= P(A and not B) +P (B and not A) +P (A and B)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$) + P(A $\cap$ B)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$) + P(A) $\times$P(B)

= $(\frac{1}{4} \times \frac{2}{3}) + (\frac{1}{3} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{1}{3})$

= $\frac{2}{12} + \frac{3}{12} + \frac{1}{12}$

= $\frac{1}{2}$

Therefore , The probability that atleast one of them will get selected is $\frac{1}{2}$

13. Question

A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that

(i) both of them are selected

(ii) only one of them is selected

(iii) none is selected

(iv) at least one of them is selected.

Answer

Given : A and B appear for an interview ,then P(A) =$\frac{1}{6}$ and P(B) = $\frac{1}{4}$ $\Rightarrow$ P($\bar{A}$) = $\frac{5}{6}$ and P($\bar{B}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: i) The probability that both of them are selected.

We know that, P(both of them are selected) = P(A $\cap$ B) = P(A) $\times$P(B)

= $\frac{1}{6} \times \frac{1}{4}$

= $\frac{1}{24}$

Therefore , The probability that both of them are selected is $\frac{1}{24}$

ii) P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= $(\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6})$

= $\frac{3}{24} + \frac{5}{24}$

= $\frac{1}{3}$

Therefore, the probability that only one of them Is selected is $\frac{1}{3}$

iii)none is selected

we know that P(none is selected) = P($\bar{A}$ $\cap$ $\bar{B}$)

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{5}{8}$

Therefore , the probability that none is selected is $\frac{5}{8}$

iv) atleast one of them is selected

Now, P(atleast one of them is selected) = P(selecting only A) + P(selecting only B) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

$= P(A \cap \bar{B}) + P(B \cap \bar{A}) + P(A \cap B)$

$= P(A) \times P(\bar{B}) + P(B) \times P(\bar{A}) + P(A) \times P(B)$

$= (\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6}) + (\frac{1}{6} \times \frac{1}{4})$

$= \frac{3}{24} + \frac{5}{24} + \frac{1}{24}$

$= \frac{3}{8}$

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$

14. Question

Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is %, find the probability that

(i)at least one of A and B will solve the problem

(ii)none of the two will solve the problem

Answer

Given : Here probability of A and B that can solve the same problem is given , i.e., P(A) $= \frac{2}{3}$ and P(B) $= \frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: i) atleast one of A and B will solve the problem

Now , P(atleast one of them will solve the problem) = 1 – P(both are unable to solve)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - (\frac{1}{3} \times \frac{2}{5})$

$= \frac{13}{15}$

Therefore , atleast one of A and B will solve the problem is $\frac{13}{15}$

ii) none of the two will solve the problem

Now, P(none of the two will solve the problem) = $P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Therefore , none of the two will solve the problem is $\frac{2}{15}$

15. Question

A problem is given to three students whose chances of solving it are 1/4, 1/5 and 1/6, respectively. Find the probability that the problem is solved.

Answer

Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) $= \frac{1}{4}$, P(B) $= \frac{1}{5}$ and P(C) $= \frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) $= \frac{3}{4}$, P($\bar{B}$) $= \frac{4}{5}$ and P($\bar{C}$)$= \frac{5}{6}$

To Find: The probability that the problem is solved .

Here , P(the problem is solved) = 1 - P(the problem is not solved)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$)]

$= 1- [\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}]$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Therefore , The probability that the problem is solved is $\frac{1}{2}$.

16. Question

The probabilities of A, B, C solving a problem are 1/3, 1/4 and 1/6, respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Answer

Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) $= \frac{1}{3}$, P(B) $= \frac{1}{4}$ and P(C) $= \frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) $= \frac{2}{3}$, P($\bar{B}$) $= \frac{3}{4}$ and P($\bar{C}$)$= \frac{5}{6}$

To Find: The probability that excatly one of them will solve it .

Now, P(excatly one of them will solve it) = P(A and not B and not c) +P (B and not A and not C) +P (C and not A and not B)

= P(A $\cap \bar{B} \cap \bar{C}$) + P(B $\cap \bar{A} \cap \bar{C}$) + P(C $\cap \bar{A} \cap \bar{B}$)

= P(A) ×P($\bar{B}$) × P($\bar{C}$) + P(B) × P($\bar{A}$) × P($\bar{C}$) + P(C) ×P($\bar{B}$) × P($\bar{A}$)

$= [\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}] + [\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}] + [\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}]$

$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72}$

$= \frac{31}{72}$

Therefore , The probability that excatly one of them will solve the problem is $\frac{31}{72}$

17. Question

A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that

(i) A, B and C all hit the target

(ii) B and C hit and A does not hit the target.

Answer

Given : let A , B and C chances of hitting a target is given i.e , P(A) $= \frac{4}{5}$, P(B) = $\frac{3}{4}$ and P(C) = $\frac{2}{3}$.

$\Rightarrow$ P($\bar{A}$) = $\frac{1}{5}$, P($\bar{B}$) = $\frac{1}{4}$ and P($\bar{C}$)= $\frac{1}{3}$

To Find: i)The probability that A , B and C all hit the target.

Now ,P(all hitting the target) = P($A \cap B \cap C$)

= P(A) $\times$P(B) $\times$ P(C)

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{2}{5}$

Hence , The probability that A , B and C all hit the target is $\frac{2}{5}$

ii) B and C hit and A does not hit the target

Here, P(B and C hit and not A) = P(B $\cap$ C $\cap$ $\bar{A}$)

= P(B) $\times$P(C) $\times$ P($\bar{A}$)

$= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}$

$= \frac{1}{10}$

Hence , the probability that B and C hit and A does not hit the target is $\frac{1}{10}$

18. Question

Neelam has offered physics, chemistry and mathematics in Class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam receives

(i) all A grades

(ii) no A grade

(iii) exactly 2 A grades.

Answer

Given : let A , B and C represent the subjects physics,chemistry and mathematics respectively ,the probability of neelam getting A grade in these three subjects is given i.e , P(A) = 0.2 , P(B) = 0.3 and P(C) =0.9

$\Rightarrow$ P($\bar{A}$) = 0.8 , P($\bar{B}$) = 0.7 and P($\bar{C}$)= 0.1

To Find: i)The probability that neelam gets all A grades

Here, P(getting all A grades) = P($A \cap$ B $\cap$ C)

= P(A) $\times$P(B) $\times$ P(C)

=0.2 $\times$ 0.3 $\times$ 0.9

= 0.054

Therefore, The probability that neelam gets all A grades is 0.054.

ii)no A grade

Here , P(getting no A grade) = P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= P($\bar{A}$) $\times$ P($\bar{B}$) $\times$ P($\bar{C}$)

= 0.8 $\times$ 0.7 $\times$0.1

= 0.056

Therefore, The probability that neelam gets no A grade is 0.056.

iii)excatly 2 a grades

P(getting excatly 2 A grades) = P(A and B and not c) +P (B and C and not A) +P (C and A and not B)

= P(A $\cap$ B $\cap$ $\bar{C}$) + P(B $\cap$ C $\cap$ $\bar{A}$) + P(C $\cap$ A $\cap$ $\bar{B}$)

= P(A) $\times$P(B) $\times$ P($\bar{C}$) + P(B) $\times$ P(C) $\times$ P($\bar{A}$) + P(C) $\times$P(A) $\times$ P($\bar{B}$)

= [0.2 $\times$ 0.3 $\times$0.1] + [0.3 $\times$ 0.9 $\times$ 0.8] + [0.9 $\times$ 0.2 $\times$0.7]

= 0.006 + 0.216 +0.126

= 0.348

Therefore, The probability that neelam gets excatly 2 A grades is 0.348.

19. Question

An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X. 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Answer

Given: X and Y are the two parts of a company that manufactures an article.

Here the probability of the parts being defective is given i.e, P(X) $=\frac{8}{100}$ and P(Y) = $\frac{5}{100}$ $\Rightarrow$ P($\bar{X}$) = $\frac{92}{100}$ and P($\bar{Y}$) = $\frac{95}{100}$

To Find: the probability that the assembled product will not be defective.

Here,

P(product assembled will not be defective) = 1 – P(product assembled to be defective)

=1 – [P(X and not Y) + P(Y and not X) +P(both)]

= 1- [P(X $\cap$ $\bar{Y}$) + P(Y $\cap$ $\bar{X}$) + P(X $\cap$ Y)]

= 1- [P(X) $\times$P($\bar{Y}$) + P(Y) $\times$ P($\bar{X}$) + P(X) $\times$P(Y)]

= 1-[$(\frac{8}{100} \times \frac{95}{100}) + (\frac{5}{100} \times \frac{92}{100}) + (\frac{8}{100} \times \frac{5}{100})$]

=1- $[\frac{760}{10000} + \frac{460}{10000} + \frac{40}{10000}]$

$= \frac{437}{500}$

Therefore, The probability that the assembled product will not be defective is $\frac{437}{500}$.

20. Question

A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that

(i) neither of them is available when needed?

(ii) an engine is available when needed?

Answer

Given: Let A and B be two fire extinguishing engines . The probability of availability of each of the two fire extinguishing engines is given i.e., P(A) = 0.95 and P(B) = 0.95 $\Rightarrow$ P($\bar{A}$) = 0.05 and P($\bar{B}$) = 0.05

To Find: i) The probability that neither of them is available when needed

Here, P(not A and not B) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) $\times$P($\bar{B}$)

=0.05 $\times$ 0.05

= 0.0025 = $\frac{1}{400}$

Therefore, The probability that neither of them is available when needed is $\frac{1}{400}$

ii)an engine is available when needed

Here, P(A and not B or B and not A) = P(A$\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= (0.95 $\times$0.05) + (0.95 $\times$ 0.05)

= 0.0475 + 0.0475

= 0.095

$= \frac{19}{200}$

Therefore, The probability that an engine is available when needed is $\frac{19}{200}$

21. Question

A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0.14, 0.10 and 0.05, respectively. What is the probability that the machine will fail?

Answer

Given: let A ,B and C be the three components of a machine which works only if all its three compenents function.the probabilities of the failures of A,B

and C respectively is given i.e, P(A) = 0.14 ,P(B) = 0.10 and P(C) = 0.05

$\Rightarrow$ P($\bar{A}$) = 0.86 and P($\bar{B}$) = 0.90 and P($\bar{C}$) = 0.95

To Find: The probability that the machine will fail.

Here, P(the machine will fail) = 1 – P(the machine will function)

= 1 – P(all three components are working)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1 - [0.86 \times 0.90 \times 0.95]$

= 1 – 0.7353

= 0.2647

Therefore, The probability that the machine will fail is 0.2647.

22. Question

An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

Answer

Given:Let A ,B , C and Dbe first second third and fourth shots whose probability of hitting the plane is given i.e, P(A) = 0.4 ,P(B) = 0.3 , P(C) = 0.2 and P(D) = 0.1 respectively

$\Rightarrow P(\bar{A}) = 0.6$ and $P(\bar{B}) = 0.7$ and $P(\bar{C}) = 0.8$ and $P(\bar{D}) = 0.9$

To Find: The probability that atleast one shot hits the plane .

Here , P(atleast one shot hits the plane) = 1 – P(none of the shots hit the plane)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D})]$

$= 1 - [0.6 \times 0.7 \times 0.8 \times 0.9]$

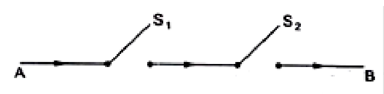
= 1 – 0.3024

= 0.6976

Therefore, The probability that atleast one shot hits the plane is 0.6976.

23. Question

Let S_1 and S_2 be the two switches and let their probabilities of working be given by $P(S_1) = 4/5$ and $P(S_2) = 9/10$. Find the probability that the current flows from the terminal A to terminal B when S_1 and S_2 are installed in series, shown as follows:



Answer

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{4}{5}$ and $P(S_2) = \frac{9}{10}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in series.

Now, since the current in series flows from end to end

⇒ the flow of current from terminal A to terminal B is given by

$P(S_1 \cap S_2) = P(S_1) \times P(S_2)$

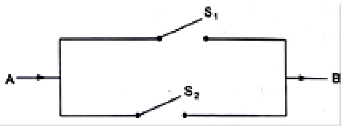
$= \frac{4}{5} \times \frac{9}{10}$

$= \frac{18}{25}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in series is $\frac{18}{25}$

24. Question

Let S_1 and S_2 be two the switches and let their probabilities of working be given by $P(S_1) = 2/3$ and $P(S_2) = 3/4$. Find the probability that the current flows from terminal A to terminal B, when S_1 and S_2 are installed in parallel, as shown below:



Answer

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{2}{3}$ and $P(S_2) = \frac{3}{4}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in parallel.

Now, since current in parallel flows in two or more paths and hence the sum of currents through each path is equal to total current that flows from the source.

$\Rightarrow$ the flow of current from terminal A to terminal B in a parallel circuit is given by

$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2)$

$= P(S_1) + P(S_2) – [P(S_1) \times P(S_2)]$

$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{11}{12}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in parallel is $\frac{11}{12}$

25. Question

A coin is tossed. If a head comes up, a die is thrown, but if a tail comes up, the coin is tossed again. Find the probability of obtaining

(i) two tails

(ii) a head and the number 6

(iii) a head and an even number.

Answer

Given : let H be head, and T be tails where as 1,2,3,4,5,6 be the numbers on the dice which are thrown when a head comes up or else coin is tossed again if its tail.

According to the question ,sample space S = {(TH),(TT) ,(H1),(H2),(H3),(H4),(H5),(H6)}

To Find: i)the probability of obtaining two tails

From sample space, it is clear that the probability of obtaining two tails is $\frac{1}{8}$

i.e., {TT} with total no of elements in sample space as 8.

ii) the probability of obtaining a head and the number 6

From sample space, it is clear that the probability of obtaining a head and the number 6 is $\frac{1}{8}$

i.e., {H6} with total no of elements in sample space as 8.

iii) the probability of obtaining a head and an even number

From sample space, it is clear that the probability of obtaining a head and an even number is $\frac{3}{8}$

i.e, {H2,H4,H6} with total no of elements in sample space as 8.