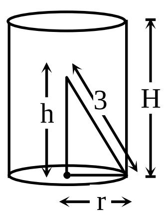
$$S=2-\frac{11}{2^{19}}$$

Q.10 The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is -

(1) $2\sqrt{3}$ (2) $\sqrt{6}$ (3) $\sqrt{3}$ (4) $\frac{2}{3}\sqrt{3}$

Ans. **[1]**

Sol.



$h^2 + r^2 = 9$

$r^2 = 9 - h^2$

$V = \pi r^2(2h)$

$v = 2\pi h\,(9 - h^2)$

$v = 2\pi\,(9 - 4^3)$

$\frac{dv}{dx}=0$ $h=\sqrt{3}$

Height of cylinder (H) = 2(h) = $2\sqrt{3}$

Q.11 If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is -

(1) 10 (2) 10^3 (3) 100 (4) 10^4

Ans. **[1, Bonus]**

Sol. $\left(\left(\frac{1}{x^{1+\log_{10}x}}\right)^{\frac{1}{2}}+x^{\frac{1}{12}}\right)^6$, $T_4 = 200$ given

$T_4 = {}^6C_3\ (x)^{-\frac{3}{2}(1+\log_{10} x)} \cdot \left(x^{\frac{1}{12}}\right)^3 = 200$

$20 \cdot x^{-\frac{3}{2}(1+\log_{10} x)} + \frac{1}{4} = 200$

$x^{-\frac{3}{2}(1+\log_{10} x)+\frac{1}{4}} = 10$

log on both side

$\left(-\frac{3}{2}(1+\log_{10} x)+\frac{1}{4}\right)\log_{10} x = 1$

let $\log_{10} x = t$

$\left(-\frac{3}{2}(1+t)+\frac{1}{4}\right)t = 1$

$-6(t^2 + t) + \frac{t}{4} = 1$

$-6(t^2 + t) + t = 4$

$-6t^2 - 6t + t = 4$

$6t^2 + 5t + 4 = 0$

$\Delta < 0$

Roots imaginary (Bonus)

Q.12 In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is -

(1) 5 (2) 6 (3) 8 (4) 10

Ans. **[1]**

Sol. Given that $2b - 2a = 10$

$b - a = 5$ ...(i)

given that $2be = 10\sqrt{3}$

$be = 5\sqrt{3}$

$b^2e^2 = 75$

$(b^2 - a^2) = 75$

$(b - a)(b + a) = 75$

$5(b + a) = 75$

$b + a = 15$ … (ii)

from equation (i) & equation (2)

$b = 10$

$a = 5$

length of lotus rectum $= \frac{2a^2}{b}$

$= \frac{2\times 25}{10} = 5$

Q.13 Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if -

(1) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (2) $r \ge 5\sqrt{\frac{3}{2}}$ (3) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (4) $0 < r \le \sqrt{\frac{3}{2}}$

Ans. **[2]**

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$\vec{a} \times \vec{b} = \hat{i}(2+x) - \hat{j}(3-x) + \hat{k}(-5)$

$\vec{a} \times \vec{b} = (2+x)\hat{i} - +(x-3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(2+x)^2 + (x-3)^2 + 25}$

$= \sqrt{2x^2 - 2x + 38}$

$\ge \sqrt{\frac{75}{2}}$

$\ge 5\sqrt{\frac{3}{2}}$

Q.14 Let S(α) = {(x, y) : $y^2 \le x, \le \alpha$} and A(α) is area of the region S(α). If for a λ, 0 < λ 4, A(λ) : A(4) = 2 : 5, then l equals :

(1) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (3) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Ans. **[1]**

Sol. $\frac{A(\lambda)}{A(4)} = \frac{2}{5}$

$\frac{\int_0^{\lambda} \sqrt{x}\,dx}{\int_0^{4} \sqrt{x}\,dx} = \frac{2}{5}$

$\frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$

$\lambda^{3/2} = \frac{2}{5} \times 8$

$\lambda = \left(\frac{16}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{2}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{4}{25}\right)^{1/3}$

Q.15 If f(1) = 1, f'(1) = 3, then the derivative of $f(f(f(x))) + (f(x))^2$ at x = 1 is -

(1) 9 (2) 12 (3) 15 (4) 33

Ans. **[4]**

Sol. f(1) = 1 f ' (1) = 3

$f(f(f(x))) + (f(x))^2$ at x = 1

$f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) f'(x)$

$f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) f'(1)$

$f'(f(f(1))) \cdot f'(1) \cdot f'(1) + 2f'(1)$

$f'(f(1) \cdot f'(1) \cdot f'(1) + 2f'(1)$

$3 \times 3 \times 3 + (2 \times 3)$

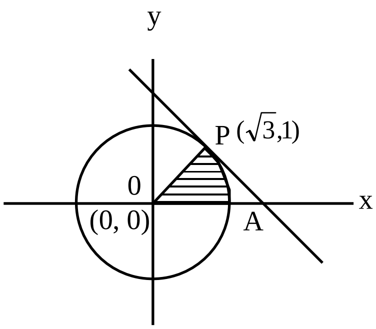
$27 + 6 = 33$

Q.16 The tangent and the normal lines at the point $(\sqrt{3},1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle, The area of this triangle (in square units) is -

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{1}{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. **[4]**

Sol.



equation of tangent

$\sqrt{3}x + y = 4$

point $A\left(\frac{4}{\sqrt{3}}, 0\right)$

Area of $\Delta OPA = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$

Q.17 The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is -

(1) 4 (2) 5 (3) 3 (4) 2

Ans. **[1]**

Sol. P (at least one hears) = 1 – [(no one heads) $\geq \frac{90}{100}$

$1 - \left(\frac{1}{2}\right)^n \geq \frac{9}{100}$

$\left(\frac{1}{2}\right)^n \leq \frac{10}{100}$ $2^n \geq 10$

least value of n is 4

Q.18 Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x + y)\, f_2(x - y)$ (2) $2f_1(x + y)\, f_1(x - y)$ (3) $2f_1(x)\, f_2(y)$ (4) $2f_1(x)\, f_1(y)$

Ans. **[4]**

Sol.

$$f(x) = \left(\frac{f(x)+f(-x)}{2}\right) + \left(\frac{f(x)-f(-x)}{2}\right)$$

↓ even ↓ odd

$$f_1(x) = \frac{f(x)+f(-x)}{2}$$

$$f_1(x+y) + f_1(x-y) = \frac{a^{x+y}+a^{-x-y}}{2} + \frac{a^{x-y}+a^{y-x}}{2}$$

$$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^{-x} + a^{-y})\right]$$

$$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y})$$

$$= \frac{1}{2}\left(\frac{a^x + a^{-x}}{2}\right)\left(\frac{a^y + a^{-y}}{2}\right)$$

$= 2f_1(x)\, f_1(y)$

Q.19 Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals -

(1) $-\frac{1}{7}$ (2) $\frac{1}{3}$ (3) 0 (4) 3

Ans. **[2]**

Sol. equation of line passes through (1, 2) & (–3, 4)

$$(y - 2) = \frac{4-2}{-3-1}(x-1)$$

$$(y - 2) = -\frac{1}{2}(x-1)$$

$2y - 4 = -x + 1$

$x + 2y = 5$ … (i)

$\perp$ line $2x - y = \lambda \rightarrow$ passes through (4, 3)

$2x - y = 5$ …. (2) $\lambda = 5$

Intersection point of line (i) & line (ii) is (3, 1)

$$\frac{k}{h} = \frac{1}{3}$$

Q.20 If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is -

(1) $4x - 3y - 4 = 0$ (2) $3x - 4y - 4 = 0$ (3) $3x - 4y - 1 = 0$ (4) $4x - 3y - 1 = 0$

Ans. **[1]**

Sol. $x - 2y + kz = 1$ …. (i)
$2x + y + z = 2$ … (ii)
$3x - y - kz = 3$ … (iii)
for locus of (x, y)
equation (i) + (iii)
$4x - 3y = 4$
$4x - 3y - 4 = 0$

Q.21 The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant , passes through the point -

(1) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (2) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Ans. **[2]**

Sol. $y^2 = 4x$ … (i)
$x^2 + y^2 = 5$ ….(ii)
for point of intersection
$x^2 + 4x - 5 = 0$
$(x + 5)(x - 1) = 0$
$x = -5$ $x = 1$
not possible $y = \pm 2$
Point in IQ (1, 2)
Tangent at (1, 2)

$$2y = 4\left(\frac{x+1}{2}\right)$$

$y = x + 1$

point $\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on tangent

Q.22 The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repretition of digits is allowed) is -

(1) 306 (2) 360 (3) 310 (4) 288

Ans. **[3]**

Sol. Given digits are 0, 1, 2, 3, 4, 5
requries four number greater than 4321

5				= 216
↓ 1	↓ 6	↓ 6	↓ 6	

4				= 72
	↓ 2	↓ 6	↓ 6	

4	3			= 24
		↓ 4	↓ 6	

total cae = 22
{substract two case 4320 & 4321}
total = 216 + 72 + 22
= 310

Q.23 The number of integral value of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + 8m) = 0$ has no real root is -

(1) 1 (2) infinitely many (3) 3 (4) 2

Ans. **[2]**

Sol.
$4(1 + 3m)^2 - 4 \times (1 + m^2)(1 + 8m) < 0$

$1 + 9m^2 + 6m - (1 + 8m + m^2 + 8m^3) < 0$

$1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3 < 0$

$-8m^3 + 8m^2 - 2m < 0$

$8m^3 - 8m^2 + 2m > 0$

$m(4m^2 - 4m + 2) > 0$

$m[(2m - 1)^2 + 1] > 0$

$m > 0$

Q.24 Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If f(x + 5) = g (x), then $\int_0^x f(t)dt$ equals-

(1) $5\int_{x+5}^{5} g(t)dt$ (2) $2\int_{5}^{x+5} g(t)dt$ (3) $\int_{5}^{x+5} g(t)dt$ (4) $\int_{x+5}^{5} g(t)dt$

Ans. **[4]**

Sol. $f(0) = \int_0^0 g(t)dt = 0$

f(0) = 0 and f(x) = g′(x) so that f(x) is odd function

f(x + 5) = f (–x + 5) = g(x) = g(–x)

$$I = \int_0^x f(t)dt$$

z = t + 5

$$I = \int_5^{x+5} f(z-5)dz$$

$$I = \int_5^{x+5} f(-(5-z))dz$$

$$I = \int_5^{x+5} f(5-z)\,dz$$

$$I = \int_{X+5}^{5} f(5-z)\,dz$$

$$I = \int_{X+5}^{5} g(z)dz$$

$$I = \int_{X+5}^{5} g(t)dt$$

Q.25 If $x = \frac{\sqrt{3}}{2} + \frac{i}{2} (i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to -

(1) 0 (2) –1 (3) $(-1 + 2i)^9$ (4) 1

Ans. **[2]**

Sol. $z = \frac{\sqrt{3}}{2} + \frac{p}{2} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$(i + iz + z^5 + i\,z^8)^9$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2} + \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + i\left(\cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6}\right)\right)^3$$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{i}{2} + i\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)^9$$

$$\left(1 + i\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^9$$

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^9$$

$$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^9$$

$\cos 3\pi + i\sin 3\pi = -1$

Q.26 If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is -

(1) $2x - y - 2 = 0$ (2) $3x - 2y = 0$ (3) $2x - 3y + 10 = 0$ (4) $x - 2y + 8 = 0$

Ans. **[1]**

Sol. Let equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \rightarrow (4, 6)$

$\frac{16}{a^2} - \frac{36}{b^2}$ …. (1)

$a^2 = 1 + \frac{b^2}{a^2}$

$4 = 1 + \frac{b^2}{a^2}$

$b^2 = 3a^2$ …(ii)

from (i) & (ii)

$a^2 = 4$

$b^2 = 12$

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

tangent at (4, 8)

$\frac{4x}{4} - \frac{6y}{12} = 1$

$x - \frac{y}{2} = 1$

$2x - y - 2 = 0$

Q.27 Let $f : R \rightarrow R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is qual to -

(1) 1 (2) e^{-1} (3) e (4) e^2

Ans. **[1]**

Sol. $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ $(1)^{\infty}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left[\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right]}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left[\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right]}\left(\frac{0}{0}\right)$

$= e^{\lim_{x\to 0}\left(\frac{f(3+z)-f(x)-f(2-z)+f(z)}{x}\right)\times\left(\frac{1}{1}\right)}$

$= e^{\lim_{x\to 0}\frac{f'(3+x)+f'(2-x)}{1}}$

$= e^{f'(3)+f'(2)} = e^0 = 1$

Q.28 If $\int \frac{dx}{x^3(1-x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$ where C is a constant of integration, then the function f(x) is equal to

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$ (3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{6x^3}$

Ans. **[1]**

Sol. $I = \int \frac{dx}{x^3(1+2x^6)^{2/3}}$

$I = \int \frac{x^{-7}dx}{(x^{-b}+1)^{2/3}}$

x – 6 + 1 = t

$-6x^{-7}$ dx = dt

$x^{-7}dx = -\frac{1}{6}dt$

$I = -\frac{1}{6}\int t^{-2/3}dt$

$I = -\frac{1}{6}\times 3(t)^{1/3} + c$

$I = -\frac{1}{2}\left(\frac{1+x^6}{x^6}\right)^{1/3} + c$

$I = -\frac{1}{2}(1+x^6)^{1/3} + c$

$xf(x) = \frac{1}{2x^2}$

$f(x) = -\frac{1}{2x^2}$

Q.29 Let the number 2, b,c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det (A) $\in$ [2, 16], then c lies in the interval -

(1) $(2 + 2^{3/4}, 4)$ (2) [4, 6] (3) $[3, 2 + 2^{3/4}]$ (4) [2, 3)

Ans. **[2]**

Sol. $|A| = (2-b)(b-c)(c-2)$

2, b, c, are in AP.

$b = \frac{2+c}{2}$

Det (A) = $\frac{1}{4}(c-2)^3$

$2 \le \frac{1}{4}(c-2)^3 \le 16$

$8 \le (c-2)^3 \le 64$

$2 \le c - 2 \le 4$

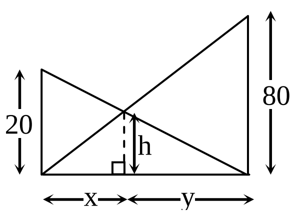
$4 \le c \le 6$

Q.30 Two vertical poles of heights, 20 m and 80 m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is -

(1) 18 (2) 16 (3) 15 (4) 12

Ans. **[2]**

Sol.



$\frac{h}{x} = \frac{80}{x+y}$

$h = \frac{80}{1+(y/x)}$ …. (i)

$h = \frac{80}{1+\frac{h}{20-h}}$

h = 4 (20 – h)

h = 80 – 4h

5h = 80

h = 16

$\frac{h}{y} = \frac{20}{x+y}$

$h = \frac{20}{\left(\frac{x}{y}\right)+1}$

$\frac{x}{y}+1 = \frac{20}{h}$

$\frac{x}{y} = \frac{20}{h} - 1$

$\frac{x}{y} = \frac{20-h}{h}$

$\frac{x}{y} = \frac{h}{20-h}$

put in … (i)